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# Exponential stability of an alternate control system with double impulses

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**Abstract**
In this paper, we propose a new system called an alternate control system with double impulses. The present system is a cyclic control system, composed of four parts in a circle: to the first and last halves of each period of the system we add different continuous controls, and at the half-period time and the end of each period of the system we add different impulses. We then investigate the exponential stability of the considered system. An example based on Chua's circuit is provided to confirm the effectiveness of the theoretical result.

**MSC:** 37N35; 49N25

**Keywords:** Exponential stability; Alternate control; Double impulses; Chua's oscillator

## 1 Introduction

Throughout this paper, let $R^n$ denote an $n$-dimensional real Euclidean space with norm $\|\cdot\|$. $R^{m\times n}$ refers to the set of all $m \times n$-dimensional real matrices. $\lambda_{\max}(A)$, $\lambda_{\min}(A)$, $A^T$, and $A^{-1}$ stand for the maximum, the minimum eigenvalue, the transpose, and the inverse of matrix $A$, respectively. $I$ is the identity matrix with proper dimension. We use $A > 0$ to mean that $A$ is a positive-definite matrix. Let $f(x(a^-)) = \lim_{t\to a^-} f(x(t))$.

There are many methods to make a nonlinear system stable, for instance, sliding mode control [1], fuzzy control [2], feedback control [3], adaptive control [4], alternate control [5, 6], impulsive control [7, 8], etc. Taking into account the engineering applications, the cost of continuous control is high. Through intermittent control, the control cost and the amount of transmitted information can be greatly reduced. As is known, impulsive control is a discontinuous control method.

A class of nonlinear systems can be described as

$$\begin{cases} \dot{x}(t) = Ax(t) + f(x(t)) + w(t), \\ x(t_0) = x_0, \end{cases} \tag{1.1}$$

where $x(t) \in R^n$ is the state vector, $A \in R^{n\times n}$ is a constant matrix, $f : R^n \to R^n$ is a continuous nonlinear function satisfying $f(0) = 0$ and $\|f(x)\| \le l\|x\|$, $l \ge 0$ is a constant. $w(t)$ is the control input. Without loss of generality, let $t_0 = 0$, $x_0 \in R^n$ is a given vector.



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In order to stabilize system (1.1) at the origin by means of an alternate control system with double impulses, we set four kinds of control in one period, i.e., $t \in (kT, kT + \frac{T}{2})$, we set $w(t) = B_1 x(t)$, where $B_1 \in R^{n\times n}$ is a known matrix, $t \in (kT + \frac{T}{2}, (k+1)T)$, we set $w(t) = B_2 x(t)$, where $B_2 \in R^{n\times n}$ is a constant matrix, at the same time, at time $t = kT + \frac{T}{2}$, an impulse $J_1$ is given, and an impulse $J_2$ is given to the system at time $t = (k+1)T$. Hence, system (1.1) is rewritten as

$$\begin{cases} \dot{x}(t) = Ax(t) + f(x(t)) + B_1 x(t), & kT < t < kT + \frac{T}{2}, \\ x(t) = x(t^-) + J_1 x(t^-), & t = kT + \frac{T}{2}, \\ \dot{x}(t) = Ax(t) + f(x(t)) + B_2 x(t), & kT + \frac{T}{2} < t < (k+1)T, \\ x(t) = x(t^-) + J_2 x(t^-), & t = (k+1)T, \\ x(t_0) = x_0, & t_0 = 0, \end{cases} \tag{1.2}$$

where $T > 0$ is a control cycle and $k$ is a nonnegative integer.

*Remark* 1.1 When $B_2 = 0$, the system (1.2) becomes the alternate continuous-control system with double impulses [9].

For more information on stability and applications of nonlinear systems that have been investigated in the literature, for instance, see [10–14].

## 2 Main result

We begin this section with two lemmas that will turn out to be useful in the proof of our main result.

**Lemma 2.1** ([15]) *Suppose that any $x, y \in R^n$, then*

$$\left|x^T y\right| \le \|x\|\|y\|.$$

**Lemma 2.2** ([15]) *Let $A \in R^{n\times n}$ be a symmetric matrix, then for all $x \in R^n$,*

$$\lambda_{\min}(A)x^T x \le x^T A x \le \lambda_{\max}(A)x^T x.$$

**Theorem 2.1** *Let $0 < P \in R^{n\times n}$ such that the following conditions are satisfied*:

(1) $u_1 < 0$,

(2) $(\frac{u_1+u_2}{2})T + \ln\beta + \ln\gamma < 0$,

*where $\beta = \lambda_{\max}(P^{-1}(I+J_1)^T P(I+J_1))$, $\gamma = \lambda_{\max}(P^{-1}(I+J_2)^T P(I+J_2))$, $\beta_1 = \lambda_{\max}(P^{-1}(PA + A^T P + PB_1 + B_1^T P))$, $\beta_2 = \lambda_{\max}(P)$, $\beta_3 = \lambda_{\min}(P)$, $\beta_4 = \lambda_{\max}(P^{-1}(PA + A^T P + +PB_2 + B_2^T P))$, $u_1 = \beta_1 + 2l\sqrt{\frac{\beta_2}{\beta_3}}$, $u_2 = \beta_4 + 2l\sqrt{\frac{\beta_2}{\beta_3}}$. Then, system (1.2) is exponentially stable at the origin.*

*Proof* Define

$$V(x(t)) = x^T(t)Px(t).$$

For $t \in (kT, kT + \frac{T}{2})$, using Lemmas 2.1 and 2.2, we obtain

$$D^+\big(V(x(t))\big) = 2x^T(t)P\big(Ax(t) + f(x(t)) + B_1 x(t)\big)$$

$$\begin{aligned}
&= 2x^T(t)PAx(t) + 2x^T(t)Pf\big(x(t)\big) + 2x^T(t)PB_1x(t) \\
&= x^T(t)\big(PA + A^TP + PB_1 + B_1^TP\big)x(t) + 2x^T(t)P^{\frac{1}{2}}P^{\frac{1}{2}}f\big(x(t)\big) \\
&\le \beta_1 x^T(t)Px(t) + 2\sqrt{x^T(t)Px(t)f^T\big(x(t)\big)Pf\big(x(t)\big)} \\
&\le \beta_1 x^T(t)Px(t) + 2\sqrt{x^T(t)Px(t)\beta_2 f^T\big(x(t)\big)f\big(x(t)\big)} \\
&\le \beta_1 x^T(t)Px(t) + 2\sqrt{x^T(t)Px(t)\beta_2 l^2 x^T(t)x(t)} \\
&\le \beta_1 x^T(t)Px(t) + 2l\sqrt{x^T(t)Px(t)\frac{\beta_2}{\beta_3}x^T(t)Px(t)} \\
&= u_1 V\big(x(t)\big),
\end{aligned}$$

which implies that

$$V\big(x(t)\big) \le V\big(x(kT)\big)e^{u_1(t-kT)}. \tag{2.1}$$

For $t = kT + \frac{T}{2}$, we obtain

$$\begin{aligned}
V\big(x(t)\big) &= \big(x(t^-) + J_1x(t^-)\big)^T P\big(x(t^-) + J_1x(t^-)\big) \\
&= x^T(t^-)(I + J_1)^T P(I + J_1)x(t^-) \\
&= x^T(t^-)P^{\frac{1}{2}}P^{-\frac{1}{2}}(I + J_1)^T P(I + J_1)P^{-\frac{1}{2}}P^{\frac{1}{2}}x(t^-) \\
&\le \beta V\big(x(t^-)\big).
\end{aligned} \tag{2.2}$$

For $t \in (kT + \frac{T}{2}, (k+1)T)$, using Lemmas 2.1 and 2.2, we obtain

$$\begin{aligned}
D^+\big(V\big(x(t)\big)\big) &= 2x^T(t)P\big(Ax(t) + f\big(x(t)\big) + B_2x(t)\big) \\
&= 2x^T(t)PAx(t) + 2x^T(t)Pf\big(x(t)\big) + 2x^T(t)PB_2x(t) \\
&= x^T(t)\big(PA + A^TP + PB_2 + B_2^TP\big)x(t) + 2x^T(t)P^{\frac{1}{2}}P^{\frac{1}{2}}f\big(x(t)\big) \\
&\le \beta_4 x^T(t)Px(t) + 2\sqrt{x^T(t)Px(t)f^T\big(x(t)\big)Pf\big(x(t)\big)} \\
&\le \beta_4 x^T(t)Px(t) + 2\sqrt{x^T(t)Px(t)\beta_2 f^T\big(x(t)\big)f\big(x(t)\big)} \\
&\le \beta_4 x^T(t)Px(t) + 2\sqrt{x^T(t)Px(t)\beta_2 l^2 x^T(t)x(t)} \\
&\le \beta_4 x^T(t)Px(t) + 2l\sqrt{x^T(t)Px(t)\frac{\beta_2}{\beta_3}x^T(t)Px(t)},
\end{aligned}$$

which implies that

$$V\big(x(t)\big) \le V\left(x\left(kT + \frac{T}{2}\right)\right)e^{u_2(t-kT-\frac{T}{2})}. \tag{2.3}$$

By (2.2) and (2.3), we deduce that

$$V\big(x(t)\big) \leq \beta V\bigg(x\bigg(\bigg(kT + \frac{T}{2}\bigg)^{-}\bigg)\bigg)e^{u_2(t-kT-\frac{T}{2})}, \tag{2.4}$$

where $t \in [kT + \frac{T}{2}, (k+1)T)$.

For $t = (k+1)T$, we obtain

$$\begin{aligned} V\big(x(t)\big) &= \big(x\big(t^{-}\big) + J_2 x\big(t^{-}\big)\big)^T P\big(x\big(t^{-}\big) + J_2 x\big(t^{-}\big)\big) \\ &= x^T\big(t^{-}\big)(I+J_2)^T P(I+J_2)x\big(t^{-}\big) \\ &= x^T\big(t^{-}\big)P^{\frac{1}{2}}P^{-\frac{1}{2}}(I+J_2)^T P(I+J_2)P^{-\frac{1}{2}}P^{\frac{1}{2}}x\big(t^{-}\big) \\ &\leq \gamma V\big(x\big(t^{-}\big)\big). \end{aligned} \tag{2.5}$$

When $k = 0$, for $t \in (0, \frac{T}{2})$, from (2.1), we can obtain

$$V\big(x(t)\big) \leq V\big(x(0)\big)e^{u_1 t}.$$

Consequently,

$$V\bigg(x\bigg(\bigg(\frac{T}{2}\bigg)^{-}\bigg)\bigg) \leq V\big(x(0)\big)e^{\frac{u_1 T}{2}}. \tag{2.6}$$

For $t \in [\frac{T}{2}, T)$, applying (2.4) and (2.6), we obtain

$$\begin{aligned} V\big(x(t)\big) &\leq \beta V\bigg(x\bigg(\bigg(\frac{T}{2}\bigg)^{-}\bigg)\bigg)e^{u_2(t-\frac{T}{2})} \\ &\leq \beta V\big(x(0)\big)e^{\frac{u_1 T}{2}+u_2(t-\frac{T}{2})}. \end{aligned}$$

Consequently,

$$V\big(x\big(T^{-}\big)\big) \leq \beta V\big(x(0)\big)e^{\frac{u_1 T+u_2 T}{2}}. \tag{2.7}$$

For $t = T$, applying (2.5) and (2.7), we obtain

$$\begin{aligned} V\big(x(T)\big) &\leq \gamma V\big(x\big(T^{-}\big)\big) \\ &\leq \beta\gamma V\big(x(0)\big)e^{\frac{u_1 T+u_2 T}{2}}. \end{aligned} \tag{2.8}$$

When $k = 1$, for $t \in (T, T + \frac{T}{2})$, applying (2.1) and (2.8), we obtain

$$\begin{aligned} V\big(x(t)\big) &\leq V\big(x\big(T^{+}\big)\big)e^{u_1(t-T)} \\ &\leq V\big(x(T)\big)e^{u_1(t-T)} \\ &\leq \beta\gamma V\big(x(0)\big)e^{u_1(t-\frac{T}{2})+\frac{u_2 T}{2}}. \end{aligned}$$

Consequently,

$$V\left(x\left(\left(T+\frac{T}{2}\right)^{-}\right)\right) \le \beta\gamma V\big(x(0)\big)e^{u_1 T+\frac{u_2 T}{2}}. \tag{2.9}$$

For $t \in [T+\frac{T}{2}, 2T)$, applying (2.4) and (2.9), we obtain

$$\begin{aligned} V\big(x(t)\big) &\le \beta V\left(x\left(\left(T+\frac{T}{2}\right)^{-}\right)\right)e^{u_2(t-\frac{3T}{2})} \\ &\le \beta^2\gamma V\big(x(0)\big)e^{u_1 T+u_2(t-T)}. \end{aligned}$$

Consequently,

$$V\big(x\big((2T)^{-}\big)\big) \le \beta^2\gamma V\big(x(0)\big)e^{(u_1+u_2)T}. \tag{2.10}$$

For $t = 2T$, applying (2.5) and (2.10), we obtain

$$\begin{aligned} V\big(x(2T)\big) &\le \gamma V\big(x\big((2T)^{-}\big)\big) \\ &\le \beta^2\gamma^2 V\big(x(0)\big)e^{(u_1+u_2)T}. \end{aligned} \tag{2.11}$$

When $k = 2$, for $t \in (2T, 2T+\frac{T}{2})$, applying (2.1) and (2.11), we obtain

$$\begin{aligned} V\big(x(t)\big) &\le V\big(x(2T)^{+}\big)e^{u_1(t-2T)} \\ &\le V\big(x(2T)\big)e^{u_1(t-2T)} \\ &\le \beta^2\gamma^2 V\big(x(0)\big)e^{u_1(t-T)+u_2 T}. \end{aligned}$$

Consequently,

$$V\left(x\left(\left(2T+\frac{T}{2}\right)^{-}\right)\right) \le \beta^2\gamma^2 V\big(x(0)\big)e^{\frac{3u_1 T}{2}+u_2 T}. \tag{2.12}$$

For $t \in [2T+\frac{T}{2}, 3T)$, applying (2.4) and (2.12), we obtain

$$\begin{aligned} V\big(x(t)\big) &\le \beta V\left(x\left(\left(2T+\frac{T}{2}\right)^{-}\right)\right)e^{u_2(t-\frac{5T}{2})} \\ &\le \beta^3\gamma^2 V\big(x(0)\big)e^{\frac{3u_1 T}{2}+u_2(t-\frac{3T}{2})}. \end{aligned}$$

Consequently,

$$V\big(x\big((3T)^{-}\big)\big) \le \beta^3\gamma^2 V\big(x(0)\big)e^{\frac{3u_1 T+3u_2 T}{2}}. \tag{2.13}$$

For $t = 3T$, applying (2.5) and (2.13), we obtain

$$\begin{aligned} V\big(x(3T)\big) &\le \gamma V\big(x\big((3T)^{-}\big)\big) \\ &\le \beta^3\gamma^3 V\big(x(0)\big)e^{\frac{3u_1 T+3u_2 T}{2}}. \end{aligned}$$

By induction, when $k = m,\ m = 0, 1, \dots$, for $t \in (mT, mT + \frac{T}{2})$, we obtain

$$V\big(x(t)\big) \le \beta^m \gamma^m V\big(x(0)\big) e^{u_1(t-\frac{mT}{2})+\frac{u_2 mT}{2}}. \tag{2.14}$$

For $t \in [mT + \frac{T}{2}, (m+1)T)$, we obtain

$$V\big(x(t)\big) \le \beta^{m+1} \gamma^m V\big(x(0)\big) e^{\frac{(m+1)u_1 T}{2}+u_2(t-\frac{(m+1)T}{2})}. \tag{2.15}$$

For $t = (m+1)T$, we obtain

$$V\big(x(t)\big) \le \beta^{m+1} \gamma^{m+1} V\big(x(0)\big) e^{\frac{(m+1)u_1 T}{2}+\frac{(m+1)u_2 T}{2}}. \tag{2.16}$$

By (2.14), for $t \in (mT, mT + \frac{T}{2})$, let $t = mT$

$$\begin{aligned} V\big(x(t)\big) &\le \beta^m \gamma^m V\big(x(0)\big) e^{u_1(t-\frac{mT}{2})+\frac{u_2 mT}{2}} \\ &\le \beta^m \gamma^m V\big(x(0)\big) e^{(\frac{u_1+u_2}{2})mT} \\ &= e^{(\ln\beta+\ln\gamma)m} V\big(x(0)\big) e^{(\frac{u_1+u_2}{2})mT} \\ &= V\big(x(0)\big) e^{((\frac{u_1+u_2}{2})T+\ln\beta+\ln\gamma)m}. \end{aligned} \tag{2.17}$$

By (2.15), for $t \in [mT + \frac{T}{2}, (m+1)T)$, we have
Case 1. When $u_2 > 0$, let $t = (m+1)T$

$$\begin{aligned} V\big(x(t)\big) &\le \beta^{m+1} \gamma^m V\big(x(0)\big) e^{\frac{(m+1)u_1 T}{2}+u_2(t-\frac{(m+1)T}{2})}. \\ &\le \beta^{m+1} \gamma^m V\big(x(0)\big) e^{(\frac{u_1+u_2}{2})(m+1)T} \\ &= V\big(x(0)\big) e^{(\frac{u_1+u_2}{2})(m+1)T+(m+1)\ln\beta+m\ln\gamma} \\ &= V\big(x(0)\big) e^{((\frac{u_1+u_2}{2})T+\ln\beta+\ln\gamma)m+(\frac{u_1+u_2}{2})T+\ln\beta}. \end{aligned} \tag{2.18}$$

Case 2. When $u_2 \le 0$, let $t = mT + \frac{T}{2}$

$$\begin{aligned} V\big(x(t)\big) &\le \beta^{m+1} \gamma^m V\big(x(0)\big) e^{\frac{(m+1)u_1 T}{2}+u_2(t-\frac{(m+1)T}{2})}. \\ &\le \beta^{m+1} \gamma^m V\big(x(0)\big) e^{\frac{(m+1)u_1 T}{2}+\frac{m u_2 T}{2}} \\ &= V\big(x(0)\big) e^{\frac{(m+1)u_1 T}{2}+\frac{m u_2 T}{2}+(m+1)\ln\beta+m\ln\gamma} \\ &= V\big(x(0)\big) e^{((\frac{u_1+u_2}{2})T+\ln\beta+\ln\gamma)m+\frac{u_1 T}{2}+\ln\beta}. \end{aligned} \tag{2.19}$$

By (2.16), for $t = (m+1)T$, we have

$$\begin{aligned} V\big(x(t)\big) &\le \beta^{m+1} \gamma^{m+1} V\big(x(0)\big) e^{\frac{(m+1)u_1 T}{2}+\frac{(m+1)u_2 T}{2}} \\ &= V\big(x(0)\big) e^{\frac{(m+1)u_1 T}{2}+\frac{(m+1)u_2 T}{2}+(m+1)\ln\beta+(m+1)\ln\gamma} \\ &= V\big(x(0)\big) e^{((\frac{u_1+u_2}{2})T+\ln\beta+\ln\gamma)(m+1)}. \end{aligned} \tag{2.20}$$

From (2.17)–(2.20), we conclude that the system (1.2) is exponentially stable at the origin.

This completes the proof. □

## 3 A numerical example

In this section, we study the control of Chua's oscillator by applying Theorem 2.1.

*Example* 3.1 Consider Chua's system [16]:

$$\begin{cases} \dot{x}_1 = \alpha(x_2 - x_1 - h(x_1)), \\ \dot{x}_2 = x_1 - x_2 + x_3, \\ \dot{x}_3 = -\beta x_2, \end{cases} \tag{3.1}$$

where $\alpha$ and $\beta$ are two parameters,

$$h(x_1) = bx_1 + \frac{1}{2}(a-b)\big(|x_1+1| - |x_1-1|\big),$$

where $a$ and $b$ are two given constants satisfying $a < b < 0$.

In order to apply Theorem 2.1, we may rewrite system (3.1) as

$$\dot{x}(t) = Ax + f(x),$$

where

$$A = \begin{bmatrix} -\alpha - \alpha b & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix},$$

$$f(x) = \begin{bmatrix} -\frac{1}{2}\alpha(a-b)(|x_1+1| - |x_1-1|) \\ 0 \\ 0 \end{bmatrix}.$$

By easy computation, we obtain

$$\begin{aligned} \|f(x)\|^2 &= \frac{1}{4}\alpha^2(a-b)^2\big[(x_1+1)^2 \\ &\quad + (x_1-1)^2 - 2\big|(x_1+1)(x_1-1)\big|\big] \\ &= \frac{1}{2}\alpha^2(a-b)^2\big(x_1^2 + 1 - \big|x_1^2 - 1\big|\big) \\ &= \begin{cases} \alpha^2(a-b)^2, & x_1^2 > 1 \\ \alpha^2(a-b)^2x^2, & x_1^2 \le 1 \end{cases} \\ &\le \alpha^2(a-b)^2x_1^2 \\ &\le \alpha^2(a-b)^2\big(x_1^2 + x_2^2 + x_3^2\big). \end{aligned}$$

Hence, we choose $l^2 = \alpha^2(a-b)^2$.

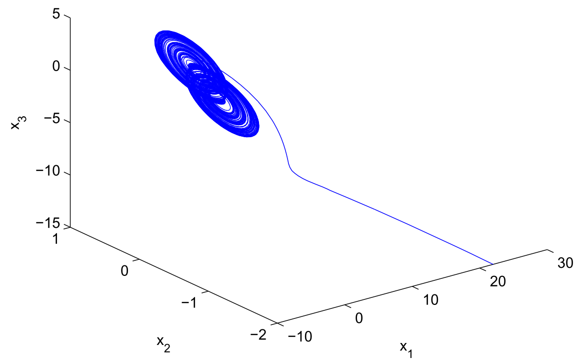


**Figure 1** The chaotic phenomenon of (3.1) with the initial condition $x(0) = (22, -2, -15)^T$

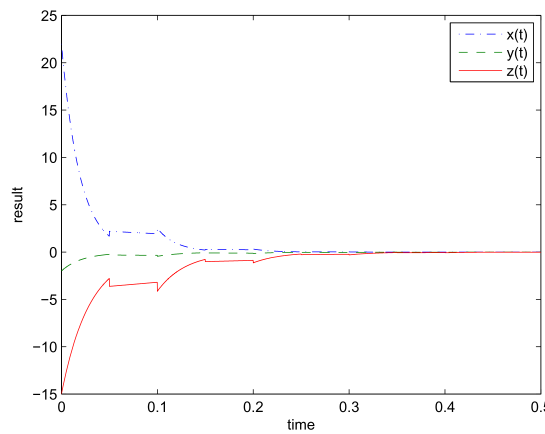


**Figure 2** Time response curves of (3.1) via an alternate control system with double impulses

In the initial condition $x(0) = (22, -2, -15)^T$, Chua's system exhibits chaotic phenomenon when

$$\alpha = 9.2156, \qquad \beta = 15.9946, \qquad a = -1.24905, \qquad b = -0.75735,$$

as shown in Fig. 1.

Meanwhile, for simplicity of calculation, we choose $P = I$, $J_1 = J_2 = \mathrm{diag}(0.3, 0.3, 0.3)$, $B_1 = \mathrm{diag}(-49, -42, -32)$, $B_2 = \mathrm{diag}(-1, -1, -1)$. A small calculation shows that $\beta = \gamma = 1.69$, $\beta_1 = -55.0889$, $\beta_2 = \beta_3 = 1$, $\beta_4 = 15.4359$, $l = 4.5313$, $u_1 = -46.0263$, $u_2 = 24.4985$. By the condition of Theorem 2.1, we have $T > 0.0975$. Thus, in the initial condition $x(0) = (22, -2, -15)^T$, system (3.1) is exponentially stable by Theorem 2.1, The simulation results with $T = 0.1000$ are shown in Fig. 2.

## 4 Conclusions

The paper presents a new model of a control system, namely an alternate control system with double impulses. Theorem 2.1 gives the exponential stability criteria of the considered system. The stability conditions avoid solving linear matrix inequalities. Moreover, the chaotic Chua's circuit can be controlled by Theorem 2.1.

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**Availability of data and materials**
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## Declarations

**Competing interests**
The author declares that he has no competing interests.

**Author contribution**
The author read and approved the final manuscript.

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