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q-ROF mappings and Suzuki type common fixed point results in b-metric spaces with application

Maliha Rashid[1], Lariab Shahid[1*], Ravi P. Agarwal[2], Aftab Hussain[3] and Hamed Al-Sulami[3]

*Correspondence:
lariab.phdma112@iiu.edu.pk
[1]Department of Mathematics and Statistics, International Islamic University, Islamabad, Pakistan
Full list of author information is available at the end of the article

Abstract

In the present paper the concepts of q-rung orthopair fuzzy mappings (q-ROF mapping) and q-rung (α, β)-cuts are introduced. Some common fixed point results for q-ROF mappings are presented in b-metric spaces using Suzuki-type contractive conditions. Examples in support of obtained results are also presented. We have also presented an application of our result for the existence of solution of nonlinear fractional integral inclusion. The results are of their own kind in the literature of q-ROF sets and will pave the way for further research in the area.

Keywords: q-ROF sets; q-ROF mapping; Suzuki contractive conditions; b-metric spaces

1 Introduction

The concept of fuzzy sets was introduced by Zadeh [55] to pave a path for the better interpretation of data in real life problems. The key concept given by him was to award a membership grade from $[0,1]$ to the specific attribute. Since then various ideas and applications of fuzzy sets towards decision making, game theory, control systems, engineering, robotics, image processing, optimization theory, etc. have been initiated. There are situations where just membership grade is not enough to deal with, and on this account a grade against the membership of an attribute to a specific trait was introduced. Such sets are defined as orthopair fuzzy sets represented by $\langle \mu_A, \nu_A \rangle$, where μ_A stands for grade of membership while ν_A for nonmembership. Generalizations of orthopair fuzzy sets have been introduced as intuitionistic fuzzy sets (IFS) and Pythagorean fuzzy sets [11, 12]. The difference between these two types is that for the first case, the sum of membership and nonmembership grades is bounded by 1, while for the second case the sum of squares of membership and nonmembership grades is bounded by 1.

Yager then, in 2017, gave a further generalization of orthopair fuzzy sets known as q-rung orthopair fuzzy sets (q-ROF sets) where $(\mu_A)^q + (\nu_A)^q \leq 1$ [54]. The main advantage of a q-ROF set is that it increases the bounding space of selection of belongingness and non-belongingness grade of a trait for a given set. Several mathematicians have further studied q-ROF sets and have applied the concept in decision making problems and artificial intelligence, especially in the filed of medicine and agriculture. Multi-attribute decision making is an important aspect of decision sciences. It is a process that can give the ranking results for the finite alternatives according to the attribute values of different alternatives. The concept of q-ROF sets is combined with many existing aggregation operators for the improved management of evaluating information and decision making [40, 45, 53, 56].

Ever since, in the history of fixed point theory, mathematicians have introduced several contractive conditions and mappings for more improved fixed point results. In this regard several contractions have been developed like Banach contraction, Chatterjea contraction [26], Kannan contraction [36], α-ψ type contractions [51], (θ, L)-weak contraction [22], etc. Suzuki [52] in 2008 introduced Suzuki-type contractive condition, which generalizes Banach contraction and characterizes the metric completeness of the underlying space. Since then the concept has been extended in various directions, and fixed point, common fixed point results along with applications have been presented, for example, [1, 6, 7, 21, 23, 34, 38, 39, 41, 42, 50]. In 2015 Saleem *et al.* [49] presented fixed point results for Suzuki-type contractive conditions utilizing multivalued mappings in fuzzy metric spaces with applications. Recently Gopal and Moreno [31] presented the concept of Suzuki-type fuzzy Z-contractive mappings, which is a generalization of Fuzzy $\mathcal{Z}$-contractive mappings, and obtained fixed point results.

The notion of fuzzy mappings was initiated by Heilpern [33], and he proved a fixed point result for fuzzy contractive mappings to generalize Nadler's result [43]. Afterwards, the idea of fuzzy mappings has been extended in various directions [3, 5, 16, 17, 19, 20, 32, 46–48].

Moreover, fixed points results for various metric spaces using contractive conditions for single-valued and multivalued mappings have been studied. Czerwik introduced b-metric spaces [28], and since then various fixed point results have been obtained using various contractive conditions, e.g., [2, 4, 8, 9, 13, 14, 29, 37, 44] in b-metric spaces.

In the following article we introduce the notion of q-rung orthopair fuzzy mapping as a generalization of fuzzy mapping and q-rung (α, β)-level sets and hence prove some common fixed point results for a pair of q-rung orthopair fuzzy mapping in b-metric space.

2 Preliminaries

Consider (X, d) to be a metric space and $CB(X)$ denotes the family of all closed and bounded subsets of X. Consider that H denotes the Hausdorff metric induced by d defined as

$$H(A,B) = \max\left\{\sup_{a\in A} d(a,B), \sup_{b\in B} d(b,A)\right\},$$

where $A, B \in CB(X)$ and $d(x,B) = \inf_{y\in B} d(x,y)$.

Lemma 1 ([43]) *If $A, B \in CB(X)$ and $x \in A$, then for any real number $l \geq 1$ there exists $y \in B$ such that $d(x,y) \leq l.H(A,B)$. Also $d(x,B) \leq H(A,B)$.*

Czerik introduced the generalized notion of b-metric space by changing the triangular inequality in a metric space.

Definition 1 ([28]) Consider $X \neq \emptyset$ and $s \geq 1$. A function $d : X \times X \to [0, \infty)$ will be b-metric on X if for all $x, y, z \in X$ the following hold:

(i) $d(x, y) = 0$ if and only if $x = y$ (indistancy);

(ii) $d(x, y) = d(y, x)$, (symmetry);

(iii) $d(x, z) \leq s[d(x, y) + d(y, z)]$ (b-triangular inequality).

Definition 2 ([35]) Consider $\{x_n\}$ to be a sequence in a b-metric space (X, d). Then,

(a) $\{x_n\}$ is called b-convergent if $x \in X$ so that $d(x_n, x) \to 0$ as $n \to \infty$.

(b) $\{x_n\}$ is a b-Cauchy sequence if $d(x_n, x_m) \to 0$ as $n, m \to \infty$.

A b-metric space is complete if and only if each b-Cauchy sequence in the space is b-convergent.

Example 1 ([24]) Let $X = 0, 1, 2$ and $d : X \times X \to \mathbb{R}_+$ such that $d(0, 1) = d(1, 0) + d(0, 2) = d(2, 0) = 1$, $d(1, 2) = d(2, 1) = \tau \geq 2$, $d(0, 0) = d(1, 1) = d(2, 2) = 0$. Then

$$d(x, y) \leq \frac{\tau}{2}[d(x, z) + d(z, y) \quad \text{for all } x, y, z \in X. \tag{2.1}$$

Then (X, d) is a b-metric space. If $\tau > 2$, the ordinary triangular inequality does not hold and (X, d) is not a metric space.

Zadeh [55] introduced fuzzy sets by defining the membership grade of an instinct to a trait which in real life does not have precisely defined criteria of membership for that particular trait.

Definition 3 Let X be a nonempty set. Then $\mu : X \to [0, 1]$ is a fuzzy set defining the grades of membership of elements of X.

Definition 4 An α-level set of a fuzzy set μ is defined as

$$\mu_\alpha = \{x \in X : \mu(x) \geq \alpha\}, \quad \text{where } \alpha \in [0, 1].$$

A fuzzy set μ is convex if and only if the sets μ_α are convex.

Atanassov [11] presented intuitionistic fuzzy sets as a generalized notion of fuzzy sets. Intuitionistic fuzzy set depicts the grade of membership of an element for a set and its grade of nonmembership. Atanassov [12] then in 1993 introduced another type of orthopair fuzzy sets known as Pythagorean fuzzy sets in which the sum of squares of grades of membership and nonmembership of element is bounded by 1. Yager [54] in 2017 generalized the class of orthopair fuzzy sets called q-rung orthopair fuzzy sets or q-ROF sets.

Following concepts are defined by Yager in [54].

Definition 5 A q-rung orthopair fuzzy subset A of X, denoted as a q-ROF set, is an orthopair.

$$A = \langle \mu_A, \eta_A \rangle_q,$$

where $\mu_A, \eta_A : X \to [0, 1]$ indicate the grade of belongingness and non-belongingness of elements in A respectively, which fulfills

1. $q \geq 1$;
2. $\mu_A(x) \in [0,1]$ and $\eta_A(x) \in [0,1]$;
3. $(\mu_A(x))^q + (\eta_A(x))^q \leq 1$.

Heilpern [33] in 1981 introduced fuzzy contractive mappings and extended Banach contraction theorem for fuzzy contractive mappings. Further this concept has been extended in various directions; for example, see [15, 16, 20, 25, 27, 46, 48]. Following are the concepts defined in [33].

Definition 6 A fuzzy subset A of X is called an approximate quantity if and only if its α-level set is a compact convex subset of X for each $\alpha \in [0,1]$ and $\sup A(x) = 1$ for all $x \in X$.

$W(X)$ denotes the collection of approximate quantities of X. When $A \in W(X)$ and $A(x_0) = 1$ for some $x_0 \in X$, then A is identified as an approximation of x_0.

Let $A, B \in W(X)$. An approximate quantity A is more accurate than B, denoted by $A \subset B$, if and only if $A(x) \leq B(x)$ for all $x \in X$.

Let $A, B \in W(X)$, $\alpha \in [0,1]$, then the distance between A and B is defined as follows:

$$p_\alpha(A,B) = \inf_{x \in A_\alpha, y \in B_\alpha} d(x,y),$$

$$D_\alpha(A,B) = H(A_\alpha, B_\alpha).$$

Let X be a set and Y be a metric linear space. F is called fuzzy mapping if F is a mapping from the set X into $W(Y)$, i.e., $F(x)$ is an approximate quantity.

Lemma 2 *Let $x \in X$, $A \in W(X)$ and $\{x\}$ be a fuzzy set with membership function equal to the characteristic function of set $\{x\}$. If $\{x\} \subset A$, then $p_\alpha(x,A) = 0$.*

Lemma 3 *For any $x, y \in X$,*

$$p_\alpha(x,A) \leq d(x,y) + p_\alpha(y,A).$$

In 2008, Suzuki [52] presented a fixed point theorem generalizing the Banach contraction theorem and characterizing the metric completeness.

Theorem 1 *Consider (X,d) to be a complete metric space and $T: X \to X$. A nonincreasing function $\theta: [0,1) \to (\frac{1}{2}, 1]$ is given by*

$$\theta(r) = \begin{cases} 1 & \text{if } 0 \leq r \leq (\sqrt{5}-1)/2, \\ (1-r)^{-2} & \text{if } (\sqrt{5}-1)/2 \leq r \leq 2^{-\frac{1}{2}}, \\ (1+r)^{-1} & \text{if } 2^{-\frac{1}{2}} \leq r < 1. \end{cases}$$

Suppose that there is $r \in [0,1)$ so that

$$\theta(r) d(x, Tx) \leq d(x,y) \quad \text{implies} \quad d(Tx, Ty) \leq r d(x,y)$$

for all $x, y \in X$. Then there exists a unique fixed point of T. Moreover, $\lim_n T^n x = z$ for all $x \in X$.

Suzuki contraction theorem is extended in various directions, e.g., [10, 23, 49]. Doric and Lazovic [30] presented the following fixed point theorem for multivalued mappings using Suzuki contraction.

Theorem 2 *Consider a nonincreasing function defined as* $\varphi : [0,1) \to (0,1]$:

$$\varphi(r) = \begin{cases} 1 & \textit{if } 0 \le r < \frac{1}{2}, \\ 1 - r & \textit{if } \frac{1}{2} \le r < 1. \end{cases}$$

Consider (X,d) *to be a complete metric space and* $T : X \to CB(X)$. *Suppose that there is* $r \in [0,1)$ *so that* $\varphi(r)d(x,Tx) \le d(x,y)$ *implies*

$$H(Tx,Ty) \le r.\max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(y,Tx)}{2}\right\}$$

for all $x, y \in X$. *Then* $z \in X$ *so that* $z \in Tz$.

3 *q*-ROF mappings and level sets

On the basis of well-known fuzzy notions existing in the literature, we have dedicated the following section to some new concepts defined for q-ROF sets such as q-rung α-level sets, q-rung (α,β)-level sets, and q-rung orthopair fuzzy mappings. A common fixed point result for a pair of q-rung orthopair fuzzy mappings in the settings of b-metric space is also presented using Suzuki-type contractive condition. An example in the support of our main result is also given.

Throughout this article the class of all q-ROF subsets of X will be denoted by $F^q(X)$ and $\varphi : [0,1) \to (0,1]$ is a nonincreasing function defined as

$$\varphi(r) = \begin{cases} 1 & \text{if } 0 \le r < \frac{1}{2}, \\ (\frac{1-r}{s}) & \text{if } \frac{1}{2} \le r < 1. \end{cases}$$

Definition 7 Let $A \in F^q(X)$ and $x \in X$, then q-rung α-level set of A is

$$[A]^q_\alpha = \{x \in X : (\mu_A(x))^q \ge \alpha \text{ and } (\eta_A(x))^q \le 1 - \alpha\}.$$

Definition 8 Consider $\alpha, \beta \in [0,1]$ and $\alpha + \beta \le 1$, then q–rung (α,β)-level sets of A is

$$[A]^q_{(\alpha,\beta)} = \{x \in X : (\mu_A(x))^q \ge \alpha \text{ and } (\eta_A(x))^q \le \beta\}$$

and

$$A^q_{(\alpha,\beta)} = \{x \in X : (\mu_A(x))^q > \alpha \text{ and } (\eta_A(x))^q < \beta\}.$$

Definition 9 Consider X to be an arbitrary set, Y a metric space. A mapping $T : X \to F^q(Y)$ is called q-rung orthopair fuzzy mapping.

Since we claim that q-rung orthopair fuzzy mapping is a generalization of intuitionistic fuzzy mapping, so below is an example in support of the claim.

Example 2 Consider $X = [0,1]$ and let $T : X \to F^q(X)$ be defined as

$$\mu_{Tx}(t) = \begin{cases} 1 & t = 0, \\ (\frac{9}{2q})^{\frac{1}{q}} & 0 < t \leq \frac{1}{70}, \\ (\frac{5}{3q})^{\frac{1}{q}} & t > \frac{1}{70}, \end{cases} \qquad \eta_{Tx}(t) = \begin{cases} 0 & t = 0, \\ (\frac{7}{8q})^{\frac{1}{q}} & 0 < t \leq \frac{1}{60}, \\ (\frac{8}{5q})^{\frac{1}{q}} & t > \frac{1}{60}. \end{cases}$$

Clearly T is a q-rung orthopair fuzzy mapping for $q = 6$ and is not an intuitionistic fuzzy mapping.

Definition 10 Let X be a metric space. A point $x^* \in X$ is called fixed point of a q-rung orthopair fuzzy mapping $T : X \to F^q(X)$ if there exist $\alpha, \beta \in [0,1]$ such that $x^* \in [Tx^*]^q_{(\alpha,\beta)}$ for some $x^* \in X$.

Definition 11 A q-ROF set $A = \langle \mu_A, \eta_A \rangle_q$ in a b-metric linear space X will be an approximate quantity if and only if $[A]^q_{(\alpha,\beta)}$ is compact and convex in X for each $\alpha, \beta \in (0,1]$ along with

$$\sup_{x \in V} \bigl(\mu_A(x)\bigr)^q = 1 \quad \text{and} \quad \inf_{x \in V} \bigl(\eta_A(x)\bigr)^q = 0.$$

$Ķ(X) = \{A \in F^q(X) : A \text{ is an approximate quantity}\}$.

Definition 12 Consider (X, d) to be a b-metric space with a constant $s \geq 1$. For $A, B \in Ķ(X)$ and $\alpha, \beta \in [0,1]$, define

$$p^q_{(\alpha,\beta)}(A,B) = d\bigl([A]^q_{(\alpha,\beta)}, [B]^q_{(\alpha,\beta)}\bigr) = \inf_{x \in [A]^q_{(\alpha,\beta)}, y \in [B]^q_{(\alpha,\beta)}} d(x,y),$$

$$p^q(A,B) = \sup_{\alpha} \inf_{\beta} p^q_{(\alpha,\beta)}(A,B),$$

$$D^q_{(\alpha,\beta)}(A,B) = H\bigl([A]^q_{(\alpha,\beta)}, [B]^q_{(\alpha,\beta)}\bigr),$$

$$D^q(A,\xi) = \sup_{\alpha} \inf_{\beta} D^q_{(\alpha,\beta)}(A,B).$$

The following results are the generalizations of the results defined in [33] which will be helpful in proving fixed point theorems for q-ROF mappings in b-metric spaces.

Lemma 4 *Let $x \in X$, $A \in Ķ(X)$, and $\{x\}$ be a q-ROF set as its membership function is equal to $\chi_{\{x\}}$ (the characteristic function of $\{x\}$) and nonmembership function is equal to $1 - \chi_{\{x\}}$ defined as*

$$\mu_A(x) = \chi_{\{x\}} = \begin{cases} 1 & \mathit{if}\ e \in \{x\}, \\ 0 & \mathit{if}\ e \notin \{x\}, \end{cases} \qquad \eta_A(x) = \begin{cases} 0 & \mathit{if}\ e \in \{x\}, \\ 1 & \mathit{if}\ e \notin \{x\} \end{cases}$$

for some $e \in \{x\}$. Clearly, $(\mu(x))^q + (\eta(x))^q \leq 1$. If $\{x\} \subset A$, then $p^q_{(\alpha,\beta)}(x, A) = 0$ for each $\alpha, \beta \in [0,1]$.

Proof If $\{x\} \subset A$, then $x \in [A]^q_{(\alpha,\beta)}$ for each $\alpha, \beta \in [0,1]$.

$$p^q_{(\alpha,\beta)}(x,A) = \inf_{y \in [A]^q_{(\alpha,\beta)}} d(x,y) = 0. \qquad \square$$

Lemma 5 *Let $x \in X$, $A \in Ḳ(X)$, then for $s \geq 1$,*

$$p^q_{(\alpha,\beta)}(x,A) \leq s\big(d(x,y) + p^q_{(\alpha,\beta)}(y,A)\big).$$

Proof

$$p^q_{(\alpha,\beta)}(x,A) = \inf_{z \in [A]^q_{(\alpha,\beta)}} d(x,z) \leq \inf_{z \in [A]^q_{(\alpha,\beta)}} s\big(d(x,y) + d(y,z)\big) = s\big(d(x,y) + p^q_{(\alpha,\beta)}(y,A)\big). \qquad \square$$

Lemma 6 *Let $A \in Ḳ(X)$ and $\{x_0\} \subseteq A$. Then*

$$d\big(x_0, [x]^q_{(\alpha,\beta)}\big) \leq D^q_{(\alpha,\beta)}(A,x)$$

for each $B \in Ḳ(X)$ and $\alpha, \beta \in [0,1]$.

Proof Since $\{x_0\} \subseteq A$, therefore $x_0 \in [A]^q_{(\alpha,\beta)}$ for all $\alpha, \beta \in [0,1]$. Hence

$$d\big(x_0, [B]^q_{(\alpha,\beta)}\big) \leq H\big([A]^q_{(\alpha,\beta)}, [B]^q_{(\alpha,\beta)}\big) = D^q_{(\alpha,\beta)}(A,B). \qquad \square$$

Lemma 7 *Consider (X,d) to be a complete b-metric linear space, $s \geq 1$, and let $T : X \to Ḳ(X)$ be a q-rung orthopair fuzzy mapping. Consider that for each $x \in X$ and each pair $(\alpha, \beta) \in [0,1]^2$, $[Tx]^q_{(\alpha,\beta)_{Tx}}$, $[Ta]^q_{(\alpha,\beta)_{Ta}}$ are nonempty. Then we have*

$$d\big(x, [Tx]^q_{(\alpha,\beta)_{Tx}}\big) \leq s\Big(\big(d\big(x, [Ta]^q_{(\alpha,\beta)_{Ta}}\big) + H\big([Tx]^q_{(\alpha,\beta)_{Tx}}, [Ta]^q_{(\alpha,\beta)_{Ta}}\big)\Big).$$

Proof

$$\begin{aligned} d\big(x, [Tx]^q_{(\alpha,\beta)_{Tx}}\big) &\leq s\big(d(x,y) + d\big(y, [Tx]^q_{(\alpha,\beta)_{Tx}}\big)\big) \\ &\leq s\Big(\big(d\big(x, [Ta]^q_{(\alpha,\beta)_{Ta}}\big) + H\big([Tx]^q_{(\alpha,\beta)_{Tx}}, [Ta]^q_{(\alpha,\beta)_{Ta}}\big)\Big). \qquad \square \end{aligned}$$

Theorem 3 *Consider (X,d) to a complete b-metric linear space, $s \geq 1$, and $T : X \to Ḳ(X)$ be a q- rung orthopair fuzzy mapping. For each element $x \in X$ and each pair $(\alpha, \beta) \in [0,1] \times [0,1]$, $[Tx]^q_{(\alpha,\beta)_{Tx}}$ is nonempty. Assume that $r \in [0,1)$ such that*

$$\varphi(r) d\big(x, [Tx]^q_{(\alpha,\beta)_{Tx}}\big) \leq d(x,y) \tag{3.1}$$

implies

$$\begin{aligned} H\big([Tx]^q_{(\alpha,\beta)_{Tx}}, [Ty]^q_{(\alpha,\beta)_{Ty}}\big) \leq r \max\Bigg\{ & d(x,y), d\big(x, [Tx]^q_{(\alpha,\beta)_{Tx}}\big), d\big(y, [Ty]^q_{(\alpha,\beta)_{Ty}}\big), \\ & \frac{d(x, [Ty]^q_{(\alpha,\beta)_{Tx}}) + d(y, [Tx]^q_{(\alpha,\beta)_{Ty}})}{2s} \Bigg\} \end{aligned} \tag{3.2}$$

for all $x, y \in X$. Then there exists $z \in X$ such that $z \in [Tz]^q_{(\alpha,\beta)_{Tz}}$.

Proof Consider $x_1 \in X$. Then $(\alpha,\beta)_{Tx_1} \in [0,1]^2$ so that $[Tx_1]^q_{(\alpha,\beta)_{Tx_1}}$ is nonempty. Let $x_2 \in [Tx_1]^q_{(\alpha,\beta)_{Tx_1}}$, then

$$d\bigl(x_2,[Tx_2]^q_{(\alpha,\beta)_{Tx_2}}\bigr) \le H\bigl([Tx_1]^q_{(\alpha,\beta)_{Tx_1}},[Tx_2]^q_{(\alpha,\beta)_{Tx_2}}\bigr).$$

As $\varphi(r) \le 1$, this implies

$$\varphi(r)d\bigl(x_1,[Tx_1]^q_{(\alpha,\beta)_{Tx_1}}\bigr) \le d(x_1,x_2).$$

Then from (3.2) we have

$$\begin{aligned} d\bigl(x_2,[Tx_2]^q_{(\alpha,\beta)_{Tx_2}}\bigr) &\le H\bigl([Tx_1]^q_{(\alpha,\beta)_{Tx_1}},[Tx_2]^q_{(\alpha,\beta)_{Tx_2}}\bigr) \\ &\le r\max\biggl\{d(x_1,x_2), d\bigl(x_1,[Tx_1]^q_{(\alpha,\beta)_{Tx_1}}\bigr), d\bigl(x_2,[Tx_2]^q_{(\alpha,\beta)_{Tx_2}}\bigr), \\ &\quad \frac{d(x_1,[Tx_2]^q_{(\alpha,\beta)_{Tx_2}}) + d(x_2,[Tx_1]^q_{(\alpha,\beta)_{Tx_1}})}{2s}\biggr\} \\ &\le r\max\biggl\{d(x_1,x_2), d\bigl(x_2,[Tx_2]^q_{(\alpha,\beta)_{Tx_2}}\bigr), \\ &\quad \frac{d(x_1,[Tx_2]^q_{(\alpha,\beta)_{Tx_2}}) + d(x_2,[Tx_1]^q_{(\alpha,\beta)_{Tx_2}})}{2s}\biggr\} \\ &\le r\max\biggl\{d(x_1,x_2), d\bigl(x_2,[Tx_2]^q_{(\alpha,\beta)_{Tx_2}}\bigr), \frac{d(x_1,[Tx_2]^q_{(\alpha,\beta)_{Tx_2}})}{2s}\biggr\} \\ &\le r\max\biggl\{d(x_1,x_2), d\bigl(x_2,[Tx_2]^q_{(\alpha,\beta)_{Tx_2}}\bigr), \\ &\quad \frac{s[d(x_1,x_2) + d(x_2,[Tx_2]^q_{(\alpha,\beta)_{Tx_2}})]}{2s}\biggr\} \\ &\le r\max\biggl\{d(x_1,x_2), d\bigl(x_2,[Tx_2]^q_{(\alpha,\beta)_{Tx_2}}\bigr), \\ &\quad \frac{d(x_1,x_2) + d(x_2,[Tx_2]^q_{(\alpha,\beta)_{Tx_2}})}{2}\biggr\}. \end{aligned}$$

Since $r < 1$, so $d(x_2,[Tx_2]^q_{(\alpha,\beta)_{Tx_2}}) \le rd(x_1,x_2)$. Hence there exists $x_3 \in X$ such that $d(x_2,x_3) \le rd(x_1,x_2)$. Thus we can construct a sequence $\{x_n\}$ in X such that $x_{n+1} \in [Tx_n]^q_{(\alpha,\beta)_{Tx_n}}$ and $d(x_n,x_{n+1}) \le rd(x_{n-1},x_n)$, and hence

$$\sum_{n=1}^{\infty} d(x_n,x_{n+1}) \le \sum_{n=1}^{\infty} r^{n-1}d(x_1,x_2) < \infty.$$

This implies $\{x_n\}$ is a Cauchy sequence. As X is complete, $z \in X$ so that $\lim_{n\to\infty} x_n = z$.

Next it is proved that

$$d\bigl(z,[Ty]^q_{(\alpha,\beta)_{Ty}}\bigr) \le r\max\bigl\{d(z,y), d\bigl(y,[Ty]^q_{(\alpha,\beta)_{Ty}}\bigr)\bigr\} \quad \forall y \in X\backslash\{z\}. \tag{3.3}$$

Since $x_n \to z$, $n_0 \in \mathbb{N}$ so that $d(z,x_n) \le \frac{1}{3s^2}d(z,y)$ $\forall n \ge n_0$. Then we have

$$\begin{aligned}\varphi(r)d\big(x_n,[Tx_n]^q_{(\alpha,\beta)_{Tx_n}}\big) &\le d(x_n,x_{n+1})\\ &\le s\big(d(x_n,z)+d(z,x_{n+1})\big)\\ &\le \frac{2}{3s}d(y,z).\end{aligned}$$

Thus

$$\varphi(r)d\big(x_n,[Tx_n]^q_{(\alpha,\beta)_{Tx_n}}\big) \le \frac{2}{3s}d(y,z).$$

Since $\frac{2}{3s}d(y,z) = \frac{1}{s}(d(y,z) - \frac{1}{3}d(y,z)) \le \frac{1}{s}(d(y,z) - sd(z,x_n)) \le d(x_n,y)$. Hence $\varphi(r)d(x_n, [Tx_n]^q_{(\alpha,\beta)_{Tx_n}}) \le d(x_n,y)$. Then from (3.2)

$$H\big([Tx_n]^q_{(\alpha,\beta)_{Tx_n}},[Ty]^q_{(\alpha,\beta)_{Ty}}\big) \le r\max\Big\{d(x_n,y),d\big(x_n,[Tx_n]^q_{(\alpha,\beta)_{Tx_n}}\big),d\big(y,[Ty]^q_{(\alpha,\beta)_{Ty}}\big), \frac{d(x_n,[Ty]^q_{(\alpha,\beta)_{Ty}})+d(y,[Tx_n]^q_{(\alpha,\beta)_{Tx_n}})}{2s}\Big\}. \tag{3.4}$$

Since $x_{n+1} \in [Tx_n]^q_{(\alpha,\beta)_{Tx_n}}$, then

$$\begin{aligned}&d\big(x_{n+1},[Ty]^q_{(\alpha,\beta)_{Ty}}\big) \le H\big([Tx_n]^q_{(\alpha,\beta)_{Tx_n}},[Ty]^q_{(\alpha,\beta)_{Ty}}\big) \quad\text{and}\\ &d\big(x_n,[Tx_n]^q_{(\alpha,\beta)_{T_n}}\big) \le d(x_n,x_{n+1}).\end{aligned}$$

Then from (3.4) we get

$$d\big(x_{n+1},[Ty]^q_{(\alpha,\beta)_{Ty}}\big) \le r\max\Big\{d(x_n,y),d\big(x_n,[Tx_n]^q_{(\alpha,\beta)_{Tx_n}}\big),d\big(y,[Ty]^q_{(\alpha,\beta)_{Ty}}\big), \frac{d(x_n,[Ty]^q_{(\alpha,\beta)_{Ty}})+d(y,[Tx_n]^q_{(\alpha,\beta)_{Tx_n}})}{2s}\Big\}$$

for all natural numbers with $n \ge n_0$. Letting $n \to \infty$, we obtain (3.3).

Next it is shown that $z \in [Tz]^q_{(\alpha,\beta)_{Tz}}$. First consider $0 \le r < \frac{1}{2}$. Suppose that $z \notin [Tz]^q_{(\alpha,\beta)_{Tz}}$. Let $\wp \in [Tz]^q_{(\alpha,\beta)_{Tz}}$, so $2srd(\wp,z) < d(z,[Tz]^q_{(\alpha,\beta)_{Tz}})$. Since $\wp \in [Tz]^q_{(\alpha,\beta)_{Tz}}$ implies $\wp$ is not equal to z, hence from (3.3) we have

$$d\big(z,[Ta]^q_{(\alpha,\beta)_{T\wp}}\big) \le r\max\big\{d(z,\wp),d\big(\wp,[T\wp]^q_{(\alpha,\beta)_{T\wp}}\big)\big\}. \tag{3.5}$$

Also, since $\varphi(r)d(z,[Tz]^q_{(\alpha,\beta)_{Tz}}) \le d(z,[Tz]^q_{(\alpha,\beta)_{Tz}}) \le d(z,\wp)$, then from (3.2) we have

$$\begin{aligned}H\big([Tz]^q_{(\alpha,\beta)_{Tz}},[T\wp]^q_{(\alpha,\beta)_{T\wp}}\big) &\le r\max\Big\{d(z,\wp),d\big(z,[Tz]^q_{(\alpha,\beta)_{Tz}}\big),d\big(\wp,[T\wp]^q_{(\alpha,\beta)_{T\wp}}\big), \frac{d(\wp,[Tz]^q_{(\alpha,\beta)_{Tz}})+d(z,[T\wp]^q_{(\alpha,\beta)_{T\wp}})}{2s}\Big\}\\ &\le r\max\big\{d(z,\wp),d\big(\wp,[T\wp]^q_{(\alpha,\beta)_{T\wp}}\big)\big\}.\end{aligned}$$

Hence

$$\begin{aligned} d\big(\wp,[T\wp]^q_{(\alpha,\beta)_{T\wp}}\big) &\le H\big([Tz]^q_{(\alpha,\beta)_{Tz}},[T\wp]^q_{(\alpha,\beta)_{T\wp}}\big) \\ &\le r\max\big\{d(z,\wp),d\big(\wp,[T\wp]^q_{(\alpha,\beta)_{T\wp}}\big)\big\}. \end{aligned}$$

Hence $d(\wp,[T\wp]^q_{(\alpha,\beta)_{T\wp}}) \le rd(z,\wp) < d(z,\wp)$ and from (3.5), $d(z,[T\wp]^q_{(\alpha,\beta)_{T\wp}}) \le rd(z,wp)$.

Therefore, by Lemma 7, we obtain

$$\begin{aligned} d\big(z,[Tz]^q_{(\alpha,\beta)_{Tz}}\big) &\le sd\big(z,[T\wp]^q_{(\alpha,\beta)_{T\wp}}\big)+sH\big([Tz]^q_{(\alpha,\beta)_{Tz}},[T\wp]^q_{(\alpha,\beta)_{T\wp}}\big) \\ &\le sd\big(z,[T\wp]^q_{(\alpha,\beta)_{T\wp}}\big)+rs\max\big\{d(z,\wp),d\big(\wp,[T\wp]^q_{(\alpha,\beta)_{T\wp}}\big)\big\} \\ &\le srd(z,\wp)+srd(z,\wp) \\ &\le 2srd(z,\wp) < d\big(z,[Tz]^q_{(\alpha,\beta)_{Tz}}\big). \end{aligned}$$

A contradiction, hence $z \in [Tz]^q_{(\alpha,\beta)_{Tz}}$.

Now, for the case $\frac{1}{2} \le r < 1$, we will first prove

$$\begin{aligned} H\big([Tx]^q_{(\alpha,\beta)_{Tx}},[Tz]^q_{(\alpha,\beta)_{Tz}}\big) \le r\max\Bigg\{& d(x,z),d\big(x,[Tx]^q_{(\alpha,\beta)_{Tx}}\big),d\big(z,[Tz]^q_{(\alpha,\beta)_{Tz}}\big), \\ & \frac{d(z,[Tx]^q_{(\alpha,\beta)_{Tx}})+d(x,[Tz]^q_{(\alpha,\beta)_{Tz}})}{2s}\Bigg\} \quad \forall x \in X. \end{aligned} \tag{3.6}$$

If $x = z$, then (3.6) holds. Let $x \neq z$, then for every n belonging to natural numbers, there is a sequence $y_n \in [Tx]^q_{(\alpha,\beta)_{Tx}}$ so that $sd(z,y_n) \le d(z,[Tx]^q_{(\alpha,\beta)_{Tx}}) + \frac{1}{n}d(x,z)$. Now from (3.3) we have

$$\begin{aligned} d\big(x,[Tx]^q_{(\alpha,\beta)_{Tx}}\big) &\le d(x,y_n) \le s\big(d(x,z)+d(z,y_n)\big) \\ &\le sd(x,z)+d\big(z,[Tx]^q_{(\alpha,\beta)_{Tx}}\big)+\frac{1}{n}d(x,z) \\ &\le sd(x,z)+r\max\big\{d(x,z),d\big(x,[Tx]^q_{(\alpha,\beta)_{Tx}}\big)\big\}+\frac{1}{n}d(x,z). \end{aligned}$$

If $d(x,z) > d(x,[Tx]^q_{(\alpha,\beta)_{Tx}})$, then

$$\begin{aligned} d\big(x,[Tx]^q_{(\alpha,\beta)_{Tx}}\big) &\le sd(x,z)+rd(x,z)+\frac{1}{n}d(x,z) \\ &= \bigg(\frac{1}{n}+s+r\bigg)d(x,z). \end{aligned}$$

Letting $n \to \infty$, we have $d(x,[Tx]^q_{(\alpha,\beta)_{Tx}}) \le (s+r)d(x,z)$.

$$\begin{aligned} \varphi(r)d\big(x,[Tx]^q_{(\alpha,\beta)_{Tx}}\big) &= \bigg(\frac{1-r}{s}\bigg)d\big(x,[Tx]^q_{(\alpha,\beta)_{Tx}}\big) \le d\big(x,[Tx]^q_{(\alpha,\beta)_{Tx}}\big) \\ &\le \frac{1}{s+r}d\big(x,[Tx]^q_{(\alpha,\beta)_{Tx}}\big) \le d(x,z). \end{aligned}$$

Using (3.2) we get (3.6).

If $d(x,z) < d(x,[Tx]^q_{(\alpha,\beta)_{Tx}})$, then

$$d\big(x,[Tx]^q_{(\alpha,\beta)_{Tx}}\big) \le sd(x,z) + rd\big(x,[Tx]^q_{(\alpha,\beta)_{Tx}}\big) + \frac{1}{n}d(x,z),$$

and therefore, we have

$$(1-r)d\big(x,[Tx]^q_{(\alpha,\beta)_{Tx}}\big) \le s\left(1+\frac{1}{sn}\right)d(x,z),$$
$$\left(\frac{1-r}{s}\right)d\big(x,[Tx]^q_{(\alpha,\beta)_{Tx}}\big) \le \left(1+\frac{1}{sn}\right)d(x,z).$$

Now letting $n \to \infty$, $\varphi(r)d(x,[Tx]^q_{(\alpha,\beta)_{Tx}}) \le d(x,z)$. Then we have (3.6). Finally, from (3.6) we have

$$\begin{aligned} d\big(z,[Tz]^q_{(\alpha,\beta)_{Tz}}\big) &= \lim_{n\to\infty} d\big(x_{n+1},[Tz]^q_{(\alpha,\beta)_{Tz}}\big) \le \lim_{n\to\infty} H\big([Tx_n]^q_{(\alpha,\beta)_{Tx_n}},[Tz]^q_{(\alpha,\beta)_{Tz}}\big) \\ &\le \lim_{n\to\infty} r\max\Bigg\{d(x_n,z), d\big(x_n,[Tx_n]^q_{(\alpha,\beta)_{Tx_n}}\big), d\big(z,[Tz]^q_{(\alpha,\beta)_{Tz}}\big), \\ &\qquad \frac{d(z,[Tx_n]^q_{(\alpha,\beta)_{Tx_n}}) + d(x_n,[Tz]^q_{(\alpha,\beta)_{Tz}})}{2s}\Bigg\} \\ &\le \lim_{n\to\infty} r\max\Bigg\{d(x_n,z), d(x_n,x_{n+1}), d\big(z,[Tz]^q_{(\alpha,\beta)_{Tz}}\big), \\ &\qquad \frac{d(z,x_{n+1}) + d(x_n,[Tz]^q_{(\alpha,\beta)_{Tz}})}{2s}\Bigg\} \\ &= rd\big(z,[Tz]^q_{(\alpha,\beta)_{Tz}}\big). \end{aligned}$$

Since $\frac{1}{2} \le r < 1$, we obtain $d(z,[Tz]^q_{(\alpha,\beta)_{Tz}}) = 0$ implying $z \in [Tz]^q_{(\alpha,\beta)_{Tz}}$. Hence this completes the proof. □

Example 3 Let $X = [0,1]$, $d : X \times X \to \mathbb{R}$ such that $d(x,y) = |x-y|$, where $x,y \in X$. $(\alpha_1,\beta_1) \in [0,1] \times [0,1]$ and $T : X \to F^q(X)$ is a q-ROF mapping defined as follows:

If $x = 0$, then we have

$$\mu_{T0}(t) = \begin{cases} 1 & t = 0, \\ (\frac{9}{10})^{\frac{1}{q}} & 0 < t \le \frac{1}{70}, \\ (\frac{1}{4})^{\frac{1}{q}} & t > \frac{1}{70}, \end{cases}$$

$$\nu_{T0}(t) = \begin{cases} 0 & t = 0, \\ (\frac{1}{20})^{\frac{1}{q}} & 0 < t \le \frac{1}{60}, \\ (\frac{11}{20})^{\frac{1}{q}} & t > \frac{1}{60}. \end{cases}$$

If $x \neq 0$, then we have

$$\mu_{Tx}(t) = \begin{cases} (\alpha_1)^{\frac{1}{q}} & 0 \le t < \frac{1}{40}, \\ (\frac{\alpha_1}{q})^{\frac{1}{q}} & \frac{1}{40} \le t < \frac{1}{20}, \\ (\frac{\alpha_1}{3q})^{\frac{1}{q}} & \frac{1}{20} \le t \le 1, \end{cases}$$

$$\nu_{Tx}(t) = \begin{cases} 0 & 0 \leq t < \frac{1}{50}, \\ (\beta_1)^{\frac{3}{q}} & \frac{1}{50} \leq t < \frac{1}{20}, \\ (\beta_1)^{\frac{2}{q}} & \frac{1}{20} \leq t \leq 1. \end{cases}$$

q-rung (α, β) of T will be:

for $x = 0$,

$$[T0]^q_{(\alpha,\beta)_{T0}} = \begin{cases} [0,1] & 0 \leq \alpha < 0.25, 0.55 \leq \beta \leq 1, \\ [0, \frac{1}{70}] & 0.25 \leq \alpha < 0.9, 0.05 \leq \beta \leq 0.55, \\ \{0\} & \alpha > 0.9, \beta < 0.05, \end{cases}$$

for $x \neq 0$,

$$[Tx]^q_{(\alpha,\beta)_{Tx}} = \begin{cases} [0, \frac{1}{50}] & \alpha_1 \leq \alpha \leq 1, 0 \leq \beta < \beta_1^3, \\ [0, \frac{1}{40}] & \alpha_1 \leq \alpha \leq 1, \beta = \beta_1^3, \\ [0, \frac{1}{20}] & \frac{\alpha_1}{q} \leq \alpha \leq \alpha_1, \beta_1^3 \leq \beta < \beta_1^2, \\ [0,1] & \alpha \leq \frac{\alpha_1}{3q}, \beta \geq \beta_1^2. \end{cases}$$

This implies that

$$[Tx]^q_{(\alpha_1,\beta_1^3)} = \left[0, \frac{1}{40}\right] \quad \text{and} \quad [Ty]^q_{(\frac{\alpha_1}{q},\beta_1^3)} = \left[0, \frac{1}{20}\right],$$

$$H\bigl([Tx]^q_{(\alpha,\beta)_{Tx}}, [Ty]^q_{(\alpha,\beta)_{Ty}}\bigr) = \begin{cases} 0 & x = y, \\ \frac{1}{40} & x \neq y. \end{cases}$$

Then, for $q = 5$, $r = 0.999$, $s = 4$ all the conditions of Theorem 3 are satisfied.

Corollary 1 *Consider (X, d) to be a complete b-metric linear space, $s \geq 1$, and let $T : X \to Ķ(X)$ be an intuitionistic fuzzy mapping. Consider T to satisfy the same contractive conditions as in Theorem 3, then T has a fixed point.*

Corollary 2 *Consider (X, d) to be a complete b-metric linear space, $s \geq 1$, and let $T : X \to Ķ(X)$ be a fuzzy mapping. Consider T to satisfy the same contractive conditions as in Theorem 3, then T has a fixed point.*

Theorem 4 *Consider (X, d) to be a complete b-metric linear space, $s \geq 1$, $S, T : X \to Ķ(X)$ be any two q-rung orthopair fuzzy mappings. For each element $x \in X$ and each pair $(\alpha, \beta) \in (0,1]^2$, $[Tx]^q_{(\alpha,\beta)_{Tx}}$, $[Sx]^q_{(\alpha,\beta)_{Sx}}$ are nonempty. Assume that $r \in [0,1)$ such that*

$$\varphi(r) \min\bigl\{d\bigl(x, [Sx]^q_{(\alpha,\beta)_{sx}}\bigr), d\bigl(y, [Ty]^q_{(\alpha,\beta)_{Ty}}\bigr)\bigr\} \leq d(x,y) \tag{3.7}$$

implies

$$H\bigl([Sx]^q_{(\alpha,\beta)_{sx}}, [Ty]^q_{(\alpha,\beta)_{Ty}}\bigr) \leq r \max\bigl\{d\bigl(x, [Sx]^q_{(\alpha,\beta)_{sx}}\bigr), d\bigl(y, [Ty]^q_{(\alpha,\beta)_{Ty}}\bigr)\bigr\}. \tag{3.8}$$

Then $z \in X$ so that $z \in [Tz]^q_{(\alpha,\beta)_{Tz}} \cap [Sz]^q_{(\alpha,\beta)_{Sz}}$.

Proof Starting with $x_0 \in X$ and since $[Tx_0]^q_{(\alpha,\beta)_{Tx_0}}$ is nonempty, there exists $x_1 \in X$ such that $x_1 \in [Tx_0]^q_{(\alpha,\beta)_{Tx_0}}$. For the ease of notation, assume $(\alpha,\beta)_{Tx_0} = (\alpha^1,\beta^1)$ and $x_1 \in [Tx_0]^q_{(\alpha^1,\beta^1)}$. Similarly, for x_1, we have $x_2 \in X$ such that $x_2 \in [Sx_1]^q_{(\alpha,\beta)_{Sx_1}}$. Let $(\alpha,\beta)_{Sx_1} = (\alpha^2,\beta^2)$, and so $x_2 \in [Sx_1]^q_{(\alpha^2,\beta^2)}$. So in general

$$x_{2n+1} \in [Tx_{2n}]^q_{(\alpha^{2n+1},\beta^{2n+1})}, \qquad x_{2n+2} \in [Sx_{2n+1}]^q_{(\alpha^{2n+2},\beta^{2n+2})}.$$

By using Lemma 1 and condition (3.7) either for $d(x_{2n-1},[Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}) \le d(x_{2n-1},x_{2n})$ or $d(x_{2n},[Tx_{2n}]^q_{(\alpha^{2n+1},\beta^{2n+1})}) \le d(x_{2n-1},x_{2n})$, we have

$$\varphi(r)\min\bigl\{d\bigl(x_{2n-1},[Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\bigr),d\bigl(x_{2n},[Tx_{2n}]^q_{(\alpha^{2n+1},\beta^{2n+1})}\bigr)\bigr\} \le d(x_{2n-1},x_{2n}).$$

This implies

$$\begin{aligned}
H\bigl([Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})},[Tx_{2n}]^q_{(\alpha^{2n+1},\beta^{2n+1})}\bigr) &\le r\max\bigl\{d\bigl(x_{2n-1},[Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\bigr),\\
&\qquad d\bigl(x_{2n},[Tx_{2n}]^q_{(\alpha^{2n+1},\beta^{2n+1})}\bigr)\bigr\},\\
d(x_{2n},x_{2n+1}) &\le kH\bigl([Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})},[Tx_{2n}]^q_{(\alpha^{2n+1},\beta^{2n+1})}\bigr)\\
&\le kr\max\bigl\{d\bigl(x_{2n-1},[Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\bigr),\\
&\quad d\bigl(x_{2n},[Tx_{2n}]^q_{(\alpha^{2n+1},\beta^{2n+1})}\bigr)\bigr\}\\
&\le \nu\max\bigl\{d(x_{2n-1},x_{2n}),\\
&\quad d(x_{2n},x_{2n+1})\bigr\},
\end{aligned}$$

where $\nu = kr < 1$. Hence

$$d(x_{2n},x_{2n+1}) \le \nu d(x_{2n-1},x_{2n}).$$

Similarly, we have $d(x_{2n+1},x_{2n+2}) \le \nu d(x_{2n},x_{2n+1})$. This implies

$$d(x_n,x_{n+1}) \le \nu d(x_{n-1},x_n),$$

and therefore $\{x_n\}$ is a Cauchy sequence such that $x_n \to \omega \in X$.

Next it will be proved that

$$d\bigl(\omega,[Ty]^q_{(\alpha,\beta)_{Ty}}\bigr) \le rd\bigl(y,[Ty]^q_{(\alpha,\beta)_{Ty}}\bigr) \quad\text{and}\quad d\bigl(\omega,[Sy]^q_{(\alpha,\beta)_{Sy}}\bigr) \le xd\bigl(y,[Sy]^q_{(\alpha,\beta)_{Sy}}\bigr) \tag{3.9}$$

for all $y \in X - \{\omega\}$.

Since $x_n \to \omega$, so $n_0 \in \mathbb{N}$ such that $d(\omega,x_n) \le \frac{1}{3s^2}d(\omega,y)$ for $\omega \ne y$. Then

$$\begin{aligned}
\varphi(r)d\bigl(x_{2n-1},[Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\bigr) &\le d\bigl(x_{2n-1},[Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\bigr) \le d(x_{2n-1},x_{2n})\\
&\le s\bigl(d(x_{2n-1},\omega)+d(\omega,x_{2n})\bigr)\\
&\le \frac{2}{3s}d(\omega,y) = \frac{1}{s}d(\omega,y) - \frac{1}{3s}d(\omega,y)
\end{aligned}$$

$$\leq \frac{1}{s}\big(d(\omega, y) - sd(\omega, x_{2n-1})\big) \leq d(x_{2n-1}, y).$$

Now either $d(x_{2n-1}, [Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}) < d(y, [Ty]^q_{(\alpha,\beta)_{Ty}})$ or $d(y, [Ty]^q_{(\alpha,\beta)_{Ty}}) < d(x_{2n-1}, [Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})})$, we have

$$\varphi(r)\min\big\{d\big(x_{2n-1}, [Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\big), d\big(y, [Ty]^q_{(\alpha,\beta)_{Ty}}\big)\big\} \leq d(x_{2n-1}, y).$$

And hence,

$$\begin{aligned} d\big(x_{2n}, [Ty]^q_{(\alpha,\beta)_{Ty}}\big) &\leq H\big([Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}, [Ty]^q_{(\alpha,\beta)_{Ty}}\big) \\ &\leq r\max\big\{d\big(x_{2n-1}, [Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\big), d\big(y, [Ty]^q_{(\alpha,\beta)_{Ty}}\big)\big\} \\ &\leq r\max\big\{d(x_{2n-1}, x_{2n}), d\big(y, [Ty]^q_{(\alpha,\beta)_{Ty}}\big)\big\}. \end{aligned}$$

Letting $n \to \infty$, we have $d(\omega, [Ty]^q_{(\alpha,\beta)_{Ty}}) \leq rd(y, [Ty]^q_{(\alpha,\beta)_{Ty}})$. Similarly, it can be shown that $d(\omega, [Sy]^q_{(\alpha,\beta)_{Sy}}) \leq rd(y, [Sy]^q_{(\alpha,\beta)_{Sy}})$ for all $y \in X - \{\omega\}$.

Now we show that $\omega \in [T\omega]^q_{(\alpha,\beta)_{T\omega}} \cap [S\omega]^q_{(\alpha,\beta)_{S\omega}}$.

Consider $0 \leq r < \frac{1}{2}$ and let $\omega \notin [T\omega]^q_{(\alpha,\beta)_{T\omega}}$ and $\omega \notin [S\omega]^q_{(\alpha,\beta)_{S\omega}}$. Then there is an element $\mu \in X$ so that $\mu \in [T\omega]^q_{(\alpha,\beta)_{T\omega}}$ and $\omega \neq \mu$. From (3.9) we have $d(\omega, [T\mu]^q_{(\alpha,\beta)_{T\mu}}) \leq rd(\mu, [T\mu]^q_{(\alpha,\beta)_{T\mu}})$. On the other hand,

$$\varphi(r)d\big(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}\big) \leq d\big(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}\big) \leq d(\omega, \mu).$$

Also

$$\varphi(r)\min\big\{d\big(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}\big), d\big(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}}\big)\big\} \leq d(\omega, \mu),$$

implying that

$$\begin{aligned} d\big(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}}\big) &\leq H\big([T\omega]^q_{(\alpha,\beta)_{T\omega}}, [S\mu]^q_{(\alpha,\beta)_{S\mu}}\big) \\ &\leq r\max\big\{d\big(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}\big), d\big(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}}\big)\big\} \leq rd\big(\omega, [T\omega]^q_{(\alpha,\beta)_{\omega}}\big). \end{aligned}$$

Also from (3.9) we have

$$d\big(\omega, [S\mu]^q_{(\alpha,\beta)_{S\mu}}\big) \leq rd\big(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}}\big). \tag{3.10}$$

Now,

$$\begin{aligned} d\big(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}\big) &\leq sd\big(\omega, [S\mu]^q_{(\alpha,\beta)_{S\mu}}\big) + sH\big([S\mu]^q_{(\alpha,\beta)_{S\mu}}, [T\omega]^q_{(\alpha,\beta)_{T\omega}}\big) \\ &\leq rsd\big(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}}\big) + rsd\big(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}\big) \end{aligned}$$

implies

$$\begin{aligned} d\big(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}\big) &\leq \frac{rs}{1-rs}d\big(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}}\big) \\ &\leq \frac{r^2s}{1-rs}d\big(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}\big), \end{aligned}$$

that is, $\frac{1-rs-r^2s}{1-rs}d(\omega,[T\omega]^q_{(\alpha,\beta)_{T\omega}})\leq 0$, and since $\frac{1-rs-r^2s}{1-rs}\geq 0$ therefore $\omega\in[T\omega]^q_{(\alpha,\beta)_{T\omega}}$. Similarly, $\omega\in[S\omega]^q_{(\alpha,\beta)_{S\omega}}$.

Now consider $\frac{1}{2}\leq r<1$. Firstly it will be proved that whenever $\omega\neq\mu$,

$$H\big([T\omega]^q_{(\alpha,\beta)_{T\omega}},[S\mu]^q_{(\alpha,\beta)_{S\mu}}\big)\leq r\max\big\{d\big(\omega,[T\omega]^q_{(\alpha,\beta)_{T\omega}}\big),d\big(\mu,[S\mu]^q_{(\alpha,\beta)_{S\mu}}\big)\big\}.$$

Consider that for $n\in\mathbb{N}$ there exists $z_n\in[S\mu]^q_{(\alpha,\beta)_{S\mu}}$ such that $sd(\omega,z_n)\leq d(\omega,[S\mu]^q_{(\alpha,\beta)_{S\mu}})+\frac{1}{n}d(\mu,\omega)$. Therefore,

$$\begin{aligned}d\big(\mu,[S\mu]^q_{(\alpha,\beta)_{S\mu}}\big)&\leq d(\mu,z_n)\leq s\big(d(\mu,\omega)+d(\omega,z_n)\big)\\&\leq sd(\mu,\omega)+d\big(\omega,[S\mu]^q_{(\alpha,\beta)_{S\mu}}\big)+\frac{1}{n}d(\mu,\omega)\\&\leq sd(\mu,\omega)+rd\big(\mu,[S\mu]^q_{(\alpha,\beta)_{S\mu}}\big)+\frac{1}{n}d(\mu,\omega)\quad\text{by using (3.10).}\end{aligned}$$

This implies

$$(1-r)d\big(\mu,[S\mu]^q_{(\alpha,\beta)_{S\mu}}\big)\leq\left(s+\frac{1}{n}\right)d(\mu,\omega).$$

Letting $n\to\infty$,

$$\left(\frac{1-r}{s}\right)d\big(\mu,[S\mu]^q_{(\alpha,\beta)_{S\mu}}\big)\leq d(\mu,\omega).$$

And hence, we have $\varphi(r)d(\mu,[S\mu]^q_{(\alpha,\beta)_{S\mu}})\leq d(\mu,\omega)$. This implies

$$H\big([T\omega]^q_{(\alpha,\beta)_{T\omega}},[S\mu]^q_{(\alpha,\beta)_{S\mu}}\big)\leq r\max\big\{d\big(\omega,[T\omega]^q_{(\alpha,\beta)_{T\omega}}\big),d\big(\mu,[S\mu]^q_{(\alpha,\beta)_{S\mu}}\big)\big\}.$$

Let $\mu=x_{2n-1}$, then we have

$$H\big([T\omega]^q_{(\alpha,\beta)_{T\omega}},[Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\big)\leq r\max\big\{d\big(\omega,[T\omega]^q_{(\alpha,\beta)_{T\omega}}\big),d\big(x_{2n-1},[Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\big)\big\}.$$

Taking $n\to\infty$,

$$\begin{aligned}\lim_{n\to\infty}d\big([T\omega]^q_{(\alpha,\beta)_{T\omega}},x_{2n}\big)&\leq\lim_{n\to\infty}H\big([T\omega]^q_{(\alpha,\beta)_{T\omega}},[Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\big)\\&\leq\lim_{n\to\infty}r\max\big\{d\big(\omega,[T\omega]^q_{(\alpha,\beta)_{T\omega}}\big),d\big(x_{2n-1},[Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\big)\big\},\\d\big([T\omega]^q_{(\alpha,\beta)_{T\omega}},\omega\big)\leq rd\big(\omega,[T\omega]^q_{(\alpha,\beta)_{T\omega}}\big)&\quad\Longrightarrow\quad d\big(\omega,[T\omega]^q_{(\alpha,\beta)_{T\omega}}\big)=0.\end{aligned}$$

Hence $\omega\in[T\omega]^q_{(\alpha,\beta)_{T\omega}}$. Similarly, we can easily prove that $\omega\in[S\omega]^q_{(\alpha,\beta)_{S\omega}}$, and hence $\omega\in[T\omega]^q_{(\alpha,\beta)_{T\omega}}\cap[S\omega]^q_{(\alpha,\beta)_{S\omega}}$. □

Example 4 Let $X=[1,2]$, $d:X\times X\to\mathbb{R}$ such that $d(x,y)=|x-y|$, where $x,y\in X$. $(\alpha_1,\beta_1),(\alpha_2,\beta_2)\in[0,1]\times[0,1]$ and $T:X\to F^q(X)$ is a q-ROF mapping defined as follows:

$$\mu_{Tx}(t)=\begin{cases}(\frac{\alpha_1}{2})^{\frac{1}{q}} & 1\leq t\leq\frac{27}{20},\\(\frac{\alpha_1}{3q})^{\frac{1}{q}} & \frac{27}{20}<t\leq\frac{31}{20},\\(\frac{\alpha_1}{4q})^{\frac{1}{q}} & \frac{31}{20}<t\leq 2,\end{cases}$$

$$\upsilon_{Tx}(t) = \begin{cases} 0 & 1 \le t \le \frac{5}{4}, \\ (\frac{\beta_1}{2})^{\frac{1}{q}} & \frac{5}{4} < t \le \frac{31}{20}, \\ (2\beta_1)^{\frac{1}{q}} & \frac{31}{20} < t \le 2, \end{cases}$$

$$\mu_{Sx}(t) = \begin{cases} (\frac{\alpha_2}{3})^{\frac{1}{q}} & 1 \le t \le \frac{33}{25}, \\ (\frac{\alpha_2}{4})^{\frac{1}{q}} & \frac{33}{25} < t \le \frac{31}{20}, \\ (\frac{\alpha_2}{3q})^{\frac{1}{q}} & \frac{31}{20} < t \le 2, \end{cases}$$

$$\upsilon_{Sx}(t) = \begin{cases} 0 & 1 \le t \le \frac{36}{25}, \\ (\frac{\beta_2}{6})^{\frac{1}{q}} & \frac{36}{25} < t \le \frac{31}{20}, \\ (\frac{\beta_2}{2})^{\frac{1}{q}} & \frac{31}{20} < t \le 2. \end{cases}$$

Then $[Tx]^q_{(\frac{\alpha_1}{2},0)} = [1, \frac{5}{4}]$, $[Sy]^q_{(\frac{\alpha_2}{3},0)} = [1, \frac{33}{25}]$, $x \in [Tx]^q_{(\frac{\alpha_1}{2},0)} \cap [Sy]^q_{(\frac{\alpha_2}{3},0)}$, and $H([Tx]^q_{(\alpha,\beta)_{Tx}}, [Sy]^q_{(\alpha,\beta)_{Sy}}) = 0.07$. Hence all conditions of Theorem 4 are satisfied for $r = 0.9$ and $q = 6$.

Corollary 3 *Consider (X, d) to be a complete b-metric linear space, $s \ge 1$, $S, T : X \to Ķ(X)$ be intuitionistic fuzzy mappings. Then S and T have a common fixed point under the contractive conditions as in Theorem 4.*

Corollary 4 *Consider (X, d) to be a complete b-metric linear space, $s \ge 1$, $S, T : X \to Ķ(X)$ be fuzzy mappings. Then S and T have a common fixed point under the contractive conditions as in Theorem 4.*

Theorem 5 *Consider (X, d) to be a complete b-metric linear space, $s \ge 1$, $S, T : X \to Ķ(X)$ be a pair of q-rung orthopair fuzzy mappings. For each element $x \in X$ and each pair $(\alpha, \beta) \in (0, 1]^2$, $[Tx]^q_{(\alpha,\beta)_{Tx}}$, $[Sx]^q_{(\alpha,\beta)_{Sx}}$ are nonempty. Assume that $r \in [0, 1)$ so that*

$$\varphi(r) \min\{d(x, [Sx]^q_{(\alpha,\beta)_{sx}}), d(y, [Ty]^q_{(\alpha,\beta)_{Ty}})\} \le d(x, y) \tag{3.11}$$

implies

$$H([Sx]^q_{(\alpha,\beta)_{sx}}, [Ty]^q_{(\alpha,\beta)_{Ty}}) \le r\{d(x, [Sx]^q_{(\alpha,\beta)_{sx}}) + d(y, [Ty]^q_{(\alpha,\beta)_{Ty}})\}. \tag{3.12}$$

Then $z \in X$ so that $z \in [Tz]^q_{(\alpha,\beta)_{Tz}} \cap [Sz]^q_{(\alpha,\beta)_{Sz}}$.

Proof Starting with $x_0 \in X$ and since $[Tx_0]^q_{(\alpha,\beta)_{Tx_0}}$ is nonempty, there exists $x_1 \in X$ such that $x_1 \in [Tx_0]^q_{(\alpha,\beta)_{Tx_0}}$. For the ease of notation, assume $(\alpha, \beta)_{Tx_0} = (\alpha^1, \beta^1)$ and $x_1 \in [Tx_0]^q_{(\alpha^1,\beta^1)}$. Similarly, for x_1, we have $x_2 \in X$ such that $x_2 \in [Sx_1]^q_{(\alpha,\beta)_{Sx_1}}$. Let $(\alpha, \beta)_{Sx_1} = (\alpha^2, \beta^2)$ and so $x_2 \in [Sx_1]^q_{(\alpha^2,\beta^2)}$. So, in general,

$$x_{2n+1} \in [Tx_{2n}]^q_{(\alpha^{2n+1},\beta^{2n+1})}, \qquad x_{2n+2} \in [Sx_{2n+1}]^q_{(\alpha^{2n+2},\beta^{2n+2})}.$$

By using Lemma 1 and condition (3.11) either for $d(x_{2n-1}, [Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}) \le d(x_{2n-1}, x_{2n})$ or $d(x_{2n}, [Tx_{2n}]^q_{(\alpha^{2n+1},\beta^{2n+1})}) \le d(x_{2n-1}, x_{2n})$, we have

$$\varphi(r) \min\{d(x_{2n-1}, [Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}), d(x_{2n}, [Tx_{2n}]^q_{(\alpha^{2n+1},\beta^{2n+1})})\} \le d(x_{2n-1}, x_{2n}).$$

This implies

$$\begin{aligned} H\big([Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})},[Tx_{2n}]^q_{(\alpha^{2n+1},\beta^{2n+1})}\big) &\le r\big\{d\big(x_{2n-1},[Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\big) \\ &\quad + d\big(x_{2n},[Tx_{2n}]^q_{(\alpha^{2n+1},\beta^{2n+1})}\big)\big\} \\ d(x_{2n},x_{2n+1}) &\le kH\big([Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})},[Tx_{2n}]^q_{(\alpha^{2n+1},\beta^{2n+1})}\big) \\ &\le kr\big\{d\big(x_{2n-1},[Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\big) \\ &\quad + d\big(x_{2n},[Tx_{2n}]^q_{(\alpha^{2n+1},\beta^{2n+1})}\big)\big\} \\ &\le \nu\big\{d(x_{2n-1},x_{2n}) \\ &\quad + d(x_{2n},x_{2n+1})\big\}, \end{aligned}$$

where $\nu = kr < 1$. Hence

$$d(x_{2n},x_{2n+1}) \le \nu d(x_{2n-1},x_{2n}).$$

Similarly, we have $d(x_{2n+1},x_{2n+2}) \le \nu d(x_{2n},x_{2n+1})$. This implies

$$d(x_n,x_{n+1}) \le \nu d(x_{n-1},x_n),$$

and therefore $\{x_n\}$ is a Cauchy sequence such that $x_n \to \omega \in X$.

Next it will be proved that

$$d\big(\omega,[Ty]^q_{(\alpha,\beta)_{Ty}}\big) \le rd\big(y,[Ty]^q_{(\alpha,\beta)_{Ty}}\big) \quad \text{and} \quad d\big(\omega,[Sy]^q_{(\alpha,\beta)_{Sy}}\big) \le xd\big(y,[Sy]^q_{(\alpha,\beta)_{Sy}}\big) \tag{3.13}$$

for all $y \in X - \{\omega\}$.

Since $x_n \to \omega$, so $n_0 \in \mathbb{N}$ such that $d(\omega,x_n) \le \frac{1}{3s^2}d(\omega,y)$ for $\omega \ne y$. Then

$$\begin{aligned} \varphi(r)d\big(x_{2n-1},[Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\big) &\le d\big(x_{2n-1},[Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\big) \le d(x_{2n-1},x_{2n}) \\ &\le s\big(d(x_{2n-1},\omega) + d(\omega,x_{2n})\big) \\ &\le \frac{2}{3s}d(\omega,y) = \frac{1}{s}d(\omega,y) - \frac{1}{3s}d(\omega,y) \\ &\le \frac{1}{s}\big(d(\omega,y) - sd(\omega,x_{2n-1})\big) \le d(x_{2n-1},y). \end{aligned}$$

Now either $d(x_{2n-1},[Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}) < d(y,[Ty]^q_{(\alpha,\beta)_{Ty}})$ or $d(y,[Ty]^q_{(\alpha,\beta)_{Ty}}) < d(x_{2n-1}, [Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})})$, we have

$$\varphi(r)\min\big\{d\big(x_{2n-1},[Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\big),d\big(y,[Ty]^q_{(\alpha,\beta)_{Ty}}\big)\big\} \le d(x_{2n-1},y).$$

And hence,

$$\begin{aligned} d\big(x_{2n},[Ty]^q_{(\alpha,\beta)_{Ty}}\big) &\le H\big([Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})},[Ty]^q_{(\alpha,\beta)_{Ty}}\big) \\ &\le r\max\big\{d\big(x_{2n-1},[Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\big),d\big(y,[Ty]^q_{(\alpha,\beta)_{Ty}}\big)\big\} \end{aligned}$$

$$\leq r \max\{d(x_{2n-1}, x_{2n}), d(y, [Ty]^q_{(\alpha,\beta)_{Ty}})\}.$$

Letting $n \to \infty$, we have $d(\omega, [Ty]^q_{(\alpha,\beta)_{Ty}}) \leq rd(y, [Ty]^q_{(\alpha,\beta)_{Ty}})$. Similarly, it can be shown that $d(\omega, [Sy]^q_{(\alpha,\beta)_{Sy}}) \leq rd(y, [Sy]^q_{(\alpha,\beta)_{Sy}})$ for all $y \in X - \{\omega\}$.

Now we show that $\omega \in [T\omega]^q_{(\alpha,\beta)_{T\omega}} \cap [S\omega]^q_{(\alpha,\beta)_{S\omega}}$.

Consider $0 \leq r < \frac{1}{2}$ and let $\omega \notin [T\omega]^q_{(\alpha,\beta)_{T\omega}}$ and $\omega \notin [S\omega]^q_{(\alpha,\beta)_{S\omega}}$. Then there is an element $\mu \in X$ so that $\mu \in [T\omega]^q_{(\alpha,\beta)_{T\omega}}$ and $\omega \neq \mu$. From (3.13) we have $d(\omega, [T\mu]^q_{(\alpha,\beta)_{T\mu}}) \leq rd(\mu, [T\mu]^q_{(\alpha,\beta)_{T\mu}})$. On the other hand,

$$\varphi(r)d(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}) \leq d(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}) \leq d(\omega, \mu).$$

Also

$$\varphi(r) \min\{d(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}), d(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}})\} \leq d(\omega, \mu),$$

implying that

$$\begin{aligned}
d(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}}) &\leq H([T\omega]^q_{(\alpha,\beta)_{T\omega}}, [S\mu]^q_{(\alpha,\beta)_{S\mu}}) \\
&\leq r\{d(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}) \\
&\quad + d(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}})\}, \\
(1-r)d(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}}) &\leq rd(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}), \\
d(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}}) &\leq \frac{r}{1-r} d(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}).
\end{aligned}$$

Also from (3.13)

$$d(\omega, [S\mu]^q_{(\alpha,\beta)_{S\mu}}) \leq rd(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}}).$$

Now,

$$\begin{aligned}
d(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}) &\leq sd(\omega, [S\mu]^q_{(\alpha,\beta)_{S\mu}}) + H([T\omega]^q_{(\alpha,\beta)_{T\omega}}, [S\mu]^q_{(\alpha,\beta)_{S\mu}}) \\
&\leq srd(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}}) + \frac{sr}{1-r} d(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}), \\
\left(1 - \frac{sr}{1-r}\right) d(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}) &\leq srd(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}}), \\
\left(1 - \frac{sr}{1-r}\right) d(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}) &\leq \left(\frac{sr^2}{1-r}\right) d(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}), \\
d(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}) &\leq \left(\frac{sr^2}{1-r-sr}\right) d(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}).
\end{aligned}$$

Now consider $\frac{1}{2} \leq r < 1$. Firstly, it will be proved that whenever $\omega \neq \mu$,

$$H([T\omega]^q_{(\alpha,\beta)_{T\omega}}, [S\mu]^q_{(\alpha,\beta)_{S\mu}}) \leq r \max\{d(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}), d(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}})\}.$$

Consider that for $n \in \mathbb{N}$, there exists $z_n \in [S\mu]^q_{(\alpha,\beta)_{S\mu}}$ such that $sd(\omega, z_n) \le d(\omega, [S\mu]^q_{(\alpha,\beta)_{S\mu}}) + \frac{1}{n}d(\mu,\omega)$. Therefore,

$$\begin{aligned} d\big(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}}\big) &\le d(\mu, z_n) \le s\big(d(\mu,\omega) + d(\omega, z_n)\big) \\ &\le sd(\mu,\omega) + d\big(\omega, [S\mu]^q_{(\alpha,\beta)_{S\mu}}\big) + \frac{1}{n}d(\mu,\omega) \\ &\le sd(\mu,\omega) + rd\big(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}}\big) + \frac{1}{n}d(\mu,\omega) \quad \text{by using (3.10)}. \end{aligned}$$

This implies

$$(1-r)d\big(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}}\big) \le \left(s + \frac{1}{n}\right)d(\mu,\omega).$$

Letting $n \to \infty$,

$$\left(\frac{1-r}{s}\right)d\big(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}}\big) \le d(\mu,\omega).$$

And hence, we have $\varphi(r)d(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}}) \le d(\mu,\omega)$. This implies

$$H\big([T\omega]^q_{(\alpha,\beta)_{T\omega}}, [S\mu]^q_{(\alpha,\beta)_{S\mu}}\big) \le r\max\big\{d\big(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}\big), d\big(\mu, [S\mu]^q_{(\alpha,\beta)_{S\mu}}\big)\big\}.$$

Let $\mu = x_{2n-1}$, then we have

$$H\big([T\omega]^q_{(\alpha,\beta)_{T\omega}}, [Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\big) \le r\max\big\{d\big(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}\big), d\big(x_{2n-1}, [Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\big)\big\}.$$

Taking $n \to \infty$,

$$\begin{aligned} \lim_{n\to\infty} d\big([T\omega]^q_{(\alpha,\beta)_{T\omega}}, x_{2n}\big) &\le \lim_{n\to\infty} H\big([T\omega]^q_{(\alpha,\beta)_{T\omega}}, [Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\big) \\ &\le \lim_{n\to\infty} r\big\{d\big(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}\big), d\big(x_{2n-1} + [Sx_{2n-1}]^q_{(\alpha^{2n},\beta^{2n})}\big)\big\}, \\ d\big([T\omega]^q_{(\alpha,\beta)_{T\omega}}, \omega\big) \le rd\big(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}\big) \quad &\Longrightarrow \quad d\big(\omega, [T\omega]^q_{(\alpha,\beta)_{T\omega}}\big) = 0. \end{aligned}$$

Hence $\omega \in [T\omega]^q_{(\alpha,\beta)_{T\omega}}$. Similarly we can easily prove that $\omega \in [S\omega]^q_{(\alpha,\beta)_{S\omega}}$ and hence $\omega \in [T\omega]^q_{(\alpha,\beta)_{T\omega}} \cap [S\omega]^q_{(\alpha,\beta)_{S\omega}}$. □

4 Application

There are known applications of fuzzy sets for the solution of integral equations (for example, see [18, 19]). In the present section, with the help of completeness property of function space $C[a,b]$ and by applying Theorem 6, we have presented an existence theorem for the solution of the class of nonlinear integral equations.

We will use Theorem 6 to show the existence of common solutions of two nonlinear integral inclusions defined as

$$x(\sigma) \in u(\sigma) + \lambda \int_a^\sigma \big[F_1\big(\sigma,\tau,x(\tau)\big)\big]\,d\tau, \quad \sigma \in [a,b],$$

$$x(\sigma) \in u(\sigma) + \lambda \int_a^\sigma \bigl[F_2\bigl(\sigma,\tau,x(\tau)\bigr)\bigr]\,d\tau, \quad \sigma \in [a,b], \tag{4.1}$$

where $x \in C[a,b]$ is unknown, $u_o \in \mathbb{R}$, and F_1, F_2 are multivalued operators having compact, convex values in $\mathbb{R}$ defined as $F_1, F_2 : [a,b] \times [a,b] \to \mathbb{R}_{cp,cv}$. By a common solution of system 1, we mean a continuous function x such that

$$x(\sigma) = u(\sigma) + \lambda \int_a^\sigma \bigl[f_1\bigl(\sigma,\tau,x(\tau)\bigr)\bigr]\,d\tau, \quad \sigma \in [a,b],$$
$$x(\sigma) = u(\sigma) + \lambda \int_a^\sigma \bigl[f_2\bigl(\sigma,\tau,x(\tau)\bigr)\bigr]\,d\tau, \quad \sigma \in [a,b],$$

where $f_1, f_2 : [a,b] \times [a,b] \to \mathbb{R}, f_1 \in F_1(\sigma,\tau,x(\tau)), f_2 \in F_2(\sigma,\tau,x(\tau))$.

Theorem 6 *Consider the system of nonlinear integral inclusions in* (2.1). *Assume that the following conditions hold*:

(*i*) *The operators* $F_1(\sigma,\tau,x(\tau))$, $F_2(\sigma,\tau,x(\tau))$ *are continuous on* $[a,b]^2$.

(*ii*) *Suppose* $r \in [0,1)$ *such that, for every* $\sigma \in [a,b]$ *and* $x, y \in X$, *the inequality holds*

$$\varphi(r)\min\bigl\{d\bigl(x,[Ax]^q_{(\alpha,\beta)_{Ax}}\bigr), d\bigl(y,[By]^q_{(\alpha,\beta)_{By}}\bigr)\bigr\} \le d(x,y)$$
$$\Longrightarrow \quad \bigl|f_1\bigl(\sigma,\tau,x(\tau)\bigr) - f_2\bigl(\sigma,\tau,y(\tau)\bigr)\bigr|^p \le \biggl(\frac{r}{(b-a)|\lambda|^p}\biggr)\inf_{\sigma\in[a,b]}\bigl|x(\sigma)-z(\sigma)\bigr|^p.$$

Proof Let $X = C[a,b]$ and define $d : X \times X \to \mathbb{R}$ by $d(x,y) = |x(\sigma)-y(\sigma)|^p$ for all $x,y \in X$. Then (X,d) is a complete b-metric space with $s = 2^{p-1}$ where $p > 1$. Assume that $U, V, E, Z : X \to (0,1]$ are four arbitrary mappings.

Now, define a pair of qth rung fuzzy mappings $A, \xi : X \to F^q(X)$ as follows:

$$\begin{aligned} A : X \to F^q(X) : A\bigl(x(\sigma)\bigr) &= \omega_x(\sigma) \\ &= \biggl\{\varrho \in X : \varrho(\sigma) \in u_o + \int_a^\sigma F_1\bigl(\sigma,\tau,x(\tau)\bigr)\,dz, \sigma \in [a,b]\biggr\}, \\ \xi : X \to F^q(X) : \xi\bigl(x(\sigma)\bigr) &= \Omega_x(\sigma) \\ &= \biggl\{\varrho \in X : \varrho(t) \in u_o + \int_a^\sigma F_2\bigl(\sigma,\tau,x(\tau)\bigr)\,dz, \sigma \in [a,b]\biggr\} \end{aligned}$$

such that

$$\mu_{Ax}(\varrho) = \begin{cases} (U(x))^{\frac{1}{q}} & \text{if } \varrho(\sigma) \in \omega_x(\sigma), \forall \sigma \in [a,b], \\ 0 & \text{otherwise,} \end{cases}$$
$$\nu_{Ax}(\varrho) = \begin{cases} 0 & \text{if } \varrho(\sigma) \in \omega_x(\sigma), \forall \sigma \in [a,b], \\ (V(x))^{\frac{1}{q}} & \text{otherwise} \end{cases}$$

and

$$\mu_{Bx}(\varrho) = \begin{cases} (E(x))^{\frac{1}{q}} & \text{if } \varrho(\sigma) \in \Omega_x(\sigma), \forall t \in [a,b], \\ 0 & \text{otherwise,} \end{cases}$$

$$\nu_{Bx}(\varrho) = \begin{cases} 0 & \text{if } \varrho(\sigma) \in \Omega_x(\sigma), \forall \sigma \in [a,b], \\ (H(x))^{\frac{1}{q}} & \text{otherwise.} \end{cases}$$

If we take $\alpha_A(x) = U(x)$, $\beta_A(x) = 0$ and $\alpha_B(x) = E(x)$, $\beta_B(x) = 0$, then we have

$$\begin{aligned} \bigcup_{x\in X} [Ax]^q_{(\alpha,\beta)_A(x)} &= \bigcup_{x\in X} \bigl\{\varrho \in X : \mu_{Ax}(\varrho) = \bigl(U(x)\bigr)^{\frac{1}{q}} \text{ and } \nu_{Ax}(\varrho) = 0\bigr\} \\ &= \bigcup_{x\in X} \bigl\{\omega_x(\sigma)\bigr\} \end{aligned}$$

and

$$\begin{aligned} \bigcup_{x\in X} [Bx]^q_{(\alpha,\beta)_B(x)} &= \bigcup_{x\in X} \bigl\{\varrho \in X : \mu_{Bx}(\varrho) = \bigl(E(x)\bigr)^{\frac{1}{q}} \text{ and } \nu_{Bx}(\varrho) = 0\bigr\} \\ &= \bigcup_{x\in X} \bigl\{\Omega_x(\sigma)\bigr\}. \end{aligned}$$

For multivalued operators $F_1(\sigma,\tau,x(\tau))$ and $F_2(t,\tau,x(\tau))$, applying Michael's selection theorem, there exist continuous operators $f_1(\sigma,\tau,x(\tau)) \in F_1(t,\tau,x(\tau))$ and $f_2(\sigma,\tau,x(\tau) \in F_2(t,\tau,x(\tau))$, therefore

$$\begin{aligned} &u(\sigma) + \lambda \int_a^\sigma \bigl[f_1\bigl(\sigma,\tau,x(\tau)\bigr)\bigr]\,d\tau \in A\bigl(x(\sigma)\bigr), \\ &u(\sigma) + \lambda \int_a^\sigma \bigl[f_2\bigl(\sigma,\tau,x(\tau)\bigr)\bigr]\,d\tau \in B\bigl(x(\sigma)\bigr). \end{aligned}$$

Thus $A(x(\sigma)) \neq \emptyset$ and $B(x(\sigma)) \neq \emptyset$. As $F_1(\sigma,\tau,x(\tau))$ and $F_2(t,\tau,x(\tau))$ are continuous on $[a,b]$, their ranges are bounded. Now, for $z(t) \in A(x(\sigma))$,

$$z(\sigma) = u(\sigma) + \lambda \int_a^\sigma \bigl[f_1\bigl(\sigma,\tau,x(\tau)\bigr)\bigr].$$

Also, for some $w(\sigma) \in B(y(\sigma))$, we have

$$w(\sigma) = u(\sigma) + \lambda \int_a^t \bigl[f_2\bigl(t,\tau,y(\tau)\bigr)\bigr].$$

Also, for $z(\sigma) \in [Ax]^q_{(\alpha,\beta)_A(x)}$ and $w(\sigma) \in [By]^q_{(\alpha,\beta)_B(x)}$,

$$\begin{aligned} \bigl|z(\sigma) - w(\sigma)\bigr|^p &= \left|u(\sigma) + \lambda \int_a^\sigma \bigl[f_1\bigl(\sigma,\tau,x(\tau)\bigr)\bigr]\,d\tau - u(\sigma) - \lambda \int_a^\sigma \bigl[f_2\bigl(\sigma,\tau,y(\tau)\bigr)\bigr]\right|^p \\ &\leq |\lambda|^p \int_a^\sigma \bigl|f_1\bigl(\sigma,\tau,x(\tau)\bigr) - f_2\bigl(\sigma,\tau,y(\tau)\bigr)\bigr|^p\,d\tau \\ &\leq |\lambda|^p \bigl|f_1\bigl(\sigma,\tau,x(\tau)\bigr) - f_2\bigl(\sigma,\tau,y(\tau)\bigr)\bigr|^p \int_a^\sigma d\tau \\ &\leq |\lambda|^p \bigl|f_1\bigl(\sigma,\tau,x(\tau)\bigr) - f_2\bigl(\sigma,\tau,y(\tau)\bigr)\bigr|^p (b-a) \\ &\leq \left(\frac{r|\lambda|^p}{(b-a)|\lambda|^p}\right)(b-a) \inf_{t\in[a,b]} \bigl|x(\sigma) - z(\sigma)\bigr|^p \end{aligned}$$

$$
\begin{aligned}
&\leq r \inf_{\sigma\in[a,b]} \left|x(\sigma) - z(\sigma)\right|^p \\
&\leq r\left\{\inf\left|x(\sigma) - z(\sigma)\right|^p + \inf\left|y(\sigma) - w(\sigma)\right|^p\right\} \\
&= r\left\{d\left(x, [Ax]^q_{(\alpha,\beta)_{Ax}}\right) + d\left(y, [By]^q_{(\alpha,\beta)_{By}}\right)\right\}.
\end{aligned}
$$

Hence, by Theorem 6, there exists a common fixed point of mappings A and B. □

Conclusion The concept of q-ROF mapping, as a generalization of fuzzy mappings, is introduced. Also the concept of q-rung (α,β)-level sets is presented and some common fixed point results utilizing this concept for q-ROF mappings are obtained in b-metric space via Suzuki-type contractive conditions. We have also presented examples in support of our results. An application of obtained results for the existence of solution of nonlinear fractional integral inclusion is also presented.

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Author details
[1]Department of Mathematics and Statistics, International Islamic University, Islamabad, Pakistan. [2]Department of Mathematics, Texas University-Kingsville, Kingsville, Texas, USA. [3]Department of Mathematics, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia.

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