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Impulsive control of a class of multiple unstable neural networks

Songfang Jia^{1*} and Yanheng Chen¹

*Correspondence:
jiasongfang@163.com

¹College of Mathematic and
Statistics, Chongqing Three Gorges
University, Wanzhou, 404100, China

Abstract

This paper addresses the issue of stability of a class of multiple unstable Cohen–Grossberg neural networks (CGNNs) under impulsive control. Some novel sufficient conditions are given to make the unstable equilibrium points of the model locally μ -stable. An example is offered to demonstrate the effectiveness of the control strategy by comprehensive computer simulations.

Keywords: Cohen–Grossberg neural network; Multiple unstable equilibrium points; Impulsive control; μ -stability

1 Introduction

Recently, the multi-stability of neural network models has attracted extensive attention because of its wide application in the pattern recognition. Many experts and scholars contributed to this topic (see [1–35]). For example, Cao et al. proved that the CGNNs with multi-stability and multi-periodicity could find 2^n locally exponentially stable equilibrium points in [1]. The paper [26] revealed the co-existence of unstable and stable equilibrium points of a class of n -neuron recurrent neural networks model with time-varying delays. In [31], Nie et al. investigated a class of n -neuron competitive neural networks and showed that the systems have exactly 5^n equilibrium points, and $5^n - 3^n$ among them are unstable. Based on the partition space method, [32] proved that a class of CGNNs with unbounded time-varying delays could have 3^n equilibrium points, of which $3^n - 2^n$ are unstable and the remaining ones are locally μ -stable. By the above-mentioned references, we can see that most literature focused on the properties of multiple stable equilibrium points of the system. Still, few papers considered the properties of those unstable equilibrium points. Hence it is a challenging problem.

It is common knowledge that impulsive control is a very effective and economical method to address the unstable or chaotic neural networks, and its main idea is to add a pulse into the network topology to control the state of the system. In the past few years, many significant results on impulsive control neural network have been proposed, see [36–52]. In [41], the authors studied the delay-dependent passivity analysis of impulsive neural networks by using functional and inequality method and compared the system model with impulsive control and without impulsive control, extended the recent results of passivity. [45] introduced new sandwich control systems with impulse time windows and illustrated

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the stability of the chaotic system by using impulsive. Li et al. in [50] added impulse inputs in unstable neural networks to keep the unstable equilibrium point or the chaotic system stable. Hence it may be a good idea to investigate the stability of unstable equilibrium points of multiple systems by way of impulsive control.

Motivated by the above discussions, we investigated the stability of multiple unstable CGNNs in [32] by introducing a pulse into the system and obtained some sufficient conditions to make unstable equilibrium points of the models locally μ -stable, which generalized the results of paper [50]. The arrangement of this article is as follows. In the second section, the Cohen–Grossberg model and some preliminary conclusions are given. The main results are given and proved in the third section. The corollaries and comparisons with the existing literature are given in the fourth section. Section 5 gives a numerical example with simulation to illustrate the effectiveness of the control strategy. At the end of this paper, the conclusion is made.

2 System description and preliminaries

This article focuses on a class of n -neuron multiple unstable neural networks under some conditions described by the following equations:

$$\begin{cases} \frac{dx_i(t)}{dt} = -a_i(x_i(t))[b_i(x_i(t)) - \sum_{j=1}^n c_{ij}g_j(x_j(t)) - \sum_{j=1}^n d_{ij}f_j(x_j(t - \tau(t))) + I_i], \\ i = 1, 2, \dots, n, t \geq 0, \end{cases} \quad (1)$$

where $x_i(t)$ represents the current state of the i th neuron; $a_i(x_i(t))$ denotes the amplification function of the i th neuron; and $b_i(x_i(t))$ is the inhibition behavior function of the i th neuron; $g_j(x_j(t))$ and $f_j(x_j(t - \tau(t)))$ are current activation functions of the j th neuron, and $\tau(t)$ is a nonnegative function and denotes the delay of transmission; c_{ij} is the connection weight of the i th neuron and j th neuron, and d_{ij} denotes their delayed feedback connection weight; I_i is a constant and denotes the external input of the i th neuron.

Suppose that model (1) has the initial condition

$$x_i(s) = \varphi_i(s), \quad s \in (-\infty, 0],$$

where $\varphi_i(s) \in C((-\infty, 0], \mathbb{R})$, $i = 1, 2, \dots, n$. Let $(x_1(t), x_2(t), \dots, x_n(t))$ and $x^* = (x_1^*, x_2^*, \dots, x_n^*)$ stand for a solution and an equilibrium point of model (1), respectively. Then x^* is said to be μ -stable if there exist a positive constant M and a nondecreasing function $\mu(t)$ with $\lim_{t \rightarrow +\infty} \mu(t) = +\infty$ such that

$$|x_i(t) - x_i^*| \leq \frac{M}{\mu(t)}, \quad i = 1, 2, \dots, n.$$

Imitating [32], we can divide the R^n into 3^n non-intersection subregions. Let Φ be a set of these subregions, and let $(-\infty, +\infty) = (-\infty, p_i) \cup [p_i, q_i] \cup (q_i, +\infty)$, $i = 1, 2, \dots, n$. One can get

$$\Phi = \left\{ \prod_{i=1}^n w_i \mid w_i = (-\infty, p_i), [p_i, q_i] \text{ or } (q_i, +\infty) \right\}.$$

We define the index subsets for each $\prod_{i=1}^n w_i \in \Phi$ as $N_1 = \{i \mid w_i = (-\infty, p_i), i = 1, 2, \dots, n\}$, $N_2 = \{i \mid w_i = [p_i, q_i], i = 1, 2, \dots, n\}$, $N_3 = \{i \mid w_i = (q_i, +\infty), i = 1, 2, \dots, n\}$, and obviously

$N_1 \cup N_2 \cup N_3 = \{1, 2, \dots, n\}$. Moreover, we also can separate the set Φ into two parts Φ_1 and Φ_2 , where $\Phi_1 = \{\prod_{i=1}^n w_i \mid w_i = (-\infty, p_i) \text{ or } (q_i, +\infty), i = 1, 2, \dots, n\}$, $\Phi_2 = \Phi - \Phi_1$. Obviously, there are 2^n and $3^n - 2^n$ elements in Φ_1 and Φ_2 , respectively.

For convenience, let $\hat{A} = \text{diag}\{\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n\}$ and $\check{A} = \text{diag}\{\underline{a}_1, \underline{a}_2, \dots, \underline{a}_n\}$ be two positive diagonal matrices. Denote by $a(x(t)) = (a_1(x_1(t)), \dots, a_n(x_n(t)))$ the amplification function of (1), where, for each i , $a_i(u)$ is nonnegative continuous and satisfies

$$\underline{a}_i \leq a_i(u) \leq \bar{a}_i, \quad u \in (-\infty, +\infty), i = 1, 2, \dots, n.$$

Denote by $b(x(t)) = (b_1(x_1(t)), \dots, b_n(x_n(t)))$ the inhibition behavior function, where $b_i(u)$ is an odd function that grows monotonically, and there exists a positive matrix $B = \text{diag}\{b_1, b_2, \dots, b_n\}$ such that

$$\frac{b_i(y) - b_i(z)}{y - z} \geq b_i, \quad z \in (-\infty, +\infty), y \neq z, i = 1, 2, \dots, n.$$

Denote by $g(x(t)) = (g_1(x_1(t)), \dots, g_n(x_n(t)))$ and $f(x(t)) = (f_1(x_1(t)), \dots, f_n(x_n(t)))$ the activation functions, where $g_i(\cdot)$ and $f_j(\cdot)$ are continuous linear nondecreasing piecewise function or continuous nonlinear nondecreasing sigmoid function, and one can find some constants $p_j \leq q_j, m_j \leq M_j, m'_j \leq M'_j, m''_j \leq M''_j$, so that

$$\begin{aligned} m'_j &= \lim_{x \rightarrow -\infty} g_j(x), & M'_j &= \lim_{x \rightarrow +\infty} g_j(x), & m''_j &= \lim_{x \rightarrow -\infty} f_j(x), & M''_j &= \lim_{x \rightarrow +\infty} f_j(x). \\ 0 \leq \underline{\sigma}_j^l &\leq \frac{g_j(u) - g_j(v)}{u - v} \leq \bar{\sigma}_j^l, & 0 \leq \underline{\delta}_j^l &\leq \frac{f_j(u) - f_j(v)}{u - v} \leq \bar{\delta}_j^l, & \forall u, v &\in (-\infty, p_j), \\ 0 \leq \underline{\sigma}_j^m &\leq \frac{g_j(u) - g_j(v)}{u - v} \leq \bar{\sigma}_j^m, & 0 \leq \underline{\delta}_j^m &\leq \frac{f_j(u) - f_j(v)}{u - v} \leq \bar{\delta}_j^m, & \forall u, v &\in [p_j, q_j], \\ 0 \leq \underline{\sigma}_j^r &\leq \frac{g_j(u) - g_j(v)}{u - v} \leq \bar{\sigma}_j^r, & 0 \leq \underline{\delta}_j^r &\leq \frac{f_j(u) - f_j(v)}{u - v} \leq \bar{\delta}_j^r, & \forall u, v &\in (q_j, +\infty). \end{aligned}$$

Let $\Sigma^g = \text{diag}\{\bar{\sigma}_1, \bar{\sigma}_2, \dots, \bar{\sigma}_n\}$ and $\Delta^f = \text{diag}\{\bar{\delta}_1, \bar{\delta}_2, \dots, \bar{\delta}_n\}$, where $m_j = \min\{m'_j, m''_j\}$, $M_j = \max\{M'_j, M''_j\}$, $\bar{\sigma}_j = \max\{\bar{\sigma}_j^l, \bar{\sigma}_j^m, \bar{\sigma}_j^r\}$, $\bar{\delta}_j = \max\{\bar{\delta}_j^l, \bar{\delta}_j^m, \bar{\delta}_j^r\}$, $j = 1, 2, \dots, n$. Obviously, both of Σ^g and Δ^f are two positive matrices.

In addition, we also denote by $C = (c_{ij})_{n \times n}$ and $D = (d_{ij})_{n \times n}$ the connection weight matrices. Other hypotheses and notations of this article are consistent with the literature [32], no more explanation.

By Theorems 1–3 of paper [32], we know that model (1) has 3^n equilibrium points, $3^n - 2^n$ among them are unstable, and others are locally μ -stable. Here, we present only the results in [32] as lemmas directly without proof.

Lemma 1 ([32], Theorem 1) *For any $\prod_{i=1}^n w_i \in \Phi$, if*

$$\begin{cases} -b_i(p_i) + c_{ii}g_i(p_i) + d_{ii}f_i(p_i) + \sum_{j=1, j \neq i}^n \max\{(c_{ij} + d_{ij})m_j, (c_{ij} + d_{ij})M_j\} - I_i < 0, \\ i \in N_1 \cup N_2, \\ -b_i(q_i) + c_{ii}g_i(q_i) + d_{ii}f_i(q_i) + \sum_{j=1, j \neq i}^n \min\{(c_{ij} + d_{ij})m_j, (c_{ij} + d_{ij})M_j\} - I_i > 0, \\ i \in N_2 \cup N_3, \end{cases} \quad (2)$$

then there exists at least an equilibrium point of (1) in $\prod_{i=1}^n w_i$.

Lemma 2 ([32], Theorem 2) *For any $\prod_{i=1}^n w_i \in \Phi_1$, given that*

$$\begin{cases} -b_i(p_i) + c_{ii}g_i(p_i) + \sum_{j=1}^n \max\{c_{ij}m_j, c_{ij}M_j\} + \sum_{j=1}^n \max\{d_{ij}m_j, d_{ij}M_j\} - I_i < 0, \\ i \in N_1, \\ -b_i(q_i) + c_{ii}g_i(q_i) + \sum_{j=1}^n \min\{c_{ij}m_j, c_{ij}M_j\} + \sum_{j=1}^n \min\{d_{ij}m_j, d_{ij}M_j\} - I_i > 0, \\ i \in N_3, \end{cases} \quad (3)$$

and the nondecreasing function $\mu(t) > 0$ with

$$\lim_{t \rightarrow +\infty} \mu(t) = +\infty, \quad 0 \leq \sup_{t \geq T^*} \frac{\dot{\mu}(t)}{\mu(t)} \leq \alpha, \quad \sup_{t \geq T^*} \frac{\mu(t)}{\mu(t - \tau(t))} \leq 1 + \beta, \quad (4)$$

where $\alpha \geq 0, \beta \geq 0$, and $T^* \geq 0$. Then x^* is μ -stable in $\prod_{i=1}^n w_i$ (locally μ -stable in Φ_1) if there exist some positive constants $\zeta_1, \zeta_2, \dots, \zeta_n$ such that

$$\begin{aligned} & (-\underline{a}_i\beta_i + \alpha)\zeta_i + \sum_{j \in N_1} \zeta_j \bar{a}_i \bar{\sigma}_j^l |c_{ij}| + \sum_{j \in N_3} \zeta_j \bar{a}_i \bar{\sigma}_j^r |c_{ij}| \\ & + (1 + \beta) \left(\sum_{j \in N_1} \zeta_j \bar{a}_i \bar{\delta}_j^l |d_{ij}| + \sum_{j \in N_3} \zeta_j \bar{a}_i \bar{\delta}_j^r |d_{ij}| \right) < 0, \end{aligned} \quad (5)$$

where $i = 1, 2, \dots, n$.

Lemma 3 ([32], Theorem 3) *For any $\prod_{i=1}^n w_i \in \Phi_2$, given that (2) holds. If there exist some positive constants ξ_1, \dots, ξ_n such that*

$$\begin{aligned} \min_{i \in N_2} \left\{ & (-\beta_i + c_{ii}\sigma_i^m)\xi_i - \sum_{j \in N_1} \xi_j |c_{ij}| \bar{\sigma}_j^l - \sum_{j \in N_2} \xi_j |c_{ij}| \bar{\sigma}_j^m - \sum_{j \in N_3} \xi_j |c_{ij}| \bar{\sigma}_j^r \right. \\ & \left. - \sum_{j \in N_1} \xi_j |d_{ij}| \bar{\delta}_j^l - \sum_{j \in N_2} \xi_j |d_{ij}| \bar{\delta}_j^m - \sum_{j \in N_3} \xi_j |d_{ij}| \bar{\delta}_j^r \right\} > \max\{\lambda, 0\}, \end{aligned} \quad (6)$$

where

$$\begin{aligned} \lambda \triangleq \max_{i \in N_1 \cup N_3} \left\{ & (-\beta_i \xi_i + \sum_{j \in N_1} \xi_j |c_{ij}| \bar{\sigma}_j^l + \sum_{j \in N_2} \xi_j |c_{ij}| \bar{\sigma}_j^m + \sum_{j \in N_3} \xi_j |c_{ij}| \bar{\sigma}_j^r \right. \\ & \left. + \sum_{j \in N_1} \xi_j |d_{ij}| \bar{\delta}_j^l + \sum_{j \in N_2} \xi_j |d_{ij}| \bar{\delta}_j^m + \sum_{j \in N_3} \xi_j |d_{ij}| \bar{\delta}_j^r \right\}, \end{aligned} \quad (7)$$

then x^* in $\prod_{i=1}^n w_i \in \Phi_2$ is unstable.

To discuss the stability under impulsive control of unstable equilibrium points of model (1), the following two lemmas are useful.

Lemma 4 ([11]) *Let Q be a positive definite matrix. Then, for any $y, z \in \mathbb{R}^n$, $2y^T z \leq y^T Q^{-1} y + z^T Q z$.*

Lemma 5 ([22]) *The LMI $Q = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{pmatrix} < 0$ with $Q_{11} = Q_{11}^T$, $Q_{22} = Q_{22}^T$ is equivalent to one of the following conditions:*

- (i) $Q_{22} < 0$, $Q_{11} - Q_{12}Q_{22}^{-1}Q_{12}^T < 0$.
- (ii) $Q_{11} < 0$, $Q_{22} - Q_{12}^TQ_{11}^{-1}Q_{12} < 0$.

3 Impulsive control strategy and main results

For the unstable equilibrium points of model (1), we consider designing an impulsive control strategy to make the unstable equilibrium points stable in each subregion of Φ_2 . For any subregion $\prod_{i=1}^n w_i \in \Phi_2$, assume that x^* is one unstable equilibrium point in $\prod_{i=1}^n w_i$ of model (1). Then we introduce the following impulsive control on account of x^* at discrete instances:

$$\Delta x(t_i) = \Upsilon_i(x(t_i^-) - x^*), \quad i \in \mathbb{Z}_+, \quad (8)$$

where $\Upsilon_i \in \mathbb{R}^{n \times n}$ is a control matrix based on the i th pulse. Let $h(t) = x(t) - x^*$. We can transform (1) and (8) into the matrix equation shown below:

$$\begin{cases} \frac{dh(t)}{dt} = A(h(t))[-B(h(t)) + CG(h(t)) + DF(h(t - \tau(t)))], & t > 0, t \neq t_i, \\ \Delta h(t_i) = \Upsilon_i h(t_i^-), & t = t_i, i \in \mathbb{Z}_+, \\ h(s) = \phi(s), & s \leq 0, \end{cases} \quad (9)$$

where $\phi(t) = \varphi(t) - x^*$, $A(h(t)) = a(h(t) + x^*)$, $B(h(t)) = b(h(t) + x^*) - b(x^*)$, $G(h(t)) = g(h(t) + x^*) - g(x^*)$, $F(h(t)) = f(h(t) + x^*) - f(x^*)$, and $\lim_{t \rightarrow \infty} \tau(t) = +\infty$.

Definition 1 Let $h(t)$ be a solution to model (9). Then model (9) is said to be locally μ -stable, if one can find a constant $M > 0$ satisfying that

$$\|h\|^2 \leq \frac{M}{\mu(t)} \|\phi\|^2, \quad t \geq 0,$$

where $\|\phi\|^2 = \sup_{s \leq 0} \|\phi(s)\|^2$, and $\mu(t)$ is a continuously differentiable and nondecreasing function on $[0, +\infty)$.

Remark 1 The definition of local μ -stability here includes some famous stabilities such as local asymptotic stability, local Lipschitz stability, and so on. Besides, we design an impulsive control strategy $\{t_i, \Upsilon_i\}_{i \in \mathbb{Z}_+}$ (8) to stabilize the unstable equilibrium points of system (1).

Theorem 1 Suppose that there are two constants $\mu_1 \geq 1$, $\mu_2 > 0$ such that

$$\frac{\mu(t_i)}{\mu(t_{i-1})} \leq \mu_1, \quad \int_{t_{i-1}}^{t_i} \frac{\mu(t)}{\mu^*(t - \tau(t))} dt \leq \mu_2, \quad i \in \mathbb{Z}_+, \quad (10)$$

where $t_0 = 0$, and $\mu^*(t) = \mu(t)$ if $t \geq 0$ and $\mu^*(t) = 1$ if $t < 0$. Besides, one can find a matrix $P > 0$, two diagonal matrices $Q_1 > 0$, $Q_2 > 0$, and three constants $\lambda_1 \geq 0$, $\lambda_2 \geq 0$, $\gamma > 1$ such

that

$$\begin{pmatrix} \Pi & P\hat{A}C & P\hat{A}D \\ * & -Q_1 & 0 \\ * & * & -Q_2 \end{pmatrix} \leq 0, \quad \Delta^f Q_2 \Delta^f \leq \lambda_2 P, \quad (11)$$

and

$$\ln \mu_1 + \lambda_1 \theta + \mu_2 \lambda_2 \gamma < \ln \gamma, \quad (12)$$

where $\Pi = -P\check{A}\beta - \beta\check{A}P + \Sigma^g Q_1 \Sigma^g - \lambda_1 P$, $\theta = \sup_{i \in \mathbb{Z}_+} \{t_i - t_{i-1}\}$.

Consider the following positive definite Lyapunov function:

$$\mathfrak{s}_P(t) = \mu^* h^T P h,$$

where $h(t)$ is an arbitrary solution in $\prod_{i=1}^n w_i \in \Phi_2$ of model (9) with the initial condition $h(s) = \phi(s)$, $s \leq 0$. If there exist $i \in \mathbb{Z}_+$ and some $v_1 < v_2 \in [t_{i-1}, t_i]$ such that $\{h(t)|h(t) = (h_1(t), h_2(t), \dots, h_n(t)), t \in [v_1, v_2]\} \subseteq \prod_{i=1}^n w_i \in \Phi_2$, $\{h(t - \tau(t))|h(t) = (h_1(t), h_2(t), \dots, h_n(t)), t \in [v_1, v_2]\} \subseteq \prod_{i=1}^n w_i \in \Phi_2$, and

$$\mathfrak{s}_P(s) \leq \gamma \mathfrak{s}_P(t), \quad \forall s \in (-\infty, t], \quad (13)$$

then

$$\mathfrak{s}_P(v_2) \leq \gamma \mathfrak{s}_P(v_1).$$

Proof For $\forall t \in [v_1, v_2]$, by Lemma 3 and Lemma 4, the right upper Dini derivative of function $\mathfrak{s}_P(t)$ can be inferred from (11) and (13):

$$\begin{aligned} D^+ \mathfrak{s}_P(t) &= \mu^{*'}(t) h^T P h + 2\mu^*(t) h^T P h' \\ &\leq \mu^{*'}(t) h^T P h + \mu(t) [h^T (-P\check{A}\beta - \beta\check{A}P) h + h^T P\hat{A}C Q_1 C^T \hat{A} P h + h^T \Sigma^g Q_1 \Sigma^g h \\ &\quad + h^T P\hat{A}D Q_2 D^T \hat{A} P h + h^T (t - \tau(t)) \Delta^f Q_2 \Delta^f h(t - \tau(t))] \\ &= \frac{\mu'(t)}{\mu(t)} \mu(t) h^T P h + \mu(t) [h^T (-P\check{A}\beta - \beta\check{A}P + P\hat{A}C Q_1 C^T \hat{A} P + \Sigma^g Q_1 \Sigma^g \\ &\quad + P\hat{A}D Q_2 D^T \hat{A} P) h + h^T (t - \tau(t)) \Delta^f Q_2 \Delta^f h(t - \tau(t))] \\ &\leq \frac{\mu'(t)}{\mu(t)} \mu(t) h^T P h + \mu(t) [h^T \lambda_1 P h + h^T (t - \tau(t)) \lambda_2 P h(t - \tau(t))] \\ &= \left(\frac{\mu'(t)}{\mu(t)} + \lambda_1 \right) \mu(t) h^T P h + \lambda_2 \frac{\mu(t)}{\mu^*(t - \tau(t))} \mu^*(t - \tau(t)) h^T (t - \tau(t)) P h(t - \tau(t)) \\ &= \left(\frac{\mu'(t)}{\mu(t)} + \lambda_1 \right) \mathfrak{s}_P(t) + \lambda_2 \frac{\mu(t)}{\mu^*(t - \tau(t))} \mathfrak{s}_P(t - \tau(t)) \\ &\leq \left(\frac{\mu'(t)}{\mu(t)} + \lambda_1 + \lambda_2 \gamma \frac{\mu(t)}{\mu^*(t - \tau(t))} \right) \mathfrak{s}_P(t). \end{aligned} \quad (14)$$

Let us take the integral of (14) for t in the interval $[v_1, v_2]$. By (10) and (12), it can be obtained that

$$\begin{aligned} \ln \frac{s_P(v_2)}{s_P(v_1)} &\leq \ln \frac{\mu(t_i)}{\mu(t_{i-1})} + \lambda_1 \theta + \lambda_2 \gamma \int_{t_{k-1}}^{t_k} \frac{\mu(t)}{\mu^*(t - \tau(t))} dt \\ &\leq \ln \mu_1 + \lambda_1 \theta + \lambda_2 \gamma \mu_2 \leq \ln \gamma. \end{aligned} \quad (15)$$

Therefore, one can get $s_P(v_2) \leq \gamma s_P(v_1)$ from (15). \square

Theorem 2 *Let x^* be one unstable equilibrium point in $\prod_{i=1}^n w_i \in \Phi_2$ of model (1). If (10), (11), and (12) hold and*

$$\gamma(I + \Upsilon_i)^T P(I + \Upsilon_i) \leq P, \quad i \in \mathbb{Z}_+, \quad (16)$$

then model (9) is locally μ -stable. Furthermore, x^ under impulsive control strategy $\{t_i, \Upsilon_i\}_{i \in \mathbb{Z}_+}$ (8) is locally μ -stable, and so model (1) can increase $3^n - 2^n$ locally μ -stable equilibrium points.*

Proof We use a similar method as that in [50] to prove the theorem. Let $\phi \neq 0$. Then we just have to prove the following inequality:

$$s_P(t) \leq \gamma M, \quad t \in [t_{i-1}, t_i], i \in \mathbb{Z}_+, \quad (17)$$

where $M = \mu(0)\lambda_{\max}(P)\|\phi\|^2$, $\{h(t)|h(t) \in R^n, t \in [t_{i-1}, t_i]\} \subset \prod_{i=1}^n w_i$ in Φ_2 . Note that

$$s_P(t) = \mu^* h^T P h = h^T P h \leq \mu(0)\lambda_{\max}(P)\|\phi\|^2 \leq \gamma M, \quad t \leq 0. \quad (18)$$

Firstly, if $s_P(t) \leq \gamma M$ is not true when $k = 1$, then there is $v_2 \in (0, t_1)$ so that $h(v_2) \in \prod_{i=1}^n w_i$, and then

$$s_P(v_2) = \gamma M, \quad s_P(t) \leq \gamma M, \quad t \leq v_2. \quad (19)$$

By (18) and (19), it can be seen that there must exist $v_1 \in [0, v_2)$ and $h(v_1) \in \prod_{i=1}^n w_i$ in Φ_2 so that

$$s_P(v_1) = M, \quad M \leq s_P(t) \leq \gamma M, \quad t \in [v_1, v_2]. \quad (20)$$

For any $t \in [v_1, v_2]$, by (19) and (20), it follows that

$$s_P(s) \leq \gamma M \leq \gamma s_P(t), \quad \forall s \in (-\infty, t]. \quad (21)$$

Meanwhile, by Theorem 1, we can get

$$\gamma M = s_P(v_2) < \gamma s_P(v_1) = \gamma M,$$

which is a contradiction. Hence $s_P(t) \leq \gamma M$, and $k = 1$ holds.

Secondly, suppose that $s_P(t) \leq \gamma M$ holds for any $k \leq N, \forall N \in \mathbb{Z}_+$. However, if $s_P(t) \leq \gamma M$ is not true when $n = N + 1$, then there exists $v_2^* \in (t_N, t_{N+1})$ such that $h(v_2^*) \in \prod_{i=1}^n w_i$, and then

$$s_P(v_2^*) = \gamma M, \quad s_P(t) \leq \gamma M, \quad t \leq v_2^*. \quad (22)$$

With respect to (16), we can obtain that

$$\begin{aligned} s_P(t_N) &= \mu^*(t_N) h^T(t_N) P h(t_N) = \mu^* h^T(t_N^-) (I + \Upsilon_N)^T P (I + \Upsilon_N) h(t_N^-) \\ &\leq \frac{1}{\gamma} s_P(t_N^-) = M. \end{aligned} \quad (23)$$

By (22) and (23), there must exist $v_1^* \in [t_N, v_2^*)$ so that $h(v_1^*) \in \prod_{i=1}^n w_i$, and then

$$s_P(v_1^*) = M, \quad M \leq s_P(t) \leq \gamma M, \quad t \in [v_1^*, v_2^*]. \quad (24)$$

Furthermore, for any $t \in [v_1^*, v_2^*]$, by (23) and (24), we have

$$s_P(s) \leq \gamma M \leq \gamma s_P(t), \quad \forall s \in (-\infty, t]. \quad (25)$$

However, we can obtain by Theorem 1

$$\gamma M = s_P(v_2^*) < \gamma s_P(v_1^*) = \gamma M,$$

which leads to a contradiction. Thus $s_P(t) \leq \gamma M$, and $k = N + 1$ holds.

Finally, by mathematical induction, we get

$$\begin{aligned} \mu(t) \lambda_{\min}(P) h^T h &\leq \mu^*(t) h^T P h = s_P(t) \leq \gamma \mu(0) \lambda_{\max}(P) \|\varphi\|^2, \\ \forall t &\in [t_{i-1}, t_i], i \in \mathbb{Z}_+, \end{aligned} \quad (26)$$

which implies that (26) satisfies Definition 1. Hence model (9) is μ -stable in $\prod_{i=1}^n w_i$ of Φ_2 . Consequently, x^* under impulsive control (8) is locally μ -stable, and so model (1) can add $3^n - 2^n$ locally μ -stable points. \square

Remark 2 Theorem 2 shows that the impulse control can make the unstable regions stable and also increases the stable equilibrium points of model (1).

4 Corollaries and comparisons

On the basis of lemmas and theorems above, the following conclusions are drawn and compared with those in the existing literature.

When $a_i(x_i(t)) = 1$, model (1) converts into the model HNN:

$$\begin{cases} \frac{dx_i(t)}{dt} = -b_i(x_i(t)) + \sum_{j=1}^n c_{ij} g_j(x_j(t)) + \sum_{j=1}^n d_{ij} f_j(x_j(t - \tau(t))) - I_i, \\ i = 1, 2, \dots, n, t \geq 0. \end{cases} \quad (27)$$

According to conditions (2)–(6), there are at least 3^n equilibrium points in model (27), 2^n of them in Φ_1 are locally μ -stable, and the remaining in Φ_2 are unstable.

Let $h(t) = x(t) - x^*$, $t \geq 0$, where x^* is an unstable equilibrium point in $\prod_{i=1}^n w_i \in \Phi_2$ and $x(t)$ is a solution of (27) with the initial condition $x(s) = \varphi(s) \in \Phi_2, s \in (-\infty, 0]$. Then model (27) and the impulsive control (8) with respect to x^* can be turned into

$$\begin{cases} \frac{dh(t)}{dt} = -B(h(t)) + CG(h(t)) + DF(h(t - \tau(t))), & t > 0, t \neq t_i, \\ h(t) = \phi(t), & t \leq 0, \\ \Delta h(t_i) = \Upsilon_i h(t_i^-), & t = t_i, i \in \mathbb{Z}_+. \end{cases} \quad (28)$$

Corollary 1 Under conditions (10), (12), (16), and

$$\begin{pmatrix} \Pi & PC & PD \\ * & -Q_1 & 0 \\ * & * & -Q_2 \end{pmatrix} \leq 0, \quad \Delta^f Q_2 \Delta^f \leq \lambda_2 P, \quad (29)$$

where $\Pi = -P\beta - \beta P + \Sigma^g Q_1 \Sigma^g - \lambda_1 P$, model (28) is μ -stable in $\prod_{i=1}^n w_i$. Furthermore, x^* under impulsive control strategy $\{t_i, \Upsilon_i\}_{i \in \mathbb{Z}_+}$ (8) is locally μ -stable, and so model (27) can increase $3^n - 2^n$ locally μ -stable points.

When $a_i(x_i(t)) = 1$ and $b_i(x_i(t)) = b_i \cdot x_i(t)$, model (1) changes into

$$\begin{cases} \frac{dx_i(t)}{dt} = -b_i \cdot x_i(t) + \sum_{j=1}^n c_{ij} g_j(x_j(t)) + \sum_{j=1}^n d_{ij} f_j(x_j(t - \tau(t))) - I_i, \\ i = 1, 2, \dots, n, t \geq 0. \end{cases} \quad (30)$$

If (30) meets (4) and the following conditions (31)–(33):

$$\begin{cases} -b_i p_i + c_{ii} g_i(p_i) + \sum_{j \neq i}^n \max\{c_{ij} m_j, c_{ij} M_j\} + \sum_{j=1}^n \max\{d_{ij} m_j, d_{ij} M_j\} - I_i < 0, \\ -b_i q_i + c_{ii} g_i(q_i) + \sum_{j \neq i}^n \min\{c_{ij} m_j, c_{ij} M_j\} + \sum_{j=1}^n \min\{d_{ij} m_j, d_{ij} M_j\} - I_i > 0, \\ i = 1, 2, \dots, n. \end{cases} \quad (31)$$

$$\begin{aligned} & (-\underline{a}_i \beta_i + \alpha) \zeta_i + \sum_{j \in N_1} \zeta_j \bar{a}_i \bar{\sigma}_j^l |c_{ij}| + \sum_{j \in N_3} \zeta_j \bar{a}_i \bar{\sigma}_j^r |c_{ij}| \\ & + (1 + \beta) \left(\sum_{j \in N_1} \zeta_j \bar{a}_i \bar{\delta}_j^l |d_{ij}| + \sum_{j \in N_3} \zeta_j \bar{a}_i \bar{\delta}_j^r |d_{ij}| \right) < 0, \end{aligned} \quad (32)$$

$$\begin{aligned} & \min_{i \in N_2} \left\{ (-b_i + c_{ii} \sigma_i^{m*}) \xi_i - \sum_{j \in N_1} \xi_j |c_{ij}| \bar{\sigma}_j^l - \sum_{j \in N_2} \xi_j |c_{ij}| \bar{\sigma}_j^m - \sum_{j \in N_3} \xi_j |c_{ij}| \bar{\sigma}_j^r - \sum_{j \in N_1} \xi_j |d_{ij}| \bar{\delta}_j^l \right. \\ & \left. - \sum_{j \in N_2} \xi_j |d_{ij}| \bar{\delta}_j^m - \sum_{j \in N_3} \xi_j |d_{ij}| \bar{\delta}_j^r \right\} > \max\{\lambda, 0\}, \end{aligned} \quad (33)$$

where $\zeta_1, \zeta_2, \dots, \zeta_n, \xi_1, \xi_2, \dots, \xi_n$ are positive constants, and

$$\begin{aligned} \lambda \triangleq & \max_{i \in N_1 \cup N_3} \left\{ (-b_i \xi_i + \sum_{j \in N_1} \xi_j |c_{ij}| \bar{\sigma}_j^l + \sum_{j \in N_2} \xi_j |c_{ij}| \bar{\sigma}_j^m + \sum_{j \in N_3} \xi_j |c_{ij}| \bar{\sigma}_j^r \right. \\ & \left. + \sum_{j \in N_1} \xi_j |d_{ij}| \bar{\delta}_j^l + \sum_{j \in N_2} \xi_j |d_{ij}| \bar{\delta}_j^m + \sum_{j \in N_3} \xi_j |d_{ij}| \bar{\delta}_j^r \right\}, \end{aligned} \quad (34)$$

then one can obtain that there exist at least 3^n equilibrium points in model (30), $3^n - 2^n$ of them in Φ_2 are unstable, and the remaining 2^n points in Φ_1 are locally μ -stable.

Let x^* be an unstable equilibrium point in $\prod_{i=1}^n w_i \in \Phi_2$ and $x(t)$ be a solution of (30) with the initial condition $x(s) = \varphi(s) \in \Phi_2, s \in (-\infty, 0]$, and let $h(t) = x(t) - x^*, t \geq 0$. Then model (30) and the impulsive control (8) with respect to x^* can transform into the following matrix form:

$$\begin{cases} \frac{dh(t)}{dt} = -Bh(t) + CG(h(t)) + DF(h(t - \tau(t))), & t > 0, t \neq t_i, \\ h(t) = \phi(t), & t \leq 0, \\ \Delta h(t_i) = \Upsilon_i h(t_i^-), & t = t_i, i \in \mathbb{Z}_+. \end{cases} \quad (35)$$

Corollary 2 Under conditions (10), (12), (16), and (29), where $\Pi = -PB - BP + \Sigma^g Q_1 \Sigma^g - \lambda_1 P$, model (35) is μ -stable in $\prod_{i=1}^n w_i$. Furthermore, x^* under impulsive control strategy $\{t_i, \Upsilon_i\}_{i \in \mathbb{Z}_+}$ (8) is locally μ -stable, and so model (30) can increase $3^n - 2^n$ locally μ -stable points.

Remark 3 The net self-inhibition function $b_i(x_i(t))$ in model (30) is monotone increasing and odd, which contains the case of Ref. [50]. Hence model (30) is more general.

Remark 4 Ref. [50] studied the stability of unstable systems with unbounded time-varying delays at some certain discrete time for HNN model (30) and derived some control results to stabilize neural networks with an unstable equilibrium point by the impulsive control. However, we studied in the present paper the stability of multiple unstable equilibrium points.

Corollary 3 When $\mu(t) = 1 + \varsigma t, \varsigma > 0$, and $\tau(t) = kt, k \in (0, 1)$, assume that (2), (3), (6) hold in Φ_1 , and satisfy

$$\begin{aligned} & (-\underline{a}_i \beta_i + \varsigma) \zeta_i + \sum_{j \in N_1} \zeta_j \bar{a}_i \bar{\sigma}_j^l |c_{ij}| + \sum_{j \in N_3} \zeta_j \bar{a}_i \bar{\sigma}_j^r |c_{ij}| \\ & + (1 + \varsigma \tau) \left(\sum_{j \in N_1} \zeta_j \bar{a}_i \bar{\delta}_j^l |d_{ij}| + \sum_{j \in N_3} \zeta_j \bar{a}_i \bar{\delta}_j^r |d_{ij}| \right) < 0. \end{aligned}$$

Let $\ln(1 + \varsigma \theta) + \lambda_1 \theta + \frac{\theta}{1 - \tau} \lambda_2 \gamma < \ln \gamma$. If (11) and (16) hold for Φ_2 , then model (1) under the impulsive control (8) is asymptotically stable in each local region of Φ_2 .

Corollary 4 When $\mu(t) = \ln(f + t), f > e$, and $\tau(t) = t - \ln t/t$, assume that (2), (3), (6) hold in Φ_1 and satisfy

$$-\underline{a}_i \beta_i \zeta_i + \sum_{j \in N_1} \zeta_j \bar{a}_i \bar{\sigma}_j^l |c_{ij}| + \sum_{j \in N_3} \zeta_j \bar{a}_i \bar{\sigma}_j^r |c_{ij}| + \left(\sum_{j \in N_1} \zeta_j \bar{a}_i \bar{\delta}_j^l |d_{ij}| + \sum_{j \in N_3} \zeta_j \bar{a}_i \bar{\delta}_j^r |d_{ij}| \right) < 0.$$

Let $\ln \frac{\ln(f + \theta)}{\ln f} + \lambda_1 \theta + \ln(1 + \theta/f) \lambda_2 \gamma < \ln \gamma$. If (11) and (16) hold for Φ_2 , then model (1) under impulsive control (8) is log-stable in each local region of Φ_2 .

Corollary 5 When $\mu(t) = \ln(f + t)$, $\tau(t) = (f + t) - (f + t)^\epsilon$, where $f > e, \epsilon \in (0, 1)$, assume that (2), (3), (6) hold in Φ_1 and satisfy

$$-a_i \beta_i \zeta_i + \sum_{j \in N_1} \zeta_j \bar{a}_i \bar{\sigma}_j^l |c_{ij}| + \sum_{j \in N_3} \zeta_j \bar{a}_i \bar{\sigma}_j^r |c_{ij}| + \frac{1}{\epsilon} \left(\sum_{j \in N_1} \zeta_j \bar{a}_i \bar{\delta}_j^l |d_{ij}| + \sum_{j \in N_3} \zeta_j \bar{a}_i \bar{\delta}_j^r |d_{ij}| \right) < 0.$$

Let $\ln \frac{\ln(f+\theta)}{\ln f} + \lambda_1 \theta + \frac{\theta}{\epsilon} \lambda_2 \gamma < \ln \gamma$. If (11) and (16) hold for Φ_2 , then model (1) under impulsive control (8) is log-log-stable in each region of Φ_2 .

5 Numerical example

Example Consider the following two-dimensional CGNNs model:

$$\begin{cases} \frac{dx_i(t)}{dt} = -a(x_i(t)) [b_i(x_i(t)) - \sum_{j=1}^n c_{ij} g(x_j(t)) - \sum_{j=1}^n d_{ij} f(x_j(t - \tau(t))) + I_i], \\ i = 1, 2, t \geq 0, \end{cases} \quad (36)$$

where $a(x) = 1 + 0.2 \sin(x)$, $b_1(x_1(t)) = x_1(t)$, $b_2(x_2(t)) = -1.2x_2(t)$, $f(x) = \frac{|x+1| - |x-1|}{2}$,

$$g(x) = \begin{cases} \tanh(0.2x) - \tanh(1) + \tanh(0.2), & x < -1, \\ \tanh(x), & -1 \leq x \leq 1, \\ \tanh(0.2x) + \tanh(1) - \tanh(0.2), & x > 1, \end{cases} \quad \tau(t) = 0.2t,$$

$$C = (c_{ij}) = \begin{pmatrix} 3.5 & 0.2 \\ 0.4 & 4.8 \end{pmatrix}, \quad D = (d_{ij}) = \begin{pmatrix} 0.4 & 0.2 \\ 0.2 & 0.5 \end{pmatrix}, \quad I_1 = I_2 = \begin{pmatrix} -0.3 \\ -0.6 \end{pmatrix}.$$

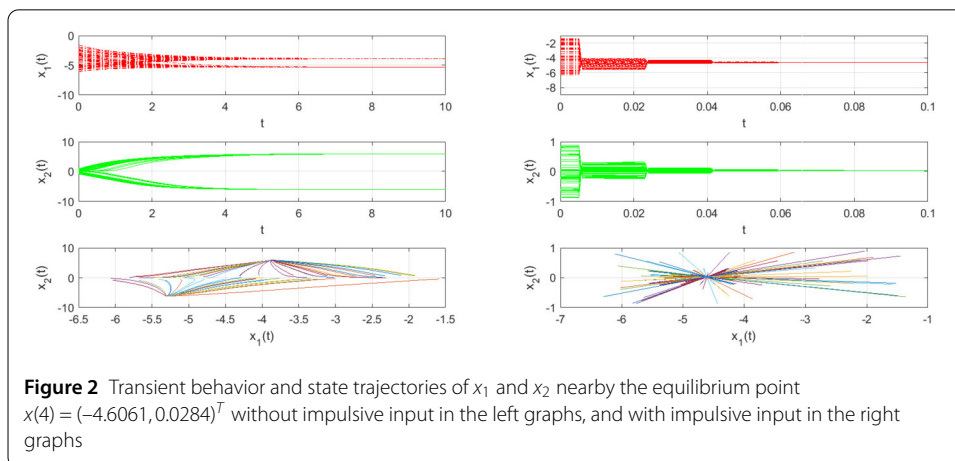
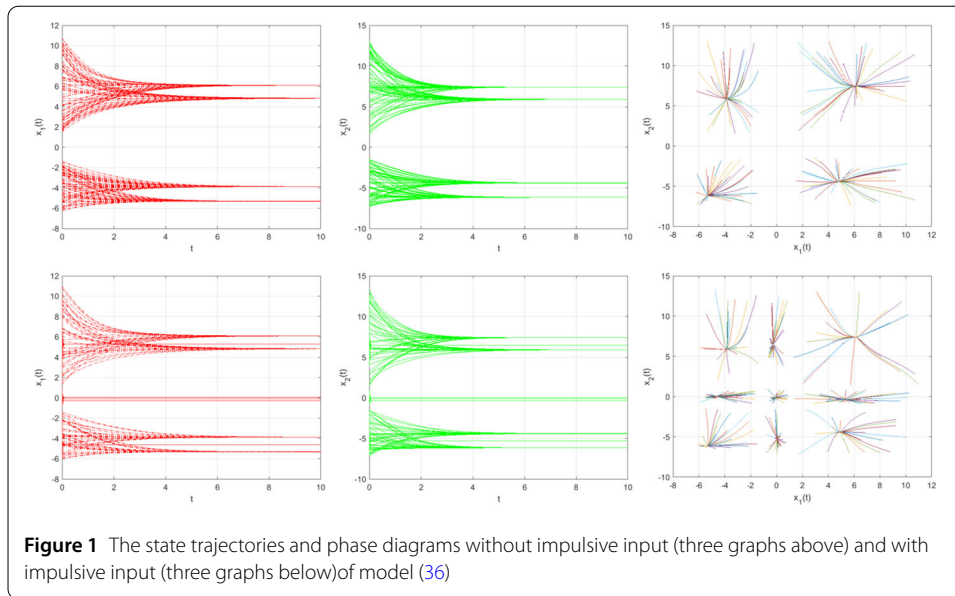
Let $\mu^*(t) = 1 + 0.2t$ if $t \geq 0$ and $\mu^*(t) = 1$ if $t < 0$. Then we know that the hypothesis of Eq. (36) and $\mu^*(t)$ satisfy condition (6) by calculation. Therefore, by Lemmas 1–3, there are nine equilibrium points in model (36), four of which are μ -stable, and others are unstable. Running program `[x, fval] = fsolve('myfun7', x0)` with Matlab software for model (36) in each subregion, one can obtain the nine equilibrium points of (36) as follows:

$$\begin{aligned} x(1) &= (-3.8676, 5.9164)^T, & x(2) &= (-0.2792, 6.4788)^T, \\ x(3) &= (6.1086, 7.4167)^T, & x(4) &= (-4.6061, 0.0284)^T, \\ x(5) &= (-0.0852, -0.1349)^T, & x(6) &= (5.2870, -0.3416)^T, \\ x(7) &= (-5.3083, -6.1639)^T, & x(8) &= (0.0586, -5.2811)^T, \\ x(9) &= (4.8374, -4.3921)^T. \end{aligned}$$

Trace the solutions of model (36) with 150 initial conditions, the dynamics of $x_1(t)$ and $x_2(t)$ are depicted in the above three graphs of Fig. 1, which show that there are four locally μ -stable equilibrium points, which is in accord with our results.

With the functions and parameters given above, we can find that $\check{A} = \text{diag}\{0.8, 0.8\}$, $\hat{A} = \text{diag}\{1.2, 1.2\}$, $\Sigma^g = \text{diag}\{1.564, 1.564\}$, $\Delta^f = \text{diag}\{1, 1\}$, $\lambda_1 = 18$. And we can obtain the following results by resorting to Matlab LMI control toolbox:

$$P = \begin{pmatrix} 0.4605 & 0.0069 \\ 0.0069 & 0.4558 \end{pmatrix}, \quad Q_1 = \begin{pmatrix} 1.7450 & 0 \\ 0 & 1.7450 \end{pmatrix},$$



$$Q_2 = \begin{pmatrix} 4.6120 & 0 \\ 0 & 4.6120 \end{pmatrix}.$$

Let $\lambda_2 = 11$, $\gamma = 3.5$, $\theta = 0.0186$, and $t_i = 0.018i$. Then we can deduce the following impulsive control matrix by (16):

$$\Upsilon_i = \begin{pmatrix} -0.7 & 0 \\ 0 & -0.7 \end{pmatrix}. \quad (37)$$

Under the impulsive control matrix (37), the state trajectory curve of model (36) can be obtained with the same 150 initial solutions, which is the below three graphs of Fig. 1. It is easy to get that the stable equilibrium points of model (36) are more than before adding impulse, and just right one equilibrium point exists in each region of model (36). Specifically, Figs. 2–6 show that the other five equilibrium points are unstable, while they are locally μ -stable after adding impulse, which verifies the effectiveness of the control strategy and the correctness of the obtained results.

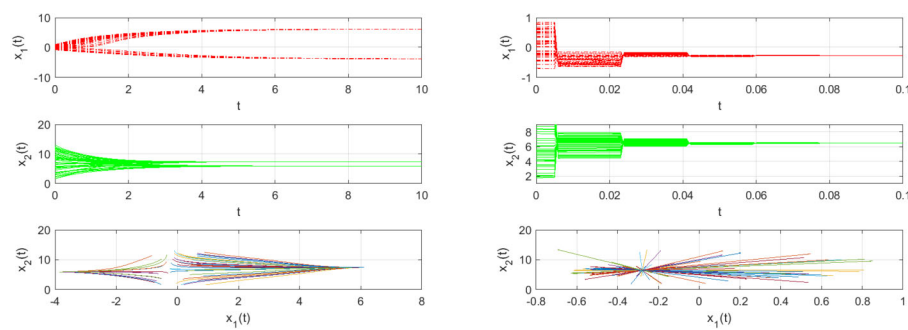


Figure 3 Transient behavior and state trajectories of x_1 and x_2 near the equilibrium point $x(2) = (-0.2792, 6.4788)^T$ without impulsive input in the left graphs, and with impulsive input in the right graphs

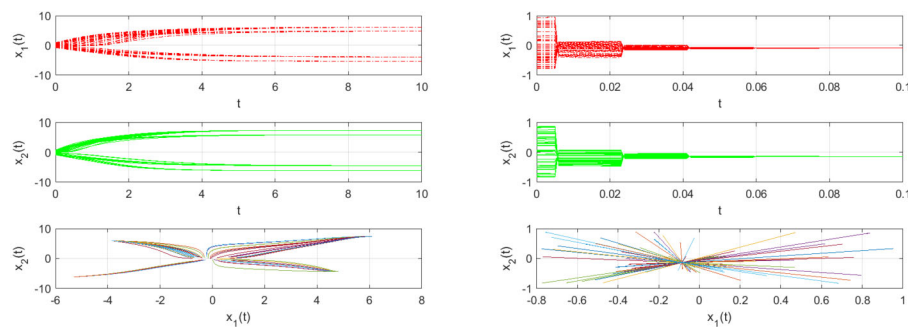


Figure 4 Transient behavior and state trajectories of x_1 and x_2 near the equilibrium point $x(5) = (-0.0852, -0.1349)^T$ without impulsive input in the left graphs, and with impulsive input in the right graphs

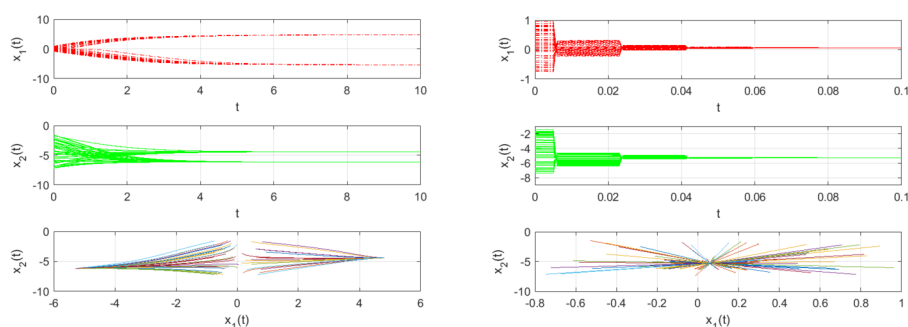
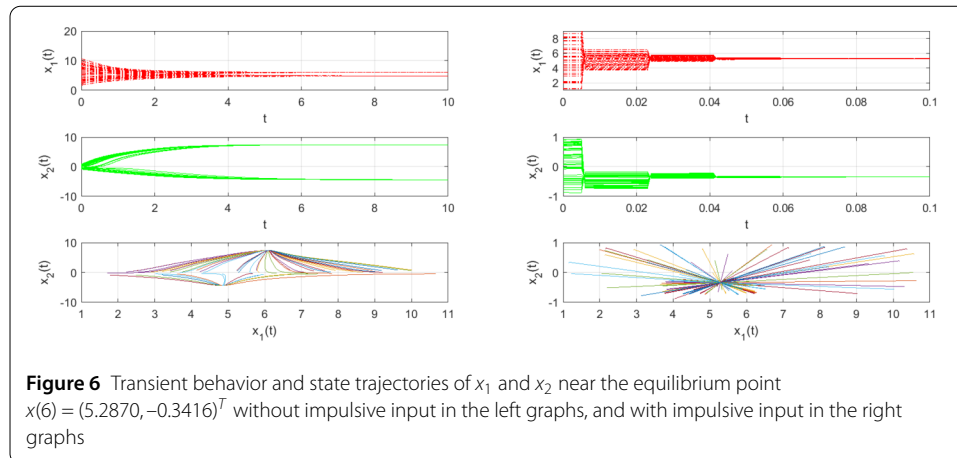


Figure 5 Transient behavior and state trajectories of x_1 and x_2 near the equilibrium point $x(8) = (0.0586, -5.2811)^T$ without impulsive input in the left graphs, and with impulsive input in the right graphs

Remark 5 The activation functions in the example of Ref. [50] without time delay and with time delay are the same, but they are different in the present paper. Therefore, the simulation of this paper is closer to the results of the theory.



6 Conclusion

Impulsive control of multiple unstable CGNNs with unbounded time-varying delays is studied in this article. Ref. [32] proved that there exist multiple equilibrium points, and some of them are unstable in model(1). For those unstable equilibrium points, we introduce an impulsive control strategy into the unstable region to ensure that system (1) is μ -stable in each local region of Φ_2 . In Sect. 4, we conclude some results of other models and point out the advantages of model (28). Meanwhile, we summarize that model (1) is μ -stable in each local region of R^n under impulsive control, including the asymptotically stable, log-stable, and log-log-stable. In addition, we also show the effectiveness of impulsive control strategy by one example and its comprehensive numerical simulations. From the results of this article, we see that it is an effective method to study the stability of multiple unstable CGNNs by introducing impulse inputs. Therefore, we can investigate the stability of other multiple unstable system by employing the impulsive control strategy further.

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Competing interests

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Authors' contributions

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