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Analysis on control of a class of uncertain stochastic system by inequality technique

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Abstract

In this paper, some theoretical results of PID control of second order nonlinear uncertain stochastic system are given via inequalities. We extend the results of the corresponding deterministic systems to stochastic systems. Specifically, as long as we have a certain understanding of the upper bound of the derivative of the unknown nonlinear drift term and diffusion term, an analytic design method can be constructed for these three PID parameters to ensure the global stability and asymptotic stability of the closed-loop control systems. In addition, the numerical simulation results verify the theoretical analysis results.

Keywords: Stochastic system; Inequality; Global stability

1 Introduction

The rapid development of control technology has an impact on every field of the control discipline. Although in the past half century, people have carried out extensive research on modern control theory, classic proportional integral differential (PID) control is still the most widely used and successful controller design method in all engineering systems [1, 2].

There are some reasons for the widely used of the PID controller: It does not need precise mathematical models and has a simple controller structure; it can not only eliminate steady state offsets via the integral action, but also anticipate the tendency through the derivative action; through the linear feedback mechanism, the influence of various uncertainties such as internal structure uncertainty and external interference can be reduced. On the contrary, the PID controller also has some shortcomings which cannot be ignored, for instance, the application of various advanced PID controls is not perfect, which is difficult to master by enterprise technicians. Specifically, one of the key problems in the realization of PID controller is how to choose three PID parameters, which are usually realized by experiment or experience. One of the famous PID parameter design methods is the Ziegler–Nichols rule. Naturally, with its extensive practical application, PID controllers have been widely studied in the academic fields, but most of them are for the linear deterministic system, less for the uncertain stochastic system [3–6].

In practical control engineering, due to the modeling error, environmental disturbances and other factors, a completely deterministic system usually does not exist. It has impor-

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tant theoretical and practical value to study the control of uncertain nonlinear systems. A stochastic nonlinear system is a kind of nonlinear system with stochastic dynamic characteristics, which has become one of the much-studied topics of nonlinear control theory in recent years. The effect of PID controller in the actual system is related to many factors. Therefore, in order to provide theoretical support for the design of PID parameters with excellent performance and improve the wide application of PID controller in engineering, the uncertain nonlinear stochastic dynamic system must be investigated [7–14].

Recently, Zhao and Guo investigated the PID control for uncertain nonlinear deterministic dynamic. They constructed a three-dimensional manifold within which the three PID parameters can be chosen arbitrarily to stabilize the nonlinear uncertain dynamical systems [15, 16]. Then Cong and Guo extended the results of [16] to stochastic system, they demonstrated the global stability and asymptotic regulation of the closed-loop control systems [17]. Motivated by these facts, we will further extend the results of [17]. By using upper bounds of the derivatives of both the nonlinear drift and the diffusion terms, we will construct a concrete three-dimensional manifold within which the three PID parameters can be chosen arbitrarily to globally stabilize the uncertain stochastic systems. We modified some inequalities to ensure that the results of our paper can be degenerated to the case of deterministic systems of [16] when the diffusion terms is zero. Also, the numerical simulation is given to verify the theoretical analysis results.

The remainder of the paper is organized as follows. Section 2 will give the preliminaries and problem description, Sect. 3 will present the main results together with mathematical proofs, Sect. 4 will show the numerical simulation results to verify the theoretical analysis results, and Sect. 5 will give the conclusion.

2 Preliminaries

Definition 2.1 (Ω, \mathcal{F}, P) is a probability space. When $0 \leq t < s < \infty$, family $\{\mathcal{F}_t\}_{t \geq 0}$ is a filtration, and it satisfies the relation that $\mathcal{F}_t \subset \mathcal{F}_s \subset \mathcal{F}$. For all $t \geq 0$, the filtration is right continuous on the premise of expression that $\mathcal{F}_t = \bigcap_{s>t} \mathcal{F}_s$. When the probability space is complete, if the filtration is right continuous and \mathcal{F}_0 contains all P -null sets, then it is considered to satisfy the usual conditions [13].

Set x as the state of the system, $x \in \mathbb{R}^n, f \in \mathbb{R}^n, g \in \mathbb{R}^n$, and $B(t)$ is a Brownian motion, then a stochastic system defined by stochastic differential equation is as follows:

$$dx(t) = f(x(t), t) dt + g(x(t), t) dB(t),$$

where the f term can be called a drift or a vector field, $g(x(t), t)$ is called the diffusion coefficient, while the noise term $g(x(t), t) dB(t)$ is a model for uncertainty. Both the external random effect and the parameter fluctuation in the mathematical model may affect the uncertainty of the model.

Ito’s Formula ([13]) Set $0 < h \leq \infty$. Using $C^{2,1}(S_h \times R_+; R_+)$ to represent the family $V(x, t)$ of all nonnegative functions defined on $S_h \times R_+$, so that they can be continuously twice differentiable in x and once in t . Define the differential operator L as follows:

$$L = \frac{\partial}{\partial t} + \frac{1}{2} \sum_{ij=1}^n [g(x, t)g^T(x, t)]_{ij} \frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^n f_i(x, t) \frac{\partial}{\partial x_i}.$$

If L acts on a function $V \in C^{2,1}(S_h \times \mathbb{R}_+; \mathbb{R}_+)$, then

$$LV(x, t) = \frac{\partial V}{\partial t}(x, t) + \frac{1}{2} \operatorname{tr}[gg^T H(V)](x, t) + f^T \nabla V(x, t).$$

If $x(t) \in S_h$, then

$$dV(x(t), t) = LV(x, t) dt + (\nabla V(x(t), t))^T g(x(t), t) dB(t),$$

where tr is the trace of the matrix, $H(V) = V_{x_i x_j}$ is the $n \times n$ symmetric Hessian matrix.

3 Main results

Let $\{B(t)\}_{t \geq 0}$ be a standard Wiener process defined on a complete probability space (Ω, \mathcal{F}, P) with a natural filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions (see Definition 2.1 in the Preliminaries).

The classical PID controller has the following standard form:

$$u(t) = k_p e(t) + k_i \int_0^t e(s) ds + k_d \dot{e}(t), \tag{1}$$

where k_p, k_i, k_d are the three controller parameters, $e(t) = y(t) - y^* \in \mathbb{R}^d$ is the regulation error.

Consider the following class of second order nonlinear uncertain stochastic systems:

$$\begin{cases} dx_1 = x_2 dt, \\ dx_2 = f(x_1, x_2, t) dt + u(t) dt + g(x_1, x_2, t) dB(t), \\ u(t) = k_p e(t) + k_i \int_0^t e(s) ds + k_d \dot{e}(t), \end{cases} \tag{2}$$

where $u(t)$ denotes the input signals, $x_1(0), x_2(0) \in \mathbb{R}; f(x_1, x_2, t)$ and $g(x_1, x_2, t)$ are both unknown nonlinear functions.

Two function spaces are defined as follows:

$$\begin{aligned} \mathcal{F}_{L_1, L_2} &= \left\{ f \in C^1(\mathbb{R}^2 \times \mathbb{R}^+) \mid \left| \frac{\partial f}{\partial x_1} \right| \leq L_1, \left| \frac{\partial f}{\partial x_2} \right| \leq L_2, \forall x_1, x_2 \in \mathbb{R}, \forall t \in \mathbb{R}^+ \right\}, \\ \mathcal{D}_{G_1, G_2} &= \left\{ W \in C^1(\mathbb{R}^2 \times \mathbb{R}^+) \mid \left| \frac{\partial g}{\partial x_1} \right| \leq G_1, \left| \frac{\partial g}{\partial x_2} \right| \leq G_2, \forall x_1, x_2 \in \mathbb{R}, \forall t \in \mathbb{R}^+ \right\}, \end{aligned}$$

where $C^1(\mathbb{R}^2 \times \mathbb{R}^+)$ represents the space of all functions from $\mathbb{R}^2 \times \mathbb{R}^+$ to \mathbb{R} , which are locally Lipschitz in (x_1, x_2) uniformly in t , piecewise continuous in t , continuous partial derivative in (x_1, x_2) , where L_1, L_2 and G_1, G_2 are known positive constants.

Theorem 1 Consider the PID controlled system (2) with any unknown functions $f \in \mathcal{F}_{L_1, L_2}$ and $g \in \mathcal{D}_{N_1, N_2}$. Assume $f(y, 0, t) = f(y, 0, 0)$, $g(y, 0, t) = g(y, 0, 0)$, and $g(y^*, 0, t) = 0$ for all $t \in \mathbb{R}^+$ and $y \in \mathbb{R}$. Then, for any $L_1, L_2 > 0$ and $G_1, G_2 > 0$, there exists a three-dimensional

manifold $\Omega_{\text{pid}} \subset \mathbb{R}^3$, the specific form is as follows:

$$\Omega_{\text{pid}} = \left\{ (k_p, k_i, k_d) \in \mathbb{R}^3 \mid k_p > L_1, k_d > L_2 + \frac{1}{2}G_2^2, k_i > 0, \right. \\ \left. (k_p - L_1) \left(k_d - L_2 - \frac{1}{2}G_2^2 \right) - k_i - \frac{1}{2}G_1^2 + \frac{1}{2}L_2G_1G_2 \right. \\ \left. > \sqrt{\left(k_d + L_2 - \frac{1}{2}G_2^2 \right) \left(L_2^2k_i + 2L_2k_pG_1G_2 + 2L_1L_2G_1G_2 \right) + L_2^2\frac{1}{2}G_1^2(k_d + L_2) + (k_p + L_1)G_1^2G_2^2} \right\}, \tag{3}$$

when the controller parameters (k_p, k_i, k_d) are taken from Ω_{pid} , the closed-loop system (2) will be globally stable and asymptotically optimal under the conditions

$$\sup_{t \geq 0} E[x_1^2(t) + x_2^2(t) + u^2(t)] < \infty \tag{4}$$

and

$$\lim_{t \rightarrow \infty} E|y^* - x_1(t)|^2 = 0, \tag{5}$$

for any initial value $(x_1(0), x_2(0)) \in \mathbb{R}^2$ and any constant setpoint $y^* \in \mathbb{R}$.

Remark 1 Obviously, the manifold Ω_{pid} of the controller parameters is an infinite open set. Theorem 1 shows that the design of PID parameters has great flexibility and the PID control system has strong robustness to unknown nonlinear dynamics and random noise.

Proof The first step: We transform the control system (3) into a standard state space equation by introducing some symbols.

Denote $x(t) = \int_0^t e(s) ds + \frac{f(y^*, 0, 0)}{k_i}$, $y(t) = e(t)$, $z(t) = \dot{e}(t)$, $h_1(y, z, t) = -f(y^* - y, -z, t) + f(y^*, 0, t)$, $h_2(y, z, t) = -g(y^* - y, -z, t)$ (see [16]), then (2) turns into

$$\begin{cases} dx = y dt, \\ dy = z dt, \\ dz = [h_1(y, z, t) - k_ix - k_p y - k_d z] dt + h_2(y, z, t) dB(t). \end{cases} \tag{6}$$

Here, $h_1(y, z, t)$ and $h_2(y, z, t)$ can be expressed as follows:

$$h_1(y, z, t) = q_1(y, t)y + p_1(y, z, t)z, \quad h_2(y, z, t) = q_2(y, t)y + p_2(y, z, t)z,$$

the functions $q_1(y, t)$, $p_1(y, z, t)$, $q_2(y, t)$ and $p_2(y, z, t)$ are defined as follows:

$$p_1(y, z, t) = \begin{cases} \frac{h_1(y, z, t) - h_1(y, 0, t)}{z}, & z \neq 0, \\ \frac{\partial h_1}{\partial z}(y, 0, t), & z = 0, \end{cases}$$

$$q_1(y, t) = \begin{cases} \frac{h_1(y, 0, t)}{y}, & y \neq 0, \\ \frac{\partial h_1}{\partial y}(0, 0, t), & y = 0, \end{cases}$$

$$p_2(y, z, t) = \begin{cases} \frac{h_2(y, z, t) - h_2(y, 0, t)}{z}, & z \neq 0, \\ \frac{\partial h_2}{\partial z}(y, 0, t), & z = 0, \end{cases}$$

$$q_2(y, t) = \begin{cases} \frac{h_2(y, 0, t) - h_2(0, 0, t)}{y}, & y \neq 0, \\ \frac{\partial h_2}{\partial y}(0, 0, t), & y = 0. \end{cases}$$

According to the mean value theorem and the definition of \mathcal{F}_{L_1, L_2} , obviously, for all y, z, t , the inequality $|q_1(y, t)| \leq L_1, |p_1(y, z, t)| \leq L_2$ can be established. This is so because, for all $t \geq 0$ and $y \in \mathbb{R}, f(y, 0, t) = f(y, 0, 0)$, obviously, $q_1(y, t) = \frac{h_1(y, 0, 0)}{y}$ is merely a function of y , denoted henceforth by $q_1(y)$, and $q_1(\cdot)$ is continuous. Similarly, $q_2(y, t)$ can be denoted by $q_2(y)$, and $|q_2(y)| \leq G_1, |p_2(y, z, t)| \leq G_2$.

Hence, the closed-loop equation (6) can be rewritten as

$$d \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y \\ z \\ F(x, y, z, t) \end{bmatrix} dt + \begin{bmatrix} 0 \\ 0 \\ h_2(y, z, t) \end{bmatrix} dB(t), \tag{7}$$

where

$$F(x, y, z, t) = -k_i x + (q_1(y) - k_p)y + (p_1(y, z, t) - k_d)z.$$

The second step: The Lyapunov function is constructed now.

Denote $\Psi = \frac{\Psi_0 + \Psi_1}{2}$, where $\Psi_0 = \inf_{y, z, t} \{-p_1(y, z, t) + k_d\}, \Psi_1 = \sup_{y, z, t} \{-p_1(y, z, t) + k_d\}$, and $\varphi_0 = \inf_y \varphi(y)$ where $\varphi(y) = -q_1(y) + k_p$. Then $\varphi_0 \geq k_p - L_1 > 0$ and $\Psi_0 \geq k_d - L_2 > 0$ under the condition of $k_p > L_1$ and $k_d > L_2 + \frac{1}{2}G_2^2$.

Similar to the cases in [16] and [17], we continue to prove that the following quadratic form plus an integral term is indeed a stochastic Lyapunov function:

$$V(x, y, z) = [x, y, z]P[x, y, z]^T + \int_0^y (\varphi(s) - \varphi_0) s ds,$$

where the constant matrix P is

$$P = \frac{1}{2} \begin{bmatrix} \mu k_i & k_i & 0 \\ k_i & \varphi_0 + \mu \Psi & \mu \\ 0 & \mu & 1 \end{bmatrix}, \tag{8}$$

and $\mu > 0$ is a constant defined by

$$\mu = \frac{2[\varphi_0(\Psi_0 - \frac{1}{2}G_2^2) + k_i + \frac{1}{2}G_1^2] - L_2G_1G_2}{4\varphi_0 + L_2^2}.$$

It can be proved that p is a positive definite matrix, so $V(x, y, z)$ is a positive definite function, and it is radially unbounded in x, y, z .

The third step: We calculate the differential operator L (see the Preliminaries) associated with (7),

$$LV(x, y, z) = \frac{\partial V}{\partial t} + [y, z, F(x, y, z, t)]^T \nabla V + \frac{1}{2} [0, 0, h_2(y, z, t)] H(V) [0, 0, h_2(y, z, t)]^T.$$

It is obvious that the first term on the right-hand side is zero, and the third term can be expressed as follows:

$$\frac{1}{2}h_2^2(y, z, t) = \frac{1}{2}q_2^2(y)y^2 + q_2(y)p_2(y, z, t)yz + \frac{1}{2}p_2^2(y, z, t)z^2,$$

according to the definition of $h_2(y, z, t)$ and the fact that Hessian matrix $H(V)$ is P . Therefore, after analyzing the second item and considering the above third term, it can be found that

$$LV(x, y, z) = -[y, z]Q(y, z, t) \begin{bmatrix} y \\ z \end{bmatrix},$$

where $Q(y, z, t)$ is a symmetric matrix, and the specific expression is

$$Q(y, z, t) = \begin{bmatrix} m_1(y, z, t) & m_2(y, z, t) \\ m_2(y, z, t) & m_3(y, z, t) \end{bmatrix},$$

and we have

$$\begin{aligned} m_1(y, z, t) &= -k_i + \mu\varphi(y) - \frac{1}{2}q_2^2(y), \\ m_2(y, z, t) &= -\frac{\mu(\Psi + p_1(y, z, t) - k_d) + q_2(y)p_2(y, z, t)}{2}, \\ m_3(y, z, t) &= -\mu - p_1(y, z, t) + k_d - \frac{1}{2}p_2^2(y, z, t). \end{aligned}$$

Now, we prove that $Q(y, z, t)$ is actually positive definite for all $y, z \in \mathbb{R}$ and $t \in \mathbb{R}^+$.

Denote $\alpha = -\frac{\mu}{2}[\Psi + p_1(y, z, t) - k_d]$, $\beta = -\mu - p_1(y, z, t) + k_d$, note that by the definitions of $\varphi_0, \Psi_0, \Psi_1$, we have

$$-k_i + \mu\varphi(y) - \frac{1}{2}q_2^2(y) \geq -k_i + \mu\varphi_0 - \frac{1}{2}G_1^2 > 0, \tag{9}$$

$$-\mu + \Psi_1 - \frac{1}{2}p_2^2(y, z, t) \geq \beta - \frac{1}{2}p_2^2(y, z, t) \geq -\mu + \Psi_0 - \frac{1}{2}G_2^2 > 0, \tag{10}$$

$$|\Psi + p_1(y, z, t) - k_d| \leq L_2, \tag{11}$$

here, the expressions $\Psi = \frac{\Psi_0 + \Psi_1}{2}$ and $|p_1(y, z, t)| \leq L_2$ hold.

Therefore, by (9) and (10), the following inequalities can be obtained:

$$\begin{aligned} &\left(-k_i + \mu\varphi(y) - \frac{1}{2}q_2^2(y)\right)\left(\beta - \frac{1}{2}p_2^2(y, z, t)\right) \\ &\geq \left(-k_i + \mu\varphi_0 - \frac{1}{2}G_1^2\right)\left(-\mu + \Psi_0 - \frac{1}{2}G_2^2\right) \\ &> \frac{1}{4}(\mu^2L_2^2 + G_1^2G_2^2 + 2|\mu L_2G_1G_2|) \\ &\geq \left[\alpha - \frac{1}{2}q_2(y)p_2(y, z, t)\right]^2. \end{aligned}$$

According to the above inequality and (9), the matrix $Q(y, z, t)$ is positive definite for all y, z, t .

The minimum eigenvalue of $Q(y, z, t)$ can be obtained as

$$\begin{aligned} \lambda_{\min}\{Q(y, z, t)\} &= \theta(y, \alpha, \beta) \\ &\triangleq \frac{1}{2} \left\{ \mu\varphi(y) - k_i - \frac{1}{2}q_2^2 + \beta - \frac{1}{2}p_2^2 \right. \\ &\quad \left. - \sqrt{\left[\left(\mu\varphi(y) - k_i - \frac{1}{2}q_2^2 \right) - \left(\beta - \frac{1}{2}p_2^2 \right) \right]^2 + 4\left(\alpha - \frac{1}{2}p_2q_2 \right)^2} \right\}. \end{aligned}$$

Define $\lambda(y) = \inf_{\alpha, \beta} \theta(y, \alpha, \beta)$, where the infimum is taken for all $|\alpha - \frac{1}{2}p_2q_2| \leq \frac{1}{2} \times \sqrt{\mu^2L_2^2 + G_1^2G_2^2 + 2|\mu L_2G_1G_2|}$ and $-\mu + \Psi_0 \leq \beta \leq -\mu + \Psi_1$.

We can derive that $\lambda(\cdot)$ is a positive function of y , and $\lambda(\cdot)$ is a continuous function by [16]. Further, by the boundedness of the function $\varphi(y)$, there exists $\underline{\lambda} > 0$ such that $\lambda(y) \geq \underline{\lambda}$.

Therefore, we have

$$LV(x, y, z) \leq -\underline{\lambda}[y^2 + z^2]. \tag{12}$$

The fourth step: By the Itô formula (see the Preliminaries), we have

$$dV(x, y, z) = LV(x, y, z) dt + (z + \mu y)h_2(y, z, t) dB(t).$$

We express the diffusion term as $G(y, z, t)$, noting the definition of $h_2(y, z, t)$, we have

$$G(y, z, t) = (z + \mu y)h_2(y, z, t) = \mu q_2(y)y^2 + p_2(y, z, t)z^2 + (q_2(y) + \mu p_2(y, z, t))yz.$$

Then, for any case where $T > 0$, the following equation holds:

$$V(x(T), y(T), z(T)) = V(x_0, y_0, z_0) + \int_0^T LV(x, y, z) dt + \int_0^T G(y, z, t) dB(t). \tag{13}$$

We wish to prove the following equation:

$$E \int_0^T G(y, z, t) dB(t) = 0.$$

We need to prove the following inequality:

$$E \int_0^T |G(y, z, t)|^2 dt < \infty. \tag{14}$$

From the above expression of $G(y, z, t)$ and the boundedness of $q_2(y)$ and $p_2(y, z, t)$, we can come to the following conclusion:

$$|G(y, z, t)|^2 = O(y^4 + z^4).$$

Therefore, by taking $p = 4$ in Theorem 4.1 in [13], we can get (14). So, considering the expectation on both sides of (13) and using Eq. (12), we get

$$EV(x(T), y(T), z(T)) \leq V(x_0, y_0, z_0) - E \int_0^T \underline{\lambda}(y^2 + z^2) dt. \tag{15}$$

Therefore, according to the definition of $V(x, y, z)$ and the positive attribute of P , for all $T \geq 0$, we get

$$E(x^2(T) + y^2(T) + z^2(T)) \leq V(x_0, y_0, z_0) \tag{16}$$

and

$$\int_0^T E(y^2(t) + z^2(t)) dt \leq V(x_0, y_0, z_0). \tag{17}$$

Therefore, we get the global stability as follows:

$$\sup_{t \geq 0} E[y^2(t) + z^2(t) + u^2(t)] < \infty,$$

this is expected result (4). In order to prove the optimality of the trace, we can get it by letting $T \rightarrow \infty$ in (17),

$$\int_0^\infty E(y^2(t) + z^2(t)) dt \leq V(x_0, y_0, z_0). \tag{18}$$

The fifth step: We need to verify the uniform continuity of $Ey^2(t)$ on $(0, \infty)$ in order to use the Barbalat lemma in Ref. [18], from which it can be concluded that $Ey^2(t) \rightarrow 0$ when $t \rightarrow \infty$.

First of all, according to the mean value theorem, there exists a random variable $\bar{y} \in [y_{t_1}, y_{t_2}]$ such that

$$|Ey^2(t_1) - Ey^2(t_2)| \leq E|y^2(t_1) - y^2(t_2)| \leq E[2\bar{y}|y_{t_1} - y_{t_2}|].$$

By the Schwarz inequality, we have

$$E[2\bar{y}|y_{t_1} - y_{t_2}|] \leq 2\sqrt{E\bar{y}^2} \sqrt{E|y_{t_1} - y_{t_2}|^2}. \tag{19}$$

There exists a constant $M_1 > 0$ such that

$$E|\bar{y}|^2 \leq 2E\{|y_{t_1}|^2 + |y_{t_2}|^2\} \leq M_1. \tag{20}$$

According to the boundedness property (4), we know that there is a constant $M_2 > 0$ such that

$$E\left\{ \sup_{t \in [t_1, t_2]} |z_t|^2 \right\} \leq M_2. \tag{21}$$

Finally, by using (19)–(21), we can get the following inequality:

$$|E y^2(t_1) - E y^2(t_2)| \leq 2\delta \sqrt{M_1 M_2} \leq \varepsilon.$$

The sixth step: We draw the conclusion that

$$\lim_{t \rightarrow \infty} E |y^* - x_1(t)|^2 = 0.$$

Then the proof of Theorem 1 has been finished. □

4 Simulation

We use a numerical simulation example to illustrate the theoretical results. We consider the following system:

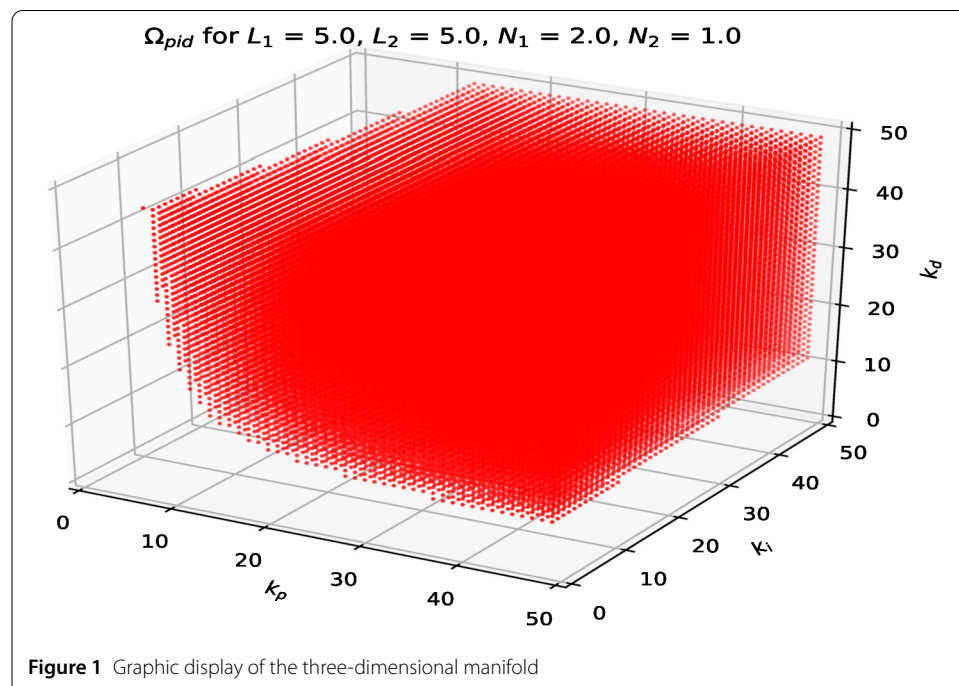
$$\begin{cases} dx_1 = x_2 dt, \\ dx_2 = f(x_1, x_2, t) dt + u(t) dt + g(x_1, x_2, t) dB(t). \end{cases}$$

We use the PID controller

$$u(t) = k_p(x_1(t) - y^*) + k_i \int_0^t (x_1(s) - y^*) ds + k_d \frac{dx_1(t)}{dt},$$

such that $x_1(t)$ converges to the given constant setpoint y^* . The two function spaces \mathcal{F}_{L_1, L_2} and \mathcal{D}_{G_1, G_2} are defined in Sect. 3. The three-dimensional manifold $\Omega_{pid} \subset \mathbb{R}^3$ can be found in Eq. (3) of Sect. 3.

Let: $L_1 = 5$ and $L_2 = 5$, $G_1 = 2$ and $G_2 = 1$, the domain of the set Ω_{pid} is restricted to $0 \leq k_p, k_i, k_d \leq 50$. Then Fig. 1 shows the graphic display of the three-dimensional manifold.



5 Conclusion

The theory and the design method of the PID controller for a class of second order nonlinear uncertain stochastic system are given in this paper. We have shown that as long as the upper bounds of the derivative of the nonlinear uncertain diffusion and drift functions are valid, the global stability and asymptotic regulation of the closed-loop stochastic control system can be guaranteed by constructing a three-dimensional manifold within which the three PID parameters can be chosen arbitrarily. Also, when the diffusion term is zero, it can be degenerated to the case of deterministic systems of [16]. Furthermore, the numerical simulation is given to verify the theoretical analysis results.

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Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors read and approved the final manuscript.

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