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Sensitivity analysis for a new class of generalized parametric nonlinear ordered variational inequality problems in ordered Banach spaces

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Abstract
In this study, we introduce a class of new generalized parametric nonlinear ordered variational inequality problems and discuss its existence result. Also, we prove the sensitivity of the solution for the parametric inequality class with the help of *B*-restricted-accretive method in ordered Banach spaces. Some special cases of the main results are also discussed.

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1 Introduction

Ordered equations or inequalities have capacious significance to many fields, including neural networks, remote sensing, optimization of structures, optimization of electromagnetic systems, and many other applied sciences. Lately, much consideration has been given to the sensitivity analysis of variational inequalities. We comment that sensitivity analysis is imperative for a few reasons.

To begin with, since assessing issue information regularly presents estimation mistakes, sensitivity analysis soothes in distinguishing touchy factors that ought to be acquired with generally high exactness. Second, sensitivity analysis may anticipate in the doom changes of the steadiness because of alters in the dictating systems. Lastly, sensitivity gives valuable input for outlining or arranging different equilibrium in various frames. Moreover, from scientific and engineering perspectives, sensitivity analysis can give new bits of knowledge with respect to issues being considered and can fortify new thoughts for critical thinking.

During the most-recent decade, there has been expanding enthusiasm to concentrate on checking the sensitivity of different inequalities and inclusion systems. We contemplate the subjective conduct of the solution of the variational inequalities when the given operator and the feasible convex set change in a parameter. Such an investigation is known as sensitivity analysis, which is vital and significant. Sensitivity analysis gives us helpful data for outlining different equilibrium systems and for anticipating the coming alters of the equilibria because of the adjustments in the dictating systems.

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Dafermos in [7] utilized the fixed-point technique to ponder the sensitivity analysis of Stampacchia's variational inequalities. Time to time the model has been changed and stretched out by numerous writers for reviewing the sensitivity analysis of different classes of variational inequalities and inclusions problems; for more details, see [1, 2, 4–6, 12–14, 16–18].

Then again, in 1972, Amman [3] proposed the idea of finding the numbers of invariants under the nonlinear mapping in an ordered Banach space. Motivated by the idea of Amman, Li [9] instigated the work on the generalized nonlinear ordered variational inequalities and equation. By the B-restricted-accretive method for map A, Li initiated the study of existence and convergence result for approximation solution for the said problems in ordered Banach space. After that, in 2009, Li [10] started the study of a new class in ordered Banach space, and it was abbreviated as GNOVI. The sensitivity analysis was also investigated by Li [11], in which an existence result was proposed for another class of problems constricted as parametric GNOVI. Persuaded by the exploration in this tendency, we present a new class of generalized parametric nonlinear ordered variational inequalities involving $\oplus$ operator in ordered Banach space. We also prove the existence and continuity of the solution for the said problem.

2 Prelude

In this part of the paper, we review some basic facets which are supplementary for further processing.

Allow $(\mathcal{E}, \text{``} \leq \text{''})$ to be a real ordered Banach space with a norm $\|\cdot\|$. Let $(\mathcal{C}, \text{``} \leq \text{''})$ be a partial ordered cone having a normal constant N, θ is the zero member of $(\mathcal{E}, \text{``} \leq \text{''})$. Presume that $\mathrm{glb}\{m,n\}$ and $\mathrm{lub}\{m,n\}$ both exist.

Definition 2.1 $\mathcal{C} \subset \mathcal{E}$, a non-void convex and closed subset, is termed cone if

(i) as $m \in \mathcal{C}$ and $\kappa > 0$, $\kappa m \in \mathcal{C}$;

(ii) if $-m, m \in \mathcal{C}$, then $m = \theta$.

Definition 2.2 ([8]) A non-void subset $\mathcal{C} \subset \mathcal{E}$ characterizes as normal cone iff $\exists\, N > 0$ with $0 \leq m \leq n$ such that $\|m\| \leq N\|n\|$.

Definition 2.3 ([15]) Define a partial order "$\leq$" for any elements $m, n \in \mathcal{E}$ as $m \leq n$ iff $m - n \in \mathcal{C}$. Then $(\mathcal{E}, \text{``} \leq \text{''})$ is a real ordered Banach space.

Definition 2.4 ([15]) Elements $m, n \in \mathcal{E}$ related by the partial order defined above are called comparable elements.

Definition 2.5 ([15]) Let $\vee$, $\wedge$, and $\oplus$ be operations named *OR*, *AND*, and *XOR* and characterized respectively as follows:

(i) $m \vee n = \mathrm{lub}\{m,n\}$;

(ii) $m \wedge n = \mathrm{glb}\{m,n\}$;

(iii) $m \oplus n = (m - n) \vee (n - m)$.

Lemma 2.1 [15] *For any members $m, n, w \in \mathcal{E}$, the following relations hold*:

(i) *if $m \leq n$, then $m \vee n = n$, $m \wedge n = m$*;

(ii) *if $m \propto n$, then $\theta \leq m \oplus n$*;

(iii) $(m + w) \vee (n + w)$ *exists and* $(m + w) \vee (n + w) = (m \vee n) + w$;
(iv) $(m \wedge n) = (m + n) - (m \vee n)$;
(v) *for* $\lambda \geq 0$, *one can have* $\lambda(m \vee n) = \lambda m \vee \lambda n$;
(vi) *for* $\lambda \leq 0$, *one can have* $\lambda(m \wedge n) = \lambda m \vee \lambda n$;
(vii) *the converse part of* (v) *and* (vi) *holds if* $m \neq n$;
(viii) *either* $m \vee n$ *or* $m \wedge n$ *exists, then* $\mathcal{E}$ *is a lattice*;
(ix) $(m + w) \wedge (n + w)$ *exists and* $(m + w) \wedge (n + w) = (m \wedge n) + w$;
(m) $(m \wedge n) = -(-m \vee -n)$;
(xi) $(-m) \wedge (m) \leq \theta \leq (-m) \vee m$.

Proposition 1 ([8]) *For the comparable members* m *and* n *in* $\mathcal{E}$, *then* $m - n \propto n - m$, *and* $\theta \leq (m - n) \vee (n - m)$.

Proposition 2 ([8]) *For any positive integer* k, *if* $m \propto n_k$ *and* $n_k \to n^*$ $(k \to \infty)$, *then* $m \propto n^*$.

Lemma 2.2 ([9]) *For any comparable elements* $m, n, z, w \in \mathcal{E}$ *and an operation* $\oplus$, *the following relations hold*:
(i) $m \oplus n = n \oplus m$;
(ii) $m \oplus m = \theta$;
(iii) $\theta \leq m \oplus \theta$;
(iv) *let* $\lambda \in \mathbb{R}$, *then* $(\lambda m) \oplus (\lambda n) = |\lambda|(m \oplus n)$;
(v) $(m \oplus n) \leq (m \oplus w) + (w \oplus n)$;
(vi) *suppose* $(m + n) \vee (p + q)$ *exists, and if* $m \propto p, q$ *and* $n \propto p, q$, *then*

$$(m + n) \oplus (p + q) \leq (m \oplus p + n \oplus q) \wedge (m \oplus q + n \oplus p);$$

(vii) *suppose* m, n, z, w *can be comparative to each other, then*

$$(m \wedge n) \oplus (z \wedge w) \leq \bigl((m \oplus z) \vee (n \oplus w)\bigr) \wedge \bigl((m \oplus w) \vee (n \oplus z)\bigr);$$

(viii) *if* $m \propto \theta$, *then* $\alpha m \oplus \beta m = |\alpha - \beta| m + (\alpha \oplus \beta) m$.

Definition 2.6 ([9]) Allow $\mathcal{A}, \mathcal{B} : \mathcal{E} \times \Sigma \to \mathcal{E}$ to be two parametric maps and $\Sigma \subset \mathcal{E}$ be a non-void open subset in which ς lives.
(i) $\mathcal{A}$ is called comparison regarding the slot ς, if for any $\varsigma \in \Sigma$ and each $m(\varsigma), n(\varsigma) \in \mathcal{E}$ we have $m(\varsigma) \propto n(\varsigma)$, then $\mathcal{A}(m(\varsigma), \varsigma) \propto \mathcal{A}(n(\varsigma), \varsigma)$, $m(\varsigma) \propto \mathcal{A}(m(\varsigma), \varsigma)$, and $n(\varsigma) \propto \mathcal{A}(n(\varsigma), \varsigma)$.
(ii) $\mathcal{A}$ and $\mathcal{B}$ are termed comparison regarding the spot ς to each other, if for each $m(\varsigma) \in \mathcal{E}$, $\mathcal{A}(m(\varsigma), \varsigma) \propto \mathcal{B}(m(\varsigma), \varsigma)$ (denoted by $\mathcal{A} \propto \mathcal{B}$).

Definition 2.7 ([9]) Allow $\mathcal{A} : \mathcal{E} \times \Sigma \to \mathcal{E}$ to be a parametric map. $\mathcal{A}$ is called β-ordered compression regarding the second slot ς, if $\mathcal{A}$ is comparative with respect to ς, and $\exists \beta \in (0, 1)$ such that, for any $\varsigma \in \Sigma$,

$$\mathcal{A}\bigl(m(\varsigma), \varsigma\bigr) \oplus \mathcal{A}\bigl(n(\varsigma), \varsigma\bigr) \leq \beta\bigl(m(\varsigma) \oplus n(\varsigma)\bigr)$$

holds.

Definition 2.8 ([9]) Let $(\mathcal{E}, \text{“} \leq \text{”})$, $\mathcal{C}$, N, and Σ have their predefined meanings. Allow $\mathcal{A}, \mathcal{B} : \mathcal{E} \times \Sigma \to \mathcal{E}$ to be two parametric maps, I stands for an identity map on $\mathcal{E} \times \mathcal{E}$.

(i) A map $\mathcal{A}$ is referred as restricted-accretive regarding the slot ς if $\mathcal{A}$ is comparative, and there exist two constants $0 < \alpha_1, \alpha_2 \leq 1$ such that, for any $\varsigma \in \Sigma$ and arbitrary $m(\varsigma), n(\varsigma) \in \mathcal{E}$,

$$\begin{aligned}&\bigl(\mathcal{A}\bigl(m(\varsigma),\varsigma\bigr) + I\bigl(m(\varsigma),\varsigma\bigr)\bigr) \oplus \bigl(\mathcal{A}\bigl(n(\varsigma),\varsigma\bigr) + I\bigl(n(\varsigma),\varsigma\bigr)\bigr)\\ &\quad\leq \alpha_1\bigl(\mathcal{A}\bigl(m(\varsigma),\varsigma\bigr) \oplus \mathcal{A}\bigl(n(\varsigma),\varsigma\bigr)\bigr) + \alpha_2\bigl(m(\varsigma) \oplus n(\varsigma)\bigr)\end{aligned}$$

holds.

(ii) A map $\mathcal{A} : \mathcal{E} \times \Sigma \to \mathcal{E}$ is called a B-restricted-accretive map regarding the slot ς if $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{A} \wedge \mathcal{B} : \mathcal{E} \times \Sigma \to \mathcal{A}(m(\varsigma),\varsigma) \wedge \mathcal{B}(m(\varsigma),\varsigma) \in \mathcal{E}$ all are comparative and they are comparison regarding the slot ς to each other, and $\exists$ constants $0 < \alpha_1, \alpha_2 \leq 1$ such that, for any $\varsigma \in \Sigma$ and any $m(\varsigma), n(\varsigma) \in \mathcal{E}$,

$$\begin{aligned}&\bigl(\mathcal{A}\bigl(m(\varsigma),\varsigma\bigr) \wedge \mathcal{B}\bigl(m(\varsigma),\varsigma\bigr) + I\bigl(m(\varsigma),\varsigma\bigr)\bigr)\\ &\qquad\oplus \bigl(\mathcal{A}\bigl(n(\varsigma),\varsigma\bigr) \wedge \mathcal{B}\bigl(n(\varsigma),\varsigma\bigr) + I\bigl(n(\varsigma),\varsigma\bigr)\bigr)\\ &\quad\leq \alpha_1\bigl(\bigl(\mathcal{A}\bigl(m(\varsigma),\varsigma\bigr) \wedge \mathcal{B}\bigl(m(\varsigma),\varsigma\bigr)\bigr) \oplus \bigl(\mathcal{A}\bigl(n(\varsigma),\varsigma\bigr) \wedge \mathcal{B}\bigl(n(\varsigma),\varsigma\bigr)\bigr)\bigr)\\ &\qquad+ \alpha_2\bigl(m(\varsigma) \oplus n(\varsigma)\bigr)\end{aligned}$$

holds.

Proposition 3 ([11]) *Allow $\mathcal{A} : \mathcal{E} \to \mathcal{E}$ to be a comparison map, then for each $m, n \in \mathcal{E}$, the following relations hold*:

(i) $\|\theta + \theta\| = \|\theta\| = \theta$;

(ii) $\|m \vee n\| \leq \|m\| \vee \|n\| \leq \|m\| + \|n\|$;

(iii) $\|m \oplus n\| \leq \|m - n\| \leq N\|m \oplus n\|$;

(iv) *if $m \propto n$, then* $\|m \oplus n\| = \|m - n\|$;

(v) $\lim_{m \to m_0} \|\mathcal{A}(m) - \mathcal{A}(m_0)\| = 0$ *iff* $\lim_{m \to m_0} \mathcal{A}(m) \oplus \mathcal{A}(m_0) = 0$.

3 Formatting of the problem

Let $\Sigma \subset \mathcal{E}$ be a non-void open subset in which the parameter ς lives. Let $\mathcal{M}, \mathcal{A}, \mathcal{F}, g, h : \mathcal{E} \times \Sigma \to \mathcal{E}$ be single-valued compression maps and range $g(\cdot,\varsigma) \cap \operatorname{dom} \mathcal{A}(\cdot,\varsigma) \neq \Phi$, range $h(\cdot,\varsigma) \cap \operatorname{dom} \mathcal{F}(\cdot,\varsigma) \neq \Phi$ for any $\varsigma \in \Sigma$.

We contemplate the following problem:

Find $m = m(\varsigma) : \Sigma \to \mathcal{E}$ such that

$$\theta \leq \mathcal{M}(m,\varsigma) + \mathcal{A}\bigl(g(m,\varsigma),\varsigma\bigr) \oplus \mathcal{F}\bigl(h(m,\varsigma),\varsigma\bigr). \tag{1}$$

Problem (1) is referred to as a new class of generalized parametric nonlinear ordered variational inequalities involving *XOR* operator (or in short GPNOVI).

3.1 Special cases

Case 1: If $\mathcal{F}(h(m,\varsigma),\varsigma) \equiv \theta$ (the zero map), then problem (1) is reverted into problem (1.1) which was encountered by Li in [11].

Case 2: If $\mathcal{F}(h(m,\varsigma),\varsigma) \equiv \theta$ and $\mathcal{M}(m,\varsigma) \equiv \theta$, then problem (1) is transformed into problem (2.1) which was encountered by Li in [9].

Lemma 3.1 *Allow $\mathcal{M}$, $\mathcal{A}$, $\mathcal{F}$, $(\mathcal{M}+\mathcal{A}\oplus\mathcal{F})$, g, h, and $(\mathcal{M}+\mathcal{A}\oplus\mathcal{F})\wedge\mathcal{B}:\mathcal{E}\times\Sigma\to\mathcal{E}$ be the comparison mappings. If $[\mathcal{M}(m,\varsigma)+\mathcal{A}(g(m,\varsigma),\varsigma)\oplus\mathcal{F}(h(m,\varsigma),\varsigma)]\wedge\mathcal{B}(m,\varsigma)=\theta$ $(\theta\in\mathcal{E})$ has an answer m^*, then m^* will also be an answer to problem (1).*

Proof With the help of definitions and conditions on the mappings $\mathcal{M}$, $\mathcal{A}$, $\mathcal{F}$, $(\mathcal{M}+\mathcal{A}\oplus\mathcal{F})$, g, h, and $(\mathcal{M}+\mathcal{A}\oplus\mathcal{F})\wedge\mathcal{B}:\mathcal{E}\times\Sigma\to\mathcal{E}$, the proof follows. □

4 Existence result for generalized parametric nonlinear ordered variational inequality problem with ⊕ operator

In this part of the paper, we will prove the existence of solution of generalized parametric nonlinear ordered variational inequality problem (1).

Theorem 4.1 *Let $(\mathcal{E},\text{“}\leq\text{”})$, $(\mathcal{C},\text{“}\leq\text{”})$, N, and $\Sigma\subset\mathcal{E}$ have their predefined meaning, and let $\mathcal{M}$, $\mathcal{A}$, $\mathcal{F}$, $\mathcal{B}$, g, h, $(\mathcal{M}+\mathcal{A}\oplus\mathcal{F})$, and $(\mathcal{M}+\mathcal{A}\oplus\mathcal{F})\wedge\mathcal{B}:\mathcal{E}\times\Sigma\to\mathcal{E}$ be comparison parametric mappings to each other. Suppose that M is λ_M-ordered compression, A is λ_A-ordered compression, F is λ_F-ordered compression, B is λ_B-ordered compression, g is λ_g-ordered compression, and h is λ_h-ordered compression mappings regarding the second slot ς, respectively. Further, if $(\mathcal{M}+\mathcal{A}\oplus\mathcal{F}):\mathcal{E}\times\Sigma\to\mathcal{E}$ is a B-restricted accretive map regarding the second slot ς for constants α_1 and α_2, and for any $\tau>0$, the given condition*

$$\tau\left[\lambda_M+(\lambda_A\lambda_g\oplus\lambda_F\lambda_h)\right]\vee\lambda_B<\frac{1-N\alpha_2}{N\alpha_1} \tag{2}$$

holds, then m^* is the answer to problem (1).

Proof For any given $\varsigma\in\Sigma$ and $m_1=m_1(\varsigma)$ and $m_2=m_2(\varsigma)$ in $\mathcal{E}$, for $\tau>0$, let $m_1(\varsigma)\propto m_2(\varsigma)$, then

$$\begin{aligned}\mathcal{F}\big(m_i(\varsigma),\varsigma\big)={}&\tau\big[\mathcal{M}\big(m_i(\varsigma),\varsigma\big)+\mathcal{A}\big(g\big(m_i(\varsigma),\varsigma\big),\varsigma\big)\oplus\mathcal{F}\big(h\big(m_i(\varsigma),\varsigma\big),\varsigma\big)\big]\\&\wedge\mathcal{B}\big(m_i(\varsigma),\varsigma\big)+I\big(m_i(\varsigma),\varsigma\big),\end{aligned} \tag{3}$$

where $i=1,2$. It follows from the conditions, $\mathcal{M}$, $\mathcal{A}$, $\mathcal{F}$, $\mathcal{B}$, $(\mathcal{M}+\mathcal{A}\oplus\mathcal{F})$, g, h, and $(\mathcal{M}+\mathcal{A}\oplus\mathcal{F})\wedge\mathcal{B}:\mathcal{E}\times\Sigma\to\mathcal{E}$ are comparison parametric mappings regarding the second slot ς to each other, and $m_1(\varsigma)\propto m_2(\varsigma)$ that $\mathcal{F}(m_1(\varsigma),\varsigma)\propto\mathcal{F}(m_2(\varsigma),\varsigma)$. Using the conditions of restricted-accretive and the ordered-compression on suitable mappings regarding the second slot ς, respectively, and Proposition 2, we have

$$\begin{aligned}\theta&\leq\mathcal{F}(m_1,\varsigma)\oplus\mathcal{F}(m_2,\varsigma)\\&\leq\big[\tau\big(\mathcal{M}\big(m_1(\varsigma),\varsigma\big)+\mathcal{A}\big(g\big(m_1(\varsigma),\varsigma\big),\varsigma\big)\oplus\mathcal{F}\big(h\big(m_1(\varsigma),\varsigma\big),\varsigma\big)\big)\\&\quad\wedge\mathcal{B}\big(m_1(\varsigma),\varsigma\big)+I\big(m_1(\varsigma),\varsigma\big)\big]\\&\quad\oplus\big[\tau\big(\mathcal{M}\big(m_2(\varsigma),\varsigma\big)+\mathcal{A}\big(g\big(m_2(\varsigma),\varsigma\big),\varsigma\big)\oplus\mathcal{F}\big(h\big(m_2(\varsigma),\varsigma\big),\varsigma\big)\big)\\&\quad\wedge\mathcal{B}\big(m_2(\varsigma),\varsigma\big)+I\big(m_2(\varsigma),\varsigma\big)\big]\end{aligned}$$

$$
\begin{aligned}
&\le \alpha_1\tau\Big[\big(\big(\mathcal{M}\big(m_1(\varsigma),\varsigma\big)+\mathcal{A}\big(g\big(m_1(\varsigma),\varsigma\big),\varsigma\big)\oplus\mathcal{F}\big(h\big(m_1(\varsigma),\varsigma\big),\varsigma\big)\big)\\
&\quad\wedge\mathcal{B}\big(m_1(\varsigma),\varsigma\big)\big)\\
&\quad\oplus\big(\big(\mathcal{M}\big(m_2(\varsigma),\varsigma\big)+\mathcal{A}\big(g\big(m_2(\varsigma),\varsigma\big),\varsigma\big)\oplus\mathcal{F}\big(h\big(m_2(\varsigma),\varsigma\big),\varsigma\big)\big)\\
&\quad\wedge\mathcal{B}\big(m_2(\varsigma),\varsigma\big)\big)\Big]+\alpha_2\big(m_1(\varsigma)\oplus m_2(\varsigma)\big)\\
&\le \tau\alpha_1\Big[\big(\mathcal{M}\big(m_1(\varsigma),\varsigma\big)+\mathcal{A}\big(g\big(m_1(\varsigma),\varsigma\big),\varsigma\big)\oplus\mathcal{F}\big(h\big(m_1(\varsigma),\varsigma\big),\varsigma\big)\big)\\
&\quad\oplus\big(\mathcal{M}\big(m_2(\varsigma),\varsigma\big)+\mathcal{A}\big(g\big(m_2(\varsigma),\varsigma\big),\varsigma\big)\oplus\mathcal{F}\big(h\big(m_2(\varsigma),\varsigma\big),\varsigma\big)\big)\\
&\quad\vee\big(\mathcal{B}\big(m_1(\varsigma),\varsigma\big)\oplus\mathcal{B}\big(m_2(\varsigma),\varsigma\big)\big)\Big]+\alpha_2\big(m_1(\varsigma)\oplus m_2(\varsigma)\big)\\
&\le \tau\alpha_1\Big[\big(\big(\mathcal{M}\big(m_1(\varsigma),\varsigma\big)\oplus\mathcal{M}\big(m_2(\varsigma),\varsigma\big)\big)\\
&\quad+\big(\mathcal{A}\big(g\big(m_1(\varsigma),\varsigma\big),\varsigma\big)\oplus\mathcal{F}\big(h\big(m_1(\varsigma),\varsigma\big),\varsigma\big)\big)\\
&\quad\oplus\big(\mathcal{A}\big(g\big(m_2(\varsigma),\varsigma\big),\varsigma\big)\oplus\mathcal{F}\big(h\big(m_2(\varsigma),\varsigma\big),\varsigma\big)\big)\big)\\
&\quad\vee\lambda_B\big(m_1(\varsigma)\oplus m_2(\varsigma)\big)\Big]+\alpha_2\big(m_1(\varsigma)\oplus m_2(\varsigma)\big)\\
&\le \tau\alpha_1\Big[\big(\lambda_M\big(m_1(\varsigma)\oplus m_2(\varsigma)\big)+(\lambda_A\lambda_g\oplus\lambda_F\lambda_h)\big(m_1(\varsigma)\oplus m_2(\varsigma)\big)\big)\\
&\quad\vee\lambda_B\big(m_1(\varsigma)\oplus m_2(\varsigma)\big)\Big]+\alpha_2\big(m_1(\varsigma)\oplus m_2(\varsigma)\big)\\
&\le \tau\alpha_1\Big[\big(\lambda_M+(\lambda_A\lambda_g\oplus\lambda_F\lambda_h)\big)\vee\lambda_B\Big]\big(m_1(\varsigma)\oplus m_2(\varsigma)\big)\\
&\quad+\alpha_2\big(m_1(\varsigma)\oplus m_2(\varsigma)\big)\\
&=\Big[\tau\alpha_1\big(\big(\lambda_M+(\lambda_A\lambda_g\oplus\lambda_F\lambda_h)\big)\vee\lambda_B\big)+\alpha_2\Big]\big(m_1(\varsigma)\oplus m_2(\varsigma)\big).
\end{aligned}
$$

Using Definition 2.2 and Lemma 2.1, we obtain

$$
\big\|\mathcal{F}\big(m_1(\varsigma),\varsigma\big)-\mathcal{F}\big(m_2(\varsigma),\varsigma\big)\big\|\le N\Psi\big\|m_1(\varsigma)-m_2(\varsigma)\big\|, \tag{4}
$$

where $\Psi=[\tau\alpha_1((\lambda_M+(\lambda_A\lambda_g\oplus\lambda_F\lambda_h))\vee\lambda_B)+\alpha_2]$. It follows from condition (2) that $0<N\Psi<1$, we realize that $\mathcal{F}(m(\varsigma),\varsigma)$ is a contraction mapping, then $\exists m^*\in\mathcal{E}$, which is invariant under $\mathcal{F}(m(\varsigma),\varsigma)$, and m^* is the answer of problem (1), i.e., m^* is the solution of the GPNOVI with $\oplus$ operator

$$
\big[\mathcal{M}(m,\varsigma)+\mathcal{A}\big(g(m,\varsigma),\varsigma\big)\oplus\mathcal{F}\big(h(m,\varsigma),\varsigma\big)\big]\wedge\mathcal{B}(m,\varsigma)=\theta
$$

for any parametric $\varsigma\in\Sigma$. By Lemma 3.1, then the generalized parametric nonlinear ordered variational inequality (1), there exists a solution m^*. □

5 Sensitivity analysis for GPNOVI with ⊕ operator

In this part of the article, we will prove sensitivity analysis for generalized parametric nonlinear ordered variational inequality problem (1).

Theorem 5.1 *Suppose $(\mathcal{E},\text{“}\le\text{”})$, $(\mathcal{C},\text{“}\le\text{”})$, N, and $\Sigma\subset\mathcal{E}$ have their predefined meanings. Let $\mathcal{M}(\cdot,\varsigma)$, $\mathcal{A}(\cdot,\varsigma)$, $\mathcal{F}(\cdot,\varsigma)$, $\mathcal{B}(\cdot,\varsigma)$, $g(\cdot,\varsigma)$, and $h(\cdot,\varsigma):\mathcal{E}\times\Sigma:\to\mathcal{E}$ be the parametric mapping continuous regarding the slot $\varsigma\in\Sigma$, $\mathcal{M}$, $\mathcal{A}$, $\mathcal{F}$, $\mathcal{B}$, $(\mathcal{M}+\mathcal{A}\oplus\mathcal{F})$, g, h, and $(\mathcal{M}+\mathcal{A}\oplus\mathcal{F})\wedge\mathcal{B}$ be comparison mappings to each other. Suppose $\mathcal{M}$ is λ_M-ordered compression, $\mathcal{A}$ is λ_A-ordered compression, $\mathcal{F}$ is λ_F-ordered compression, $\mathcal{B}$ is λ_B-ordered compression,*

g is λ_g-ordered compression, and h is λ_h-ordered compression mappings regarding second slot ς, respectively. Further, if $(\mathcal{M}+\mathcal{A}\oplus\mathcal{F}):\mathcal{E}\times\Sigma\to\mathcal{E}$ is a B-restricted accretive mapping regarding the second slot ς, with constants α_1 and α_2 and for any $\tau>0$,

$$\tau\big[\lambda_M+(\lambda_A\lambda_g\oplus\lambda_F\lambda_h)\big]\vee\lambda_B<\frac{1-\alpha_2}{\alpha_1} \tag{5}$$

holds, then the answer $m(\varsigma)$ of problem (1) is continuous in Σ.

Proof For given $\varsigma,\bar{\varsigma}\in\Sigma$, let $m(\varsigma)$ and $m(\bar{\varsigma})$ be two solutions of problem (1), then for any $\tau>0$, we have

$$\begin{aligned} m(\varsigma)&=\mathcal{F}\big(m(\varsigma),\varsigma\big)\\ &=\tau\big[\mathcal{M}\big(m(\varsigma),\varsigma\big)+\mathcal{A}\big(g\big(m(\varsigma),\varsigma\big),\varsigma\big)\oplus\mathcal{F}\big(h\big(m(\varsigma),\varsigma\big),\varsigma\big)\big]\\ &\quad\wedge\mathcal{B}\big(m(\varsigma),\varsigma\big)+I\big(m(\varsigma),\varsigma\big),\\ m(\bar{\varsigma})&=\mathcal{F}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\\ &=\tau\big[\mathcal{M}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)+\mathcal{A}\big(g\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\bar{\varsigma}\big)\oplus\mathcal{F}\big(h\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\bar{\varsigma}\big)\big]\\ &\quad\wedge\mathcal{B}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)+I\big(m(\bar{\varsigma}),\bar{\varsigma}\big). \end{aligned} \tag{6}$$

As per conditions that $\mathcal{M}$, $\mathcal{A}$, $\mathcal{B}$, $(\mathcal{M}+\mathcal{A}\oplus\mathcal{F})$, g, h, and $(\mathcal{M}+\mathcal{A}\oplus\mathcal{F})\wedge\mathcal{B}$ are comparison mappings regarding the second slot ς with each other, respectively, and Lemma 2.2, we obtain

$$\begin{aligned} \theta&=m(\varsigma)\oplus m(\bar{\varsigma})\\ &=\mathcal{F}\big(m(\varsigma),\varsigma\big)\oplus\mathcal{F}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\\ &\le\mathcal{F}\big(m(\varsigma),\varsigma\big)\oplus\theta\oplus\mathcal{F}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\\ &=\big[\mathcal{F}\big(m(\varsigma),\varsigma\big)\oplus\mathcal{F}\big(m(\bar{\varsigma}),\varsigma\big)\big]\oplus\big[\mathcal{F}\big(m(\bar{\varsigma}),\varsigma\big)\oplus\mathcal{F}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\big]. \end{aligned} \tag{7}$$

Further, since $(\mathcal{M}+\mathcal{A}\oplus\mathcal{F})$ is a B-restricted-accretive mapping with constants α_1, α_2, $\mathcal{M}$ is λ_M-ordered compression, $\mathcal{A}$ is λ_A-ordered compression, $\mathcal{F}$ is λ_F-ordered compression, $\mathcal{B}$ is λ_B-ordered compression, g is λ_g-ordered compression, h is λ_h-ordered compression, regarding the slot ς, respectively, so by Theorem 4.1, we obtain

$$\begin{aligned} &\mathcal{F}\big(m(\varsigma),\varsigma\big)\oplus\mathcal{F}\big(m(\bar{\varsigma}),\varsigma\big)\\ &\quad\le\big[\tau\big(\mathcal{M}\big(m(\varsigma),\varsigma\big)+\mathcal{A}\big(g\big(m(\varsigma),\varsigma\big),\varsigma\big)\oplus\mathcal{F}\big(h\big(m(\varsigma),\varsigma\big),\varsigma\big)\big)\\ &\qquad\wedge\mathcal{B}\big(m(\varsigma),\varsigma\big)+I\big(m(\varsigma),\varsigma\big)\big]\oplus\big[\tau\big(\mathcal{M}\big(m(\bar{\varsigma}),\varsigma\big)+\mathcal{A}\big(g\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big)\\ &\qquad\oplus\mathcal{F}\big(h\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big)\big)\wedge\mathcal{B}\big(m(\bar{\varsigma}),\varsigma\big)+I\big(m(\bar{\varsigma}),\varsigma\big)\big]\\ &\quad\le\alpha_1\big[\big(\tau\big[\mathcal{M}\big(m(\varsigma),\varsigma\big)+\mathcal{A}\big(g\big(m(\varsigma),\varsigma\big),\varsigma\big)\oplus\mathcal{F}\big(h\big(m(\varsigma),\varsigma\big),\varsigma\big)\big]\wedge\mathcal{B}\big(m(\varsigma),\varsigma\big)\big)\\ &\qquad\times\big\{\tau\big[\mathcal{M}\big(m(\bar{\varsigma}),\varsigma\big)+\mathcal{A}\big(g\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big)\oplus\mathcal{F}\big(h\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big)\big]\wedge\mathcal{B}\big(m(\bar{\varsigma}),\varsigma\big)\big\}\big]\\ &\qquad+\alpha_2\big(m(\varsigma)\oplus m(\bar{\varsigma})\big)\\ &\quad\le\Psi\big(m(\varsigma)\oplus m(\bar{\varsigma})\big), \end{aligned} \tag{8}$$

where $\Psi = \alpha_1[\tau(\lambda_M + (\lambda_A\lambda_g \oplus \lambda_F\lambda_h)) \vee \lambda_B] + \alpha_2 < 1$ for condition (5), and

$$\begin{aligned}
&\mathcal{F}\big(m(\bar{\varsigma}),\varsigma\big) \oplus \mathcal{F}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\\
&\quad\le \big[\tau\big(\mathcal{M}\big(m(\bar{\varsigma}),\varsigma\big) + \mathcal{A}\big(g\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big) \oplus \mathcal{F}\big(h\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big)\big)\\
&\qquad \wedge \mathcal{B}\big(m(\bar{\varsigma}),\varsigma\big) + I\big(m(\bar{\varsigma}),\varsigma\big)\big]\\
&\qquad \oplus \big[\tau\big(\mathcal{M}\big(m(\bar{\varsigma}),\bar{\varsigma}\big) + \mathcal{A}\big(g\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\bar{\varsigma}\big) \oplus \mathcal{F}\big(h\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\bar{\varsigma}\big)\big)\\
&\qquad \wedge \mathcal{B}\big(m(\bar{\varsigma}),\bar{\varsigma}\big) + I\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\big]\\
&\quad\le \alpha_1\big[\big(\tau\big[\mathcal{M}\big(m(\bar{\varsigma}),\varsigma\big) + \mathcal{A}\big(g\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big) \oplus \mathcal{F}\big(h\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big)\big]\\
&\qquad \wedge \mathcal{B}\big(m(\bar{\varsigma}),\varsigma\big)\big)\\
&\qquad \oplus \big(\tau\big[\mathcal{M}\big(m(\bar{\varsigma}),\bar{\varsigma}\big) + \mathcal{A}\big(g\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\bar{\varsigma}\big) \oplus \mathcal{F}\big(h\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\bar{\varsigma}\big)\big]\\
&\qquad \wedge \mathcal{B}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\big)\big] + \alpha_2\big(m(\bar{\varsigma}) \oplus m(\bar{\varsigma})\big)\\
&\quad\le \alpha_1\big[\tau\big(\big[\mathcal{M}\big(m(\bar{\varsigma}),\varsigma\big) + \mathcal{A}\big(g\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big) \oplus \mathcal{F}\big(h\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big)\big]\\
&\qquad \oplus \big[\mathcal{M}\big(m(\bar{\varsigma}),\bar{\varsigma}\big) + \mathcal{A}\big(g\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\bar{\varsigma}\big) \oplus \mathcal{F}\big(h\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\bar{\varsigma}\big)\big]\big)\big]\\
&\qquad \vee \big[\mathcal{B}\big(m(\bar{\varsigma}),\varsigma\big) \oplus \mathcal{B}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\big]\\
&\quad\le \alpha_1\big[\tau\big(\big[\mathcal{M}\big(m(\bar{\varsigma}),\varsigma\big) \oplus \mathcal{M}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\big] + \big[\mathcal{A}\big(g\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big)\\
&\qquad \oplus \mathcal{A}\big(g\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\bar{\varsigma}\big)\big] \oplus \big[\mathcal{F}\big(h\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big) \oplus \mathcal{F}\big(h\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\bar{\varsigma}\big)\big]\big)\big]\\
&\qquad \vee \big[\mathcal{B}\big(m(\bar{\varsigma}),\varsigma\big) \oplus \mathcal{B}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\big].
\end{aligned}\tag{9}$$

Combining equations (7), (8), and (9), and by making use of Lemma 2.2, we obtain

$$\begin{aligned}
m(\varsigma) \oplus m(\bar{\varsigma}) &\le \big[\Psi\big(m(\varsigma) \oplus m(\bar{\varsigma})\big)\big] \oplus \alpha_1\big[\tau\big(\big(\mathcal{M}\big(m(\bar{\varsigma}),\varsigma\big) \oplus \mathcal{M}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\big)\\
&\quad + \big(\mathcal{A}\big(g\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big) \oplus \mathcal{A}\big(g\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\bar{\varsigma}\big)\big)\\
&\quad \oplus \big(\mathcal{F}\big(h\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big) \oplus \mathcal{F}\big(h\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\bar{\varsigma}\big)\big)\big)\\
&\quad \vee \big(\mathcal{B}\big(m(\bar{\varsigma}),\varsigma\big) \oplus \mathcal{B}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\big)\big],\\
(1 \oplus \Psi)\big(m(\varsigma) \oplus m(\bar{\varsigma})\big) &\le \alpha_1\big[\tau\big(\big(\mathcal{M}\big(m(\bar{\varsigma}),\varsigma\big) \oplus \mathcal{M}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\big)\\
&\quad + \big(\mathcal{A}\big(g\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big) \oplus \mathcal{A}\big(g\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\bar{\varsigma}\big)\big)\\
&\quad \oplus \big(\mathcal{F}\big(h\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big) \oplus \mathcal{F}\big(h\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\bar{\varsigma}\big)\big)\big)\\
&\quad \vee \big(\mathcal{B}\big(m(\bar{\varsigma}),\varsigma\big) \oplus \mathcal{B}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\big)\big],\\
\big(m(\varsigma) \oplus m(\bar{\varsigma})\big) &\le \left(\frac{\alpha_1}{1 \oplus \Psi}\right)\big[\tau\big(\big(\mathcal{M}\big(m(\bar{\varsigma}),\varsigma\big) \oplus \mathcal{M}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\big)\\
&\quad + \big(\mathcal{A}\big(g\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big) \oplus \mathcal{A}\big(g\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\bar{\varsigma}\big)\big)\\
&\quad \oplus \big(\mathcal{F}\big(h\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big) \oplus \mathcal{F}\big(h\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\bar{\varsigma}\big)\big)\big)\\
&\quad \vee \big(\mathcal{B}\big(m(\bar{\varsigma}),\varsigma\big) \oplus \mathcal{B}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\big)\big]\\
&\le \left(\frac{\alpha_1}{1 \oplus \Psi}\right)\big[\tau\big(\big(\mathcal{M}\big(m(\bar{\varsigma}),\varsigma\big) \oplus \mathcal{M}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\big)
\end{aligned}\tag{10}$$

$$
\begin{aligned}
&+ \big(\mathcal{A}\big(g\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big) \oplus \mathcal{A}\big(g\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\varsigma\big) \oplus \mathcal{A}\big(g\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\varsigma\big) \\
&\oplus \mathcal{A}\big(g\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\bar{\varsigma}\big)\big) \oplus \big(\mathcal{F}\big(h\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big) \oplus \mathcal{F}\big(h\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\varsigma\big) \\
&\oplus \mathcal{F}\big(h\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\varsigma\big) \oplus \mathcal{F}\big(h\big(m(\bar{\varsigma}),\bar{\varsigma}\big),\bar{\varsigma}\big)\big)\big) \\
&\vee \big(\mathcal{B}\big(m(\bar{\varsigma}),\varsigma\big) \oplus \mathcal{B}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\big)\big].
\end{aligned}
$$

Using the continuity of the parametric mappings regarding the second slot $\varsigma \in \Sigma$, we have

$$
\begin{aligned}
&\lim_{\varsigma\to\bar{\varsigma}} \big\|g\big(m(\bar{\varsigma}),\varsigma\big) - g\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\big\| = 0, \\
&\lim_{\varsigma\to\bar{\varsigma}} \big\|h\big(m(\bar{\varsigma}),\varsigma\big) - h\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\big\| = 0, \\
&\lim_{\varsigma\to\bar{\varsigma}} \big\|\mathcal{B}\big(m(\bar{\varsigma}),\varsigma\big) - \mathcal{B}\big(m(\bar{\varsigma}),\bar{\varsigma}\big)\big\| = 0, \\
&\lim_{\varsigma\to\bar{\varsigma}} \big\|\mathcal{A}\big(g\big(m(\bar{\varsigma}),\varsigma\big),\varsigma\big) - \mathcal{A}\big(g\big(m(\bar{\varsigma}),\varsigma\big),\bar{\varsigma}\big)\big\| = 0, \\
&\lim_{\varsigma\to\bar{\varsigma}} \big\|\mathcal{F}(\cdot,\varsigma) - \mathcal{F}(\cdot,\bar{\varsigma})\big\| = 0, \\
&\lim_{\varsigma\to\bar{\varsigma}} \big\|\mathcal{M}(\cdot,\varsigma) - \mathcal{M}(\cdot,\bar{\varsigma})\big\| = 0.
\end{aligned}
$$

From Proposition 3, we have

$$\lim_{\varsigma\to\bar{\varsigma}} \big(m(\varsigma) \oplus m(\bar{\varsigma})\big) = \theta$$

and

$$\lim_{\varsigma\to\bar{\varsigma}} \big\|m(\varsigma) - m(\bar{\varsigma})\big\| = 0. \tag{11}$$

It ensures that the answer $m(\varsigma)$ of problem (1) is continuous at $\varsigma = \bar{\varsigma}$. □

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