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Intuitionistic fuzzy I-convergent Fibonacci difference sequence spaces

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Abstract

Fibonacci difference matrix was defined by Kara in his paper (Kara in J. Inequal. Appl. 2013:38 2013). Recently, Khan et al. (Adv. Differ. Equ. 2018:199, 2018) using the Fibonacci difference matrix $\hat{F}$ and ideal convergence defined the notion of $c_0^I(\hat{F})$, $c^I(\hat{F})$ and $l_\infty^I(\hat{F})$. In this paper, we give the ideal convergence of Fibonacci difference sequence space in intuitionistic fuzzy normed space with respect to fuzzy norm (μ, ν). Moreover, we investigate some basic properties of the said spaces such as linearity, hausdorffness.

Keywords: Difference sequence space; Fibonacci numbers; Fibonacci difference matrix; Intuitionistic fuzzy normed space; *I*-convergence

1 Introduction and preliminaries

Let ω, c, c_0, l_∞ denote sequence space, convergent, null and bounded sequences respectively, with norm $\|x\|_\infty = \sup_{k\in\mathbb{N}} |x_k|$. The idea of difference sequence spaces was defined by Kizmaz as follows:

$$\lambda(\Delta) = \{x = (x_n) \in \omega : (x_n - x_{n+1}) \in \lambda\}, \quad \text{for } \lambda \in \{l_\infty, c, c_0\}.$$

Recently, many authors have made a new approach to construct sequence spaces using matrix domain [2, 3, 8, 10]. Lately, Kara [9] has investigated difference sequence space.

$$l_\infty(\hat{F}) = \left\{x = (x_n) \in \omega : \sup_{n\in\mathbb{N}} \left|\frac{f_n}{f_{n+1}} x_n - \frac{f_{n+1}}{f_n} x_{n-1}\right| < \infty\right\},$$

which is derived from the Fibonacci difference matrix $\hat{F} = (\hat{f_{nk}})$ as follows:

$$\hat{f_{nk}} = \begin{cases} -\frac{f_{n+1}}{f_n} & k = n-1, \\ \frac{f_n}{f_{n+1}} & k = n, \\ 0 & 0 \le k < n-1 \text{ or } k > n, \end{cases}$$

where $(f_n), n \in \mathbb{N}$ is the sequence of Fibonacci numbers given by the linear recurrence relation as $f_0 = 1 = f_1$ and $f_{n-1} + f_{n-2} = f_n$ for $n \ge 2$. Quite recently, Khan et al. [13] defined the notion of I-convergent Fibonacci difference sequence spaces as $c_0^I(\hat{F})$, $c^I(\hat{F})$ and $l_\infty^I(\hat{F})$.

Fibonacci numbers have various applications in the fields of arts, science and architecture. For further details, refer to [4, 11].

Fuzzy logic was first introduced by Zadeh in 1965 [24] and it found its applications in various fields like control theory, artificial intelligence, robotics. Later on many authors [7, 16] investigated fuzzy topology to define fuzzy metric space. As a generalisation of fuzzy sets, Atanassov [1] defined the view of intuitionistic fuzzy sets. Intuitionistic fuzzy normed space [18] and 2-normed space [17] are the recent studies in fuzzy theory.

Kostyrko et al. [15] in 1999 generalised the idea of statistical convergence [6, 21] to ideal convergence. Further this idea was investigated by Salat, Tripathy and Ziman [19, 20], Tripathy and Hazarika [22, 23] and many others.

We recall certain definitions which will be useful in this paper.

Definition 1.1 ([21]) A sequence $x = (x_n) \in \omega$ is statistically convergent to $\xi \in \mathbb{R}$ if for every $\epsilon > 0$ the set $\{n \in \mathbb{N} : |x_n - \xi| \geq \epsilon\}$ has asymptotic density zero. We write $st\text{-}\lim x = \xi$. If $\xi = 0$, then $x = (x_n)$ is called st–null.

Definition 1.2 A sequence $x = (x_n) \in \omega$ is called statistically Cauchy sequence if, for every $\epsilon > 0$, $\exists$ a number $N = N(\epsilon)$ such that

$$\lim_n \frac{1}{n}\left|\left\{j \leq n : |x_j - x_N| \geq \epsilon\right\}\right| = 0.$$

Definition 1.3 ([14]) An ideal means a family of sets $I \subset P(X)$ satisfying the following conditions:

(i) $\phi \in I$,

(ii) $C \cup D \in I$ for all $C, D \in I$,

(iii) for each $C \in I$ and $D \subset C$, we have $D \in I$.

An ideal is said to be non-trivial if $I \neq 2^X$ and admissible if $\{\{x\} : x \in X\} \subset I$.

Definition 1.4 ([13]) A family of sets $\mathcal{F} \subset P(X)$ is called filter if and only if it satisfies the following conditions:

(i) $\phi \notin \mathcal{F}$,

(ii) $C, D \in \mathcal{F} \Rightarrow C \cap D \in \mathcal{F}$,

(iii) for each $C \in \mathcal{F}$ with $C \subset D$, we have $D \in \mathcal{F}$.

Definition 1.5 ([15]) A sequence $x = (x_n)$ is called I-convergent to $\xi \in \mathbb{R}$ if, for every $\epsilon > 0$, the set $\{n \in \mathbb{N} : |x_n - \xi| \geq \epsilon\} \in I$. We write $I\text{-}\lim x = \xi$. If $\xi = 0$, then $x = (x_n)$ is said to be I-null.

Definition 1.6 ([13]) A sequence $x = (x_n)$ is said to be I-Cauchy if for every $\epsilon > 0$ $\exists$ a number $N = N(\epsilon)$ such that the set $\{n \in \mathbb{N} : |x_n - x_N| \geq \epsilon\} \in \mathcal{I}$.

Definition 1.7 ([14]) A sequence $x = (x_k)$ is convergent to ξ with respect to the intuitionistic fuzzy norm (μ, ν) if for every $\epsilon, t > 0$ $\exists N \in \mathbb{N}$ with $\mu(x_k - \xi, t) > 1 - \epsilon$ and $\nu(x_k - \xi, t) < \epsilon$ for all $k \geq N$. We write $(\mu, \nu) - \lim x = \xi$.

Definition 1.8 ([5]) Consider intuitionistic fuzzy normed space (IFNS) $(X, \mu, \nu, *, \diamond)$. A sequence $x = (x_k)$ is said to be Cauchy sequence with respect to norm (μ, ν) if, for each $\epsilon > 0$, $t > 0$, there exists $N \in \mathbb{N}$ such that $\mu(x_k - x_l, t) > 1 - \epsilon$ and $\nu(x_k - x_l) < \epsilon$ for all $k, l \geq N$.

Definition 1.9 ([12]) Let $(X,\mu,\nu,*,\diamond)$ be IFNS. A sequence $x=(x_k)\in\omega$ is called I-convergent to ξ with respect to the intuitionistic norm (μ,ν) if for every ϵ, $t>0$ if the set $\{k\in\mathbb{N}:\mu(x_k-\xi,t)\le 1-\epsilon$ or $\nu(x_k-\xi,t)\ge\epsilon\}\in I$. We write $I^{(\mu,\nu)}$- $\lim x=\xi$.

Definition 1.10 ([13]) A sequence $x=(x_k)\in\omega$ is said to be Fibonacci I-convergent to $\xi\in\mathbb{R}$ if, for every $\epsilon>0$, the set $\{k\in\mathbb{N}:|\hat{F}_k(x)-\xi|\ge\epsilon\}\in I$, where I is an admissible ideal.

Definition 1.11 ([13]) Consider an admissible ideal I. Sequence $x=(x_k)\in\omega$ is Fibonacci I-Cauchy if, for every $\epsilon>0$, $\exists N=N(\epsilon)$ such that $\{k\in\mathbb{N}:|\hat{F}_k(x)-\hat{F}_N(x)|\ge\epsilon\}\in I$.

2 Intuitionistic fuzzy *I*-convergent Fibonacci difference sequence spaces

In the following section, we introduce a new type of sequence spaces whose $\hat{F}$ transform is I-convergent with respect to the intuitionistic norm (μ,ν). Further we prove certain properties of these spaces such as hausdorfness, first countability. Throughout this paper, I is an admissible ideal. We define

$$S^I_{0(\mu,\nu)}(\hat{F})=\bigl\{x=(x_k)\in l_\infty:\bigl\{k\in\mathbb{N}:\mu\bigl(\hat{F}_k(x),t\bigr)\le 1-\epsilon \text{ or } \nu\bigl(\hat{F}_k(x),t\bigr)\ge\epsilon\bigr\}\in I\bigr\},$$
$$S^I_{(\mu,\nu)}(\hat{F})=\bigl\{x=(x_k)\in l_\infty:\bigl\{k\in\mathbb{N}:\mu\bigl(\hat{F}_k(x)-l,t\bigr)\le 1-\epsilon \text{ or } \nu\bigl(\hat{F}_k(x)-l,t\bigr)\ge\epsilon\bigr\}\in I\bigr\}.$$

We introduce an open ball with centre x and radius r with respect to t as follows:

$$\begin{aligned}B_x(r,t)(\hat{F})=\{&y=(y_k)\in l_\infty:\\ &\bigl\{k\in\mathbb{N}:\mu\bigl(\hat{F}_k(x)-\hat{F}_k(y),t\bigr)>1-r \text{ and } \nu\bigl(\hat{F}_k(x)-\hat{F}_k(y),t\bigr)<r\bigr\}.\end{aligned}$$

Remark 2.1

(i) For $p_1,p_2\in(0,1)$ such that $p_1>p_2$, there exist $p_3,p_4\in(0,1)$ with $p_1*p_3\ge p_2$ and $p_1\ge p_4\diamond p_2$.

(ii) For $p_5\in(0,1)$, there exist $p_6,p_7\in(0,1)$ such that $p_6*p_6\ge p_5$ and $p_7\diamond p_7\le p_5$.

Theorem 2.1 *The spaces $S^I_{0(\mu,\nu)}(\hat{F})$ and $S^I_{(\mu,\nu)}(\hat{F})$ are vector spaces over $\mathbb{R}$.*

Proof Let us show the result for $S^I_{(\mu,\nu)}(\hat{F})$ and the proof for another space will follow on the similar lines. Let $x=(x_k)$ and $y=(y_k)\in S^I_{(\mu,\nu)}(\hat{F})$. Then by definition there exist ξ_1 and ξ_2, and for every ϵ, $t>0$, we have

$$A=\left\{k\in\mathbb{N}:\mu\left(\hat{F}_k(x)-\xi_1,\frac{t}{2|\alpha|}\right)\le 1-\epsilon \text{ or } \nu\left(\hat{F}_k(x)-\xi_1,\frac{t}{2|\alpha|}\right)\ge\epsilon\right\}\in,$$
$$B=\left\{k\in\mathbb{N}:\mu\left(\hat{F}_k(y)-\xi_2,\frac{t}{2|\beta|}\right)\le 1-\epsilon \text{ or } \nu\left(\hat{F}_k(y)-\xi_2,\frac{t}{2|\beta|}\right)\ge\epsilon\right\}\in I,$$

where α and β are scalars.

$$A^c=\left\{k\in\mathbb{N}:\mu\left(\hat{F}_k(x)-\xi_1,\frac{t}{2|\alpha|}\right)>1-\epsilon \text{ or } \nu\left(\hat{F}_k(x)-\xi_1,\frac{t}{2|\alpha|}\right)<\epsilon\right\}\in\mathcal{F}(I),$$
$$B^c=\left\{k\in\mathbb{N}:\mu\left(\hat{F}_k(y)-\xi_2,\frac{t}{2|\beta|}\right)>1-\epsilon \text{ or } \nu\left(\hat{F}_k(y)-\xi_2,\frac{t}{2|\beta|}\right)<\epsilon\right\}\in\mathcal{F}(I).$$

Define $E = A \cup B$ so that $E \in I$. Thus $E^c \in \mathcal{F}(I)$ and therefore is non-empty. We will show

$$E^c \subset \big\{k \in \mathbb{N} : \mu\big(\alpha \hat{F}_k(x) + \beta \hat{F}_k(y) - (\alpha\xi_1 + \beta\xi_2), t\big) > 1 - \epsilon \text{ or}$$
$$\nu\big(\alpha \hat{F}_k(x) + \beta \hat{F}_k(y) - (\alpha\xi_1 + \beta\xi_2), t\big) < \epsilon\big\}.$$

Let $n \in E^c$. Then

$$\mu\left(\hat{F}_n(x) - \xi_1, \frac{t}{2|\alpha|}\right) > 1 - \epsilon \text{ or } \nu\left(\hat{F}_n(x) - \xi_1, \frac{t}{2|\alpha|}\right) < \epsilon,$$
$$\mu\left(\hat{F}_n(y) - \xi_2, \frac{t}{2|\beta|}\right) > 1 - \epsilon \text{ or } \nu\left(\hat{F}_n(y) - \xi_2, \frac{t}{2|\beta|}\right) < \epsilon.$$

Consider

$$\begin{aligned}
&\mu\big(\alpha \hat{F}_n(x) + \beta \hat{F}_n(x) - (\alpha\xi_1 + \beta\xi_2), t\big) \\
&\quad \geq \mu\left(\alpha \hat{F}_n(x) - \alpha\xi_1, \frac{t}{2}\right) * \mu\left(\beta \hat{F}_n(y) - \beta\xi_2, \frac{t}{2}\right) \\
&\quad = \mu\left(\hat{F}_n(x) - \xi_1, \frac{t}{2|\alpha|}\right) * \mu\left(\hat{F}_n(y) - \xi_2, \frac{t}{|\beta|}\right) \\
&\quad > (1-\epsilon) * (1-\epsilon) = 1 - \epsilon
\end{aligned}$$

and

$$\begin{aligned}
&\nu\big(\alpha \hat{F}_n(y) + \beta \hat{F}_n(y) - (\alpha\xi_1 + \beta\xi_2)\big) \\
&\quad \leq \nu\left(\alpha \hat{F}_n(x) - \alpha\xi_1, \frac{t}{2}\right) \diamond \nu\left(\beta \hat{F}_n(y) - \beta\xi_2, \frac{t}{2}\right) \\
&\quad = \nu\left(\hat{F}_n(x) - \xi_1, \frac{t}{2|\alpha|}\right) \diamond \nu\left(\hat{F}_n(y) - \xi_2, \frac{t}{2|\beta|}\right) \\
&\quad < \epsilon \diamond \epsilon = \epsilon.
\end{aligned}$$

Thus $E^c \subset \{k \in \mathbb{N} : \mu(\alpha \hat{F}_k(x) + \beta \hat{F}_k(y) - (\alpha\xi_1 + \beta\xi_2), t) > 1 - \epsilon$ or $\nu(\alpha \hat{F}_k(x) + \beta \hat{F}_k(y) - (\alpha\xi_1 + \beta\xi_2), t) < \epsilon\}$. $E^c \in \mathcal{F}(I)$, therefore by definition of filter, the set on the right-hand side of the above equation belongs to $\mathcal{F}(I)$ so that its complement belongs to I. This implies $(\alpha x + \beta y) \in S^I_{(\mu,\nu)}(\hat{F})$. Hence $S^I_{(\mu,\nu)}(\hat{F})$ is a vector space over $\mathbb{R}$. □

Theorem 2.2 *Every open ball $B_x(r,t)(\hat{F})$ is an open set in $S^I_{(\mu,\nu)}(\hat{F})$.*

Proof We have defined open ball as follows:

$$B_x(r,t)(\hat{F}) = \{y = (y_k) \in l_\infty :$$
$$\big\{k \in \mathbb{N} : \mu\big(\hat{F}_k(x) - \hat{F}_k(y), t\big) > 1 - r \text{ and } \nu\big(\hat{F}_k(x) - \hat{F}_k(y), t\big) < r\big\}.$$

Let $z = (z_k) \in B_x(r,t)(\hat{F})$ so that $\mu(\hat{F}_k(x) - \hat{F}_k(z), t) > 1 - r$ and $\nu(\hat{F}_k(x) - \hat{F}_k(z), t) < r$. Then there exists $t_0 \in (0,t)$ with $\mu(\hat{F}_k(x) - \hat{F}_k(z), t_0) > 1 - r$ and $\nu(\hat{F}_k(x) - \hat{F}_k(z), t_0) < r$. Put $p_0 =$

$\mu(\hat{F}_k(x)-\hat{F}_k(z),t_0)$, so we have $p_0 > 1-r$, there exists $s \in (0,1)$ such that $p_0 > 1-s > 1-r$. Using Remark 2.1(i), given $p_0 > 1-s$, we can find $p_1, p_2 \in (0,1)$ with $p_0 * p_1 > 1-s$ and $(1-p_0) \diamond (1-p_2) < s$. Put $p_3 = \max(p_1,p_2)$. We will prove $B_z(1-p_3, t-t_0)(\hat{F}) \subset B_x(r,t)(\hat{F})$. Let $w = (w_k) \in B_z(1-p_3, t-t_0)(\hat{F})$. Hence

$$\begin{aligned}\mu\big(\hat{F}_k(x)-\hat{F}_k(w),t\big) &\geq \mu\big(\hat{F}_k(x)-\hat{F}_k(z),t_0\big) * \mu\big(\hat{F}_k(z)-\hat{F}_k(w),t-t_0\big)\\ &> (p_0 * p_3) \geq (p_0 * p_1) > 1-s > 1-r,\end{aligned}$$

and

$$\begin{aligned}\nu\big(\hat{F}_k(x)-\hat{F}_k(w),t\big) &\leq \nu\big(\hat{F}_k(x)-\hat{F}_k(z),t_0\big) \diamond \nu\big(\hat{F}_k(z)-\hat{F}_k(w),t-t_0\big)\\ &< (1-p_0)) \diamond (1-p_3) \leq (1-p_0) \diamond (1-p_2) < r.\end{aligned}$$

Hence $w \in B_x(r,t)(\hat{F})$ and therefore $B_z(1-p_3, t-t_0)(\hat{F}) \subset B_x(r,t)(\hat{F})$. □

Remark 2.2 Let $S^I_{(\mu,\nu)}(\hat{F}$ be IFNS. Define $\tau^I_{(\mu,\nu)}(\hat{F}) = \{A \subset S^I_{(\mu,\nu)}(\hat{F})$: for given $x \in A$, we can find $t > 0$ and $0 < r < 1$ such that $B_x(r,t)(\hat{F}) \subset A\}$. Then $\tau^I_{(\mu,\nu)}(\hat{F})$ is a topology on $S^I_{(\mu,\nu)}(\hat{F})$.

Remark 2.3 Since $\{B_x(\frac{1}{n},\frac{1}{n})(\hat{F}) : n \in \mathbb{N}\}$ is a local base at x, the topology $\tau^I_{(\mu,\nu)}(\hat{F})$ is first countable.

Theorem 2.3 *The spaces $S^I_{(\mu,\nu)}(\hat{F})$ and $S^I_{0(\mu,\nu)}(\hat{F})$ are Hausdorff.*

Proof Let $x, y \in S^I_{(\mu,\nu)}(\hat{F})$ with x and y to be different. Then $0 < \mu(\hat{F}_k(x)-\hat{F}(y),t) < 1$ and $0 < \nu(\hat{F}(x)-\hat{F}_k(y),t) < 1$. Put $\mu(\hat{F}_k(x)-\hat{F}_k(y),t) = p_1$ and $\nu(\hat{F}_k(x)-\hat{F}k(y),t) = p_2$ and $r = \max(p_1, 1-p_2)$. Using Remark (**2.1(ii)**) for $p_0 \in (r,1)$, we can find $p_3, p_4 \in (0,1)$ such that $p_3 * p_3 \geq p_0$ and $(1-p_4) \diamond (1-p_4) \leq 1-p_0$. Put $p_5 = \max(p_3,p_4)$. Clearly $B_x(1-p_5,\frac{t}{2})(\hat{F}) \cap B_y(1-p_5,\frac{t}{2})(\hat{F}) = \phi$. Let on the contrary $z \in B_x(1-p_5,\frac{t}{2})(\hat{F}) \cap B_y(1-p_5,\frac{t}{2})(\hat{F})$. Then we have

$$\begin{aligned}p_1 &= \mu\big(\hat{F}_k(x)-\hat{F}_k(y),t\big) \geq \mu\left(\hat{F}_k(x)-\hat{F}_k(z),\frac{t}{2}\right) * \mu\left(\hat{F}_k(z)-\hat{F}_k(y),\frac{t}{2}\right)\\ &\geq p_5 * p_5 \geq p_3 * p_3 > p_0 > p_1\end{aligned}$$

and

$$\begin{aligned}p_2 &= \nu\big(\hat{F}_k(x)-\hat{F}_y,t\big) \leq \nu\left(\hat{F}_k(x)-\hat{F}_k(z),\frac{t}{2}\right) \diamond \nu\left(\hat{F}_k(z)-\hat{F}_k(y),\frac{t}{2}\right)\\ &\leq (1-p_5) \diamond (1-p_5) \leq (1-p_4) \diamond (1-p_4) \leq 1-p_0 < p_2,\end{aligned}$$

which is a contradiction. Therefore $S^I_{(\mu,\nu)}(\hat{F})$ is a Hausdorff space. The proof for $S^I_{0(\mu,\nu)}(\hat{F})$ follows similarly. □

Theorem 2.4 *Let $S^I_{(\mu,\nu)}(\hat{F})$ be IFNS and $\tau^I_{(\mu,\nu)}(\hat{F})$ be a topology on $S^I_{\mu,\nu)}(\hat{F})$. A sequence $(x_k) \in S^I_{(\mu,\nu)}(\hat{F})$ converges to ξ iff $\mu(\hat{F}_k(x)-\xi,t) \to 1$ and $\nu(\hat{F}_k(x)-\xi,t) \to 0$ as $k \to \infty$.*

Proof Suppose $x_k \to \xi$, then given $0 < r < 1$ there exists $k_0 \in \mathbb{N}$ such that $(x_k) \in B_x(r,t)(\hat{F})$ for all $k \geq k_0$ given $t > 0$. Hence, we have $1 - \mu(\hat{F}_k(x) - \xi, t) < r$ and $\nu(\hat{F}_k(x) - \xi, t) < r$. Therefore $\mu(\hat{F}_k(x) - \xi, t) \to 1$ and $\nu(\hat{F}_k(x) - \xi, t) \to 0$ as $k \to \infty$.

Conversely, if $\mu(\hat{F}_k(x) - \xi, t) \to 1$ and $\nu(\hat{F}_k(x) - \xi, t) \to 0$ as $k \to \infty$ holds for each $t > 0$. For $0 < r < 1$, there exists $k_0 \in \mathbb{N}$ such that $1 - \mu(\hat{F}_k(x) - \xi, t) < r$ and $\nu(\hat{F}_k(x) - \xi, t) < r$ for all $k \geq k_0$, which implies $\mu(\hat{F}_k(x) - \xi, t) > 1 - r$ and $\nu(\hat{F}_k(x) - \xi, t) < r$. Thus $x_k \in B_x(r,t)(\hat{F})$ for all $k \geq k_0$ and hence $x_k \to \xi$. □

3 Conclusion

In the present article, we have defined a new kind of sequence spaces $S^I_{0(\mu,\nu)}(\hat{F})$ and $S^I_{(\mu,\nu)}(\hat{F})$ using Fibonacci difference matrix $\hat{F}$. We studied certain elementary properties and topological properties like linearity, first countability, hausdorfness. These results will give new approach to deal with the problems in science and engineering. The present article is a useful tool to define ideal convergence of generalised Fibonacci difference sequence in intuitionistic fuzzy normed space given by

$$c_0\big(\hat{F}(r,s)\big) = \left\{x = (x_k) \in \omega : \lim_{n\to\infty}\left(r\frac{f_n}{f_{n+1}}x_n + s\frac{f_{n+1}}{f_n}x_{n-1}\right) = 0\right\},$$
$$c\big(\hat{F}(r,s)\big) = \left\{x = (x_k) \in \omega : \lim_{n\to\infty}\left(r\frac{f_n}{f_{n+1}}x_n + s\frac{f_{n+1}}{f_n}x_{n-1}\right) = l\right\},$$

where $\hat{F}(r,s) = \{f_{nk}(r,s)\}$ is a double generalised matrix defined as follows:

$$f_{nk}(r,s) = \begin{cases} s\frac{f_{n+1}}{f_n}, & k = n-1, \\ r\frac{f_n}{f_{n+1}}, & k = n, \\ 0, & 0 \leq k < n-1 \text{ or } k > n \end{cases}$$

$k, n \in \mathbb{N}$ and $r, s \in \mathbb{R} \setminus \{0\}$. We can study the topological properties of these spaces which will provide a better method to deal with vagueness and inexactness occurring in various fields of science, engineering and economics. Moreover, this theory can be helpful in dealing with problems in population dynamics, quantum particle physics particularly in connections with string and ϵ^∞ theory of El-Naschie.

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Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All the authors of this paper have read and agreed to its content and are responsible for all aspects of the accuracy and integrity of the manuscript.

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