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Sufficient conditions to solve two systems of integral equations via fixed point results

Tahair Rasham[1*], Abdullah Shoaib[2], Giuseppe Marino[3], Badriah A.S. Alamri[4] and Muhammad Arshad[1]

*Correspondence:
tahir_resham@yahoo.com
[1]Department of Mathematics and Statistics, International Islamic University, Islamabad, Pakistan
Full list of author information is available at the end of the article

Abstract

The purpose of this paper is to study the solution of two systems of nonlinear integral equations via fixed point results in a complete dislocated b-metric space. Also the notion of graphic contractions on a closed set for two families of graph dominated multivalued mappings is introduced. Our results generalize some previous results in the existing literature.

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1 Introduction and preliminaries

Fixed point theory plays a fundamental role in functional analysis. Nadler [18] started the investigation of fixed point results for the set-valued functions. Due to its significance, a large number of authors have proved many interesting multiplications of his result (see [1–4, 6, 8–29]).

Nazir et al. [19] showed common fixed point results for the family of generalized multivalued F-contraction mappings in ordered metric spaces. Recently Shoaib et al. [27] discussed some theorems for a family of set-valued functions. Rasham et al. [22] proved multivalued fixed point theorems for new F-contractive functions on dislocated metric spaces.

In this paper, we have obtained a common fixed point of two families of multivalued mappings satisfying generalized rational type $\alpha_* - \psi$-dominated contractive conditions on a closed set in a complete dislocated b-metric space. We have used a weaker class of strictly increasing mappings A rather than the class of mappings F used by Wardowski [29]. Examples have been given to demonstrate the variety of our results. Moreover, we investigate our results in a better framework of dislocated b-metric space. New results in ordered spaces, partial b-metric space, dislocated metric space, partial metric space, b-metric space, and metric space can be obtained as corollaries of our results. We give the following concepts which will be helpful to understand the paper.

Definition 1.1 ([14]) Let M be a nonempty set, and let $d_b : M \times M \to [0,\infty)$ be a function. If, for any $x,y,z \in M$, the following conditions hold:

(i) $d_b(x,y) \leq b[d_b(x,z) + d_b(z,y)]$, (where $b \geq 1$).

(ii) $d_b(x,y) = 0$ implies $x = y$;

(iii) $d_b(x,y) = d_b(y,x)$.

Then d_b is called a dislocated b-metric with coefficient b (or simply d_b-metric) and the pair (M,d_b) is called a dislocated b-metric space. It should be noted that every dislocated metric is a dislocated b-metric with $b = 1$.

Note that, if $x = y$, then $d_b(x,y)$ may not be 0. For $x \in M$ and $\varepsilon > 0$, $\overline{B(x,\varepsilon)} = \{y \in M : d_b(x,y) \leq \varepsilon\}$ is a closed ball in (M,d_b). We use a $D.B.M$ space instead of a dislocated b-metric space.

Definition 1.2 ([14]) Let (M,d_b) be a $D.B.M$ space.

(i) A sequence $\{x_n\}$ in (M,d_b) is called Cauchy sequence if, given $\varepsilon > 0$, there corresponds $n_0 \in N$ such that for all $n,m \geq n_0$ we have $d_b(x_m,x_n) < \varepsilon$ or $\lim_{n,m\to\infty} d_b(x_n,x_m) = 0$.

(ii) A sequence $\{x_n\}$ dislocated b-converges (for short d_b-converges) to x if $\lim_{n\to\infty} d_b(x_n,x) = 0$. In this case x is called a d_b-limit of $\{x_n\}$.

(iii) (M,d_b) is called complete if every Cauchy sequence in M converges to a point $x \in M$ such that $d_b(x,x) = 0$.

Definition 1.3 Let K be a nonempty subset of the $D.B.M$ space M, and let $x \in M$. An element $y_0 \in K$ is called a best approximation in K if

$$d_b(x,K) = d_b(x,y_0), \quad \text{where } d_b(x,K) = \inf_{y\in K} d_b(x,y).$$

If each $x \in M$ has at least one best approximation in K, then K is called a proximinal set. Let Ψ_b, where b is the coefficient of the $D.B.M$ space M. Denote the family of all nondecreasing functions $\psi_b : [0,+\infty) \to [0,+\infty)$ such that $\sum_{k=1}^{+\infty} b^k \psi_b^k(t) < +\infty$ and $b\psi_b(t) < t$ for all $t > 0$, where ψ_b^k is the kth iterate of ψ_b. Also $b^{n+1}\psi_b^{n+1}(t) = b^n b\psi_b(\psi_b^n(t)) < b^n\psi_b^n(t)$. We denote $P(M)$ to be the set of all closed proximinal subsets of M.

Definition 1.4 ([28]) The function $H_{d_b} : P(M) \times P(M) \to R^+$, defined by

$$H_{d_b}(N,R) = \max\Big\{\sup_{n\in N} d_b(n,R), \sup_{r\in R} d_b(N,r)\Big\},$$

is called dislocated Hausdorff b-metric on $P(M)$.

Definition 1.5 Let (M,d_b) be a $D.B.M$ space. Let $S : M \to P(M)$ be a multivalued mapping, $\alpha : M \times M \to [0,+\infty)$ and $\alpha_*(i,Si) = \inf\{\alpha(i,l) : l \in Si\}$. Let $H \subseteq M$, then S is said to be α_*-dominated on H, whenever $\alpha_*(i,Si) \geq 1$ for all $i \in H$. If $H = M$, then we say that the S is α_*-dominated. If $S : M \to M$ is a self mapping, then S is α-dominated on H, whenever $\alpha(i,Si) \geq 1$ for all $i \in H$.

Lemma 1.6 ([17]) *Let (Z,d_b) be a D.B.M space. Let $(P(Z),H_{d_b})$ be a dislocated Hausdorff b-metric space on $P(Z)$. For all G,H in $P(Z)$ and for any $g \in G$, let $h_g \in H$ such that $d_b(g,H) = d_b(g,h_g)$. Then $H_{d_b}(G,H) \geq d_b(g,h_g)$ holds.*

2 Main result

Let (M, d_b) be a *D.B.M* space, $c_0 \in M$, let $\{S_\sigma : \sigma \in \Omega\}$ and $\{T_\beta : \beta \in \Phi\}$ be two families of multifunctions from M to $P(M)$. Let $a \in \Omega$ and $c_1 \in S_a c_0$ be an element such that $d_b(c_0, S_a c_0) = d_b(c_0, c_1)$. Let $c_2 \in T_z c_1$ be such that $d_b(c_1, T_z c_1) = d_b(c_1, c_2)$ where $z \in \Phi$. Let $y \in \Omega$ and $c_3 \in S_y c_2$ be such that $d_b(c_2, S_y c_2) = d_b(c_2, c_3)$. In this way, we get a sequence $\{T_\beta S_\sigma(c_n)\}$ in M, where $c_{2n+1} \in S_i c_{2n}$, $c_{2n+2} \in T_j c_{2n+1}$, $n \in \mathbb{N}$, $i \in \Omega$, and $j \in \Phi$. Also $d_b(c_{2n}, S_i c_{2n}) = d_b(c_{2n}, c_{2n+1})$, $d_b(c_{2n+1}, T_j c_{2n+1}) = d_b(c_{2n+1}, c_{2n+2})$. $\{T_\beta S_\sigma(c_n)\}$ is said to be a sequence in M generated by c_0. If $\{S_\sigma : \sigma \in \Omega\} = \{T_\beta : \beta \in \Phi\}$, then we say $\{MS_\sigma(c_n)\}$ instead of $\{T_\beta S_\sigma(c_n)\}$. For $u, v \in M$, $a > 0$, we define $D_{(\sigma,\beta)}(u, v)$ as

$$D_{(\sigma,\beta)}(u,v) = \max\left\{d_b(u,v), \frac{d_b(u,S_\sigma u).d_b(v,T_\beta v)}{a + d_b(u,v)}, d_b(u,S_\sigma u), d_b(v,T_\beta v)\right\}.$$

Theorem 2.1 *Let (M, d_b) be a complete D.B.M space. Suppose that there exists a function $\alpha : M \times M \to [0,\infty)$. Let $r > 0$, $c_0 \in \overline{B_{d_b}(c_0, r)}$, $A : \mathbb{R}_+ \to \mathbb{R}$ be a strictly increasing function and $\{S_\sigma : \sigma \in \Omega\}$, $\{T_\beta : \beta \in \Phi\}$ be two families of α_*-dominated multivalued mappings from M to $P(M)$ on $\overline{B_{d_b}(c_0, r)}$. Suppose that, for some $\psi_b \in \Psi_b$, there exists $\tau > 0$ such that the following holds*:

$$\tau + A\big(H_{d_b}(S_\sigma e, T_\beta y)\big) \le A\big(\psi_b\big(D_{(\sigma,\beta)}(e,y)\big)\big) \tag{2.1}$$

for all $e, y \in \overline{B_{d_b}(c_0, r)} \cap \{T_\beta S_\sigma(c_n)\}$, $\alpha(e, y) \ge 1$, $\sigma \in \Omega$, $\beta \in \Phi$, and $H_{d_b}(S_\sigma e, T_\beta y) > 0$. Also

$$\sum_{i=0}^{n} b^{i+1}\big\{\psi_b^i\big(d_b(c_0, S_a c_0)\big)\big\} \le r \quad \text{for all } n \in \mathbb{N} \cup \{0\}. \tag{2.2}$$

Then $\{T_\beta S_\sigma(c_n)\}$ is a sequence in $\overline{B_{d_b}(c_0, r)}$, $\alpha(c_n, c_{n+1}) \ge 1$ for all $n \in \mathbb{N} \cup \{0\}$ and $\{T_\beta S_\sigma(c_n)\} \to u \in \overline{B_{d_b}(c_0, r)}$. Also, if inequality (2.1) holds, if $\overline{B_{d_b}(c_0, r)}$ is a closed set for $e, y \in \{u\}$ and either $\alpha(c_n, u) \ge 1$ or $\alpha(u, c_n) \ge 1$ for all $n \in \mathbb{N} \cup \{0\}$, then S_σ and T_β have a common fixed point u in $\overline{B_{d_b}(c_0, r)}$ for all $\sigma \in \Omega$ and $\beta \in \Phi$.

Proof Consider a sequence $\{T_\beta S_\sigma(c_n)\}$. From (2.2), we get

$$d_b(c_0, c_1) \le b d_b(c_0, S_a c_0) < \sum_{i=0}^{n} b^{i+1}\big\{\psi_b^i\big(d_b(c_0, S_a c_0)\big)\big\} \le r.$$

It follows that

$$c_1 \in \overline{B_{d_b}(c_0, r)}.$$

Let $c_2, \dots, c_j \in \overline{B_{d_b}(c_0, r)}$ for some $j \in \mathbb{N}$. If j is odd, then $j = 2\grave{\imath} + 1$ for some $\grave{\imath} \in \mathbb{N}$. Since $\{S_\sigma : \sigma \in \Omega\}$ and $\{T_\beta : \beta \in \Phi\}$ are two families of α_*-dominated multivalued mappings on $\overline{B_{d_b}(c_0, r)}$, so $\alpha_*(c_{2\grave{\imath}}, S_\sigma c_{2\grave{\imath}}) \ge 1$ and $\alpha_*(c_{2\grave{\imath}+1}, T_\beta c_{2\grave{\imath}+1}) \ge 1$ for all $\sigma \in \Omega$ and $\beta \in \Phi$. As $\alpha_*(c_{2\grave{\imath}}, S_\sigma c_{2\grave{\imath}}) \ge 1$, this implies $\inf\{\alpha(c_{2\grave{\imath}}, b) : b \in S_\sigma c_{2\grave{\imath}}\} \ge 1$. Also $c_{2\grave{\imath}+1} \in S_f c_{2\grave{\imath}}$ for some $f \in \Omega$, so $\alpha(c_{2\grave{\imath}}, c_{2\grave{\imath}+1}) \ge 1$. Also $c_{2\grave{\imath}+1} \in T_g c_{2\grave{\imath}+1}$ for some $g \in \Phi$. Now, by using Lemma 1.6, we have

$$\tau + A\big(d_b(c_{2\grave{\imath}+1}, c_{2\grave{\imath}+2})\big) \le \tau + A\big(H_{d_b}(S_f c_{2\grave{\imath}}, T_g c_{2\grave{\imath}+1})\big) \le A\big(\psi_b\big(D_{(f,g)}(c_{2\grave{\imath}}, c_{2\grave{\imath}+1})\big)\big)$$

$$\leq A\left(\psi_b\left(\max\left\{d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1}), \frac{d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1}).d_b(c_{2\grave{\imath}+1}, c_{2\grave{\imath}+2})}{a + d_b(c_{2i}, c_{2i+1})},\right.\right.\right.$$
$$\left.\left.\left. d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1}), d_b(c_{2\grave{\imath}+1}, c_{2\grave{\imath}+2})\right\}\right)\right)$$
$$\leq A\left(\psi_b\left(\max\left\{d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1}), d_b(c_{2\grave{\imath}+1}, c_{2\grave{\imath}+2})\right\}\right)\right).$$

If $\max\{d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1}), d_b(c_{2\grave{\imath}+1}, c_{2\grave{\imath}+2})\} = d_b(c_{2\grave{\imath}+1}, c_{2\grave{\imath}+2})$, then

$$\tau + A\left(d_b(c_{2\grave{\imath}+1}, c_{2\grave{\imath}+2})\right) \leq A\left(\psi_b\left(d_b(c_{2\grave{\imath}+1}, c_{2\grave{\imath}+2})\right)\right).$$

As $\tau > 0$ and $A : \mathbb{R}_+ \to \mathbb{R}$ is a strictly increasing function, then

$$d_b(c_{2\grave{\imath}+1}, c_{2\grave{\imath}+2}) < \psi_b\left(d_b(c_{2\grave{\imath}+1}, c_{2\grave{\imath}+2})\right) < b\psi_b\left(d_b(c_{2\grave{\imath}+1}, c_{2\grave{\imath}+2})\right).$$

This is a contradiction to the fact that $b\psi_b(t) < t$ for all $t > 0$. So

$$\max\left\{d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1}), d_b(c_{2\grave{\imath}+1}, c_{2\grave{\imath}+2})\right\} = d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1}).$$

Hence, we obtain

$$d_b(c_{2\grave{\imath}+1}, c_{2\grave{\imath}+2}) < \psi_b\left(d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1})\right). \tag{2.3}$$

As $\alpha_*(c_{2\grave{\imath}-1}, T_h c_{2\grave{\imath}-1}) \geq 1$ and $c_{2\grave{\imath}} \in T_h c_{2\grave{\imath}-1}$, so $\alpha(c_{2\grave{\imath}-1}, c_{2\grave{\imath}}) \geq 1$ where $h \in \Phi$ and $p \in \Omega$. Now, by using Lemma 1.6, we have

$$\tau + A\left(d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1})\right) \leq \tau + A\left(H_{d_b}(T_h c_{2\grave{\imath}-1}, S_p c_{2\grave{\imath}})\right) \leq A\left(\psi_b\left(D_{(h,p)}(c_{2\grave{\imath}}, c_{2\grave{\imath}-1})\right)\right)$$
$$\leq A\left(\psi_b\left(\max\left\{d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}-1}), \frac{d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1}).d_b(c_{2\grave{\imath}-1}, c_{2\grave{\imath}})}{a + d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}-1})},\right.\right.\right.$$
$$\left.\left.\left. d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1}), d_b(c_{2\grave{\imath}-1}, c_{2\grave{\imath}})\right\}\right)\right)$$
$$\leq A\left(\psi_b\left(\max\left\{d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}-1}), d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1})\right\}\right)\right).$$

Since $A : \mathbb{R}_+ \to \mathbb{R}$ is a strictly increasing function, then we have

$$d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1}) < \psi_b\left(\max\left\{d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}-1}), d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1})\right\}\right).$$

If $\max\{d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}-1}), d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1})\} = d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1})$, then

$$d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1}) < \psi_b\left(d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1})\right) < b\psi_b\left(d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1})\right).$$

This is a contradiction to the fact that $b\psi_b(t) < t$ for all $t > 0$. Hence, we obtain

$$d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1}) < \psi_b\left(d_b(c_{2\grave{\imath}-1}, c_{2\grave{\imath}})\right). \tag{2.4}$$

As ψ_b is nondecreasing, it follows

$$\psi_b\left(d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1})\right) < \psi_b\left(\psi_b\left(d_b(c_{2\grave{\imath}-1}, c_{2\grave{\imath}})\right)\right).$$

By using the above inequality in (2.3), we have

$$d_b(c_{2\grave{\imath}+1}, c_{2\grave{\imath}+2}) < \psi_b^2\big(d_b(c_{2\grave{\imath}-1}, c_{2\grave{\imath}})\big)).$$

Continuing in this way, we obtain

$$d_b(c_{2\grave{\imath}+1}, c_{2\grave{\imath}+2}) < \psi_b^{2\grave{\imath}+1}\big(d_b(c_0, c_1)\big). \tag{2.5}$$

Now, if $j = 2\grave{\imath}$, where $\grave{\imath} = 1, 2, \dots, \frac{j}{2}$. By using (2.4) and a similar procedure as above, we have

$$d_b(c_{2\grave{\imath}}, c_{2\grave{\imath}+1}) < \psi_b^{2\grave{\imath}}\big(d_b(c_0, c_1)\big). \tag{2.6}$$

Now, by combining (2.5) and (2.6), we get

$$d_b(c_j, c_{j+1}) < \psi_b^j\big(d_b(c_0, c_1)\big) \quad \text{for all } j \in \mathbb{N}. \tag{2.7}$$

Now, by using the triangle inequality and by (2.7), we have

$$\begin{aligned} d_b(c_0, c_{j+1}) &\leq bd_b(c_0, c_1) + b^2 d_b(c_1, c_2) + \cdots + b^{j+1} d_b(c_j, c_{j+1}) \\ &< bd_b(c_0, c_1) + b^2\psi_b\big(d_b(c_0, c_1)\big) + \cdots + b^{j+1}\psi_b^j\big(d_b(c_0, c_1)\big) \\ &< \sum_{\grave{\imath}=0}^{j} b^{\grave{\imath}+1}\big\{\psi_b^{\grave{\imath}}\big(d_b(c_0, c_1)\big)\big\} < r. \end{aligned}$$

Thus $c_{j+1} \in \overline{B_{d_b}(c_0, r)}$. Hence $c_n \in \overline{B_{d_b}(c_0, r)}$ for all $n \in \mathbb{N}$, therefore $\{T_\beta S_\sigma(c_n)\}$ is a sequence in $\overline{B_{d_b}(c_0, r)}$. As S_σ, T_β are α_*-dominated on $\overline{B_{d_b}(c_0, r)}$, so $\alpha_*(c_{2n}, S_\sigma c_{2n}) \geq 1$ and $\alpha_*(c_{2n+1}, T_\beta c_{2n+1}) \geq 1$. This implies $\alpha(c_n, c_{n+1}) \geq 1$. Also inequality (2.7) can be written as

$$d_b(c_n, c_{n+1}) < \psi_b^n\big(d_b(c_0, c_1)\big) \quad \text{for all } n \in \mathbb{N}. \tag{2.8}$$

As $\sum_{k=1}^{+\infty} b^k\psi_b^k(t) < +\infty$, then for some $p \in \mathbb{N}$ the series $\sum_{k=1}^{+\infty} b^k\psi_b^k(\psi_b^{p-1}(d_b(c_0, c_1)))$ converges. As $b\psi_b(t) < t$, so

$$b^{n+1}\psi_b^{n+1}\big(\psi_b^{p-1}\big(d_b(c_0, c_1)\big)\big) < b^n\psi_b^n\big(\psi_b^{p-1}\big(d_b(c_0, c_1)\big)\big) \quad \text{for all } n \in \mathbb{N}.$$

Fix $\varepsilon > 0$, then there exists $p(\varepsilon) \in \mathbb{N}$ such that

$$b\psi_b\big(\psi_b^{p(\varepsilon)-1}\big(d_b(c_0, c_1)\big)\big) + b^2\psi_b^2\big(\psi_b^{p(\varepsilon)-1}\big(d_b(c_0, c_1)\big)\big) + \cdots < \varepsilon.$$

Let $n, m \in \mathbb{N}$ with $m > n > p(\varepsilon)$, then we have

$$\begin{aligned} d_b(c_n, c_m) &\leq bd_b(c_n, c_{n+1}) + b^2 d_b(c_{n+1}, c_{n+2}) + \cdots + b^{m-n} d_b(c_{m-1}, c_m) \\ &< b\psi_b^n\big(d_b(c_0, c_1)\big) + b^2\psi_b^{n+1}\big(d_b(c_0, c_1)\big) + \cdots + b^{m-n}\psi_b^{m-1}\big(d_b(c_0, c_1)\big) \\ &= b\psi_b\big(\psi_b^{n-1}\big(d_b(c_0, c_1)\big)\big) + \cdots + b^{m-n}\psi_b^{m-n}\big(\psi_b^{n-1}\big(d_b(c_0, c_1)\big)\big) \\ &< b\psi_b\big(\psi_b^{p(\varepsilon)-1}\big(d_b(c_0, c_1)\big)\big) + b^2\psi_b^2\big(\psi_b^{p(\varepsilon)-1}\big(d_b(c_0, c_1)\big)\big) + \cdots < \varepsilon. \end{aligned}$$

Thus we proved that $\{T_\beta S_\sigma(c_n)\}$ is a Cauchy sequence in $\overline{B_{d_b}(c_0,r)}$. As (X,d) is complete and $\overline{B_{d_b}(c_0,r)}$ is closed, so $(\overline{B_{d_b}(c_0,r)},d_b)$ is complete. This implies that there exist $u\in\overline{B_{d_b}(c_0,r)}$ such that $\{T_\beta S_\sigma(c_n)\}\to u$ as $n\to\infty$, then

$$\lim_{n\to\infty} d_b(c_n,u)=0. \tag{2.9}$$

By assumption, $\alpha(c_n,u)\geq 1$. Suppose that $d_b(u,T_\beta u)>0$, then there exists a positive integer k such that $d_b(c_n,T_\beta u)>0$ for all $n\geq k$. For $n\geq k$, we have

$$d_b(u,T_\beta u)\leq bd_b(u,c_{2n+1})+bd_b(c_{2n+1},T_\beta u).$$

Now, there exists some $e\in\Omega$ such that $c_{2n+1}\in S_e c_{2n}$ and $d_b(c_{2n},S_e c_{2n})=d_b(c_{2n},c_{2n+1})$. By using Lemma 1.6 and inequality (2.1), we have

$$\begin{aligned} d_b(u,T_\beta u) &\leq bd_b(u,c_{2n+1})+bH_{d_b}(S_e c_{2n},T_\beta u), \quad \text{for some } \beta\in\Phi \\ &< bd_b(u,c_{2n+1})+b\psi_b\Bigg(\max\Bigg\{d_b(c_{2n},u),d_b(c_{2n},S_e c_{2n}), \\ &\qquad \frac{d_b(c_{2n},S_e c_{2n}).d_b(u,T_\beta u)}{a+d_b(c_{2n},u)},d_b(u,T_\beta u)\Bigg\}\Bigg). \end{aligned}$$

Letting $n\to\infty$, and by using (2.9), we get

$$d_b(u,T_\beta u)<b\psi_b\big(d_b(u,T_\beta u)\big)<d_b(u,T_\beta u),$$

which is a contradiction. So our supposition is wrong. Hence $d_b(u,T_\beta u)=0$ or $u\in T_\beta u$ for all $\beta\in\Phi$. Similarly, by using Lemma 1.6 and inequality (2.1), we can show that $d_b(u,S_\sigma u)=0$ or $u\in S_\sigma u$ for all $\sigma\in\Omega$. Hence S_σ and T_β have a common fixed point u in $\overline{B_{d_b}(c_0,r)}$ for all $\sigma\in\Omega$ and $\beta\in\Phi$. Now,

$$d_b(u,u)\leq bd_b(u,T_\beta u)+bd_b(T_\beta u,u)\leq 0.$$

This implies that $d_b(u,u)=0$. □

Example 2.2 Let $M=\mathbb{Q}^+\cup\{0\}$ and let $d_b:M\times M\to M$ be the complete *D.B.M* space defined by

$$d_b(\imath,\jmath)=(\imath+\jmath)^2 \quad \text{for all } \imath,\jmath\in M$$

with coefficient $b=2$. Define $S_\sigma,T_\beta:M\times M\to P(M)$ to be two families of multivalued mappings by

$$S_m x=\begin{cases}[\frac{x}{3m},\frac{2}{3m}x] & \text{if } x\in[0,14]\cap M,\\ [xm,2mx] & \text{if } x\in(14,\infty)\cap M\end{cases} \quad \text{where } m=1,2,3,\ldots$$

and

$$T_n x=\begin{cases}[\frac{x}{4n},\frac{3}{4n}x] & \text{if } x\in[0,14]\cap M,\\ [2nx,3nx] & \text{if } x\in(14,\infty)\cap M.\end{cases} \quad \text{where } n=1,2,3,\ldots.$$

Suppose that, $x_0 = 1$, $r = 225$, $a = 1$, then $\overline{B_{d_b}(x_0, r)} = [0, 14] \cap M$. Now, $d_b(x_0, S_1 x_0) = d_b(1, S_1 1) = d_b(1, \frac{1}{3})$. So $x_1 = \frac{1}{3}$. Now, $d_b(x_1, T_1 x_1) = d_b(\frac{1}{3}, T_1 \frac{1}{3}) = d_b(\frac{1}{3}, \frac{1}{12})$. So $x_2 = \frac{1}{12}$. Now, $d_b(x_2, S_2 x_2) = d_b(\frac{1}{12}, S_2 \frac{1}{12}) = d_b(\frac{1}{12}, \frac{1}{72})$. So $x_3 = \frac{1}{72}$. Continuing in this way, we have $\{T_n S_m(x_n)\} = \{1, \frac{1}{3}, \frac{1}{12}, \frac{1}{72}, \ldots\}$. Let $\psi_b(t) = \frac{4t}{10}$, then $b\psi_b(t) < t$. Consider the mapping $\alpha : M \times M \to [0, \infty)$ by

$$\alpha(j,k) = \begin{Bmatrix} 1 & \text{if } j > k \\ \frac{1}{2} & \text{otherwise} \end{Bmatrix}.$$

Now, if $x, y \in \overline{B_{d_b}(x_0, r)} \cap \{T_\beta S_\sigma(x_n)\}$ with $\alpha(x, y) \geq 1$, we have

$$\begin{aligned} H_{d_b}(S_m x, T_n y) &= \max\Big\{\sup_{a \in S_m x} d_b(a, T_n y), \sup_{b \in T_n y} d_b(S_m x, b)\Big\} \\ &= \max\left\{\sup_{a \in S_m x} d_b\left(a, \left[\frac{y}{4n}, \frac{3y}{4n}\right]\right), \sup_{b \in T_n y} d_b\left(\left[\frac{x}{3m}, \frac{2x}{3m}\right], b\right)\right\} \\ &= \max\left\{d_b\left(\frac{2x}{3m}, \left[\frac{y}{4n}, \frac{3y}{4n}\right]\right), d_b\left(\left[\frac{x}{3m}, \frac{2x}{3m}\right], \frac{3y}{4n}\right)\right\} \\ &= \max\left\{d_b\left(\frac{2x}{3m}, \frac{y}{4n}\right), d_b\left(\frac{x}{3m}, \frac{3y}{4n}\right)\right\} \\ &= \max\left\{\left(\frac{2x}{3m} + \frac{y}{4n}\right)^2, \left(\frac{x}{3m} + \frac{3y}{4n}\right)^2\right\} \\ &< \psi_b\left\{\max\left((x+y)^2, \frac{(x + \frac{x}{3m})^4.(y + \frac{y}{4n})^2}{\{1 + (x+y)^4\}},\right.\right. \\ &\quad \left.\left.\left(x + \frac{x}{3m}\right)^2, \left(x + \frac{y}{4n}\right)^2\right)\right\} \\ &< \psi_b\left\{\max\left(d_b(x,y), \frac{d_b(x, [\frac{x}{3m}, \frac{2}{3m}x]).d_b(y, [\frac{y}{4n}, \frac{3}{4n}y])}{1 + d_b(x,y)},\right.\right. \\ &\quad \left.\left. d_b\left(x, \left[\frac{x}{3m}, \frac{2}{3m}x\right]\right), d_b\left(x, \left[\frac{y}{4n}, \frac{3}{4n}y\right]\right)\right)\right\}. \end{aligned}$$

Thus,

$$H_{d_b}(Sx, Ty)) < \psi_b\big(D_{(\sigma,\beta)}(x,y)\big),$$

which implies that, for any $\tau \in (0, \frac{12}{95}]$ and for a strictly increasing mapping $A(s) = \ln s$, we have

$$\tau + A\big(H_{d_l}(S_m x, T_n y)\big) \leq A\big(\psi_b\big(D_{(\sigma,\beta)}(x,y)\big)\big).$$

Note that, for $15, 16 \in M$, then $\alpha(16, 15) \geq 1$. However, we have

$$\tau + A\big(H_{d_l}(S_2 16, T_1 15)\big) > A\big(\psi_b\big(D_{(\sigma,\beta)}(16, 15)\big)\big).$$

So condition (2.1) does not hold on M. Also, for all $n \in \mathbb{N} \cup \{0\}$, we have

$$\sum_{i=0}^{n} b^{i+1}\left\{\psi_b^i\left(d_b(x_0, S_1 x_0)\right)\right\} = \frac{16}{9} \times 2 \sum_{i=0}^{n} \left(\frac{4}{5}\right)^i < 225 = r.$$

Thus all the conditions of Theorem 2.1 are satisfied. Hence S_σ and T_β have a common fixed point for all $\sigma \in \Omega$ and $\beta \in \Phi$, that is, 0.

Corollary 2.3 *Let (M, d_b) be a complete D.B.M space. Suppose that there exists a function $\alpha : M \times M \to [0, \infty)$. Let $r > 0$, $c_0 \in \overline{B_{d_b}(c_0, r)}$, A be a strictly increasing function, and $\{S_\sigma : \sigma \in \Omega\}$, $\{T_\beta : \beta \in \Phi\}$ be two families of α_*-dominated self mappings from M to M on $\overline{B_{d_b}(c_0, r)}$. Suppose that, for some $\psi_b \in \Psi_b$, there exists $\tau > 0$ such that the following holds:*

$$\tau + A\left(H_{d_b}(S_\sigma e, T_\beta y)\right) \le A\left(\psi_b\left(D_{(\sigma,\beta)}(e, y)\right)\right) \tag{2.10}$$

for all $e, y \in \overline{B_{d_b}(c_0, r)} \cap \{T_\beta S_\sigma(c_n)\}$, $\alpha(e, y) \ge 1$, $\sigma \in \Omega$, $\beta \in \Phi$, and $H_{d_b}(S_\sigma e, T_\beta y) > 0$. Also

$$\sum_{i=0}^{n} b^{i+1}\left\{\psi_b^i\left(d_b(c_0, S_a c_0)\right)\right\} \le r \quad \text{for all } n \in \mathbb{N} \cup \{0\}.$$

Then $\{T_\beta S_\sigma(c_n)\}$ is a sequence in $\overline{B_{d_b}(c_0, r)}$, $\alpha(c_n, c_{n+1}) \ge 1$ for all $n \in \mathbb{N} \cup \{0\}$ and $\{T_\beta S_\sigma(c_n)\} \to u \in \overline{B_{d_b}(c_0, r)}$. Also, u satisfies (2.10), if $\overline{B_{d_b}(c_0, r)}$ is a closed set and either $\alpha(c_n, u) \ge 1$ or $\alpha(u, c_n) \ge 1$ for all $n \in \mathbb{N} \cup \{0\}$, then S_σ and T_β have a common fixed point u in $\overline{B_{d_b}(c_0, r)}$ for all $\sigma \in \Omega$ and $\beta \in \Phi$.

Corollary 2.4 *Let (M, d_b) be a complete D.B.M space. Suppose that there exists a function $\alpha : M \times M \to [0, \infty)$. Let $r > 0$, $c_0 \in \overline{B_{d_b}(c_0, r)}$, A be a strictly increasing function, and $\{S_\sigma : \sigma \in \Omega\}$ be the family of α_*-dominated multivalued mappings from M to $P(M)$ on $\overline{B_{d_b}(c_0, r)}$. Suppose that, for some $\psi_b \in \Psi_b$, there exists $\tau > 0$ such that the following holds:*

$$\tau + A\left(H_{d_b}(S_\sigma e, S_\beta y)\right) \le A\left(\psi_b\left(D_{(\sigma,\beta)}(e, y)\right)\right) \tag{2.11}$$

for all $e, y \in \overline{B_{d_b}(c_0, r)} \cap \{MS_\sigma(c_n)\}$, $\alpha(e, y) \ge 1$, $\sigma, \beta \in \Omega$, and $H_{d_b}(S_\sigma e, S_\beta y) > 0$. Also

$$\sum_{i=0}^{n} b^{i+1}\left\{\psi_b^i\left(d_b(c_0, S_a c_0)\right)\right\} \le r \quad \text{for all } n \in \mathbb{N} \cup \{0\}.$$

Then $\{MS_\sigma(c_n)\}$ is a sequence in $\overline{B_{d_b}(c_0, r)}$, $\alpha(c_n, c_{n+1}) \ge 1$ for all $n \in \mathbb{N} \cup \{0\}$ and $\{MS_\sigma(c_n)\} \to u \in \overline{B_{d_b}(c_0, r)}$. Also, u satisfies (2.11), if $\overline{B_{d_b}(c_0, r)}$ is a closed set and either $\alpha(c_n, u) \ge 1$ or $\alpha(u, c_n) \ge 1$ for all $n \in \mathbb{N} \cup \{0\}$, then S_σ has a common fixed point u in $\overline{B_{d_b}(c_0, r)}$ for all $\sigma \in \Omega$.

3 Results for families of multigraph dominated mappings

In this section we present an application of Theorem 2.1 in graph theory. Jachymski [16] proved the result concerning for contraction mappings on metric space with a graph. Hussain et al. [13], introduced the fixed points theorem for graphic contraction and gave an application.

Definition 3.1 Let X be a nonempty set and $G = (V(G), E(G))$ be a graph such that $V(G) = X$, $A \subseteq X$. A mapping $F : X \to P(X)$ is said to be multigraph dominated on A if $(x, y) \in E(G)$ for all $y \in Fx$ and $x \in A$.

Theorem 3.2 *Let (M, d_b) be a complete D.B.M space endowed with a graph G with constant $b \geq 1$. Let $r > 0$, $c_0 \in \overline{B_{d_b}(c_0, r)}$, and $\{S_\sigma : \sigma \in \Omega\}$, $\{T_\beta : \beta \in \Phi\}$ be two families of multivalued mappings from M to $P(M)$. Suppose that*

(i) *$\{S_\sigma : \sigma \in \Omega\}$, $\{T_\beta : \beta \in \Phi\}$ are two families of multigraph dominated on $\overline{B_{d_b}(c_0, r)} \cap \{T_\beta S_\sigma(c_n)\}$.*

(ii) *There exist $\tau > 0$ and a strictly increasing mapping A satisfying*

$$\tau + A\bigl(H_{d_b}(S_\sigma e, T_\beta y)\bigr) \leq A\bigl(\psi_b\bigl(D_{(\sigma,\beta)}(e, y)\bigr)\bigr), \tag{3.1}$$

whenever $e, y \in \overline{B_{d_b}(c_0, r)} \cap \{T_\beta S_\sigma(c_n)\}$, $(e,y) \in E(G)$, $\sigma \in \Omega$, $\beta \in \Phi$, and $H_{d_b}(S_\sigma e, T_\beta y) > 0$.

(iii) *$\sum_{i=0}^{n} b^{i+1}\{\psi_b^i(d_b(x_0, S_\sigma c_0))\} \leq r$ for all $n \in N$.*

Then $\{T_\beta S_\sigma(c_n)\}$ is a sequence in $\overline{B_{d_b}(c_0, r)}$ if $\overline{B_{d_b}(c_0, r)}$ is a closed set $(c_n, c_{n+1}) \in E(G)$ and $\{T_\beta S_\sigma(c_n)\} \to m^*$. Also, if m^* satisfies (3.1) and $(c_n, m^*) \in E(G)$ or $(m^*, c_n) \in E(G)$ for all $n \in N \cup \{0\}$, then S_σ and T_β have a common fixed point m^* in $\overline{B_{d_b}(c_0, r)}$ for all $\sigma \in \Omega$ and $\beta \in \Phi$.

Proof Define $\alpha : M \times M \to [0, \infty)$ by

$$\alpha(e, y) = \begin{cases} 1 & \text{if } e \in \overline{B_{d_b}(c_0, r)}, (e, y) \in E(G), \\ 0 & \text{otherwise.} \end{cases}$$

As S_σ and T_β are two families of graphs dominated on $\overline{B_{d_b}(c_0, r)}$, then for $e \in \overline{B_{d_b}(c_0, r)}$, $(e, y) \in E(G)$ for all $y \in S_\sigma e$ and $(e, y) \in E(G)$ for all $y \in T_\beta e$. So, $\alpha(e, y) = 1$ for all $y \in S_\sigma e$ and $\alpha(e, y) = 1$ for all $y \in T_\beta e$. This implies that $\inf\{\alpha(e, y) : y \in S_\sigma e\} = 1$ and $\inf\{\alpha(e, y) : y \in T_\beta e\} = 1$. Hence $\alpha_*(e, S_\sigma e) = 1$, $\alpha_*(e, T_\beta e) = 1$ for all $e \in \overline{B_{d_b}(c_0, r)}$. So, $S_\sigma, T_\beta : M \to P(M)$ are two families of α_*-dominated mappings on $\overline{B_{d_b}(c_0, r)}$. Moreover, inequality (3.1) can be written as

$$\tau + A\bigl(H_{d_b}(S_\sigma e, T_\beta y)\bigr) \leq A\bigl(\psi_b\bigl(D_{(\sigma,\beta)}(e, y)\bigr)\bigr)$$

whenever $e, y \in \overline{B_{d_b}(c_0, r)} \cap \{T_\beta S_\sigma(c_n)\}$, $\alpha(e, y) \geq 1$, and $H_{d_b}(S_\sigma e, T_\beta y) > 0$. Also, (iii) holds. Then, by Theorem 2.1, we have $\{T_\beta S_\sigma(c_n)\}$ is a sequence in $\overline{B_{d_b}(c_0, r)}$ and $\{T_\beta S_\sigma(c_n)\} \to m^* \in \overline{B_{d_b}(c_0, r)}$. Now, $c_n, m^* \in \overline{B_{d_b}(c_0, r)}$ and either $(c_n, m^*) \in E(G)$ or $(m^*, c_n) \in E(G)$ implies that either $\alpha(c_n, m^*) \geq 1$ or $\alpha(m^*, c_n) \geq 1$. So, all the conditions of Theorem 2.1 are satisfied. Hence, by Theorem 2.1, S_σ and T_β have a common fixed point m^* in $\overline{B_{d_b}(c_0, r)}$ and $d_b(m^*, m^*) = 0$. □

4 Application to the systems of integral equations

Theorem 4.1 *Let (M, d_b) be a complete D.B.M space with coefficient $b \geq 1$. Let $c_0 \in M$ and $\{S_\sigma : \sigma \in \Omega\}$, $\{T_\beta : \beta \in \Phi\}$ be two families of mappings from M to M. Assume that* *there exists* $\tau > 0$ *and* $A : \mathbb{R}_+ \to \mathbb{R}$ *is a strictly increasing mapping such that the following holds*:

$$\tau + A\big(d_b(S_\sigma e, T_\beta y)\big) \le A\big(\psi_b\big(D_{(\sigma,\beta)}(e,y)\big)\big), \tag{4.1}$$

whenever $e, y \in \{T_\beta S_\sigma(c_n)\}$, $\sigma \in \Omega$, $\beta \in \Phi$, *and* $d_b(S_\sigma e, T_\beta y) > 0$. *Then* $\{T_\beta S_\sigma(c_n)\} \to u \in M$. *Also, if inequality* (4.1) *holds for* $e, y \in \{u\}$, *then* S_σ *and* T_β *have a unique common fixed point* u *in* M *for all* $\sigma \in \Omega$ *and* $\beta \in \Phi$.

Proof The proof of this theorem is similar as that of Theorem 2.1. We have to prove the uniqueness only. Let v be another common fixed point of S_σ and T_β. Suppose $d_b(S_\sigma u, T_\beta v) > 0$. Then we have

$$\tau + A\big(d_b(S_\sigma u, T_\beta v)\big) \le A\big(\psi_b\big(D_{(\sigma,\beta)}(u,v)\big)\big).$$

This implies that

$$d_b(u,v) < \psi_b d_b(u,v) < b\psi_b d_b(u,v) < d_b(u,v),$$

which is a contradiction. So $d_b(S_\sigma u, T_\beta v) = 0$. Hence $u = v$.

In this section, we discuss the application of fixed point Theorem 4.1 in a form of unique solution of two families of Volterra type integral equations given below:

$$u(k) = \int_0^k H_\sigma(k,h,u)\,dh, \tag{4.2}$$

$$c(k) = \int_0^k G_\beta(k,h,c))\,dh \tag{4.3}$$

for all $k \in [0,1]$, $\sigma \in \Omega$, $\beta \in \Phi$, and H_σ, G_β be the mappings from $[0,1] \times [0,1] \times C([0,1],\mathbb{R}_+)$ to $\mathbb{R}$. We find the solution of (4.2) and (4.3). Let $M = C([0,1],\mathbb{R}_+)$ be the set of all continuous functions on $[0,1]$ with nonnegative values endowed with the complete dislocated b-metric. For $u \in C([0,1],\mathbb{R}_+)$, define supremum norm as $\|u\|_\tau = \sup_{k\in[0,1]}\{|u(k)|e^{-\tau k}\}$, where $\tau > 0$ is taken arbitrarily. Then define

$$d_\tau(u,c) = \Big[\sup_{k\in[0,1]}\big\{\big|u(k)+c(k)\big|e^{-\tau k}\big\}\Big]^2 = \|u+c\|_\tau^2$$

for all $u, c \in C([0,1],\mathbb{R}_+)$, with these settings, $(C([0,1],\mathbb{R}_+), d_\tau)$ becomes a complete $D.B.M$ space. □

Now we prove the following theorem to ensure the existence of solution of integral equations.

Theorem 4.2 *Assume that the following conditions are satisfied*:

(i) $\{H_\sigma, \sigma \in \Omega\}$, $\{G_\beta, \beta \in \Phi\}$ *are two families of mappings from* $[0,1] \times [0,1] \times C([0,1],\mathbb{R}_+)$ *to* $\mathbb{R}$;

(ii) *Define*

$$(S_\sigma u)(k) = \int_0^k H_\sigma(k,h,u)\,dh,$$
$$(T_\beta c)(k) = \int_0^k G_\beta(k,h,c)\,dh.$$

Suppose that there exists $\tau > 0$ *such that*

$$\bigl|H_\sigma(k,h,u) + G_\beta(k,h,c)\bigr| \le \frac{\tau D_{(\sigma,\beta)}(u,c)}{\tau D_{(\sigma,\beta)}(u,c)+1}$$

for all $k,h \in [0,1]$ *and* $u,c \in C([0,1],\mathbb{R})$, *where*

$$D_{(\sigma,\beta)}(u,c) = \max\left\{\psi_b\left(\|u+c\|_\tau^2, \frac{\|u+S_\sigma u\|_\tau^2.\|c+T_\beta c\|_\tau^2}{1+\|u(h)+c(h)\|_\tau^2},\right.\right.$$
$$\left.\left.\|u+S_\sigma u\|_\tau^2, \|c+T_\beta c\|_\tau^2\right)\right\}.$$

Then integral Eqs. (4.2) *and* (4.3) *have a unique solution in* $C([0,1],\mathbb{R}_+)$.

Proof By assumption (ii)

$$\begin{aligned}
|S_\sigma u + T_\beta c| &= \int_0^k \bigl|H_\sigma(k,h,u) + G_\beta(k,h,c)\bigr|\,dh \\
&\le \int_0^k \frac{\tau D_{(\sigma,\beta)}(u,c)}{\tau D_{(\sigma,\beta)}(u,c)+1} e^{\tau h}\,dh \\
&\le \frac{\tau D_{(\sigma,\beta)}(u,c)}{\tau D_{(\sigma,\beta)}(u,c)+1} \int_0^k e^{\tau h}\,dh \\
&\le \frac{D_{(\sigma,\beta)}(u,c)}{\tau D_{(\sigma,\beta)}(u,c)+1} e^{\tau k}.
\end{aligned}$$

This implies

$$\begin{aligned}
&|S_\sigma u + T_\beta c| e^{-\tau k} \le \frac{D_{(\sigma,\beta)}(u,c)}{\tau D_{(\sigma,\beta)}(u,c)+1}. \\
&\|S_\sigma u + T_\beta c\|_\tau \le \frac{D_{(\sigma,\beta)}(u,c)}{\tau D_{(\sigma,\beta)}(u,c)+1}. \\
&\frac{\tau D_{(\sigma,\beta)}(u,c)+1}{D_{(\sigma,\beta)}(u,c)} \le \frac{1}{\|S_\sigma u + T_\beta c\|_\tau}. \\
&\tau + \frac{1}{D_{(\sigma,\beta)}(u,c)} \le \frac{1}{\|S_\sigma u + T_\beta c\|_\tau},
\end{aligned}$$

which further implies

$$\tau - \frac{1}{\|S_\sigma u + T_\beta c\|_\tau} \le \frac{-1}{D_{(\sigma,\beta)}(u,c)}.$$

So all the conditions of Theorem 4.1 are satisfied for $A(c) = \frac{-1}{\sqrt{c}}$; $c > 0$ and $d_\tau(u,c) = \|u+c\|_\tau^2$. Hence two families of integral equations given in (4.2) and (4.3) have a unique common solution. □

Example 4.3 Consider the integral equations

$$g(k) = \frac{1}{5}\int_0^k g(h)\,dh, \qquad p(k) = \frac{1}{7}\int_0^k p(h))\,dh, \quad \text{where } k \in [0,1].$$

Define $\{H_\sigma, \sigma \in \Omega\}$, $\{H_\beta, \beta \in \Phi\}$ to be two families of mappings from $[0,1] \times [0,1] \times C([0,1],\mathbb{R}_+)$ to $\mathbb{R}$; by $H_\sigma = \frac{1}{5}g(h)$, $H_\beta = \frac{1}{7}p(h)$. Now,

$$(S_\sigma g)(k) = \frac{1}{5}\int_0^k g(h)\,dh, \qquad (T_\beta p)(k) = \frac{1}{7}\int_0^k p(h))\,dh.$$

Take $\tau = \frac{6}{97}$, $\|u\|_\tau = \sup_{k\in[0,1]}\{|u(k)|e^{-\tau k}\}$ and $\psi_b(t) = \frac{4t}{10}$. Then all the conditions of Theorem 4.2 are satisfied and $g(k) = p(k) = 0$ for all k is a unique common solution to the above equations.

5 Conclusion

In the present paper, we achieved fixed point results for a pair of families of multivalued generalized $\alpha_* - \psi$-dominated contractive mappings on an intersection of a closed ball and a sequence for a more general class of α_*-dominated mappings rather than α_*-admissible mappings and for a weaker class of strictly increasing mappings A rather than the class of mappings F used by Wardowski [29]. The notion of multigraph dominated mapping is introduced. Fixed point results with graphic contractions on a closed ball for such families of mappings are established. Examples are given to demonstrate the variety of our results. An application is given to approximate the unique common solution of two families of nonlinear integral equations. Moreover, we investigated our results in a better new framework. New results in ordered spaces, partial b-metric space, dislocated metric space, partial metric space, b-metric space, and metric space can be obtained as corollaries of our results.

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Authors' contributions

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Author details

[1]Department of Mathematics and Statistics, International Islamic University, Islamabad, Pakistan. [2]Department of Mathematics and Statistics, Riphah International University, Islamabad, Pakistan. [3]Dipartimento di Matematica e Informatica, Universita della Calabria, Arcavacata di Rende (CS), Italy. [4]Department of Mathematics, King Abdulaziz University, Jeddah, Saudi Arabia.

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