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Stability analysis for uncertain neutral-type stochastic nonlinear systems with mixed time-varying delays

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Abstract

This study investigates the robust stability problem in the presence of uncertain parameters for a class of stochastic neutral-type systems with mixed time-varying delays, where external disturbance and nonlinearity are considered together. The nonlinear function is assumed to satisfy the one-sided Lipschitz condition and the quadratic inner-boundedness condition. By constructing a modified Lyapunov–Krasovskii functional and using the free-weighting matrix technique, some new delay-dependent criteria for the stability of the problem are presented. In particular, the derivatives of the time-varying delays are no longer limited to being less than one. Finally, numerical examples are given to illustrate the effectiveness of the derived results.

Keywords: Uncertainty; Neutral-type stochastic systems; Mixed time-varying delays; Linear matrix inequality

1 Introduction

The stability analysis and stabilization of time-delay systems have been tackled over because time delays occur in many practical systems, such as those in the fields of aeronautics, chemistry, and mechanics [1]. There are many valuable results regarding the stability analysis and stabilization of time delay systems [2–4]. Generally speaking, delay-dependent stability conditions are less conservative than delay-independent ones, especially when the time delays are relatively small. Thus, the present study is focused on delay-dependent stability [5].

Systems in many branches of science and industry are often subject to various types of noise and uncertainty [6]. Stochastic systems governed by Itô stochastic differential equations have attracted considerable attention, this being where the noise is described by Brownian motion [7, 8]. A great number of results have been successfully extended to stochastic time delay systems (e.g. [9–12]). Zhou et al. investigated the problem of stability analysis of a class of delayed genetic regulatory networks with stochastic disturbances, where the delays are assumed to be time-varying and bounded. Based on Itô's differential formula and free-weighting matrix method [13], delay-range-dependent and rate-dependent (or independent) stability criteria are obtained [14]. By constructing a generalized free-weighting-matrix approach, Zhang et al. investigated the delay-dependent stability problem of continuous neural networks with a bounded time-varying delay. The developed approach can estimate the single integral term arising in the derivative of the Lyapunov–Krasovskii functional more accurately [15].

Neutral-type stochastic differential equations, depend on the delays of state and state derivative simultaneously, are often encountered in various fields, such as automatic control, aircraft stabilization, lossless transmission lines, and turbojet engines [16–23]. Cheng et al. investigated the problem of robust stability criteria delay-dependent for neutral systems with interval time-varying delays and nonlinear perturbation [24]. Basic on a piecewise delay method, the authors obtained some new sufficient conditions to guarantee the asymptotic stability for neutral time-delay systems. Kao et al. have probed the problem of H_∞ sliding mode control for nonlinear uncertain neutral stochastic systems with Markovian switching parameters. By utilizing a sliding mode control strategy, they got some criteria on asymptotic stability of the error system and sliding mode dynamics [23]. In [25], Yao et al. investigated the problem of robust adaptive sliding mode control for uncertain neutral Markovian jump systems with unknown nonlinearity and unmeasured states. To the authors' best knowledge, the problem of delay-dependent stability criteria for uncertain neutral-type with mixed time-varying delays has not been well probed yet, which still is an open problem. This being because the neutral item and the nonlinearity complicate the problem.

Motivated by the aforementioned discussion, the present paper investigates the problem of robust stability for a class of stochastic nonlinear neutral-type systems with mixed time-varying delays. The main contributions of the present work are as follows.

i. Some delay-dependent sufficient conditions are proposed by constructing an appropriate Lyapunov–Krasovskii functional and using the free-weighting matrix method.
ii. The derivatives of the time-varying delays no longer must be less than one, thereby generalizing the existing results.
iii. Some free-weighting matrices are introduced to avoid using any inequality to deal with the cross terms. Therefore, our results are less conservative.

The remainder of the paper is organized as follows. Section 2 outlines the required mathematical preliminaries. The main results are presented in Sect. 3. In Sect. 4, two practical examples are provided to demonstrate the effectiveness of the proposed methods. Finally, Sect. 5 concludes the paper.

Notations Throughout this paper, let $(\Omega, \mathcal{F}, \mathcal{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions. $B(t)$ is a one-dimensional Brownian motion defined on the probability space adapted to the filtration. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the n-dimensional Euclidean space and the set of all $m\times n$ real matrix, respectively. $\|\cdot\|$ is the usual Euclidean norm in $\mathbb{R}^n$. The inner product of vectors x and y in $\mathbb{R}^n$ is denoted by $\langle x, y\rangle$ or $x^T y$. Let $C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+; \mathbb{R}_+)$ denote the family of all real-valued functions $V(x(t), t)$ defined on $\mathbb{R}^n \times \mathbb{R}_+$ such that they are continuously twice differentiable in x and once in t. $C([-r, 0]; \mathbb{R}^n)$ denotes the space of all continuous $\mathbb{R}^n$-valued functions φ defined on $[-r, 0]$ with a norm $\|\varphi\| = \sup_{-r\leq\theta\leq 0}|\varphi(\theta)|$. For a real symmetric matrix X, $X > 0$ ($X \geq 0$) means that X is positive definite (positive semi-definite). The asterisk $*$ denotes a matrix that can be inferred by symmetry and the superscript T represents the transpose of a matrix or a vector. In a matrix, (i, j) denotes an (i, j)-block element of the matrix. The notation

$\mathbf{E}\{\cdot\}$ represents the mathematical expectation operator. I denotes the identity matrix of compatible dimension. e_i denotes index matrices that consist of the unit matrix on the ith position and zero blocks on other positions.

2 Preliminaries

In this section, several basic assumptions and conclusions are offered that are of use regarding the main results. These basic facts can be found in any introductory book on stochastic differential equations (e.g. [5, 6, 26–29]).

Assumption 2.1 (One-sided Lipschitz condition [5]) The nonlinear function $f(x,y)$ is said to be one-sided Lipschitz if there exist $\alpha_1, \alpha_2 \in \mathbb{R}$ satisfying

$$\langle f(x,y), x\rangle \le \alpha_1 x^T x + \alpha_2 y^T y \tag{1}$$

for $\forall x, y \in \mathbb{R}^n$, where constant α_1 and α_2 are positive, zero, or even negative, and they are called one-sided Lipschitz constants for $f(x,y)$ with respect to x and y.

Assumption 2.2 (Quadratic inner-boundedness condition [5]) The nonlinear function $f(x,y)$ is called quadratic inner-boundedness in the region $\mathcal{C}$, if there exist constants β_1, β_2, and κ such that

$$f(x,y)^T f(x,y) \le \beta_1 x^T x + \beta_2 y^T y + \kappa\langle x, f(x,y)\rangle, \quad \text{for any } x, y \in \mathcal{C}. \tag{2}$$

For stochastic systems, Itô's formula plays an important role in the stability analysis. We cite the following result here.

Lemma 2.1 (Itô's formula [6]) *Let $x(t)$ be an n-dimensional Itô process on $t \ge 0$ with the stochastic differential*

$$\mathrm{d}x(t) = f(t)\,\mathrm{d}t + g(t)\,\mathrm{d}w(t),$$

where $f(t) \in \mathbb{R}^n$ and $g(t) \in \mathbb{R}^{n\times m}$. Let $V(x(t),t) \in \mathcal{C}^{2,1}(\mathbb{R}^n \times \mathbb{R}^+;\mathbb{R}^+)$. Then $V(x(t),t)$ is a real-valued Itô process with its stochastic differential given by

$$\mathrm{d}V\bigl(x(t),t\bigr) = \mathfrak{L}V\bigl(x(t),t\bigr)\,\mathrm{d}t + V_x\bigl(x(t),t\bigr)g(t)\,\mathrm{d}w(t),$$
$$\mathfrak{L}V\bigl(x(t),t\bigr) = V_t\bigl(x(t),t\bigr) + V_x\bigl(x(t),t\bigr)f(t) + \frac{1}{2}\operatorname{trace}\bigl(g^T(t)V_{xx}\bigl(x(t),t\bigr)g(t)\bigr),$$

where $\mathcal{C}^{2,1}(\mathbb{R}^n \times \mathbb{R}^+;\mathbb{R}^+)$ denotes the family of all real-valued functions $V(x(t),t)$ such that they are continuously twice differentiable in x and t. If $V(x(t),t) \in \mathcal{C}^{2,1}(\mathbb{R}^n \times \mathbb{R}^+;\mathbb{R}^+)$, we set

$$V_t\bigl(x(t),t\bigr) = \frac{\partial V(x(t),t)}{\partial t},$$
$$V_x\bigl(x(t),t\bigr) = \left(\frac{\partial V(x(t),t)}{\partial x_1},\dots,\frac{\partial V(x(t),t)}{\partial x_n}\right),$$
$$V_{xx}\bigl(x(t),t\bigr) = \left(\frac{\partial^2 V(x(t),t)}{\partial x_i \partial x_j}\right)_{n\times n} = \begin{pmatrix} \frac{\partial^2 V(x(t),t)}{\partial x_1\partial x_1} & \cdots & \frac{\partial^2 V(x(t),t)}{\partial x_1\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial^2 V(x(t),t)}{\partial x_n\partial x_1} & \cdots & \frac{\partial^2 V(x(t),t)}{\partial x_n\partial x_n} \end{pmatrix}.$$

Lemma 2.2 (S-procedure [29]) *Let $\mathcal{Z}$ be a linear vector space, and $\mathcal{F}(z)$, $y_1(z)$, $y_1(z)$, $\ldots$, $y_k(z)$ be some real-valued functionals over $\mathcal{Z}$. Furthermore, define the domain $\mathcal{D}$ as follows*:

$$\mathcal{D} = \bigl\{z \in \mathcal{Z} : y_1(z) \geq 0, y_2(z) \geq 0, \ldots, y_k(z) \geq 0\bigr\},$$

and the two following conditions:

(1) $\mathcal{F}(z) > 0$, $\forall z \in \mathcal{D}$,

(2) $\exists$ *scalars* $\varepsilon_1 \geq 0, \varepsilon_2 \geq 0, \ldots, \varepsilon_k \geq 0$ *such that*

$$\mathcal{S}(\varepsilon, z) = \mathcal{F}(z) - \sum_{i=1}^{k} \varepsilon_i y_i(z) > 0, \quad \forall z \in \mathcal{Z}.$$

Then (2) *implies* (1). *The procedure of replacing* (1) *by* (2) *is called the S-procedure*.

Lemma 2.3 (Schur complement [28]) *For a given symmetric matrix $S = \left[\begin{smallmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{smallmatrix}\right]$, the following conditions are equivalent*:

(1) $S < 0$;

(2) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(3) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 2.4 (Matrix inequality [30]) *Let E, G and F be real matrices of appropriate dimensions with $F^T F \leq I$, then we have, for any scalar $\varepsilon > 0$,*

$$EFG + G^T F^T E^T \leq \varepsilon^{-1} E E^T + \varepsilon G^T G.$$

3 Robust stability analysis

Consider the following uncertain neutral stochastic mixed time-varying delays neutral-type system described in Itô's form:

$$\begin{cases} \mathrm{d}[x(t) - Dx(t - \lambda(t))] \\ \quad = [(A + \Delta A(t))x(t) + (A_\tau + \Delta A_\tau(t))x(t - \tau(t)) \\ \qquad + f(x(t), x(t - \tau(t)))]\,\mathrm{d}t \\ \qquad + [(H + \Delta H(t))x(t) + (H_\tau + \Delta H_\tau(t))x(t - \tau(t))]\,\mathrm{d}w(t), \\ x(t) = \varphi(t), \quad t \in [-r, 0], r = \max\{\tau_1, \lambda_1\}, \\ y(t) = Cx(t), \end{cases} \tag{3}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $\tau(t)$ is the unknown time-varying delay satisfying $0 \leq \tau(t) < \tau_1$, $0 \leq \lambda(t) < \lambda_1$, $\dot{\tau}(t) \leq d$ and $\dot{\lambda}(t) \leq \mu$ with real constants τ_1, λ_1, d and μ. $f(x(t), x(t - \tau(t))) \in \mathbb{R}^n$ is a nonlinear function with respect to the state $x(t)$ and the delayed state $x(t - \tau(t))$, $f(0, 0) = 0$. $\varphi(t) \in C([-r, 0]; \mathbb{R}^n)$ is a vector valued continuous function, and $w(t)$ is the standard one-dimensional Brownian motion satisfying

$$\mathbf{E}\bigl\{\mathrm{d}w(t)\bigr\} = 0, \qquad \mathbf{E}\bigl\{\mathrm{d}w(t)\bigr\}^2 = \mathrm{d}t,$$

Here $A \in \mathbb{R}^{n\times n}$, $A_\tau \in \mathbb{R}^{n\times n}$, $D \in \mathbb{R}^{n\times n}$, $H \in \mathbb{R}^{n\times n}$ and $H_\tau \in \mathbb{R}^{n\times n}$. Moreover, $\Delta A(t)$, $\Delta A_\tau(t)$, $\Delta H(t)$ and $\Delta H_\tau(t)$ are unknown matrices representing time-varying parameter uncertainties and are assumed to be of the form

$$\begin{bmatrix} \Delta A(t) & \Delta A_\tau(t) \\ \Delta H(t) & \Delta H_\tau(t) \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} F(t) \begin{bmatrix} G_1 & G_2 \end{bmatrix}, \tag{4}$$

where E_1, E_2, G_1 and G_2 are known real constant matrices of appropriate dimensions, and $F(t)$ is an unknown time-varying matrix function satisfying

$$F^T(t)F(t) \le I, \quad \forall t \in \mathbb{R}. \tag{5}$$

The parameter uncertainties $\Delta A(t)$, $\Delta A_\tau(t)$, $\Delta H(t)$ and $\Delta H_\tau(t)$ are said to be admissible if both (4) and (5) hold.

Let

$$h(t) = F(t)\bigl[G_1x(t) + G_2x\bigl(t - \tau(t)\bigr)\bigr],$$
$$h_1(t) = Ax(t) + A_\tau x\bigl(t - \tau(t)\bigr) + f\bigl(x(t), x\bigl(t - \tau(t)\bigr)\bigr) + E_1h(t).$$

System (3) can be rewritten as follows:

$$\begin{cases} \mathrm{d}[x(t) - Dx(t - \lambda(t))] = h_1(t)\,\mathrm{d}t + [Hx(t) + H_\tau x(t - \tau(t)) + E_2h(t)]\,\mathrm{d}w(t), \\ h(t)^Th(t) \le [G_1x(t) + G_2x(t - \tau(t))]^T[G_1x(t) + G_2x(t - \tau(t))]. \end{cases} \tag{6}$$

In this section, we will solve the problem of robust stability for uncertain stochastic time-delay system (6) by constructing the appropriate Lyapunov–Krasovskii functional and introducing free-weighting matrix. A new delay-dependent stability criteria is derived.

Theorem 3.1 *Consider the neutral stochastic time-delay system* (6). *The nonlinear function $f(x(t), x(t - \tau(t)))$ satisfies Assumptions 2.1 and 2.2. For given scalars τ_1, λ_1, μ and d, if there exist symmetric positive definite matrices $P \in \mathbb{R}^{n\times n}$, $Q_1 \in \mathbb{R}^{n\times n}$, $Q_2 \in \mathbb{R}^{n\times n}$, $S_1 \in \mathbb{R}^{n\times n}$, $S_2 \in \mathbb{R}^{n\times n}$, $R_1 \in \mathbb{R}^{n\times n}$, $R_2 \in \mathbb{R}^{n\times n}$, $M_i > 0$ $(i = 1, \ldots, 10)$ and N_j $(j = 1, \ldots, 8)$ of appropriate dimensions and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ and $\varepsilon_3 > 0$ satisfying the following LMIs*:

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12}R_1 & \Psi_{13}R_2 & \Psi_{14} & \Psi_{15} \\ * & -\tau_1^{-1}R_1 & 0 & 0 & 0 \\ * & * & -\lambda_1^{-1}R_2 & 0 & 0 \\ * & * & * & -\bar{M} & 0 \\ * & * & * & * & -\bar{\bar{M}} \end{bmatrix} < 0, \tag{7}$$

$$\Sigma_1 = \begin{bmatrix} -M_1 & 0 & 0 & 0 & 0 & 0 \\ * & -M_2 & 0 & 0 & 0 & -N_1 \\ * & * & -M_3 & 0 & 0 & 0 \\ * & * & * & -M_4 & 0 & -N_2 \\ * & * & * & * & -M_5 & 0 \\ * & * & * & * & * & -R_1 \end{bmatrix} < 0, \tag{8}$$

$$\Sigma_2 = \begin{bmatrix} -M_1 & 0 & 0 & 0 & 0 & 0 \\ * & -M_2 & 0 & 0 & 0 & 0 \\ * & * & -M_3 & 0 & 0 & -N_3 \\ * & * & * & -M_4 & 0 & 0 \\ * & * & * & * & -M_5 & -N_4 \\ * & * & * & * & * & -R_1 \end{bmatrix} < 0, \tag{9}$$

$$\Pi_1 = \begin{bmatrix} -M_6 & 0 & 0 & 0 & 0 & 0 \\ * & -M_7 & 0 & 0 & 0 & -N_5 \\ * & * & -M_8 & 0 & 0 & 0 \\ * & * & * & -M_9 & 0 & -N_6 \\ * & * & * & * & -M_{10} & 0 \\ * & * & * & * & * & -R_2 \end{bmatrix} < 0, \tag{10}$$

$$\Pi_2 = \begin{bmatrix} -M_6 & 0 & 0 & 0 & 0 & 0 \\ * & -M_7 & 0 & 0 & 0 & 0 \\ * & * & -M_8 & 0 & 0 & -N_7 \\ * & * & * & -M_9 & 0 & 0 \\ * & * & * & * & -M_{10} & -N_8 \\ * & * & * & * & * & -R_2 \end{bmatrix} < 0, \tag{11}$$

with

$$\Psi_{11} = \begin{bmatrix} \Omega_{11} & \Omega_{12} & 0 & \Omega_{14} & 0 & N_2^T & 0 & N_6^T & 0 & \Omega_{1,10} & \Omega_{1,11} \\ * & \Omega_{22} & N_3^T & \Omega_{24} & 0 & \Omega_{26} & N_4^T & 0 & 0 & 0 & \Omega_{2,11} \\ * & * & \Omega_{33} & 0 & 0 & -N_3D & \Omega_{37} & 0 & 0 & 0 & 0 \\ * & * & * & \Omega_{44} & N_7^T & -D^TN_2^T & 0 & \Omega_{48} & N_8^T & -D^TP & \Omega_{4,11} \\ * & * & * & * & \Omega_{55} & 0 & 0 & -N_7D & \Omega_{59} & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & -D^TN_4 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Omega_{77} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Omega_{88} & -D^TN_8^T & 0 & 0 \\ * & * & * & * & * & * & * & * & \Omega_{99} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon_3 I & 0 \\ * & * & * & * & * & * & * & * & * & * & \Omega_{11,11} \end{bmatrix}, \tag{12}$$

$$\Psi_{12} = \begin{bmatrix} A & A_\tau & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & E_1 \end{bmatrix}^T,$$

$$\Psi_{13} = \begin{bmatrix} A & A_\tau & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & E_1 \end{bmatrix}^T,$$

$$\Psi_{14} = \begin{bmatrix} \bar{M}_1 & \bar{M}_2 & \bar{M}_3 & \bar{M}_7 & \bar{M}_8 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$\Psi_{15} = \begin{bmatrix} \bar{M}_6 & 0 & 0 & 0 & 0 & \bar{M}_4 & \bar{M}_5 & \bar{M}_9 & \bar{M}_{10} & 0 & 0 \end{bmatrix}^T,$$

$$\bar{M} = \operatorname{diag}\{\tau_1^{-1}M_1, \tau_2^{-1}M_2, \tau_3^{-1}M_3, \lambda_1^{-1}M_7, \lambda_1^{-1}M_8\},$$

$$\bar{\bar{M}} = \operatorname{diag}\{\lambda_1^{-1}M_6, \tau_1^{-1}M_4, \tau_1^{-1}M_5, \lambda_1^{-1}M_9, \lambda_1^{-1}M_{10}\},$$

$$\bar{M}_1 = M_1e_1, \qquad \bar{M}_2 = M_2e_2, \qquad \bar{M}_3 = M_3e_3,$$

$$\bar{M}_4 = M_4e_4, \qquad \bar{M}_5 = M_5e_5,$$

$$\bar{M}_6 = M_6e_1, \qquad \bar{M}_7 = M_7e_2, \qquad \bar{M}_8 = M_8e_3,$$

$$\bar{M}_9 = M_9e_4, \qquad \bar{M}_{10} = M_{10}e_5,$$

$$\Omega_{11} = PA + A^TP + H^TPH + Q_1 + Q_2 + S_1 + S_2 + \varepsilon_1 G_1^T G_1 + \varepsilon_2\alpha_1 I + \varepsilon_3\beta_1 I, \tag{13}$$

$$\Omega_{12} = PA_\tau + H^T PH_\tau + N_1^T + \varepsilon_1 G_1^T G_2, \qquad \Omega_{14} = N_5^T - A^T PD,$$
$$\Omega_{1,10} = P - \frac{1}{2}\varepsilon_2 I + \frac{1}{2}\varepsilon_3 \kappa I, \qquad \Omega_{1,11} = PE_1 + H^T PE_2,$$
$$\Omega_{22} = H_\tau^T PH_\tau - (1-d)Q_1 - N_1 - N_1^T + \varepsilon_1 G_2^T G_2 + \varepsilon_2 \alpha_2 I + \varepsilon_3 \beta_2 I,$$
$$\Omega_{24} = -A_\tau^T PD - N_1 D, \qquad \Omega_{26} = N_1 D - N_2^T,$$
$$\Omega_{2,11} = H_\tau^T PE_2, \qquad \Omega_{33} = -Q_2 - N_3 - N_3^T,$$
$$\Omega_{37} = N_3 D - N_4^T, \qquad \Omega_{44} = -(1-\mu)S_1 - N_5 D - D^T N_5^T - N_5 - N_5^T,$$
$$\Omega_{48} = N_5 D - D^T N_6^T - N_6^T, \qquad \Omega_{4,11} = -D^T PE_1,$$
$$\Omega_{55} = -N_7 - N_7^T - S_2,$$
$$\Omega_{59} = N_7 D - N_8^T, \qquad \Omega_{66} = N_2 D + D^T N_2^T, \qquad \Omega_{77} = N_4 D + D^T N_4^T,$$
$$\Omega_{88} = N_6 D + D^T N_6^T, \qquad \Omega_{99} = N_8 D + D^T N_8^T, \qquad \Omega_{11,11} = E_2^T PE_2 - \varepsilon_1 I,$$

then the null solution of the stochastic time-delay system (6) *is asymptotically stable in the mean square.*

Proof Choose the following Lyapunov–Krasovskii functional:

$$V\big(x(t),t\big) = \sum_{i=1}^{4} V_i, \tag{14}$$

with

$$V_1 = \big[x(t) - Dx\big(t-\lambda(t)\big)\big]^T P\big[x(t) - Dx\big(t-\lambda(t)\big)\big],$$
$$V_2 = \int_{t-\tau(t)}^{t} x^T(s)Q_1 x(s)\,\mathrm{d}s + \int_{t-\lambda(t)}^{t} x^T(s)S_1 x(s)\,\mathrm{d}s,$$
$$V_3 = \int_{t-\tau_1}^{t} x^T(s)Q_2 x(s)\,\mathrm{d}s + \int_{t-\lambda_1}^{t} x^T(s)S_2 x(s)\,\mathrm{d}s,$$
$$V_4 = \int_{-\tau_1}^{0}\int_{t+\theta}^{t} h_1^T(s)R_1 h_1(s)\,\mathrm{d}s\,\mathrm{d}\theta + \int_{-\lambda_1}^{0}\int_{t+\theta}^{t} h_1^T(s)R_2 h_1(s)\,\mathrm{d}s\,\mathrm{d}\theta.$$

Using Itô's formula, Lemma 2.1, we obtain the stochastic differential as follows:

$$\begin{aligned}
&\mathrm{d}V\big(x(t),t\big) \\
&\quad = \mathcal{L}V\big(x(t),t\big)\,\mathrm{d}t \\
&\qquad + \big\{2\big[x(t) - Dx\big(t-\lambda(t)\big)\big]^T P\big[Hx(t) + H_\tau x\big(t-\tau(t)\big) + E_2 h(t)\big]\big\}\,\mathrm{d}w(t),
\end{aligned} \tag{15}$$

with

$$\begin{aligned}
&\mathcal{L}V\big(x(t),t\big) \\
&\quad = V_t\big(x(t),t\big) + V_x\big(x(t),t\big)\big[Ax(t) + A_\tau x\big(t-\tau(t)\big) + f\big(x(t),x\big(t-\tau(t)\big)\big) + E_1 h(t)\big] \\
&\qquad + \big[Hx(t) + H_\tau x\big(t-\tau(t)\big) + E_2 h(t)\big]^T P\big[Hx(t) + H_\tau x\big(t-\tau(t)\big) + E_2 h(t)\big],
\end{aligned} \tag{16}$$

$$\begin{aligned}
V_t\bigl(x(t),t\bigr) &= V_{2t} + V_{3t} + V_{4t}, &(17)\\
V_{2t} &= x^T(t)Q_1x(t) - (1-\dot{\tau}(t)x^T\bigl(t-\tau(t)\bigr)Q_1x\bigl(t-\tau(t)\bigr)\\
&\quad + x^T(t)S_1x(t) - (1-\dot{\lambda}(t)x^T\bigl(t-\lambda(t)\bigr)S_1x\bigl(t-\lambda(t)\bigr), &(18)\\
V_{3t} &= x^T(t)Q_2x(t) - x^T(t-\tau_1)Q_2x(t-\tau_1)\\
&\quad + x^T(t)S_2x(t) - x^T(t-\lambda_1)S_2x(t-\lambda_1), &(19)\\
V_{4t} &= \tau_1h_1^T(t)R_1h_1(t) - \int_{t-\tau_1}^{t} h_1^T(s)R_1h_1(s)\,\mathrm{d}s\\
&\quad + \lambda_1h_1^T(t)R_2h_1(t) - \int_{t-\lambda_1}^{t} h_1^T(s)R_2h_1(s)\,\mathrm{d}s\}\,\mathrm{d}t, &(20)\\
V_x\bigl(x(t),t\bigr) &= 2\bigl[x(t)-Dx\bigl(t-\lambda(t)\bigr)\bigr]^TP. &(21)
\end{aligned}$$

Substituting (16)–(21) into (15), one can obtain

$$\begin{aligned}
\mathrm{d}V\bigl(x(t),t\bigr) = \Bigl\{&2\bigl[x(t)-Dx\bigl(t-\lambda(t)\bigr)\bigr]^TP\bigl[Ax(t)+A_\tau x\bigl(t-\tau(t)\bigr)\\
&+f\bigl(x(t),x\bigl(t-\tau(t)\bigr)\bigr)+E_1h(t)\bigr]\\
&+\bigl[Hx(t)+H_\tau x\bigl(t-\tau(t)\bigr)+E_2h(t)\bigr]^TP\bigl[Hx(t)+H_\tau x\bigl(t-\tau(t)\bigr)+E_2h(t)\bigr]\\
&+x^T(t)Q_1x(t)-(1-\dot{\tau}(t)x^T\bigl(t-\tau(t)\bigr)Q_1x\bigl(t-\tau(t)\bigr)\\
&+x^T(t)S_1x(t)-(1-\dot{\lambda}(t)x^T\bigl(t-\lambda(t)\bigr)S_1x\bigl(t-\lambda(t)\bigr)\\
&+x^T(t)Q_2x(t)-x^T(t-\tau_1)Q_2x(t-\tau_1)\\
&+x^T(t)S_2x(t)-x^T(t-\lambda_1)S_2x(t-\lambda_1)\\
&+\tau_1h_1^T(t)R_1h_1(t)-\int_{t-\tau_1}^{t}h_1^T(s)R_1h_1(s)\,\mathrm{d}s\\
&+\lambda_1h_1^T(t)R_2h_1(t)-\int_{t-\lambda_1}^{t}h_1^T(s)R_2h_1(s)\,\mathrm{d}s\Bigr\}\,\mathrm{d}t\\
&+\bigl\{2\bigl[x(t)-Dx\bigl(t-\lambda(t)\bigr)\bigr]^T\\
&\times P\bigl[Hx(t)+H_\tau x\bigl(t-\tau(t)\bigr)+E_2h(t)\bigr]\bigr\}\,\mathrm{d}w(t). \qquad (22)
\end{aligned}$$

Taking the expectation of both sides of (22), we have

$$\mathbf{E}\,\mathrm{d}V\bigl(x(t),t\bigr) = \mathbf{E}\mathcal{L}V\bigl(x(t),t\bigr)\,\mathrm{d}t. \qquad (23)$$

Set

$$\begin{aligned}
&x_\lambda\bigl(t-\lambda(t)\bigr) \equiv x\bigl(t-\lambda(t)-\lambda\bigl(t-\lambda(t)\bigr)\bigr),\\
&x_\lambda(t-\lambda_1) \equiv x\bigl(t-\lambda_1-\lambda(t-\lambda_1)\bigr),\\
&x_\lambda\bigl(t-\tau(t)\bigr) \equiv x\bigl(t-\tau(t)-\lambda\bigl(t-\tau(t)\bigr)\bigr),\\
&x_\lambda(t-\tau_1) \equiv x\bigl(t-\tau_1-\lambda(t-\tau_1)\bigr).
\end{aligned}$$

We can derive the following equations by using the Newton–Leibniz formula:

$$\begin{aligned}
&2\big[x^T\big(t-\tau(t)\big)N_1 + x_\lambda^T\big(t-\tau(t)\big)N_2\big] \\
&\quad\times\bigg[x(t) - Dx\big(t-\lambda(t)\big) - x\big(t-\tau(t)\big) + Dx_\lambda\big(t-\tau(t)\big) \\
&\quad-\int_{t-\tau(t)}^{t} h_1(s)\,\mathrm{d}s - \int_{t-\tau(t)}^{t}\big(Hx(s) + H_\tau x\big(s-\tau(s)\big) + E_2h(s)\big)\,\mathrm{d}w(s)\bigg] = 0, \qquad (24)\\
&2\big[x^T(t-\tau_1)N_3 + x_\lambda^T(t-\tau_1)N_4\big] \\
&\quad\times\bigg[x\big(t-\tau(t)\big) - Dx_\lambda\big(t-\tau(t)\big) - x(t-\tau_1) + Dx_\lambda(t-\tau_1) \\
&\quad-\int_{t-\tau_1}^{t-\tau(t)} h_1(s)\,\mathrm{d}s - \int_{t-\tau_1}^{t-\tau(t)}\big(Hx(s) + H_\tau x\big(s-\tau(s)\big) + E_2h(s)\big)\,\mathrm{d}w(s)\bigg] = 0, \qquad (25)\\
&2\big[x^T\big(t-\lambda(t)\big)N_5 + x_\lambda^T\big(t-\lambda(t)\big)N_6\big] \\
&\quad\times\bigg[x(t) - Dx\big(t-\lambda(t)\big) - x\big(t-\lambda(t)\big) + Dx_\lambda\big(t-\lambda(t)\big) \\
&\quad-\int_{t-\lambda(t)}^{t} h_1(s)\,\mathrm{d}s - \int_{t-\lambda(t)}^{t}\big(Hx(s) + H_\tau x\big(s-\tau(s)\big) + E_2h(s)\big)\,\mathrm{d}w(s)\bigg] = 0, \qquad (26)\\
&2\big[x^T(t-\lambda_1)N_7 + x_\lambda^T(t-\lambda_1)N_8\big] \\
&\quad\times\bigg[x\big(t-\lambda(t)\big) - Dx_\lambda\big(t-\lambda(t)\big) - x(t-\lambda_1) + Dx_\lambda(t-\lambda_1) \\
&\quad-\int_{t-\lambda_1}^{t-\lambda(t)} h_1(s)\,\mathrm{d}s - \int_{t-\lambda_1}^{t-\lambda(t)}\big(Hx(s) + H_\tau x\big(s-\tau(s)\big) + E_2h(s)\big)\,\mathrm{d}w(s)\bigg] = 0, \qquad (27)
\end{aligned}$$

where N_j ($j = 1,\dots,4$) are arbitrary matrices with appropriate dimensions. Adding the left-hand sides of (24), (25), (26) and (27) on to $\mathcal{L}V(x(t),t)$, and noticing the properties of the stochastic integral, we have

$$\begin{aligned}
&\mathbf{E}\bigg\{\big[x^T\big(t-\tau(t)\big)N_1 + x_\tau^T\big(t-\tau(t)\big)N_2\big]\int_{t-\tau(t)}^{t}\big(Hx(s) + H_\tau x\big(s-\tau(s)\big) + E_2h(s)\big)\,\mathrm{d}w(s)\bigg\} \\
&\quad= 0, \\
&\mathbf{E}\bigg\{\big[x^T(t-\tau_1)N_3 + x_\tau^T(t-\tau_1)N_4\big]\int_{t-\tau_1}^{t-\tau(t)}\big(Hx(s) + H_\tau x\big(s-\tau(s)\big) + E_2h(s)\big)\,\mathrm{d}w(s)\bigg\} = 0, \\
&\mathbf{E}\bigg\{\big[x^T\big(t-\lambda(t)\big)N_5 + x_\lambda^T\big(t-\lambda(t)\big)N_6\big]\int_{t-\lambda(t)}^{t}\big(Hx(s) + H_\tau x\big(s-\tau(s)\big) + E_2h(s)\big)\,\mathrm{d}w(s)\bigg\} \\
&\quad= 0, \\
&\mathbf{E}\bigg\{\big[x^T(t-\lambda_1)N_7 + x_\lambda^T(t-\lambda_1)N_8\big]\int_{t-\lambda_1}^{t-\lambda(t)}\big(Hx(s) + H_\tau x\big(s-\tau(s)\big) + E_2h(s)\big)\,\mathrm{d}w(s)\bigg\} = 0.
\end{aligned}$$

Thus (23) is transformed to

$$\mathbf{E}\,\mathrm{d}V\big(x(t),t\big) = \mathbf{E}\mathcal{L}\tilde{V}\big(x(t),t\big)\,\mathrm{d}t, \qquad (28)$$

where

$$\begin{aligned}
&\mathcal{L}\tilde{V}\big(x(t),t\big) = \mathcal{L}V\big(x(t),t\big) + U(t), \qquad (29)\\
&U(t) = 2\big[x^T\big(t-\tau(t)\big)N_1 + x_\lambda^T\big(t-\tau(t)\big)N_2\big]\\
&\qquad \times \left[x(t) - Dx\big(t-\lambda(t)\big) - x\big(t-\tau(t)\big) + Dx_\lambda\big(t-\tau(t)\big) - \int_{t-\tau(t)}^{t} h_1(s)\,\mathrm{d}s\right]\\
&\qquad + 2\big[x^T(t-\tau_1)N_3 + x_\lambda^T(t-\tau_1)N_4\big]\\
&\qquad \times \left[x\big(t-\tau(t)\big) - Dx_\lambda\big(t-\tau(t)\big) - x(t-\tau_1) + Dx_\lambda(t-\tau_1) - \int_{t-\tau_1}^{t-\tau(t)} h_1(s)\,\mathrm{d}s\right]\\
&\qquad + 2\big[x^T\big(t-\lambda(t)\big)N_5 + x_\lambda^T\big(t-\lambda(t)\big)N_6\big]\\
&\qquad \times \left[x(t) - Dx\big(t-\lambda(t)\big) - x\big(t-\lambda(t)\big) + Dx_\lambda\big(t-\lambda(t)\big) - \int_{t-\lambda(t)}^{t} h_1(s)\,\mathrm{d}s\right]\\
&\qquad + 2\big[x^T(t-\lambda_1)N_7 + x_\lambda^T(t-\lambda_1)N_8\big]\\
&\qquad \times \Bigg[x\big(t-\lambda(t)\big) - Dx_\lambda\big(t-\lambda(t)\big) - x(t-\lambda_1) + Dx_\lambda(t-\lambda_1)\\
&\qquad - \int_{t-\lambda_1}^{t-\lambda(t)} h_1(s)\,\mathrm{d}s\Bigg]. \qquad (30)
\end{aligned}$$

Recalling that $\dot{\tau}(t) \le d$ and $\dot{\lambda}(t) \le \mu$, and subdividing the integration interval, we have

$$\begin{aligned}
\mathcal{L}\tilde{V}\big(x(t),t\big) &= \mathcal{L}V\big(x(t),t\big) + U(t)\\
&\le \Big\{2\big[x(t) - Dx\big(t-\lambda(t)\big)\big]^T P\big[Ax(t) + A_\tau x\big(t-\tau(t)\big)\\
&\quad + f\big(x(t), x\big(t-\tau(t)\big)\big) + E_1 h(t)\big]\\
&\quad + \big[Hx(t) + H_\tau x\big(t-\tau(t)\big) + E_2 h(t)\big]^T P\\
&\quad \times \big[Hx(t) + H_\tau x\big(t-\tau(t)\big) + E_2 h(t)\big]\\
&\quad + x^T(t)Q_1 x(t) - (1-d)x^T\big(t-\tau(t)\big)Q_1 x\big(t-\tau(t)\big)\\
&\quad + x^T(t)S_1 x(t) - (1-\mu)x^T\big(t-\lambda(t)\big)S_1 x\big(t-\lambda(t)\big)\\
&\quad + x^T(t)Q_2 x(t) - x^T(t-\tau_1)Q_2 x(t-\tau_1)\\
&\quad + x^T(t)S_2 x(t) - x^T(t-\lambda_1)S_2 x(t-\lambda_1) + \tau_1 h_1^T(t)R_1 h_1(t)\\
&\quad - \int_{t-\tau_1}^{t-\tau(t)} h_1^T(s)R_1 h_1(s)\,\mathrm{d}s - \int_{t-\tau(t)}^{t} h_1^T(s)R_1 h_1(s)\,\mathrm{d}s\Big\}\\
&\quad + \lambda_1 h_1^T(t)R_2 h_1(t)\\
&\quad - \int_{t-\lambda_1}^{t-\lambda(t)} h_1^T(s)R_2 h_1(s)\,\mathrm{d}s - \int_{t-\lambda(t)}^{t} h_1^T(s)R_2 h_1(s)\,\mathrm{d}s\} + U(t). \qquad (31)
\end{aligned}$$

On the other hand, by using the one-sided Lipschitz (1) and the quadratically inner-bounded conditions (2), we obtain the following inequality:

$$\alpha_1 x^T(t)x(t) + \alpha_2 x^T\big(t-\tau(t)\big)x\big(t-\tau(t)\big) - x^T(t)f\big(x(t), x\big(t-\tau(t)\big)\big) \ge 0, \qquad (32)$$

$$\beta_1 x^T(t)x(t) + \beta_2 x^T\big(t-\tau(t)\big)x\big(t-\tau(t)\big) - f\big(x(t),x\big(t-\tau(t)\big)\big)^T f\big(x(t),x\big(t-\tau(t)\big)\big) \\ + \kappa x^T(t) f\big(x(t),x\big(t-\tau(t)\big)\big) \geq 0. \tag{33}$$

Using the S-procedure Lemma 2.2 in (31), we can see that $\mathcal{L}\tilde{V}(x(t),t) < 0$ is implied if there exist positive scalars ε_1, ε_2 and ε_3 satisfying

$$\begin{aligned} &\mathcal{L}\tilde{V}\big(x(t),t\big) + \varepsilon_1\big[G_1x(t) + G_2x\big(t-\tau(t)\big)\big]^T\big[G_1x(t) + G_2x\big(t-\tau(t)\big)\big] - \varepsilon_1 h(t)^T h(t) \\ &\quad + \varepsilon_2\alpha_1 x^T(t)x(t) + \varepsilon_2\alpha_2 x^T\big(t-\tau(t)\big)x\big(t-\tau(t)\big) - \varepsilon_2 x^T(t) f\big(x(t),x\big(t-\tau(t)\big)\big) \\ &\quad + \varepsilon_3\beta_1 x^T(t)x(t) + \varepsilon_3\beta_2 x^T\big(t-\tau(t)\big)x\big(t-\tau(t)\big) \\ &\quad - \varepsilon_3 f\big(x(t),x\big(t-\tau(t)\big)\big)^T f\big(x(t),x\big(t-\tau(t)\big)\big) \\ &\quad + \varepsilon_3\kappa x^T(t) f\big(x(t),x\big(t-\tau(t)\big)\big) < 0. \end{aligned} \tag{34}$$

Moreover, the following formula holds for any positive definite matrices M_i ($i = 1,\ldots,10$) of appropriate dimensions:

$$\tau_1 x^T(t)M_1x(t) - \int_{t-\tau_1}^{t} x^T(t)M_1x(t)\,\mathrm{d}s = 0, \tag{35}$$

$$\tau_1 x^T\big(t-\tau(t)\big)M_2x\big(t-\tau(t)\big) - \int_{t-\tau_1}^{t} x^T\big(t-\tau(t)\big)M_2\big(t-\tau(t)\big)\,\mathrm{d}s = 0, \tag{36}$$

$$\tau_1 x^T(t-\tau_1)M_3x(t-\tau_1) - \int_{t-\tau_1}^{t} x^T(t-\tau_1)M_3(t-\tau_1)\,\mathrm{d}s = 0, \tag{37}$$

$$\tau_1 x_\lambda^T\big(t-\tau(t)\big)M_4x_\lambda\big(t-\tau(t)\big) - \int_{t-\tau_1}^{t} x_\lambda^T\big(t-\tau(t)\big)M_4x_\lambda\big(t-\tau(t)\big)\,\mathrm{d}s = 0, \tag{38}$$

$$\tau_1 x_\lambda^T(t-\tau_1)M_5x_\lambda(t-\tau_1) - \int_{t-\tau_1}^{t} x_\lambda^T(t-\tau_1)M_5x_\lambda(t-\tau_1)\,\mathrm{d}s = 0, \tag{39}$$

$$\lambda_1 x^T(t)M_6x(t) - \int_{t-\lambda_1}^{t} x^T(t)M_6x(t)\,\mathrm{d}s = 0, \tag{40}$$

$$\lambda_1 x^T\big(t-\lambda(t)\big)M_7x\big(t-\lambda(t)\big) - \int_{t-\lambda_1}^{t} x^T\big(t-\lambda(t)\big)M_7\big(t-\lambda(t)\big)\,\mathrm{d}s = 0, \tag{41}$$

$$\lambda_1 x^T(t-\lambda_1)M_8x(t-\lambda_1) - \int_{t-\lambda_1}^{t} x^T(t-\lambda_1)M_8(t-\lambda_1)\,\mathrm{d}s = 0, \tag{42}$$

$$\lambda_1 x_\lambda^T\big(t-\lambda(t)\big)M_9x_\lambda\big(t-\lambda(t)\big) - \int_{t-\lambda_1}^{t} x_\lambda^T\big(t-\lambda(t)\big)M_9x_\lambda\big(t-\lambda(t)\big)\,\mathrm{d}s = 0, \tag{43}$$

$$\lambda_1 x_\lambda^T(t-\lambda_1)M_{10}x_\lambda(t-\lambda_1) - \int_{t-\lambda_1}^{t} x_\lambda^T(t-\lambda_1)M_{10}x_\lambda(t-\lambda_1)\,\mathrm{d}s = 0. \tag{44}$$

Let

$$\begin{aligned} \xi^T(t) = \big[&x^T(t)\ x^T\big(t-\tau(t)\big)\ x^T(t-\tau_1)\ x^T\big(t-\lambda(t)\big)\ x^T(t-\lambda_1) \\ &x_\lambda^T\big(t-\tau(t)\big)x_\lambda^T(t-\tau_1)x_\lambda^T\big(t-\lambda(t)\big)x_\lambda^T(t-\lambda_1)f^T\big(x(t),x\big(t-\tau(t)\big)\big)\ h^T(t)\big], \end{aligned}$$

$$\eta_1^T(t,s) = \big[x^T(t)\ x^T\big(t-\tau(t)\big)\ x^T(t-\tau_1)\ x_\lambda^T\big(t-\tau(t)\big)\ x_\lambda^T(t-\tau_1)\ h_1^T(s)\big],$$

$$\eta_2^T(t,s) = \big[x^T(t)\ x^T\big(t-\lambda(t)\big)\ x^T(t-\lambda_1)\ x_\lambda^T\big(t-\lambda(t)\big)\ x_\lambda^T(t-\lambda_1)\ h_1^T(s)\big].$$

Combining the above formulas (35)–(44) and rearranging (34), if $\Psi_{11} < 0$, we have the following inequality:

$$\begin{aligned} &\xi^T(t)\Psi_{11}\xi(t) + \tau_1 h_1^T(t)R_1 h_1(t) + \lambda_1 h_1^T(t)R_2 h_1(t) \\ &\quad + \int_{t-\tau(t)}^{t} \eta_1^T(t,s)\Sigma_1\eta_1(t,s)\,\mathrm{d}s + \int_{t-\tau_1}^{t-\tau(t)} \eta_1^T(t,s)\Sigma_2\eta_1(t,s)\,\mathrm{d}s \\ &\quad + \int_{t-\lambda(t)}^{t} \eta_2^T(t,s)\Pi_1\eta_2(t,s)\,\mathrm{d}s + \int_{t-\lambda_1}^{t-\lambda(t)} \eta_2^T(t,s)\Pi_2\eta_2(t,s)\,\mathrm{d}s < 0, \end{aligned} \tag{45}$$

with Ψ_{11}, Σ_1, Σ_2, Π_1 and Π_2 being defined as in (8), (9), (10), (11) and (12). Utilizing the Schur complement Lemma 2.3, (45) is equivalent to the following LMI:

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} \\ * & -\tau_1^{-1}R_1^{-1} & 0 & 0 & 0 \\ * & * & -\lambda_1^{-1}R_2^{-1} & 0 & 0 \\ * & * & * & -\bar{M} & 0 \\ * & * & * & * & -\bar{\bar{M}} \end{bmatrix} < 0, \tag{46}$$

with Ψ_{11}, Ψ_{12}, Ψ_{13}, Ψ_{14}, Ψ_{15}, $\bar{M}$ and $\bar{\bar{M}}$ being defined as in (12) and (13).

Pre-and post-multiplying (46) by $\operatorname{diag}\{\overbrace{I,\ldots,I}^{11},R_1,R_2,\overbrace{I,\ldots,I}^{10}\}$, we obtain LMI (7). Combining with LMIs (8), (9), (10) and (11), we find that $\mathbf{E}\mathcal{L}\tilde{V}(\xi(t),t) < 0$, i.e., it guarantees the asymptotic stability of system (6) in the mean square. □

Remark 3.1 In Theorem 3.1, we do not use any inequality techniques for the cross terms. Therefore, our result is less conservative.

Remark 3.2 In the proof of Theorem 3.1, the purpose of introducing $M_1,\ldots,M_{10}$ is to expand the dimension of the matrices Σ_1, Σ_2, Π_1 and Π_2 so that we can solve them by LMIs technology.

If uncertain parameters $\Delta A(t)$, $\Delta A_\tau(t)$, $\Delta H(t)$ and $\Delta H_\tau(t)$ in system (6) are equal to zero, the system is simplified to the following deterministic stochastic system:

$$\begin{cases} \mathrm{d}[x(t) - Dx(t-\lambda(t))] = [Ax(t) + A_\tau x(t-\tau(t)) \\ \qquad\qquad\qquad\qquad\qquad + f(x(t),x(t-\tau(t)))]\,\mathrm{d}t \\ \qquad\qquad\qquad\qquad\qquad + [Hx(t) + H_\tau x(t-\tau(t))]\,\mathrm{d}w(t), \\ x(t) = \varphi(t), t \in [-r,0], r = \max\{\tau_1,\lambda_1\}, \\ y(t) = Cx(t). \end{cases} \tag{47}$$

The following conclusion of the robust asymptotic stability is obtained by Theorem 3.1 for the deterministic stochastic system (47).

Corollary 3.1 *Consider the neutral stochastic time-delay system* (47). *The nonlinear function function* $f(x(t),x(t-\tau(t)))$ *satisfies Assumptions* 2.1 *and* 2.2. *For given scalars* τ_1, λ_1, μ *and* d, *if there exist symmetric positive definite matrices* $P \in \mathbb{R}^{n\times n}$, $Q_1 \in \mathbb{R}^{n\times n}$, $Q_2 \in \mathbb{R}^{n\times n}$,

$S_1 \in \mathbb{R}^{n\times n}$, $S_2 \in \mathbb{R}^{n\times n}$, $R_1 \in \mathbb{R}^{n\times n}$, $R_2 \in \mathbb{R}^{n\times n}$, $M_i > 0$ $(i = 1,\ldots,10)$ and N_j $(j = 1,\ldots,8)$ of appropriate dimensions and scalars $\varepsilon_2 > 0$ and $\varepsilon_3 > 0$ satisfying the following LMI:

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12}R_1 & \Xi_{13}R_2 & \Xi_{14} & \Xi_{15} \\ * & -\tau_1^{-1}R_1 & 0 & 0 & 0 \\ * & * & -\lambda_1^{-1}R_2 & 0 & 0 \\ * & * & * & -\bar{M} & 0 \\ * & * & * & * & -\bar{\bar{M}} \end{bmatrix} < 0, \tag{48}$$

with

$$\Xi_{11} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & 0 & \Phi_{14} & 0 & N_2^T & 0 & N_6^T & 0 & \Phi_{1,10} \\ * & \Phi_{22} & N_3^T & \Phi_{24} & 0 & \Phi_{26} & N_4^T & 0 & 0 & 0 \\ * & * & \Phi_{33} & 0 & 0 & -N_3D & \Phi_{37} & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & N_7^T & -D^TN_2^T & 0 & \Phi_{48} & N_8^T & -D^TP \\ * & * & * & * & \Phi_{55} & 0 & 0 & -N_7D & \Phi_{59} & 0 \\ * & * & * & * & * & \Phi_{66} & -D^TN_4 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Phi_{77} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Phi_{88} & -D^TN_8^T & 0 \\ * & * & * & * & * & * & * & * & \Phi_{99} & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon_3 I \end{bmatrix}, \tag{49}$$

$$\begin{aligned}
&\Xi_{12} = \begin{bmatrix} A & A_\tau & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}^T, \\
&\Xi_{13} = \begin{bmatrix} A & A_\tau & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}^T, \\
&\Xi_{14} = \begin{bmatrix} \bar{M}_1 & \bar{M}_2 & \bar{M}_3 & \bar{M}_7 & \bar{M}_8 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
&\Xi_{15} = \begin{bmatrix} \bar{M}_6 & 0 & 0 & 0 & 0 & \bar{M}_4 & \bar{M}_5 & \bar{M}_9 & \bar{M}_{10} & 0 \end{bmatrix}^T, \\
&\bar{M} = \operatorname{diag}\{\tau_1^{-1}M_1, \tau_2^{-1}M_2, \tau_3^{-1}M_3, \lambda_1^{-1}M_7, \lambda_1^{-1}M_8\}, \\
&\bar{\bar{M}} = \operatorname{diag}\{\lambda_1^{-1}M_6, \tau_1^{-1}M_4, \tau_1^{-1}M_5, \lambda_1^{-1}M_9, \lambda_1^{-1}M_{10}\}, \\
&\bar{M}_1 = M_1e_1, \qquad \bar{M}_2 = M_2e_2, \qquad \bar{M}_3 = M_3e_3, \\
&\bar{M}_4 = M_4e_4, \qquad \bar{M}_5 = M_5e_5, \\
&\bar{M}_6 = M_6e_1, \qquad \bar{M}_7 = M_7e_2, \qquad \bar{M}_8 = M_8e_3, \\
&\bar{M}_9 = M_9e_4, \qquad \bar{M}_{10} = M_{10}e_5, \qquad\qquad (50) \\
&\Phi_{11} = PA + A^TP + H^TPH + Q_1 + Q_2 + S_1 + S_2 + \varepsilon_2\alpha_1 I + \varepsilon_3\beta_1 I, \\
&\Phi_{12} = PA_\tau + H^TPH_\tau + N_1^T, \qquad \Phi_{14} = N_5^T - A^TPD, \\
&\Phi_{1,10} = P - \frac{1}{2}\varepsilon_2 I + \frac{1}{2}\varepsilon_3\kappa I, \\
&\Phi_{22} = H_\tau^TPH_\tau - (1-d)Q_1 - N_1 - N_1^T + \varepsilon_2\alpha_2 I + \varepsilon_3\beta_2 I, \\
&\Phi_{24} = -A_\tau^TPD - N_1D, \qquad \Phi_{26} = N_1D - N_2^T, \\
&\Phi_{33} = -Q_2 - N_3 - N_3^T, \qquad \Phi_{37} = N_3D - N_4^T, \\
&\Phi_{44} = -(1-\mu)S_1 - N_5D - D^TN_5^T - N_5 - N_5^T, \qquad \Phi_{48} = N_5D - D^TN_6^T - N_6^T, \\
&\Phi_{55} = -N_7 - N_7^T - S_2, \qquad \Phi_{59} = N_7D - N_8^T, \qquad \Phi_{66} = N_2D + D^TN_2^T, \\
&\Phi_{77} = N_4D + D^TN_4^T, \qquad \Phi_{88} = N_6D + D^TN_6^T, \qquad \Phi_{99} = N_8D + D^TN_8^T.
\end{aligned}$$

Combining with LMIs (8), (9), (10) *and* (11), *then the null solution of the stochastic time-delay system* (47) *is asymptotically stable in the mean square*.

4 Two illustrative numerical examples

In this section, we give some examples to demonstrate the effectiveness of the method proposed herein.

Example 4.1 Consider the neutral-type nonlinear mixed time-varying delays system (47). We choose the same system matrix as those in Refs. [31, 32] and nonlinear function as this in Ref. [9], i.e. $\alpha_1 = 0.5$, $\alpha_2 = 0.005$, $\beta_1 = -2.5$, $\beta_2 = -0.015$ and $\kappa = 5$. Let $\lambda(t) = \tau(t)$ and give parameter matrix D. Table 1 lists the maximal allowable upper bound of the time delay for different parameter matrix D. From Table 1, it is easily seen that, when $d > 1$, our result still holds, that our proposed method is not constrained by the condition of $0 < \dot{\tau}(t) < 1$. Therefore, we can conclude that our proposed method in this paper is a generalization of the existing results of neutral-type stochastic nonlinear time-varying delays systems.

$$A = \begin{bmatrix} -1.2 & 0.1 \\ -0.1 & -1 \end{bmatrix}, \qquad A_\tau = \begin{bmatrix} -0.6 & 0.7 \\ -1 & -0.1 \end{bmatrix}.$$

Example 4.2 Consider the neutral-type stochastic mixed time-varying delays nonlinear system (6) with the same system matrix parameters as those in Refs. [33–35]:

$$A = \begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{bmatrix}, \qquad A_\tau = \begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{bmatrix},$$
$$D = \begin{bmatrix} -0.2 & 0.0 \\ 0.2 & -0.1 \end{bmatrix}.$$

Choosing the same nonlinear function parameter constants as those in Example 4.1. By using Theorem 3.1 with $H = H_\tau = 0$, we can obtain the maximal allowable upper bounds of the time delays in Tables 2 and 3. Tables 2 and 3 give the allowable upper bounds for λ_1 and τ_1 when $d > 1$ or $\mu > 1$, respectively. It can be seen that when d increases, the maximum allowable upper bounds τ_1 descend with $\mu = 1.2$. By contrast, when τ_1 descends, the maximum allowable upper bounds λ_1 increase with $d = 0.5$.

Table 1 Maximal allowable upper bounds of τ_1 for different parameter D

	$d = 0.4$	$d = 0.9$	$d = 1.1$	$d = 2.0$	$d = 4.0$
$D = \begin{bmatrix} -0.1 & 1.09 \\ -0.02 & -1.0 \end{bmatrix}$	19.6223	13.2923	4.2053	3.2464	4.6988
$D = \begin{bmatrix} -3.0 & 3.1 \\ -29.0 & 1.0 \end{bmatrix}$	14.9681	14.9743	14.9783	13.4113	13.9748
$D = \begin{bmatrix} -0.6 & 8 \\ 0.0 & -2.0 \end{bmatrix}$	12.2443	11.7322	11.8577	11.9342	13.0935

Table 2 Maximal allowable upper bounds τ_1 for different d and λ_1, when $\mu = 1.2$

	$\lambda_1 = 5$	$\lambda_1 = 4$	$\lambda_1 = 3$	$\lambda_1 = 2$	$\lambda_1 = 1$
$d = 0.5$	0.3771	0.5169	0.7510	1.2143	2.5901
$d = 0.7$	0.3772	0.5170	0.7469	1.2107	2.5925
$d = 0.9$	0.3750	0.5152	0.7471	1.2093	2.5802

Table 3 Maximal allowable upper bounds λ_1 for different τ_1 and μ, when $d = 0.5$

	$\tau_1 = 1.5$	$\tau_1 = 1.3$	$\tau_1 = 1.1$	$\tau_1 = 0.9$	$\tau_1 = 0.7$	$\tau_1 = 0.5$
$\mu = 1.2$	1.6596	1.8819	2.1769	2.5739	3.1693	4.0981
$\mu = 1.3$	1.6551	1.8796	2.1765	2.5782	3.1748	4.1014
$\mu = 1.5$	1.6579	1.8840	2.1796	2.5790	3.1746	4.1006

Let

$$E_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad E_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad E_3 = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad G_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad G_3 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

If we choose the time-varying delays as $\tau_1 = 1.5$, $d = 0.5$, $\mu = 1.2$ and $\lambda_1 = 1.6596$ then according to Theorem 3.1 we can obtain a set of solutions as follows:

$$P = 1.0 \times e^{-4} \begin{bmatrix} 0.2351 & 0.1888 \\ 0.1888 & 0.2285 \end{bmatrix}, \qquad Q_1 = \begin{bmatrix} 0.0660 & -0.0040 \\ -0.0040 & 0.1092 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 0.0350 & 0.0014 \\ 0.0014 & 0.0204 \end{bmatrix},$$

$$S_1 = 1.0 \times e^{-7} \begin{bmatrix} 0.0310 & -0.0028 \\ -0.0028 & 0.4031 \end{bmatrix}, \qquad S_2 = \begin{bmatrix} 0.0350 & 0.0014 \\ 0.0014 & 0.0204 \end{bmatrix},$$

$$R_1 = 1.0 \times e^{-8} \begin{bmatrix} 0.1316 & -0.0318 \\ -0.0318 & 0.1615 \end{bmatrix}, \qquad R_2 = 1.0 \times e^{-8} \begin{bmatrix} 0.1362 & -0.0746 \\ -0.0746 & 0.1895 \end{bmatrix},$$

$$\varepsilon_1 = 0.0748, \qquad \varepsilon_2 = 2.7442, \qquad \varepsilon_3 = 0.6889.$$

5 Conclusions

This paper provides sufficient findings for the stability criteria of uncertain neutral-type stochastic nonlinear systems with mixed time-varying delays. Some delay-dependent stability criteria are obtained by using a suitable Lyapunov–Krasovskii functional and linear matrix inequality (LMI) techniques. The derived outcome is expressed via LMIs that can be calculated through the MATLAB LMI Control Toolbox. Compared with some previous studies, the present study has two novel aspects. First, because no inequality technology is used to deal with the cross terms, our results are less conservative. Second, the derivatives of the time-varying delays are no longer limited to being less than one, thereby broadening the study scope. Numerical examples have been provided to illustrate the effectiveness of our main results.

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Authors' contributions
TM carried out the robust stability analysis. LL participated in the design and coordination and helped to analyze the manuscript. All authors read and approved the final manuscript.

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