Öztunç and Aslan *Journal of Inequalities and Applications* (2019) 2019:145
https://doi.org/10.1186/s13660-019-2096-5

Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH Open Access

# Jungck type fixed point results for weakly compatible mappings in a rectangular soft metric space

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**Abstract**
The main objectives of the current study are to explore and to express the proof of some common fixed point theorems by using commuting maps in rectangular soft metric spaces. We obtain some common fixed point results by utilization of rectangular soft metric and scalar-valued parametric functions.

**Keywords:** Soft rectangular metric; Common fixed points; Weakly compatible mappings

## 1 Introduction

Soft set theory, which was asserted by Molodtsov [12] in 1999, contains alternative tools for state mathematical problems with different viewpoint. The theory has many applications in engineering, modeling problems, medical and social sciences, economics, etc. In recent years many mathematicians have studied soft topological and soft algebraic structures [2, 11, 20, 23].

Otherwise the theory of fixed point has a big role in different branches of mathematics. Soft metric spaces were defined by Das and Samanta [5], and Hosseinzadeh [8] developed the theoretical properties of soft metric spaces by initiating new definition of soft metric. Also Yazar, Aras, and Bayramov studied some fixed point theorems for soft contractive mappings [22]. In recent papers we defined the soft rectangular metric spaces by using Branciari's [4] rectangular metric and acquired some fixed point results such as Banach contraction theorem for rectangular soft metric spaces [17], and the other properties are investigated in [18, 19]. Immediately afterwards, in this paper we focus on Jungck's common fixed point consequences for commuting mappings [9] and present some theorems in rectangular soft metric spaces. More details on fixed point theorems and common fixed point theorems can be found at [1, 3, 6, 7, 10, 13–16, 21].

## 2 Preliminaries

**Definition 2.1** ([12]) Let $\mathbb{E}$ be a parameter set. A pairwise $(\mathcal{S},\mathbb{E})$ is called to be a soft set on the universal set $X$, where $\mathcal{S}$ is a transformation supplied with $\mathcal{S}:\mathbb{E}\to\mathcal{P}(X)$.

That is to say, a soft set over $X$ is a parameterized family of subsets of the universal set $X$. For any parameter $x\in\mathbb{E}, \mathcal{S}(x)$ could be regarded as the set of $x$-approximate members of the soft set $(\mathcal{S},\mathbb{E})$.

**Definition 2.2** ([11]) Assume that $(\mathcal{S},A)$ and $(\mathcal{G}',A')$ are two soft sets on $\mathcal{U}$. $(\mathcal{S},A)$ is named to be a soft subset of $(\mathcal{G}',A')$, and we denote it by $(\mathcal{S},A)\tilde{\subset}(\mathcal{G}',A')$ if

(1) $A \subseteq A'$ and

(2) $\mathcal{S}(x) \subseteq \mathcal{G}'(x)$ for all $x \in A$.

Then $(\mathcal{S},A)$ is a super set of in soft manner $(\mathcal{G}',A')$, if $(\mathcal{G}',A')$ is a soft subset of $(\mathcal{S},A)$. We indicate it by $(\mathcal{S},A)\tilde{\supset}(\mathcal{G}',A')$.

**Definition 2.3** ([11]) Suppose that $(\mathcal{S},\mathbb{E})$ is a soft set on $X$. Then

(1) $(\mathcal{S},\mathbb{E})$ is named to be a null set in soft manner, symbolized by $\check{\emptyset}$ if, for every $e \in \mathbb{E}$, $\mathcal{S}(e) = \emptyset$.

(2) $(\mathcal{S},\mathbb{E})$ is called to be absolute soft set, signified by $\check{\mathbb{E}}$ if, for every $e \in \mathbb{E}$, $\mathcal{S}(e) = X$.

**Definition 2.4** ([8]) Presume that $A \subseteq \mathbb{E}$ is a parameter set. The binary $(\alpha,t)$ is called to be a soft parametric scalar if $t \in \mathbb{R}$ and $\alpha \in A$. The parametric scalar $(\alpha,t)$ is entitled not negative if $t \geq 0$. Let $(\alpha,t)$ and $(\beta,t')$ be two soft parametric scalars. $(\alpha,t)$ is said to be no less than $(\beta,t')$, and it is written as $(\alpha,t) \succeq (\beta,t')$, if $t \geq t'$.

**Definition 2.5** ([8]) Let $A \subseteq \mathbb{E}$ be a parameter set. Suppose that $(\gamma,t)$ and $(\beta,t')$ are two parametric scalars in soft manner. So the addition between soft parametric scalars and scalar multiplication on soft parametric scalars is described as follows:

$$(\gamma,t) \dot{+} \left(\beta,t'\right) = \left(\{\gamma,\beta\}, t+t'\right)$$

and

$$\lambda(\gamma,t) = (\gamma,\lambda t) \quad \text{for every } \lambda \in \mathbb{R}.$$

**Definition 2.6** ([8]) Suppose that $(\mathcal{S},\mathbb{E})$ is a soft set on the universal $X$. The function $f$ is called on $(\mathcal{S},\mathbb{E})$ parametric scalar-valued if there are mappings $f_1 : \mathbb{E} \to \mathbb{E}$ and $f_2 : \mathcal{S}(\mathbb{E}) \to \mathbb{R}$ so that $f(\mathcal{S},\mathbb{E}) = (f_1,f_2)(\mathbb{E},\mathcal{S}(\mathbb{E}))$.

In a similar manner, we can amplify the parametric scalar-valued function defined above as $f : (\mathcal{S},\mathbb{E}) \times (\mathcal{S},\mathbb{E}) \to (\mathbb{E},\mathbb{R})$ by $f(\mathbb{E} \times \mathbb{E}, \mathcal{S}(\mathbb{E}) \times \mathcal{S}(\mathbb{E})) = (f_1,f_2)(\mathbb{E} \times \mathbb{E}, \mathcal{S}(\mathbb{E}) \times \mathcal{S}(\mathbb{E}))$, where $f_1 : \mathbb{E} \times \mathbb{E} \to \mathbb{E}$ and $f_2 : \mathcal{S}(\mathbb{E}) \times \mathcal{S}(\mathbb{E}) \to \mathbb{R}$.

**Definition 2.7** ([8]) Let $(\mathcal{S},\mathbb{E})$ be a soft set over $X$, and let $\check{\varphi} : \mathbb{E} \times \mathbb{E} \to \mathbb{E}$ be a parametric mapping. The parametric scalar-valued transformation $\check{\mathcal{D}} : (\mathcal{S},\mathbb{E}) \times (\mathcal{S},\mathbb{E}) \to (\mathbb{E}, \mathbb{R}^{+}\{0\})$ is called to be a soft metric on $(\mathcal{S},\mathbb{E})$ if $\check{\mathcal{D}}$ supplies the following requirements:

(1) $\check{\mathcal{D}}((x,\mathcal{S}(x)),(x',\mathcal{S}(x'))) \succeq (\check{\varphi}(x,x'),0)$ and if $x = x'$, then the equality holds.

(2) $\check{\mathcal{D}}((x,\mathcal{S}(x)),(x',\mathcal{S}(x'))) = \check{\mathcal{D}}((x',\mathcal{S}(x')),(x,\mathcal{S}(x)))$ for all $x,x' \in \mathbb{E}$.

(3) $\check{\mathcal{D}}((x,\mathcal{S}(x)),(x'',\mathcal{S}(x''))) \preceq \check{\mathcal{D}}(((x,\mathcal{S}(x)),(x',\mathcal{S}(x'))) \dot{+} \check{\mathcal{D}}((x',\mathcal{S}(x')),(x'',\mathcal{S}(x'')))$ for all $x,x',x'' \in \mathbb{E}$.

The pairwise $((\mathcal{S},\mathbb{E}),\check{\mathcal{D}})$ is reputed to be a soft metric space on $X$.

**Definition 2.8** ([4]) Assume that $A$ is a not null set, and let $d : A \times A \to [0,\infty]$ carry out the following conditions for all $u,v \in A$ and all different $w,t \in A$, each of which is dissimilar from $u$ and $v$:

(RM1) $d(u,v) = 0 \Leftrightarrow u = v$,

(RM2) $d(u,v) = d(v,u)$,
(RM3) $d(u,v) \leq d(u,w) + d(w,t) + d(t,v)$.
At that time $d$ is named a rectangular metric and the pairwise $(A,d)$ is named a rectangular metric space(RMS).

**Definition 2.9** ([3]) Let $A \neq \emptyset$. An element $a \in A$ is a fixed point of $f : A \to A$ if $f(a) = a$.

**Definition 2.10** ([3]) Let $T$ be a transformation of a metric space $(M,\sigma)$ into $(M,\sigma)$. We express that $T$ is a contraction transformation if there exists a number $\alpha$ such that $0 < \alpha < 1$ and $\sigma(Tu, Tv) \leq \alpha\sigma(u,v)$ $(\forall u, v \in M)$.

**Theorem 2.11** ([3]) *Any contraction transformation of a complete non-empty metric space $M$ into $M$ has only one fixed point in $M$.*

**Definition 2.12** ([17, 19]) Suppose that $\check{\varphi} : \mathbb{E} \times \mathbb{E} \to \mathbb{E}$ is a scalar-valued parametric function. The parametric scalar-valued mapping $\check{\mathcal{D}}_R : (\mathcal{S},\mathbb{E}) \times (\mathcal{S},\mathbb{E}) \to (\mathbb{E}, R^+ \cup \{0\})$ is called to be a rectangular soft metric on $(\mathcal{S},\mathbb{E})$ if $\check{\mathcal{D}}_R$ carries through the following properties:
(RSM1) $\check{\mathcal{D}}_R((u,\mathcal{S}(u)),(v,\mathcal{S}(v))) \succeq (\check{\varphi}(u,v),0)$, if $u = v$, then the equality holds.
(RSM2) $\check{\mathcal{D}}_R((u,\mathcal{S}(u)),(v,\mathcal{S}(v))) = (\check{\varphi}(u,v),0) \Leftrightarrow$ for all $((u,\mathcal{S}(u)),(v,\mathcal{S}(v))) \in (\mathcal{S},\mathbb{E})$, $(u,\mathcal{S}(u)) = (v,\mathcal{S}(v))$ [for all $u,v \in \mathbb{E}, u = v$].
(RSM3) $\check{\mathcal{D}}_R((u,\mathcal{S}(u)),(v,\mathcal{S}(v))) = \check{\mathcal{D}}_R((v,\mathcal{S}(v)),(u,\mathcal{S}(u)))$ for all $u,v \in \mathbb{E}$.
(RSM4) $\check{\mathcal{D}}_R((u,\mathcal{S}(u)),(v,\mathcal{S}(v))) \preceq \check{\mathcal{D}}_R((u,\mathcal{S}(u)),(w,\mathcal{S}(w))) \dotplus \check{\mathcal{D}}_R((w,\mathcal{S}(w)),(t,\mathcal{S}(t))) \dotplus \check{\mathcal{D}}_R((t,\mathcal{S}(t)),(v,\mathcal{S}(v)))$ for all $u,v,w,t \in \mathbb{E}$.
Then we express that the binary $((\mathcal{S},\mathbb{E}),\check{\mathcal{D}}_R)$ is a rectangular soft metric space over $X$.

**Definition 2.13** ([17, 19]) Assume that $(\mathcal{S},\mathbb{E})$ is a soft set on $X$. A soft sequence in $(\mathcal{S},\mathbb{E})$ is a mapping $f : \mathbb{N} \to (\mathcal{S},\mathbb{E})$ equipped with $f(n) = (\mathcal{S}_n,\mathbb{E})$ so that $(\mathcal{S}_n,\mathbb{E})$ is a soft subset of $(\mathcal{S},\mathbb{E})$ for $n \in \mathbb{N}$, and this is symbolized by $\{(\mathcal{S}_n,\mathbb{E})\}_{n=1}^{\infty}$.

**Definition 2.14** ([17, 19]) Presume that $(\mathcal{S},\mathbb{E})$ is a soft set on $X$. Let $\check{\mathcal{D}}_R$ be a rectangular soft metric on $(\mathcal{S},\mathbb{E})$, $\{(\mathcal{S}_n,\mathbb{E})\}_{n=1}^{\infty}$ be a soft sequence in $(\mathcal{S},\mathbb{E})$, and $(x,\mathcal{S}(x)) \in (\mathcal{S},\mathbb{E})$. Then we state that the $\{(\mathcal{S}_n,\mathbb{E})\}_{n=1}^{\infty}$ converges to $(x,\mathcal{S}(x))$ if, for every positive number $\epsilon$, there exists a natural number $N$ so that, for all $n \in \mathbb{N}$ which $n \geq N$, we have

$$\check{\mathcal{D}}_R\bigl((a,\mathcal{S}_n(a)),(x,\mathcal{S}(x))\bigr) \preceq \bigl(\check{\varphi}(a,x),\epsilon\bigr).$$

**Definition 2.15** ([17, 19]) Suppose that $(\mathcal{S},\mathbb{E})$ is a soft set on $X$. Let $\check{\mathcal{D}}_R$ be a rectangular soft metric on $(\mathcal{S},\mathbb{E})$ and $\{(\mathcal{S}_n,\mathbb{E})\}_{n=1}^{\infty}$ be a soft sequence in $(\mathcal{S},\mathbb{E})$. Then we express that $\{(\mathcal{S}_n,\mathbb{E})\}_{n=1}^{\infty}$ is a Cauchy soft sequence if, for every positive number $\epsilon$, there exists a natural number $N$ so that, for every natural number $n$, $m$ which $n,m \geq N$, we have

$$\check{\mathcal{D}}_R\bigl((a,\mathcal{S}_n(a)),(a,\mathcal{S}_m(a))\bigr) \preceq \bigl(\check{\varphi}(a,a),\epsilon\bigr).$$

**Theorem 2.16** ([17, 19]) *Assume that $(\mathcal{S},\mathbb{E})$ is a soft set on $X$, let $\check{\mathcal{D}}_R$ be a metric on $(\mathcal{S},\mathbb{E})$ and $(\mathcal{S}_n,\mathbb{E})_{n=1}^{\infty}$ be a Cauchy soft sequence in $(\mathcal{S},\mathbb{E})$. If $(\mathcal{S}_n,\mathbb{E})_{n=1}^{\infty}$ is convergent in $(\mathcal{S},\mathbb{E})$, in that case it converges to the unique member of $(\mathcal{S},\mathbb{E})$.*

**Definition 2.17** ([17, 19]) Let $(\mathcal{S},\mathbb{E})$ be a soft set on $X$, let $\check{\mathcal{D}}_R$ be a rectangular soft metric on $(\mathcal{S},\mathbb{E})$. $(\mathcal{S},\mathbb{E})$ is said to be a complete rectangular soft metric space if every Cauchy soft sequence converges in $(\mathcal{S},\mathbb{E})$.

**Theorem 2.18** ([17, 19]) *Let $((\mathcal{S},\mathbb{E}),\check{\mathcal{D}}_R)$ and $((\mathcal{S}',\mathbb{E}'),\check{\mathcal{D}}'_R)$ be two rectangular soft metric spaces on $X$ and $Y$, respectively. Let $f = (f_1,f_2) : ((\mathcal{S},\mathbb{E}),\check{\mathcal{D}}_R) \to ((\mathcal{S}',\mathbb{E}'),\check{\mathcal{D}}'_R)$ be a soft mapping. Then $f$ is soft continuous iff, for all $(a,\mathcal{S}(a)) \in (\mathcal{S},\mathbb{E})$ and every positive number $\epsilon$, there exists a positive number $\delta$ so that, for every $(b,\mathcal{S}(b)) \in (\mathcal{S},\mathbb{E})$,*

$$\begin{aligned}&\check{\mathcal{D}}'_R\big(\big(f\big(a,\mathcal{S}(a)\big)\big),f\big(\big(b,\mathcal{S}(b)\big)\big)\big) \preceq \big(\check{\varphi}'\big(\check{\varphi}(a,b)\big),\epsilon\big) \quad \textit{whenever}\\ &\check{\mathcal{D}}_R\big(\big(a,\mathcal{S}(a)\big),\big(b,\mathcal{S}(b)\big)\big) \preceq \big(\check{\varphi}(a,b)\big),\delta\big).\end{aligned}$$

**Definition 2.19** ([17, 19]) Let $((\mathcal{S},\mathbb{E}),\check{\mathcal{D}}_R)$ be a rectangular soft metric space on $X$ and

$$f : \big((\mathcal{S},\mathbb{E}),\check{\mathcal{D}}_R\big) \to \big((\mathcal{S},\mathbb{E}),\check{\mathcal{D}}_R\big)$$

be a soft mapping. Then $f$ is called to be soft contractive if there is a positive number $\lambda$ with $0 < \lambda < 1$ such that

$$\check{\mathcal{D}}_R\big(\big(f\big(a,\mathcal{S}(a)\big)\big),f\big(\big(b,\mathcal{S}(b)\big)\big)\big) \preceq \lambda\check{\mathcal{D}}_R\big(\big(a,\mathcal{S}(a)\big),\big(b,\mathcal{S}(b)\big)\big) \quad \text{for all } a,b \in \mathbb{E}.$$

**Theorem 2.20** ([17, 19]) *Soft contractive mapping is soft continuous in a rectangular soft metric space $((\mathcal{S},\mathbb{E}),\check{\mathcal{D}}_R)$.*

**Definition 2.21** ([17, 19]) Let $((\mathcal{S},\mathbb{E}),\check{\mathcal{D}}_R)$ be a complete rectangular soft metric space on $X$, and let $f : ((\mathcal{S},\mathbb{E}),\check{\mathcal{D}}_R) \to ((\mathcal{S},\mathbb{E}),\check{\mathcal{D}}_R)$ be a soft transformation. A fixed soft set for $f$ is a soft subset of $(\mathcal{S},\mathbb{E})$ such as $(a,\mathcal{S}(a))$ such that $f((a,\mathcal{S}(a))) = (a,\mathcal{S}(a))$.

**Theorem 2.22** ([17] (Banach contraction theorem for rectangular soft metric space)) *Let $((\mathcal{S},\mathbb{E}),\check{\mathcal{D}}_R)$ be a complete rectangular soft metric space on $X$, and let*

$$f : \big((\mathcal{S},\mathbb{E}),\check{\mathcal{D}}_R\big) \to \big((\mathcal{S},\mathbb{E}),\check{\mathcal{D}}_R\big)$$

*be a rectangular soft contractive mapping. Hence $f$ has a unique fixed soft set.*

## 3 Main results

**Definition 3.1** Assume that $f$ and $g$ are mappings from $(\mathcal{S},\mathbb{E})$ to $(\mathcal{S},\mathbb{E})$. Then we say $f$ commutes with $g$ if $f(g(x,\mathcal{S}(x))) = g(f(x,\mathcal{S}(x)))$ for all $(x,\mathcal{S}(x))$ in $(\mathcal{S},\mathbb{E})$.

**Definition 3.2** Let $f : (\mathcal{S},\mathbb{E}) \to (\mathcal{S},\mathbb{E})$ and $g : (\mathcal{S},\mathbb{E}) \to (\mathcal{S},\mathbb{E})$ be mappings in $((\mathcal{S},\mathbb{E}),\check{\mathcal{D}}_R)$. A soft point $(x,\mathcal{S}(x)) \in (\mathcal{S},\mathbb{E})$ is said to be a coincidence point of $g$ and $f$ iff $g(x,\mathcal{S}(x)) = f(x,\mathcal{S}(x)) = (x,\mathcal{S}(x))$. $g$ and $f$ are named to be weakly compatible if they commute at all coincidence points.

**Theorem 3.3** *Presume that $g$ and $f$ are commuting transformations of a complete rectangular soft metric space $((\mathcal{S},\mathbb{E}),\check{\mathcal{D}}_R)$ into itself satisfying the following inequality:*

$$\check{\mathcal{D}}_R\big(g\big(x,\mathcal{S}(x)\big),g\big(y,\mathcal{S}(y)\big)\big) \preceq \lambda\check{\mathcal{D}}_R\big(f\big(x,\mathcal{S}(x)\big),f\big(y,\mathcal{S}(y)\big)\big) \tag{1}$$

*for all* $(x,\mathcal{S}(x)),(y,\mathcal{S}(y)) \in (\mathcal{S},\mathbb{E})$, *where* $0 < \lambda < 1$ *and where* $(\mathcal{S},\mathbb{E})$ *is a soft set on* $X$. *If* $g((\mathcal{S},\mathbb{E})) \subset f((\mathcal{S},\mathbb{E}))$ *and* $f$ *is continuous in soft manner, then* $g$ *and* $f$ *have only one common fixed point*.

*Proof* Let $(\mathcal{S}_0,\mathbb{E}) \in (\mathcal{S},\mathbb{E})$ be arbitrary. Then $g((\mathcal{S}_0,\mathbb{E}))$ and $f((\mathcal{S}_0,\mathbb{E}))$ are well defined. Since $g((\mathcal{S}_0,\mathbb{E})) \in f((\mathcal{S},\mathbb{E}))$, there is $(\mathcal{S}_1,\mathbb{E}) \in (\mathcal{S},\mathbb{E})$ so that $f((\mathcal{S}_1,\mathbb{E})) = g((\mathcal{S}_0,\mathbb{E}))$. As usually, if $(\mathcal{S}_n,\mathbb{E})$ is selected, then we put a point $f((\mathcal{S}_{n+1},\mathbb{E}))$ in $(\mathcal{S},\mathbb{E})$ so that $f((\mathcal{S}_{n+1},\mathbb{E})) = g((\mathcal{S}_n,\mathbb{E}))$.

We show that $\{f((\mathcal{S}_n,\mathbb{E}))\}$ is a Cauchy sequence.

From (1) we get

$$\begin{aligned}\check{\mathcal{D}}_R\big(f(\mathcal{S}_{m+k},\mathbb{E}),f(\mathcal{S}_{n+k},\mathbb{E})\big) &= \check{\mathcal{D}}_R\big(g(\mathcal{S}_{m+k-1},\mathbb{E}),g(\mathcal{S}_{n+k-1},\mathbb{E})\big)\\ &\preceq \lambda\check{\mathcal{D}}_R\big(f(\mathcal{S}_{m+k-1},\mathbb{E}),f(\mathcal{S}_{n+k-1},\mathbb{E})\big).\end{aligned}$$

So,

$$\check{\mathcal{D}}_R\big(f(\mathcal{S}_{m+k},\mathbb{E}),f(\mathcal{S}_{n+k},\mathbb{E})\big) \preceq \lambda^k\check{\mathcal{D}}_R\big(f(\mathcal{S}_m,\mathbb{E}),f(\mathcal{S}_n,\mathbb{E})\big) \tag{2}$$

for all $k \in \mathbb{N}$.

Now, we ensure the following two situations.

*Case 1*: If $f((\mathcal{S}_n,\mathbb{E})) = f((\mathcal{S}_{n+1},\mathbb{E}))$ for some $n$, then $g((\mathcal{S}_n,\mathbb{E})) = f((\mathcal{S}_n,\mathbb{E})) = (w,\mathcal{S}(w))$. We will verify that $(w,\mathcal{S}(w))$ is a unique common fixed point of $g$ and $f$. Essentially,

$g((w,\mathcal{S}(w))) = g(f(\mathcal{S}_n,\mathbb{E})) = f(g(\mathcal{S}_n,\mathbb{E})) = f((w,\mathcal{S}(w)))$.

Let $\check{\mathcal{D}}_R((w,\mathcal{S}(w)),g((w,\mathcal{S}(w)))) \succ 0$. In this case we have

$$\begin{aligned}\check{\mathcal{D}}_R\big(\big(w,\mathcal{S}(w)\big),g\big(w,\mathcal{S}(w)\big)\big) &= \check{\mathcal{D}}_R\big(g(\mathcal{S}_n,\mathbb{E}),g\big(\big(w,\mathcal{S}(w)\big)\big)\big)\\ &\preceq \lambda\check{\mathcal{D}}_R\big(f(\mathcal{S}_n,\mathbb{E}),f\big(\big(w,\mathcal{S}(w)\big)\big)\big),\end{aligned}$$

which is a contradiction. Since condition (1) implies that $g((\mathcal{S}_n,\mathbb{E})) = f((\mathcal{S}_n,\mathbb{E})) = (w,\mathcal{S}(w))$ is a unique common fixed point $g$ and $f$, the proof of *Case 1* is finished.

*Case 2*: If $f((\mathcal{S}_n,\mathbb{E})) \neq f((\mathcal{S}_{n+1},\mathbb{E}))$ for all $n \geq 0$, then $f((\mathcal{S}_n,\mathbb{E})) \neq f((\mathcal{S}_{n+k},\mathbb{E}))$ for all $n \geq 0$, $k \geq 1$. Namely, if $f((\mathcal{S}_n,\mathbb{E}))) = f((\mathcal{S}_{n+k},\mathbb{E}))$ for some $n \geq 0$ and $k \geq 1$, we have that

$$\begin{aligned}\check{\mathcal{D}}_R\big(f(\mathcal{S}_{n+1},\mathbb{E}),f(\mathcal{S}_{n+k+1},\mathbb{E})\big) &= \check{\mathcal{D}}_R\big(g(\mathcal{S}_n,\mathbb{E}),g(\mathcal{S}_{n+k},\mathbb{E})\big)\\ &\preceq \lambda\check{\mathcal{D}}_R\big(f(\mathcal{S}_n,\mathbb{E}),f(\mathcal{S}_{n+k},\mathbb{E})\big)\\ &= \big(\check{\varphi}\big(f(\mathcal{S}_{n+1}),f(\mathcal{S}_{n+k+1})\big),0\big).\end{aligned}$$

So, $f((\mathcal{S}_{n+1},\mathbb{E})) = f((\mathcal{S}_{n+k+1},\mathbb{E}))$. Then (2) implies that

$$\begin{aligned}\check{\mathcal{D}}_R\big(f(\mathcal{S}_{n+1},\mathbb{E}),f(\mathcal{S}_n,\mathbb{E})\big) &= \check{\mathcal{D}}_R\big(f(\mathcal{S}_{n+k+1},\mathbb{E}),f(\mathcal{S}_{n+k},\mathbb{E})\big)\\ &\preceq \lambda^k\check{\mathcal{D}}_R\big(f(\mathcal{S}_{n+1},\mathbb{E}),f(\mathcal{S}_n,\mathbb{E})\big)\\ &\prec \check{\mathcal{D}}_R\big(f(\mathcal{S}_{n+1},\mathbb{E}),f(\mathcal{S}_n,\mathbb{E})\big),\end{aligned}$$

which is a contradiction. Thus we assume that $f((\mathcal{S}_n,\mathbb{E})) \neq f((\mathcal{S}_m,\mathbb{E}))$ for all distinct $(\mathcal{S}_n,\mathbb{E}),(\mathcal{S}_m,\mathbb{E}) \in (\mathcal{S},\mathbb{E})$. Note that $f((\mathcal{S}_{m+k},\mathbb{E})) \neq f((\mathcal{S}_{n+k},\mathbb{E}))$ for all natural numbers $k$

and for all distinct $n, m \in \mathbb{N}$ and $(\mathcal{S}_{n+k}, \mathbb{E}), (\mathcal{S}_{m+k}, \mathbb{E}) \in (\mathcal{S}, \mathbb{E}) \backslash \{f((\mathcal{S}_n, \mathbb{E})), f((\mathcal{S}_m, \mathbb{E}))\}$. Since $((\mathcal{S}, \mathbb{E}), \check{\mathcal{D}}_R)$ is a rectangular soft metric space from the rectangular property of soft metric, we obtain that

$$\begin{aligned}\check{\mathcal{D}}_R\big(f(\mathcal{S}_m,\mathbb{E}),f(\mathcal{S}_n,\mathbb{E})\big) = {} & \check{\mathcal{D}}_R\big(f(\mathcal{S}_m,\mathbb{E}),f(\mathcal{S}_{m+n_0},\mathbb{E})\big) \\ & + \check{\mathcal{D}}_R\big(f(\mathcal{S}_{m+n_0},\mathbb{E}),f(\mathcal{S}_{n+n_0},\mathbb{E})\big) \\ & + \check{\mathcal{D}}_R\big(f(\mathcal{S}_{n+n_0},\mathbb{E}),f(\mathcal{S}_n,\mathbb{E})\big),\end{aligned}$$

where $n_0 \in \mathbb{N}$ such that $n_0 < m, n$.

Then

$$\begin{aligned}\check{\mathcal{D}}_R\big(f(\mathcal{S}_m,\mathbb{E}),f(\mathcal{S}_n,\mathbb{E})\big) = {} & \lambda^m \check{\mathcal{D}}_R\big(f(\mathcal{S}_0,\mathbb{E}),f(\mathcal{S}_{n_0},\mathbb{E})\big) + \lambda^{n_0} \check{\mathcal{D}}_R\big(f(\mathcal{S}_m,\mathbb{E}),f(\mathcal{S}_n,\mathbb{E})\big) \\ & + \lambda^n \check{\mathcal{D}}_R\big(f(\mathcal{S}_0,\mathbb{E}),f(\mathcal{S}_{n_0},\mathbb{E})\big) \\ \big(1-\lambda^{n_0}\big)\check{\mathcal{D}}_R\big(f(\mathcal{S}_m,\mathbb{E}),f(\mathcal{S}_n,\mathbb{E})\big) = {} & \big(\lambda^m+\lambda^n\big)\check{\mathcal{D}}_R\big(f(\mathcal{S}_0,\mathbb{E}),f(\mathcal{S}_{n_0},\mathbb{E})\big) \\ \check{\mathcal{D}}_R\big(f(\mathcal{S}_m,\mathbb{E}),f(\mathcal{S}_n,\mathbb{E})\big) \preceq {} & \frac{\lambda^m+\lambda^n}{1-\lambda^{n_0}} \check{\mathcal{D}}_R\big(g(\mathcal{S}_0,\mathbb{E}),g(\mathcal{S}_{n_0},\mathbb{E})\big).\end{aligned} \tag{3}$$

Thus $\{f(\mathcal{S}_n, \mathbb{E})\}$ is a Cauchy sequence in $(\mathcal{S}, \mathbb{E})$. By completeness of $f(\mathcal{S}, \mathbb{E})$, there exists $(a, \mathcal{S}(a)) \in (\mathcal{S}, \mathbb{E})$ such that

$$\begin{aligned}\lim_{n\to\infty} f\big((\mathcal{S}_n,\mathbb{E})\big) &= \lim_{n\to\infty} g\big((\mathcal{S}_{n-1},\mathbb{E})\big) \\ &= \big(a,\mathcal{S}(a)\big).\end{aligned} \tag{4}$$

Because of the soft continuity of $f$, (1) requires that both $f$ and $g$ are soft continuous. Because of the commuting property of $g$ and $f$, we provide

$$\begin{aligned}f\big(\big(a,\mathcal{S}(a)\big)\big) &= f\Big(\lim_{n\to\infty} g(\mathcal{S}_n,\mathbb{E})\Big) \\ &= \lim_{n\to\infty} f\big(g(\mathcal{S}_n,\mathbb{E})\big) = \lim_{n\to\infty} g\big(f(\mathcal{S}_n,\mathbb{E})\big) \\ &= g\Big(\lim_{n\to\infty} f(\mathcal{S}_n,\mathbb{E})\Big) \\ &= g\big(a,\mathcal{S}(a)\big).\end{aligned} \tag{5}$$

Let $(b, \mathcal{S}(b)) = f((a, \mathcal{S}(a))) = g((a, \mathcal{S}(a)))$.

We get if

$$g\big(a,\mathcal{S}(a)\big) = g\big(b,\mathcal{S}(b)\big) \tag{6}$$

from (1), we obtain

$$\begin{aligned}\check{\mathcal{D}}_R\big(g\big(\big(a,\mathcal{S}(a)\big)\big),g\big(b,\mathcal{S}(b)\big)\big) &\preceq \lambda \check{\mathcal{D}}_R\big(f\big(\big(a,\mathcal{S}(a)\big)\big),f\big(b,\mathcal{S}(b)\big)\big) \\ &= \lambda \check{\mathcal{D}}_R\big(g\big(\big(a,\mathcal{S}(a)\big)\big),g\big(b,\mathcal{S}(b)\big)\big) \\ &\prec \check{\mathcal{D}}_R\big(g\big(\big(a,\mathcal{S}(a)\big)\big),g\big(b,\mathcal{S}(b)\big)\big),\end{aligned}$$

which is a contradiction. So we have $g(a,\mathcal{S}(a)) = g(b,\mathcal{S}(b))$, hence we obtain $g(b,\mathcal{S}(b)) = f(b,\mathcal{S}(b)) = (b,\mathcal{S}(b))$ and $(b,\mathcal{S}(b))$ is a common fixed point for $g$ and $f$. Situation (1) alludes that $(b,\mathcal{S}(b))$ is a unique common fixed point. □

**Lemma 3.4** *Let $((\mathcal{S},\mathbb{E}),\check{\mathcal{D}}_R)$ be a rectangular soft metric space, and let $f,g:(\mathcal{S},\mathbb{E})\to(\mathcal{S},\mathbb{E})$ be two self mappings so that $f((\mathcal{S},\mathbb{E}))\check{\subseteq}g((\mathcal{S},\mathbb{E}))$, i.e., $f(\mathbb{E})\subset g(\mathbb{E})$, and for all $e\in\mathbb{E}$, $\mathcal{S}(f(e))\subset\mathcal{S}(g(e))$.*

*If Jungck type soft sequence $(\mathcal{S}'_n,\mathbb{E}) = f((\mathcal{S}_n,\mathbb{E})) = g((\mathcal{S}_{n+1},\mathbb{E}))$, $(\mathcal{S}_0,\mathbb{E})\in(\mathcal{S},\mathbb{E})$, and $(\mathcal{S}'_n,\mathbb{E})\neq(\mathcal{S}'_{n+1},\mathbb{E})$ for all $n\in\mathbb{N}$ satisfies*

$$\check{\mathcal{D}}_R\big((\mathcal{S}'_{n+1},\mathbb{E}),(\mathcal{S}'_n,\mathbb{E})\big)\preceq\lambda\check{\mathcal{D}}_R\big((\mathcal{S}'_n,\mathbb{E}),(\mathcal{S}'_{n-1},\mathbb{E})\big) \tag{7}$$

*for all $n\in\mathbb{N}$, where $0<\lambda<1$, then $(\mathcal{S}'_n,\mathbb{E}) = (\mathcal{S}'_m,\mathbb{E})$ whenever $n\neq m$.*

*Proof* Suppose that $(\mathcal{S}'_n,\mathbb{E}) = (\mathcal{S}'_m,\mathbb{E})$ for some $n>m$. Then we select $(\mathcal{S}_{n+1},\mathbb{E}) = (\mathcal{S}_{m+1},\mathbb{E})$ and hereby also $(\mathcal{S}'_{n+1},\mathbb{E}) = (\mathcal{S}'_{m+1},\mathbb{E})$. Then (7) implies that

$$\begin{aligned}\check{\mathcal{D}}_R\big((\mathcal{S}'_{n+1},\mathbb{E}),(\mathcal{S}'_n,\mathbb{E})\big) &\prec \check{\mathcal{D}}_R\big((\mathcal{S}'_n,\mathbb{E}),(\mathcal{S}'_{n-1},\mathbb{E})\big)\\ &< \cdots < \check{\mathcal{D}}_R\big((\mathcal{S}'_{m+1},\mathbb{E}),(\mathcal{S}'_m,\mathbb{E})\big)\\ &= \check{\mathcal{D}}_R\big((\mathcal{S}'_{n+1},\mathbb{E}),(\mathcal{S}'_n,\mathbb{E})\big).\end{aligned}$$

But this is a contradiction. We obtain that $n\neq m$ implies $(\mathcal{S}'_n,\mathbb{E})\neq(\mathcal{S}'_m,\mathbb{E})$. □

Let $\psi$ demonstrate the set of all soft continuous parametric scalar-valued mappings $\psi:((\mathcal{S},[0,\infty)))\to((\mathcal{S},[0,\infty)))$ such that $\psi_1:[0,\infty)\to[0,\infty)$, $\psi_2:\mathcal{S}([0,\infty))\to\mathbb{R}$, and $\psi(\mathcal{S}([0,\infty))) = (\psi_1,\psi_2)(([0,\infty),\mathcal{S}([0,\infty)))$, for which $\psi(t,\mathcal{S}(t)) = 0$ iff $(t,\mathcal{S}(t)) = (0,\{0\})$.

**Theorem 3.5** *Let $((\mathcal{S},\mathbb{E}),\check{\mathcal{D}}_R)$ be a rectangular soft metric space, and let $f,g:(\mathcal{S},\mathbb{E})\to(\mathcal{S},\mathbb{E})$ be two self maps so that $f((\mathcal{S},\mathbb{E}))\check{\subset}g((\mathcal{S},\mathbb{E}))$, i.e., $f(\mathbb{E})\subset g(\mathbb{E})$, and for all $e\in\mathbb{E}$ $\mathcal{S}(f(e))\subset\mathcal{S}(g(e))$, one of these two soft subsets of $(\mathcal{S},\mathbb{E})$ being complete. If for some $\psi = (\psi_1,\psi_2)$, $\phi = (\phi_1,\phi_2)$, $L\geq 0$, the functions $\psi_1$ and $\psi_2$ are non-decreasing*

$$\begin{aligned}(\psi_1,\psi_2)\big(\check{\mathcal{D}}_R\big(f\big((x,\mathcal{S}(x))\big),f\big(y,\mathcal{S}(y)\big)\big)\big) \preceq{}& (\psi_1,\psi_2)\big(M\big((x,\mathcal{S}(x)),\big(y,\mathcal{S}(y)\big)\big)\\ &-(\phi_1,\phi_2)\big(M\big((x,\mathcal{S}(x)),\big(y,\mathcal{S}(y)\big)\big)\\ &+L(\psi_1,\psi_2)\big(N\big((x,\mathcal{S}(x)),\big(y,\mathcal{S}(y)\big)\big)\end{aligned} \tag{8}$$

*for all $(x,\mathcal{S}(x)),(y,\mathcal{S}(y))\in(\mathcal{S},\mathbb{E})$, where*

$$\begin{aligned}M\big((x,\mathcal{S}(x)),\big(y,\mathcal{S}(y)\big)\big) = \max\big\{&\check{\mathcal{D}}_R\big(g\big((x,\mathcal{S}(x))\big),g\big(y,\mathcal{S}(y)\big)\big),\\ &\check{\mathcal{D}}_R\big(g\big((x,\mathcal{S}(x))\big),f\big(x,\mathcal{S}(x)\big)\big),\\ &\check{\mathcal{D}}_R\big(g\big((y,\mathcal{S}(y))\big),f\big(y,\mathcal{S}(y)\big)\big)\big\}\end{aligned} \tag{9}$$

$$\begin{aligned}N\big((x,\mathcal{S}(x)),\big(y,\mathcal{S}(y)\big)\big) = \min\big\{&\check{\mathcal{D}}_R\big(g\big((x,\mathcal{S}(x))\big),f\big(x,\mathcal{S}(x)\big)\big)\\ &+\check{\mathcal{D}}_R\big(g\big((y,\mathcal{S}(y))\big),f\big(y,\mathcal{S}(y)\big)\big),\check{\mathcal{D}}_R\big(g\big((x,\mathcal{S}(x))\big),f\big(y,\mathcal{S}(y)\big)\big),\\ &\check{\mathcal{D}}_R\big(g\big((y,\mathcal{S}(y))\big),f\big(x,\mathcal{S}(x)\big)\big)\big\},\end{aligned} \tag{10}$$

*then $f$ and $g$ have a unique point of coincidence. If, furthermore, $f,g$ are weakly compatible, so they possess only one common fixed point.*

*Proof* Firstly, it is straightforward to see that terms (8), (9), and (10) suggest that the point of coincidence of $f$ and $g$ is unique(if it exists). On account of proving that $f$ and $g$ have a point of coincidence, take a discretionary point $(x_0,\mathcal{S}(x_0))\in(\mathcal{S},\mathbb{E})$ and, using $f(\mathbb{E})\subset g(\mathbb{E})$ and $\mathcal{S}(f(x_0))\subset\mathcal{S}(g(x_0))$, choose sequences $\{(\mathcal{S}_n,\mathbb{E})\}$ and $\{(\mathcal{S}'_n,\mathbb{E})\}$ in $(\mathcal{S},\mathbb{E})$ such that

$$\big(\mathcal{S}'_n,\mathbb{E}\big)=f\big((\mathcal{S}_n,\mathbb{E})\big)=g\big((\mathcal{S}_{n+1},\mathbb{E})\big)\quad\text{for } n=0,1,2\ldots \tag{11}$$

If $(\mathcal{S}'_k,\mathbb{E})=(\mathcal{S}'_{k+1},\mathbb{E})$ for some $k\in\mathbb{N}$, then $g(\mathcal{S}'_{k+1},\mathbb{E})=(\mathcal{S}'_k,\mathbb{E})=(\mathcal{S}'_{k+1},\mathbb{E})=f((\mathcal{S}'_{k+1},\mathbb{E}))$ and $f$ and $g$ have a point of coincidence. Assume further that $(\mathcal{S}'_n,\mathbb{E})\neq(\mathcal{S}'_{n+1},\mathbb{E})$ for all $n\in\mathbb{N}$. Setting $(x,\mathcal{S}(x))=(\mathcal{S}_{n+1},\mathbb{E})$, $(y,\mathcal{S}(y))=(\mathcal{S}_n,\mathbb{E})$ in (8), we obtain that

$$\begin{aligned}(\psi_1,\psi_2)\big(\check{\mathcal{D}}_R\big((\mathcal{S}'_{n+1},\mathbb{E}),(\mathcal{S}_n,\mathbb{E})\big)\big)&\preceq(\psi_1,\psi_2)\big(\check{\mathcal{D}}_R\big(f(\mathcal{S}_{n+1},\mathbb{E}),f(\mathcal{S}_n,\mathbb{E})\big)\big)\\&\preceq(\psi_1,\psi_2)\big(M\big((\mathcal{S}_{n+1},\mathbb{E}),(\mathcal{S}_n,\mathbb{E})\big)\big)\\&\quad-(\phi_1,\phi_2)\big(M\big((\mathcal{S}_{n+1},\mathbb{E}),(\mathcal{S}_n,\mathbb{E})\big)\big)\\&\quad+L(\psi_1,\psi_2)\big(N\big((\mathcal{S}_{n+1},\mathbb{E}),(\mathcal{S}_n,\mathbb{E})\big)\big),\end{aligned}\tag{12}$$

where

$$M\big((\mathcal{S}_{n+1},\mathbb{E}),(\mathcal{S}_n,\mathbb{E})\big)=\max\big\{\check{\mathcal{D}}_R\big((\mathcal{S}'_n,\mathbb{E}),(\mathcal{S}'_{n-1},\mathbb{E})\big),\check{\mathcal{D}}_R\big((\mathcal{S}'_n,\mathbb{E}),(\mathcal{S}'_{n+1},\mathbb{E})\big)\big\}\tag{13}$$

$$\begin{aligned}N\big((x,\mathcal{S}(x)),(y,\mathcal{S}(y))\big)=\ &\min\big\{\check{\mathcal{D}}_R\big((\mathcal{S}'_n,\mathbb{E}),(\mathcal{S}'_{n+1},\mathbb{E})\big)\\&+\check{\mathcal{D}}_R\big((\mathcal{S}'_{n-1},\mathbb{E}),(\mathcal{S}'_n,\mathbb{E})\big),\\&\check{\mathcal{D}}_R\big((\mathcal{S}'_n,\mathbb{E}),(\mathcal{S}'_n,\mathbb{E})\big),\\&\check{\mathcal{D}}_R\big((\mathcal{S}'_{n-1},\mathbb{E}),(\mathcal{S}'_{n+1},\mathbb{E})\big)\big\}\\=\ &\big(\check{\varphi}(x,y),0\big),\end{aligned}\tag{14}$$

where $\check{\varphi}:\mathbb{E}\times\mathbb{E}\to\mathbb{E}$ is a parametric function.

Further from (12), (13), and (14) we obtain that

$$\begin{aligned}(\psi_1,\psi_2)\big(\check{\mathcal{D}}_R\big((\mathcal{S}'_{n+1},\mathbb{E}),(\mathcal{S}_n,\mathbb{E})\big)\big)\preceq\ &(\psi_1,\psi_2)\max\big\{\check{\mathcal{D}}_R\big((\mathcal{S}'_n,\mathbb{E}),(\mathcal{S}'_{n-1},\mathbb{E})\big),\\&\check{\mathcal{D}}_R\big((\mathcal{S}'_{n+1},\mathbb{E}),(\mathcal{S}'_n,\mathbb{E})\big)\big\}\\&-(\phi_1,\phi_2)\max\big\{\check{\mathcal{D}}_R\big((\mathcal{S}'_n,\mathbb{E}),(\mathcal{S}'_{n-1},\mathbb{E})\big),\\&\check{\mathcal{D}}_R\big((\mathcal{S}'_{n+1},\mathbb{E}),(\mathcal{S}'_n,\mathbb{E})\big)\big\}.\end{aligned}\tag{15}$$

If $\check{\mathcal{D}}_R((\mathcal{S}'_n,\mathbb{E}),(\mathcal{S}'_{n-1},\mathbb{E}))\prec\check{\mathcal{D}}_R((\mathcal{S}'_{n+1},\mathbb{E}),(\mathcal{S}'_n,\mathbb{E}))$, then from (15) it follows that

$$\begin{aligned}(\psi_1,\psi_2)\big(\check{\mathcal{D}}_R\big((\mathcal{S}'_{n+1},\mathbb{E}),(\mathcal{S}'_n,\mathbb{E})\big)\big)&\preceq(\psi_1,\psi_2)\check{\mathcal{D}}_R\big((\mathcal{S}'_{n+1},\mathbb{E}),(\mathcal{S}'_n,\mathbb{E})\big)\\&\quad-(\phi_1,\phi_2)\max\big\{\check{\mathcal{D}}_R\big((\mathcal{S}'_{n+1},\mathbb{E}),(\mathcal{S}'_n,\mathbb{E})\big)\big\}\\&\prec(\psi_1,\psi_2)\max\big\{\check{\mathcal{D}}_R\big((\mathcal{S}'_{n+1},\mathbb{E}),(\mathcal{S}'_n,\mathbb{E})\big)\big\},\end{aligned}\tag{16}$$

which is a contradiction. Hence we have that

$$\begin{aligned}(\psi_1,\psi_2)\big(\check{\mathcal{D}}_R\big((\mathcal{S}'_{n+1},\mathbb{E}),(\mathcal{S}'_n,\mathbb{E})\big)\big) &\preceq (\psi_1,\psi_2)\check{\mathcal{D}}_R\big((\mathcal{S}'_n,\mathbb{E}),(\mathcal{S}'_{n-1},\mathbb{E})\big) \\ &\quad - (\phi_1,\phi_2)\max\big\{\check{\mathcal{D}}_R\big((\mathcal{S}'_n,\mathbb{E}),(\mathcal{S}'_{n-1},\mathbb{E})\big)\big\} \\ &\prec (\psi_1,\psi_2)\max\big\{\check{\mathcal{D}}_R\big((\mathcal{S}'_n,\mathbb{E}),(\mathcal{S}'_{n-1},\mathbb{E})\big)\big\} \end{aligned} \tag{17}$$

or $\check{\mathcal{D}}_R((\mathcal{S}'_{n+1},\mathbb{E}),(\mathcal{S}'_n,\mathbb{E})) \prec \check{\mathcal{D}}_R((\mathcal{S}'_n,\mathbb{E}),(\mathcal{S}'_{n-1},\mathbb{E}))$ for all $n \in \mathbb{N}$. Inasmuch as $(\psi_1,\psi_2)$ is non-decreasing, thereby there exists

$$\lim_{n\to\infty}\check{\mathcal{D}}_R\big((\mathcal{S}'_{n+1},\mathbb{E}),(\mathcal{S}'_n,\mathbb{E})\big) = \big(u,\mathcal{S}(u)\big) \succeq \big(\check{\varphi}(\mathcal{S}'_{n+1},\mathcal{S}'_n),0\big) \quad \text{as } n\to\infty.$$

From (16) it follows that

$$\begin{aligned}(\psi_1,\psi_2)\big(u,\mathcal{S}(u)\big) &\preceq (\psi_1,\psi_2)\big(u,\mathcal{S}(u)\big) - (\phi_1,\phi_2)\big(u,\mathcal{S}(u)\big) \\ &\preceq (\psi_1,\psi_2)\big(u,\mathcal{S}(u)\big),\end{aligned}$$

that is, $(u,\mathcal{S}(u)) = \check{0}$.

Now we easily get that $(\mathcal{S}'_n,\mathbb{E}) \neq (\mathcal{S}'_m,\mathbb{E})$ whenever $n \neq m$. Indeed, if $(\mathcal{S}'_n,\mathbb{E}) = (\mathcal{S}'_m,\mathbb{E})$ for some $n > m$, then we select $(\mathcal{S}_{n+1},\mathbb{E}) = (\mathcal{S}_{m+1},\mathbb{E})$ (and hereby also $(\mathcal{S}'_{n+1},\mathbb{E}) = (\mathcal{S}'_{m+1},\mathbb{E})$). So we have

$$\begin{aligned}\check{\mathcal{D}}_R\big((\mathcal{S}'_{n+1},\mathbb{E}),(\mathcal{S}'_n,\mathbb{E})\big) &\prec \check{\mathcal{D}}_R\big((\mathcal{S}'_n,\mathbb{E}),(\mathcal{S}'_{n-1},\mathbb{E})\big) \\ &\prec \cdots \prec \check{\mathcal{D}}_R\big((\mathcal{S}'_{m+1},\mathbb{E}),(\mathcal{S}'_m,\mathbb{E})\big) \\ &= \check{\mathcal{D}}_R\big((\mathcal{S}'_{n+1},\mathbb{E}),(\mathcal{S}'_n,\mathbb{E})\big).\end{aligned} \tag{18}$$

The other part of the proof that $\{(\mathcal{S}'_n,\mathbb{E})\}$ is a Cauchy soft sequence.

Now, let us show that $(\mathcal{S}'_n,\mathbb{E})$ is a Cauchy soft sequence. Suppose otherwise. Then there exist $\epsilon > 0$, subsequences $\{(\mathcal{S}'_{n(i)},\mathbb{E})\}$ and $\{(\mathcal{S}'_{m(i)},\mathbb{E})\}$ of $\{(\mathcal{S}'_n,\mathbb{E})\}$, $n(i) > m(i) > i$, so that

$$\check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{m(i)}(u)\big),\big(u,\mathcal{S}'_{n(i)}(u)\big)\big) \succeq \big(\check{\varphi}(u,u),\epsilon\big), \tag{19}$$

where $n(i)$ is the smallest positive integer satisfying (19), i.e.,

$$\check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{m(i)}(u)\big),\big(u,\mathcal{S}'_{n(i)-1}(u)\big)\big) \succeq \big(\check{\varphi}(u,u),\epsilon\big).$$

By using the rectangular property of rectangular soft metric in (19), we have

$$\begin{aligned}\big(\check{\varphi}(u,u),\epsilon\big) &\preceq \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{m(i)}(u)\big),\big(u,\mathcal{S}'_{n(i)}(u)\big)\big) \\ &\preceq \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{m(i)}(u)\big),\big(u,\mathcal{S}'_{n(i)-2}(u)\big)\big) + \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{n(i)-2}(u)\big),\big(u,\mathcal{S}'_{n(i)-1}(u)\big)\big) \\ &\quad + \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{n(i)-1}(u)\big),\big(u,\mathcal{S}'_{n(i)}(u)\big)\big) \\ &\preceq \big(\check{\varphi}(u,u),\epsilon\big) + \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{n(i)-2}(u)\big),\big(u,\mathcal{S}'_{n(i)-1}(u)\big)\big) \\ &\quad + \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{n(i)-1}(u)\big),\big(u,\mathcal{S}'_{n(i)}(u)\big)\big).\end{aligned}$$

As $n \to \infty$ and using (19), we get

$$\big(\check{\varphi}(u,u),\epsilon\big) \preceq \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{m(i)}(u)\big),\big(u,\mathcal{S}'_{n(i)}(u)\big)\big) \preceq \epsilon,$$

i.e.,

$$\lim_{n\to\infty} \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{m(i)}(u)\big),\big(u,\mathcal{S}'_{n(i)}(u)\big)\big) = \big(\check{\varphi}(u,u),\epsilon\big).$$

Also

$$\begin{aligned}\check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{n(i)}(u)\big),\big(u,\mathcal{S}'_{m(i)}(u)\big)\big) &\preceq \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{n(i)}(u)\big),\big(u,\mathcal{S}'_{n(i)-1}(u)\big)\big)\\ &\preceq \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{m(i)}(u)\big),\big(u,\mathcal{S}'_{n(i)-2}(u)\big)\big)\\ &\quad + \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{n(i)-2}(u)\big),\big(u,\mathcal{S}'_{n(i)-1}(u)\big)\big)\\ &\quad + \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{n(i)-1}(u)\big),\big(u,\mathcal{S}'_{m(i)-1}(u)\big)\big)\\ &\quad + \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{m(i)-1}(u)\big),\big(u,\mathcal{S}'_{m(i)}(u)\big)\big)\end{aligned}$$

and

$$\begin{aligned}\check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{n(i)-1}(u)\big),\big(u,\mathcal{S}'_{m(i)-1}(u)\big)\big) &\preceq \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{n(i)-1}(u)\big),\big(u,\mathcal{S}'_{n(i)}(u)\big)\big)\\ &\preceq \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{m(i)}(u)\big),\big(u,\mathcal{S}'_{n(i)-2}(u)\big)\big)\\ &\quad + \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{n(i)-2}(u)\big),\big(u,\mathcal{S}'_{n(i)-1}(u)\big)\big)\\ &\quad + \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{n(i)}(u)\big),\big(u,\mathcal{S}'_{m(i)}(u)\big)\big)\\ &\quad + \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{m(i)}(u)\big),\big(u,\mathcal{S}'_{m(i)-1}(u)\big)\big).\end{aligned}$$

As $n \to \infty$, we get

$$\begin{aligned}&\big(\check{\varphi}(u,u),\epsilon\big) \preceq \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{n(i)-1}(u)\big),\big(u,\mathcal{S}'_{m(i)-1}(u)\big)\big) \preceq \big(\check{\varphi}(u,u),\epsilon\big),\\ &\lim_{n\to\infty} \check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{n(i)-1}(u)\big),\big(u,\mathcal{S}'_{m(i)-1}(u)\big)\big) = \big(\check{\varphi}(u,u),\epsilon\big).\end{aligned}$$

Now using (12)

$$\begin{aligned}&(\psi_1,\psi_2)(\check{\mathcal{D}}_R\big(\big(u,\mathcal{S}'_{n(i)}(u)\big),\big(u,\mathcal{S}'_{m(i)}(u)\big)\big)\\ &\quad\preceq (\psi_1,\psi_2)\big(\check{\mathcal{D}}_R\big(f\big(u,\mathcal{S}_{n(i)-1}(u)\big),f\big(u,\mathcal{S}_{m(i)-1}(u)\big)\big)\big)\\ &\quad\preceq (\psi_1,\psi_2)\big(M\big(\big(u,\mathcal{S}_{n(i)-1}(u)\big),\big(u,\mathcal{S}_{m(i)-1}(u)\big)\big)\big)\\ &\qquad - (\phi_1,\phi_2)\big(M\big(\big(u,\mathcal{S}_{n(i)-1}(u)\big),\big(u,\mathcal{S}_{m(i)-1}(u)\big)\big)\big)\\ &\qquad + L(\psi_1,\psi_2)(N\big(\big(\big(u,\mathcal{S}_{n(i)-1}(u)\big),\big(u,\mathcal{S}_{m(i)-1}(u)\big)\big)\big)\\ &N\big(\big(\big(u,\mathcal{S}_{n(i)-1}(u)\big),\big(u,\mathcal{S}_{m(i)-1}(u)\big)\big)\big) = \min\big\{\check{\mathcal{D}}_R\big(g\big(\big(u,\mathcal{S}_{n(i)-1}(u)\big),f\big(u,\mathcal{S}_{m(i)-1}(u)\big)\big)\big)\\ &\qquad + \check{\mathcal{D}}_R\big(g\big(\big(u,\mathcal{S}_{m(i)}(u)\big),f\big(u,\mathcal{S}_{m(i)-1}(u)\big)\big)\big),\\ &\qquad \check{\mathcal{D}}_R\big(g\big(\big(u,\mathcal{S}_{n(i)-1}(u)\big),f\big(u,\mathcal{S}_{m(i)-1}(u)\big)\big)\big),\end{aligned}$$

$$\begin{aligned}
&\check{\mathcal{D}}_R\big(g\big((u,\mathcal{S}_{m(i)-1}(u)\big),f\big(u,\mathcal{S}_{n(i)-1}(u)\big)\big)\big\}\\
&=\min\big\{\check{\mathcal{D}}_R\big(\big(\big(u,\mathcal{S}'_{n(i)-1}(u)\big),\big(u,\mathcal{S}_{n(i)}(u)\big)\big)\big)\\
&\quad+\check{\mathcal{D}}_R\big(\big(\big(u,\mathcal{S}'_{m(i)-1}(u)\big),\big(u,\mathcal{S}'_{m(i)}(u)\big)\big)\big),\\
&\check{\mathcal{D}}_R\big(\big(\big(u,\mathcal{S}'_{n(i)-1}(u)\big),\big(u,\mathcal{S}_{m(i)}(u)\big)\big)\big),\\
&\check{\mathcal{D}}_R\big(\big(\big(u,\mathcal{S}'_{m(i)-1}(u)\big),\big(u,\mathcal{S}'_{n(i)}(u)\big)\big)\big)\big\}
\end{aligned}$$

$N(((u,\mathcal{S}_{n(i)-1}(u)),(u,\mathcal{S}_{m(i)-1}(u))))\to\check{0}$ as $n\to\infty$.

$$\begin{aligned}
M\big(\big(\big(u,\mathcal{S}_{n(i)-1}(u)\big),\big(u,\mathcal{S}_{m(i)-1}(u)\big)\big)\big)&=\max\big\{\check{\mathcal{D}}_R\big(g\big(\big(u,\mathcal{S}_{n(i)-1}(u)\big),f\big(u,\mathcal{S}_{m(i)-1}(u)\big)\big)\big),\\
&\quad\check{\mathcal{D}}_R\big(g\big(\big(u,\mathcal{S}_{n(i)-1}(u)\big),f\big(u,\mathcal{S}_{n(i)-1}(u)\big)\big)\big),\\
&\quad\check{\mathcal{D}}_R\big(g\big(\big(u,\mathcal{S}_{n(i)-1}(u)\big),f\big(u,\mathcal{S}_{m(i)-1}(u)\big)\big)\big)\big\}\\
&=\max\big\{\check{\mathcal{D}}_R\big(\big(\big(u,\mathcal{S}'_{n(i)-1}(u)\big),\big(u,\mathcal{S}'_{m(i)}(u)\big)\big)\big),\\
&\quad\check{\mathcal{D}}_R\big(\big(\big(u,\mathcal{S}'_{n(i)-1}(u)\big),\big(u,\mathcal{S}'_{n(i)}(u)\big)\big)\big),\\
&\quad\check{\mathcal{D}}_R\big(\big(\big(u,\mathcal{S}'_{n(i)-1}(u)\big),\big(u,\mathcal{S}'_{m(i)}(u)\big)\big)\big)\big\}\\
&=\max\big\{\big(\check{\varphi}(u,u),\epsilon\big),\check{0},\big(\check{\varphi}(u,u),\epsilon\big)\big\}\\
&=\big(\check{\varphi}(u,u),\epsilon\big).
\end{aligned}$$

Thus we have

$$\begin{aligned}
&(\psi_1,\psi_2)\big(\check{\varphi}(u,u),\epsilon\big)\preceq(\psi_1,\psi_2)\big(\check{\varphi}(u,u),\epsilon\big)-(\phi_1,\phi_2)\big(\check{\varphi}(u,u),\epsilon\big),\\
&(\phi_1,\phi_2)\big(\check{\varphi}(u,u),\epsilon\big)=0\Rightarrow\epsilon=0.
\end{aligned}$$

This contradicts our assumption that $\epsilon>0$. Therefore $\{(\mathcal{S}'_n,\mathbb{E})\}$ is a soft Cauchy sequence.

Assume that the subspace $g((\mathcal{S},\mathbb{E}))$ is complete. Therefore $(\mathcal{S}'_n,\mathbb{E})$ tends to some $g((z,\mathcal{S}(z)))$ for some $(z,\mathcal{S}(z))\in(\mathcal{S},\mathbb{E})$. In order to prove that $f((z,\mathcal{S}(z)))=g((z,\mathcal{S}(z)))$, suppose that $f((z,\mathcal{S}(z)))\neq g((z,\mathcal{S}(z)))$. By (12), we have

$$\begin{aligned}
(\psi_1,\psi_2)\big(\check{\mathcal{D}}_R\big(f\big((\mathcal{S}_n,\mathbb{E})\big),f\big(z,\mathcal{S}(z)\big)\big)\big)&\preceq(\psi_1,\psi_2)\big(M\big((\mathcal{S}_n,\mathbb{E}),\big(z,\mathcal{S}(z)\big)\big)\big)\\
&\quad-(\phi_1,\phi_2)\big(M\big((\mathcal{S}_n,\mathbb{E}),\big(z,\mathcal{S}(z)\big)\big)\big)\\
&\quad+L(\psi_1,\psi_2)\big(N\big((\mathcal{S}_n,\mathbb{E}),\big(z,\mathcal{S}(z)\big)\big)\big),
\end{aligned}\tag{20}$$

where

$$\begin{aligned}
M\big(\big((\mathcal{S}_n,\mathbb{E}),\big(z,\mathcal{S}(z)\big)\big)\big)&=\max\big\{\check{\mathcal{D}}_R\big(g(\mathcal{S}_n,\mathbb{E}),g\big(z,\mathcal{S}(z)\big)\big),\\
&\quad\check{\mathcal{D}}_R\big(g(\mathcal{S}_n,\mathbb{E}),f(\mathcal{S}_n,\mathbb{E})\big),\\
&\quad\check{\mathcal{D}}_R\big(g\big(z,\mathcal{S}(z)\big),f\big(z,\mathcal{S}(z)\big)\big)\big\}\\
&\to\check{\mathcal{D}}_R\big(g\big(z,\mathcal{S}(z)\big),f\big(z,\mathcal{S}(z)\big)\big)
\end{aligned}\tag{21}$$

as $n \to \infty$ and

$$\begin{aligned} N\big(((\mathcal{S}_n,\mathbb{E}),(z,\mathcal{S}(z)))\big) &= \min\big\{\check{\mathcal{D}}_R\big(g(\mathcal{S}_n,\mathbb{E}),f(\mathcal{S}_n,\mathbb{E})\big), \\ &\quad \check{\mathcal{D}}_R\big(g\big(v,\mathcal{S}(v)\big),f\big(v,\mathcal{S}(v)\big)\big), \\ &\quad \check{\mathcal{D}}_R\big(g\big(z,\mathcal{S}(z)\big),f(\mathcal{S}_n,\mathbb{E})\big)\big\} \\ &\to \big(\check{\varphi}\big((\mathcal{S}_n,z),0\big) \end{aligned} \tag{22}$$

as $n \to \infty$.

Applying upper limit as $n \to \infty$ in (20), we acquire

$$\begin{aligned} &(\psi_1,\psi_2)\Big(\lim_{n\to\infty}\sup\check{\mathcal{D}}_R\big(f\big((\mathcal{S}_n,\mathbb{E})\big),f\big(z,\mathcal{S}(z)\big)\big)\Big) \\ &\quad \preceq (\psi_1,\psi_2)\check{\mathcal{D}}_R\big(g\big((z,\mathcal{S}(z))\big),f\big(z,\mathcal{S}(z)\big)\big) \\ &\qquad - (\phi_1,\phi_2)\check{\mathcal{D}}_R\big(g\big(z,\mathcal{S}(z)\big),f\big((z,\mathcal{S}(z))\big)\big) \\ &\quad \prec (\psi_1,\psi_2)\check{\mathcal{D}}_R\big(g\big((z,\mathcal{S}(z))\big),f\big(z,\mathcal{S}(z)\big)\big), \end{aligned} \tag{23}$$

and using the non-decreasing property of mapping $(\psi_1,\psi_2)$, we have

$$\lim_{n\to\infty}\sup\check{\mathcal{D}}_R\big(f\big((\mathcal{S}_n,\mathbb{E})\big),f\big(z,\mathcal{S}(z)\big)\big) \prec \check{\mathcal{D}}_R\big(g\big((z,\mathcal{S}(z))\big),f\big(z,\mathcal{S}(z)\big)\big). \tag{24}$$

On the other hand, by Theorem 1.17. $(\mathcal{S}'_n,\mathbb{E})$ is different from both $f((z,\mathcal{S}(z)))$ and $g((z,\mathcal{S}(z)))$ for $n$ big enough. Hereby we can enforce the soft rectangular inequality to maintain

$$\begin{aligned} \check{\mathcal{D}}_R\big(f\big((z,\mathcal{S}(z)\big),g\big(z,\mathcal{S}(z)\big)\big) &\preceq \check{\mathcal{D}}_R\big(f\big(z,\mathcal{S}(z)\big),f\big((\mathcal{S}_n,\mathbb{E})\big)\big) \\ &\quad + \check{\mathcal{D}}_R\big(f(\mathcal{S}_n,\mathbb{E}),f\big((\mathcal{S}_{n+1},\mathbb{E})\big)\big) \\ &\quad + \check{\mathcal{D}}_R\big(g\big(((\mathcal{S}_{n+1},\mathbb{E})\big),g\big(z,\mathcal{S}(z)\big)\big). \end{aligned} \tag{25}$$

Therefore it is maintained that

$$\check{\mathcal{D}}_R\big(f\big((z,\mathcal{S}(z))\big)\big) \prec \lim_{n\to\infty}\sup\check{\mathcal{D}}_R\big(f\big((\mathcal{S}_n,\mathbb{E})\big),f\big(z,\mathcal{S}(z)\big)\big). \tag{26}$$

Because $\check{\mathcal{D}}_R(f((\mathcal{S}_n,\mathbb{E})),f((\mathcal{S}_{n+1},\mathbb{E}))) \to \check{0}$ and $\check{\mathcal{D}}_R(f((\mathcal{S}_{n+1},\mathbb{E})),g((\mathcal{S}_{n+1},\mathbb{E}))) \to \check{0}$ as $n \to \infty$. Now (23),(24), and (26) become

$$\begin{aligned} (\psi_1,\psi_2)\big(\check{\mathcal{D}}_R\big(g\big((z,\mathcal{S}(z))\big),f\big(z,\mathcal{S}(z)\big)\big)\big) &\preceq (\psi_1,\psi_2)\check{\mathcal{D}}_R\big(g\big((z,\mathcal{S}(z))\big),f\big(z,\mathcal{S}(z)\big)\big) \\ &\quad - (\phi_1,\phi_2)\check{\mathcal{D}}_R\big(g\big(z,\mathcal{S}(z)\big),f\big((z,\mathcal{S}(z))\big)\big) \end{aligned} \tag{27}$$

or $(\phi_1,\phi_2)\check{\mathcal{D}}_R(g(z,\mathcal{S}(z)),f((z,\mathcal{S}(z)))) = \check{0}$, that is, $f((z,\mathcal{S}(z))) = g((z,\mathcal{S}(z)))$, a contradiction to the assumption $f((z,\mathcal{S}(z))) \neq g((z,\mathcal{S}(z)))$.

In this situation $f$ and $g$ are weakly compatible Jungck's conclusion from rectangular soft metric involves that $f$ and $g$ have only one common fixed point. □

## 4 An application

In this section we present an application of Jungck type fixed point theorem for rectangular soft metric space to taxicab metric.

Consider the usual metric $d(x_s, y_s) = |x_s - y_s|$ on $X = [0, \infty]$ , $A = \mathbb{Q}^+ = \{x \in \mathbb{Q} : x > 0\}$, and assume $(\check{\mathcal{S}}, \check{\mathbb{E}})$ is a soft set over $X$ such that every $(x_s, \check{\mathcal{S}}(x_s)) \in (\check{\mathcal{S}}, E)$ described as $(x_s, \check{\mathcal{S}}(x_s)) = [0, \infty]$. Let $g, f : ((\check{\mathcal{S}}, \check{\mathbb{E}}), \check{\mathcal{D}}_R) \to ((\check{\mathcal{S}}, \check{\mathbb{E}}), \check{\mathcal{D}}_R)$ be commuting mappings such that $g(\check{\mathcal{S}}, \check{\mathbb{E}}) \subset f(\check{\mathcal{S}}, \check{\mathbb{E}}) = [0, \infty]$. Take into account the mapping $\check{\mathcal{D}}_R : (\check{\mathcal{S}}, \check{\mathbb{E}}) \times (\check{\mathcal{S}}, \check{\mathbb{E}}) \to (\check{\mathbb{E}}, \mathbb{R}^+ \cup \{0\})$ described by

$$\check{\mathcal{D}}_R\bigl(\bigl(x_s, \check{\mathcal{S}}(x_s)\bigr), \bigl(y_s, \check{\mathcal{S}}(y_s)\bigr)\bigr) = \bigl(\max\{x_s, y_s\}, |x_{s(1)} - y_{s(1)}| + |x_{s(2)} - y_{s(2)}|\bigr).$$

Let $g : ((\check{\mathcal{S}}, \check{\mathbb{E}}), \check{\mathcal{D}}_R) \to ((\check{\mathcal{S}}, \check{\mathbb{E}}), \check{\mathcal{D}}_R)$ be a soft function given by

$$g\bigl(\bigl(x_s, \check{\mathcal{S}}(x_s)\bigr)\bigr) = \left(\frac{1}{4}x_s, \check{\mathcal{S}}\left(\frac{1}{4}x_s\right)\right) = \left[0, \frac{1}{4}x_s\right]$$

and

$$\begin{aligned}
&\check{\mathcal{D}}_R\bigl(g\bigl(\bigl(x_s, \check{\mathcal{S}}(x_s)\bigr)\bigr), g\bigl(\bigl(y_s, \check{\mathcal{S}}(y_s)\bigr)\bigr)\bigr)\\
&\quad = \check{\mathcal{D}}_R\left(\left(\frac{1}{4}x_s, \check{\mathcal{S}}\left(\frac{1}{4}y_s\right)\right), \left(\frac{1}{4}y_s, \check{\mathcal{S}}\left(\frac{1}{4}y_s\right)\right)\right)\\
&\quad = \left(\max\left\{\frac{1}{4}x_s, \frac{1}{4}y_s\right\}, \left|\frac{1}{4}x_{s(1)} - \frac{1}{4}y_{s(1)}\right| + \left|\frac{1}{4}x_{s(2)} - \frac{1}{4}y_{s(2)}\right|\right)\\
&\quad = \frac{1}{2}\left(\max\left\{\frac{1}{4}x_s, \frac{1}{4}y_s\right\}, \frac{1}{2}\bigl(|x_{s(1)} - y_{s(1)}| + |x_{s(2)} - y_{s(2)}|\bigr)\right)\\
&\quad \preceq \frac{1}{2}\bigl(\max\{x_s, y_s\}, |x_{s(1)} - y_{s(1)}| + |x_{s(2)} - y_{s(2)}|\bigr).
\end{aligned}$$

Since $g(\check{\mathcal{S}}, \check{\mathbb{E}}) \subset f(\check{\mathcal{S}}, \check{\mathbb{E}})$, it requires that $g(x_s, \check{\mathcal{S}}(x_s)) \leq f(x_s, \check{\mathcal{S}}(x_s))$. Hence

$$\frac{1}{2}\bigl(\max\{x_s, y_s\}, |x_{s(1)} - y_{s(1)}| + |x_{s(2)} - y_{s(2)}|\bigr) \leq \frac{1}{2}\check{\mathcal{D}}_R\bigl(f\bigl(x_s, \check{\mathcal{S}}(x_s)\bigr), f\bigl(y_s, \check{\mathcal{S}}(y_s)\bigr)\bigr).$$

Then, as a consequence of Theorem 3.3, $f$ and $g$ have a unique common fixed point.

## 5 Conclusion

In this paper we conclude that the rectangular soft metric which is used in the text involves that two weakly compatible mappings have a unique fixed point due to Jungck's main results.

**Acknowledgements**
This article is dedicated to Professor Gradimir V. Milovanovic on the occasion of his 70th anniversary.

**Funding**
Not applicable.

**Availability of data and materials**
Not applicable.

**Competing interests**
The authors declare that they have no competing interests.

**Authors' contributions**
Both authors contributed in drafting this manuscript. Both authors read and approved the final manuscript.

## Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 18 March 2019 Accepted: 10 May 2019 Published online: 22 May 2019

**References**

1. Altıntaş, İ., Şimşek, D., Taşköprü, K.: Topology of Soft Cone Metric Spaces. AIP Conference Proceding **1880**(1), 030006 (2017). https://doi.org/10.1063/1.50006056 aa
2. Aras, Ç.G.G., Poşul, H.: On some new operations in probabilistic soft set theory. Eur. J. Pure Appl. Math. **9**(3), 333–339 (2016)
3. Banach, S.: Sur les operations dans les ensembles abstraits et leur application aux equations integrales. Fundam. Math. **3**, 133–181 (1922)
4. Branciari, A.: A fixed point theorem of Banach-Caccioppoli type on a class of generalized metric spaces. Publ. Math. (Debr.) **57**, 31–37 (2000)
5. Das, S., Samanta, S.K.: Soft metric. Ann. Fuzzy Math. Inform. **6**(1), 77–94 (2013)
6. Ding, H.S., Imdad, M., Radenović, S., Vujaković, J.: On some fixed point results in b-metric, rectangular and b-rectangular metric spaces. Arab J. Math. Sci. **22**, 151–164 (2016)
7. George, R., Rajagopalan, R.: Common fixed point results for $\psi - \varphi$ contractions in rectangular metric spaces. Bull. Math. Anal. Appl. **5**(1), 44–52 (2013)
8. Hosseinzadeh, H.: Fixed point theorems on soft metric spaces. J. Fixed Point Theory Appl. **19**(2), 1625–1647 (2017)
9. Jungck, G.: Commuting mappings and fixed points. Am. Math. Mon. **83**, 261–263 (1976)
10. Jungck, G.: Compatible mappings and fixed points. Int. J. Math. Math. Sci. **9**(4), 771–779 (1986)
11. Maji, P.K., Biswas, R., Roy, A.: R: soft set theory. Comput. Math. Appl. **45**, 555–562 (2003)
12. Molodtsov, D.: Soft set theory-first result. Comput. Math. Appl. **37**, 19–31 (1999)
13. Mutlu, A., Yolcu, N.: Fixed point theorems for $\phi_p$ operator in cone Banach spaces. Fixed Point Theory Appl. **2013**, 56 (2013). https://doi.org/10.1186/1687-1812-2013-56
14. Mutlu, A., Yolcu, N., Mutlu, B.: Fixed point theorems in partially ordered rectangular metric spaces. Br. J. Math. Comput. Sci. **15**(2), 1–9 (2016)
15. Mutlu, A., Yolcu, N., Mutlu, B.: A survey on Kannan mappings in universalized metric spaces. Br. J. Math. Comput. Sci. **15**(6), 1–6 (2016)
16. Mutlu, A., Yolcu, N., Mutlu, B., Bildik, N.: On common coupled fixed point theorems for comparable mappings in ordered partially metric spaces. Abstr. Appl. Anal. **2014**, Article ID 486384 (2014)
17. Öztunç, S., Mutlu, A., Aslan, S.: Soft fixed point theorems for rectangular soft metric spaces, 2. International Students Science Conference, İzmir/Turkey. Abstract Book p. 102 (2018)
18. Öztunç, S., Mutlu, A., Aslan, S.: Jungck type fixed point results in rectangular soft metric spaces. In: The Mediterranean International Conference of Pure & Applied Mathematics and Related Areas (MICOPAM 2018), Antalya/Turkey, Abstract Booklet of MICOPAM 2018, pp. 51–52 (2018)
19. Öztunç, S., Mutlu, A., Sedat, A.: Some Kannan type fixed point results in rectangular soft metric space and an application of fixed point for thermal science problem. Therm. Sci. **23**(Suppl. 1), S215–S225 (2019)
20. Shabir, M., Naz, M.: On soft topological spaces. Comput. Math. Appl. **61**(7), 1786–1799 (2011)
21. Şimşek, D., Altıntaş, İ., Ersoy, S., Abdullayev, F., İmaş Kızı, M., Taşköprü, K., Kesik, D., Abılayeva, E., Ogul, B., Çemşitov, N.: An introduction to soft cone metric spaces and some fixed point theorems. MANAS Journal of Engineering (MJEN) **6**(1), 59–89 (2018)
22. Yazar, M.I., Aras, Ç.G., Bayramov, S.: Fixed point theorems of soft contractive mappings. Filomat **30**(2), 269–279 (2016)
23. Zorlutuna, İ., Çakır, H.: On continuity of soft mappings. Appl. Math. Inf. Sci. **9**(1), 403–409 (2015)