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Some characterizations of disjoint topological transitivity on Orlicz spaces

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Abstract

In this paper, we study the disjointness of topological transitivity on Orlicz spaces endowed with the Orlicz norm. We first give the characterization for weighted translations to be topologically transitive. Based on this, a sufficient condition and a necessary condition for powers of weighted translations to be disjoint topologically transitive are obtained as well.

MSC: Primary 54H20; secondary 46E30; 47A16

Keywords: Topological transitivity; Disjoint topological transitivity; Weighted translation; Orlicz space; Locally compact group

1 Introduction

In [1, 2], we gave sufficient and necessary conditions for weighted translations to be topologically transitive and disjoint topologically transitive on the Lebesgue space of locally compact groups, which generalizes some results about weighted shifts on the discrete group \mathbb{Z} in [3, 4], respectively. We note that the investigation of linear dynamics on locally compact groups has attracted a lot of attention. Indeed, let G be a locally compact group. The disjoint hypercyclicity of weighted translations on $L^p(G)$ was studied in [5]. The existence of transitive weighted translations on $L^p(G)$ is obtained in [6]. Also, Abakumov and Kuznetsova in [7] focused on the density of translates in the weighted Lebesgue space $L_w^p(G)$ where w is a weight on G . Recently, Azimi and Akbarbaglu in [8] extended the work of [2] from the Lebesgue space $L^p(G)$ to the Orlicz space $L^\Phi(G)$ endowed with the Luxemburg norm, where Φ is a Young function. It is well known that the Orlicz space is a generalization of the usual Lebesgue space. Inspired by these, we initial the study of linear dynamics in the setting of the Orlicz space $L^\Phi(G)$, but with respect to the Orlicz norm. Although the Orlicz norm and the Luxemburg norm are equivalent, they deduce the totally different descriptions of topological transitivity in [8, Theorem 2.3] and Theorem 2.1 in Sect. 2 by applying different approaches and methods. In fact, we also investigate deeper dynamical notions, namely, disjoint topological transitivity and disjoint topological mixing in Sect. 3, which were not studied in [8].

A bounded linear operator T on a separable Banach space X is said to be *topologically transitive* if, given two non-empty open sets $U, V \subset X$, there exists $n \in \mathbb{N}$ such that $T^n(U) \cap V \neq \emptyset$. If $T^n(U) \cap V \neq \emptyset$ from some n onwards, then T is *topologically mixing*. It should be noted [9] that topological transitivity and hypercyclicity are equivalent on

separable Banach spaces. The latter notion arises from the invariant subset problem in analysis. An operator T is called *hypercyclic* if there is $x \in X$ such that its orbit under T , denoted by $Orb(T, x) := \{T^n x : n \in \mathbb{N}\}$, is dense in X . Hypercyclicity and linear dynamics have been studied intensely. We refer to these classic books [9–11] on this subject. About one decade ago, Bernal-González, Bès and Peris introduced new notions of linear dynamics, namely, disjoint topological transitivity and disjoint hypercyclicity in [12] and [3], respectively. Since then, disjoint topological transitivity and disjoint hypercyclicity were studied by many authors [11, 13–18]. These new notions, disjoint topological transitivity and disjoint hypercyclicity, are kinds of generalizations of topological transitivity and hypercyclicity, respectively. We first recall some definitions of disjointness [3, 12] for further discussion.

Definition 1.1 Given $N \geq 2$, the operators T_1, T_2, \dots, T_N acting on a separable Banach space X are *disjoint hypercyclic*, or *diagonally hypercyclic* (in short, *d-hypercyclic*) if there is some vector (x, x, \dots, x) in the diagonal of $X^N = X \times X \times \dots \times X$ such that

$$\{(x, x, \dots, x), (T_1 x, T_2 x, \dots, T_N x), (T_1^2 x, T_2^2 x, \dots, T_N^2 x), \dots\}$$

is dense in X^N ; if this is the case, then we say $x \in X$ is a *d-hypercyclic vector* associated to the operators T_1, T_2, \dots, T_N .

For topological dynamics, several new notions were given accordingly [3] as follows.

Definition 1.2 Given $N \geq 2$, the operators T_1, T_2, \dots, T_N on a separable Banach space X are *disjoint topologically transitive* or *diagonally topologically transitive* (in short, *d-topologically transitive*) if, given non-empty open sets $U, V_1, \dots, V_N \subset X$, there is some $n \in \mathbb{N}$ such that

$$\emptyset \neq U \cap T_1^{-n}(V_1) \cap T_2^{-n}(V_2) \cap \dots \cap T_N^{-n}(V_N).$$

If the above condition is satisfied from some n onwards, then T_1, T_2, \dots, T_N are called *disjoint topologically mixing* (in short, *d-topologically mixing*).

In this note, the characterizations for weighted translation operators on the Orlicz space to be topologically transitive and disjoint topologically transitive will be demonstrated. We introduce the Orlicz space briefly for further study. It should be noted the Orlicz space is defined by a Young function which highly relies on the Young inequality. Indeed, a continuous, even and convex function $\Phi : \mathbb{R} \rightarrow \mathbb{R}$ is called a *Young function* if it satisfies $\Phi(0) = 0$, $\Phi(t) > 0$ for $t > 0$, and $\lim_{t \rightarrow \infty} \Phi(t) = \infty$. For a Young function Φ , the complementary function Ψ of Φ is given by

$$\Psi(y) = \sup\{x|y| - \Phi(x) : x \geq 0\} \quad (y \in \mathbb{R}),$$

which is also a Young function. If Ψ is the complementary function Φ , then Φ is the complementary function Ψ , and they both satisfy the Young inequality,

$$xy \leq \Phi(x) + \Psi(y) \quad (x, y \geq 0).$$

Let G be a locally compact group with identity e and a right Haar measure λ . Then the Orlicz space $L^\Phi(G)$ is defined by

$$L^\Phi(G) = \left\{ f : G \rightarrow \mathbb{C} : \int_G \Phi(\alpha|f|) d\lambda < \infty \text{ for some } \alpha > 0 \right\},$$

where f is a Borel measurable function. Moreover, the Orlicz space is a Banach space under the Orlicz norm defined for $f \in L^\Phi(G)$ by

$$\|f\|_\Phi = \sup \left\{ \int_G |fv| d\lambda : \int_G \Psi(|v|) d\lambda \leq 1 \right\}.$$

One can also define the Luxemburg norm on $L^\Phi(G)$ by

$$N_\Phi(f) = \inf \left\{ k > 0 : \int_G \Phi\left(\frac{|f|}{k}\right) d\lambda \leq 1 \right\}.$$

It is well known that these two norms above are equivalent.

We note that the Lebesgue space is a special case of the Orlicz space. The interesting properties and structures of Orlicz spaces have been investigated intensely over the last several decades. For instance, the properties (T_{L^Φ}) and (F_{L^Φ}) for Orlicz spaces L^Φ were studied by Tanaka in [19] recently. In [20], Piaggio also considered Orlicz spaces and the large scale geometry of Heintze groups. Hence it is significant to tackle hypercyclicity and linear dynamics on Orlicz spaces. For more discussion and recent work as regards Orlicz spaces, see [21–23].

We note that a Banach space admits a hypercyclic operator if, and only if, it is separable and infinite-dimensional [24, 25]. Hence we assume that G is second countable and Φ is Δ_2 -regular in this paper. A Young function is said to be Δ_2 -regular in [23] if there exist a constant $M > 0$ and $t_0 > 0$ such that $\Phi(2t) \leq M\Phi(t)$ for $t \geq t_0$ when G is compact, and $\Phi(2t) \leq M\Phi(t)$ for all $t > 0$ when G is noncompact. For example, the Young functions Φ given by

$$\Phi(t) = \frac{|t|^p}{p} \quad (1 \leq p < \infty) \quad \text{and} \quad \Phi(t) = |t|^\alpha (1 + |\log |t||) \quad (\alpha > 1)$$

are both Δ_2 -regular in [8, 23]. If Φ is Δ_2 -regular, then the space $C_c(G)$ of all continuous functions on G with compact support is dense in $L^\Phi(G)$, and the dual space $(L^\Phi(G), \|\cdot\|_\Phi)$ is $(L^\Psi(G), N_\Psi(\cdot))$.

A bounded continuous function $w : G \rightarrow (0, \infty)$ is called a *weight* on G . Let $a \in G$ and let δ_a be the unit point mass at a . A *weighted translation* on G is a weighted convolution operator $T_{a,w} : L^\Phi(G) \rightarrow L^\Phi(G)$ defined by

$$T_{a,w}(f) = wT_a(f) \quad (f \in L^\Phi(G)),$$

where w is a weight on G and $T_a(f) = f * \delta_a \in L^\Phi(G)$ is the convolution:

$$(f * \delta_a)(x) = \int_{y \in G} f(xy^{-1})\delta_a(y) = f(xa^{-1}) \quad (x \in G).$$

If $w^{-1} \in L^\infty(G)$, then we can define a self-map $S_{a,w}$ on $L^\Phi(G)$ by

$$S_{a,w}(h) = \frac{h}{w} * \delta_{a^{-1}} \quad (h \in L^\Phi(G))$$

so that

$$T_{a,w}S_{a,w}(h) = h \quad (h \in L^\Phi(G)).$$

In what follows, we assume $w, w^{-1} \in L^\infty(G)$.

In Sect. 2, we will characterize topological transitivity of weighted translations on $L^\Phi(G)$ in terms of the weight, the Haar measure and the group element. Applying similar arguments, in Sect. 3, we will obtain a sufficient condition and a necessary condition for powers of weighted translations to be disjoint topologically transitive. In particular, the Minkowski inequality and some estimates with inequalities play important roles in demonstrating the results in the two sections.

2 Topological transitivity

Now we are ready to give the result of topological transitivity. Applying the same argument, the characterization for topological mixing follows immediately.

Theorem 2.1 *Let G be a locally compact group and let $a \in G$. Let w be a weight on G , and let Φ be a Young function. Let $T_{a,w}$ be a weighted translation on $L^\Phi(G)$. Then the following conditions are equivalent.*

- (i) $T_{a,w}$ is topologically transitive on $L^\Phi(G)$.
- (ii) For each compact subset K of G with $\lambda(K) > 0$, there exist a sequence of Borel sets (E_k) in K and a strictly increasing sequence $(n_k) \subset \mathbb{N}$ such that

$$\lim_{k \rightarrow \infty} \sup_{v \in \Omega} \int_{K \setminus E_k} |v(x)| d\lambda(x) = 0,$$

and the two sequences

$$\varphi_n := \prod_{j=1}^n w * \delta_{a^{-1}}^j \quad \text{and} \quad \tilde{\varphi}_n := \left(\prod_{j=0}^{n-1} w * \delta_a^j \right)^{-1}$$

satisfy

$$\lim_{k \rightarrow \infty} \sup_{v \in \Omega} \int_{E_k} \varphi_{n_k}(x) |v(xa^{n_k})| d\lambda(x) = 0$$

and

$$\lim_{k \rightarrow \infty} \sup_{v \in \Omega} \int_{E_k} \tilde{\varphi}_{n_k}(x) |v(xa^{-n_k})| d\lambda(x) = 0,$$

where Ω is the set of all Borel functions v on G satisfying $\int_G \Psi(|v|) d\lambda \leq 1$.

Proof (ii) \Rightarrow (i). Suppose that U and V are non-empty open subsets of $L^\Phi(G)$. Since the space $C_c(G)$ of all continuous functions on G with compact support is dense in $L^\Phi(G)$, there are $f, g \in C_c(G)$ such that $f \in U$ and $g \in V$. Let K be the union of supports of f and g . Given $\varepsilon > 0$, by condition (ii), there exist (E_k) and (n_k) such that, for some k , we have

$$\begin{aligned} \|f\|_\infty \cdot \sup_{v \in \Omega} \int_{K \setminus E_k} |v(x)| \, d\lambda(x) &< \varepsilon, \\ \|g\|_\infty \cdot \sup_{v \in \Omega} \int_{K \setminus E_k} |v(x)| \, d\lambda(x) &< \varepsilon, \\ \|f\|_\infty \cdot \sup_{v \in \Omega} \int_{E_k} \varphi_{n_k}(x) |v(xa^{n_k})| \, d\lambda(x) &< \varepsilon \end{aligned}$$

and

$$\|g\|_\infty \cdot \sup_{v \in \Omega} \int_{E_k} \tilde{\varphi}_{n_k}(x) |v(xa^{-n_k})| \, d\lambda(x) < \varepsilon.$$

Therefore

$$\begin{aligned} \|T_{a,w}^{n_k}(f \chi_{E_k})\|_\Phi &= \sup_{v \in \Omega} \int_G |T_{a,w}^{n_k}(f \chi_{E_k})(x)v(x)| \, d\lambda(x) \\ &= \sup_{v \in \Omega} \int_G |w(x)w(xa^{-1}) \cdots w(xa^{-n_k+1})f(xa^{-n_k})\chi_{E_k}(xa^{-n_k})v(x)| \, d\lambda(x) \\ &= \sup_{v \in \Omega} \int_G |w(xa^{n_k})w(xa^{n_k-1}) \cdots w(xa)f(x)\chi_{E_k}(x)v(xa^{n_k})| \, d\lambda(x) \\ &\leq \|f\|_\infty \cdot \sup_{v \in \Omega} \int_{E_k} \varphi_{n_k}(x) |v(xa^{n_k})| \, d\lambda(x) < \varepsilon. \end{aligned}$$

Likewise,

$$\begin{aligned} \|S_{a,w}^{n_k}(g \chi_{E_k})\|_\Phi &= \sup_{v \in \Omega} \int_G |S_{a,w}^{n_k}(g \chi_{E_k})(x)v(x)| \, d\lambda(x) \\ &= \sup_{v \in \Omega} \int_G \frac{1}{|w(xa)w(xa^2) \cdots w(xa^{n_k})|} |g(xa^{n_k})\chi_{E_k}(xa^{n_k})v(x)| \, d\lambda(x) \\ &= \sup_{v \in \Omega} \int_G \frac{1}{|w(xa^{-n_k+1})w(xa^{-n_k+2}) \cdots w(x)|} |g(x)\chi_{E_k}(x)v(xa^{-n_k})| \, d\lambda(x) \\ &\leq \|g\|_\infty \cdot \sup_{v \in \Omega} \int_{E_k} \tilde{\varphi}_{n_k}(x) |v(xa^{-n_k})| \, d\lambda(x) < \varepsilon. \end{aligned}$$

Also,

$$\begin{aligned} \|f - f \chi_{E_k}\|_\Phi &= \sup_{v \in \Omega} \int_G |f(x) - f(x)\chi_{E_k}(x)| |v(x)| \, d\lambda(x) \\ &= \sup_{v \in \Omega} \int_G |f(x)\chi_{K \setminus E_k}(x)| |v(x)| \, d\lambda(x) \\ &= \sup_{v \in \Omega} \int_{K \setminus E_k} |f(x)| |v(x)| \, d\lambda(x) \\ &\leq \|f\|_\infty \cdot \int_{K \setminus E_k} |v(x)| \, d\lambda(x) < \varepsilon. \end{aligned}$$

Similarly,

$$\|g - g\chi_{E_k}\|_{\Phi} \leq \|g\|_{\infty} \cdot \int_{K \setminus E_k} |v(x)| d\lambda(x) < \varepsilon.$$

Now let

$$v_k = f\chi_{E_k} + S_{a,w}^{n_k}(g\chi_{E_k}).$$

Then, by the Minkowski inequality,

$$\|v_k - f\|_{\Phi} \leq \|f\chi_{E_k} - f\|_{\Phi} + \|S_{a,w}^{n_k}(g\chi_{E_k})\|_{\Phi} < 2\varepsilon$$

and

$$\|T_{a,w}^{n_k}v_k - g\|_{\Phi} \leq \|T_{a,w}^{n_k}(f\chi_{E_k})\|_{\Phi} + \|g\chi_{E_k} - g\|_{\Phi} < 2\varepsilon,$$

which implies

$$T_{a,w}^{n_k}(U) \cap V \neq \emptyset$$

for some k . Therefore $T_{a,w}$ is topologically transitive.

(i) \Rightarrow (ii). Let $\varepsilon \in (0, 1)$. Let K be a compact subset of G , and let χ_K be the characteristic function of K . By the assumption of topological transitivity, there exist $f \in L^{\Phi}(G)$ and some $m \in \mathbb{N}$ such that

$$\|f - \chi_K\|_{\Phi} < \varepsilon^2 \quad \text{and} \quad \|T_{a,w}^m f + \chi_K\|_{\Phi} < \varepsilon^2.$$

Without loss of generality, we may assume that f is real-valued by the continuity of the mapping $h \in L^{\Phi}(G, \mathbb{C}) \mapsto \text{Re } h \in L^{\Phi}(G, \mathbb{R})$ and the fact that $T_{a,w}$ commutes with it. Also, the mapping $h \in L^{\Phi}(G, \mathbb{R}) \mapsto h^+ \in L^{\Phi}(G, \mathbb{R})$ commutes with $T_{a,w}$ where $h^+ = \max\{0, h\}$. Therefore, for a Borel set $F \subset G$, we have

$$\begin{aligned} \|(T_{a,w}^m f^+) \chi_F\|_{\Phi} &\leq \|(T_{a,w}^m f)^+\|_{\Phi} = \|(T_{a,w}^m f - (-\chi_K) + (-\chi_K))^+\|_{\Phi} \\ &\leq \|(T_{a,w}^m f - (-\chi_K))^+\|_{\Phi} + \|(-\chi_K)^+\|_{\Phi} \\ &= \|(T_{a,w}^m f - (-\chi_K))^+\|_{\Phi} \leq \|T_{a,w}^m f + \chi_K\|_{\Phi} < \varepsilon^2 \end{aligned}$$

and

$$\begin{aligned} \|f^- \chi_F\|_{\Phi} &\leq \|f^-\|_{\Phi} = \|(f - \chi_K + \chi_K)^-\|_{\Phi} \\ &\leq \|(f - \chi_K)^-\|_{\Phi} + \|\chi_K^-\|_{\Phi} \\ &= \|f - \chi_K\|_{\Phi} < \varepsilon^2, \end{aligned}$$

where $f^- = \max\{0, -f\}$.

Let $A = \{x \in K : |f(x) - 1| \geq \varepsilon\}$. Then

$$f(x) > 1 - \varepsilon > 0 \quad (x \in K \setminus A) \quad \text{and} \quad \sup_{v \in \Omega} \int_A |v(x)| \, d\lambda(x) < \varepsilon$$

by

$$\begin{aligned} \varepsilon^2 &> \|f - \chi_K\|_\Phi \\ &= \sup_{v \in \Omega} \int_G |f(x) - \chi_K(x)| |v(x)| \, d\lambda(x) \\ &\geq \sup_{v \in \Omega} \int_A |f(x) - 1| |v(x)| \, d\lambda(x) \\ &> \sup_{v \in \Omega} \int_A \varepsilon |v(x)| \, d\lambda(x). \end{aligned}$$

Let $B_m = \{x \in K : |T_{a,w}^m f(x) + 1| \geq \varepsilon\}$. Then

$$T_{a,w}^m f(x) < \varepsilon - 1 < 0 \quad (x \in K \setminus B_m) \quad \text{and} \quad \sup_{v \in \Omega} \int_{B_m} |v(x)| \, d\lambda(x) < \varepsilon$$

by the following estimate:

$$\begin{aligned} \varepsilon^2 &> \|T_{a,w}^m f + \chi_K\|_\Phi \\ &= \sup_{v \in \Omega} \int_G |T_{a,w}^m f(x) + \chi_K(x)| |v(x)| \, d\lambda(x) \\ &\geq \sup_{v \in \Omega} \int_{B_m} |T_{a,w}^m f(x) + 1| |v(x)| \, d\lambda(x) \\ &> \sup_{v \in \Omega} \int_{B_m} \varepsilon |v(x)| \, d\lambda(x). \end{aligned}$$

Let $E_m = K \setminus (A \cup B_m)$. Then

$$\begin{aligned} \varepsilon^2 &> \|(T_{a,w}^m f^+) \chi_{E_m a^m}\|_\Phi \\ &= \sup_{v \in \Omega} \int_{E_m a^m} |T_{a,w}^m f^+(x)| |v(x)| \, d\lambda(x) \\ &= \sup_{v \in \Omega} \int_{E_m a^m} |w(x)w(xa^{-1}) \cdots w(xa^{-m+1})f^+(xa^{-m})| |v(x)| \, d\lambda(x) \\ &= \sup_{v \in \Omega} \int_{E_m} |w(xa^m)w(xa^{m-1}) \cdots w(xa)f^+(x)| |v(xa^m)| \, d\lambda(x) \\ &= \sup_{v \in \Omega} \int_{E_m} \varphi_m(x)f^+(x) |v(xa^m)| \, d\lambda(x) \\ &> \sup_{v \in \Omega} \int_{E_m} (1 - \varepsilon)\varphi_m(x) |v(xa^m)| \, d\lambda(x). \end{aligned}$$

Hence

$$\sup_{v \in \Omega} \int_{E_m} \varphi_m(x) |v(xa^m)| \, d\lambda(x) < \frac{\varepsilon^2}{1 - \varepsilon}.$$

Similarly,

$$\sup_{v \in \Omega} \int_{E_m} \tilde{\varphi}_m(x) |v(xa^{-m})| d\lambda(x) < \frac{\varepsilon^2}{1 - \varepsilon}$$

by

$$\begin{aligned} \varepsilon^2 &> \|f^- \chi_{E_m a^{-m}}\|_{\Phi} \\ &= \sup_{v \in \Omega} \int_{E_m a^{-m}} |S_{a,w}^m(T_{a,w}^m f^-)(x)| |v(x)| d\lambda(x) \\ &= \sup_{v \in \Omega} \int_{E_m a^{-m}} |w(xa)w(xa^2) \cdots w(xa^m)(T_{a,w}^m f^-)(xa^m)| |v(x)| d\lambda(x) \\ &= \sup_{v \in \Omega} \int_{E_m} |w(xa^{-m+1})w(xa^{-m+2}) \cdots w(x)(T_{a,w}^m f^-)(x)| |v(xa^{-m})| d\lambda(x) \\ &= \sup_{v \in \Omega} \int_{E_m} \tilde{\varphi}_m(x) (T_{a,w}^m f^-)(x) |v(xa^{-m})| d\lambda(x) \\ &> \sup_{v \in \Omega} \int_{E_m} (1 - \varepsilon) \tilde{\varphi}_m(x) |v(xa^{-m})| d\lambda(x). \end{aligned}$$

Also, we have

$$\begin{aligned} &\sup_{v \in \Omega} \int_{K \setminus E_m} |v(x)| d\lambda(x) \\ &= \sup_{v \in \Omega} \int_{A \cup B_m} |v(x)| d\lambda(x) \\ &\leq \sup_{v \in \Omega} \int_A |v(x)| d\lambda(x) + \sup_{v \in \Omega} \int_{B_m} |v(x)| d\lambda(x) \\ &< \varepsilon + \varepsilon = 2\varepsilon. \end{aligned}$$

Combining all these, condition (ii) follows. □

Replacing the subsequence (n_k) by the full sequence (n) in condition (ii) of Theorem 2.1, the characterization for $T_{a,w}$ to be topologically mixing on $L^\Phi(G)$ is obtained below.

Corollary 2.2 *Let G be a locally compact group and let $a \in G$. Let w be a weight on G , and let Φ be a Young function. Let $T_{a,w}$ be a weighted translation on $L^\Phi(G)$. Then the following conditions are equivalent.*

- (i) $T_{a,w}$ is topologically mixing on $L^\Phi(G)$.
- (ii) For each compact subset K of G with $\lambda(K) > 0$, there exists a sequence of Borel sets (E_n) in K such that

$$\lim_{n \rightarrow \infty} \sup_{v \in \Omega} \int_{K \setminus E_n} |v(x)| d\lambda(x) = 0,$$

and the two sequences

$$\varphi_n := \prod_{j=1}^n w * \delta_{a^{-1}}^j \quad \text{and} \quad \tilde{\varphi}_n := \left(\prod_{j=0}^{n-1} w * \delta_a^j \right)^{-1}$$

satisfy

$$\limsup_{n \rightarrow \infty} \int_{E_n} \varphi_n(x) |v(xa^n)| d\lambda(x) = 0$$

and

$$\limsup_{n \rightarrow \infty} \int_{E_n} \tilde{\varphi}_n(x) |v(xa^{-n})| d\lambda(x) = 0,$$

where Ω is the set of all Borel functions v on G satisfying $\int_G \Psi(|v|) d\lambda \leq 1$.

Proof (ii) \Rightarrow (i). As in the proof of Theorem 2.1, let

$$v_n = f\chi_{E_n} + S_{a,w}^n(g\chi_{E_n}).$$

Then

$$\|v_n - f\|_\Phi \rightarrow 0 \quad \text{and} \quad \|T_{a,w}^n v_n - g\|_\Phi \rightarrow 0$$

as $n \rightarrow \infty$. Hence $T_{a,w}$ is topologically mixing.

(i) \Rightarrow (ii). Let $\varepsilon \in (0, 1)$. Let K be a compact subset of G , and let χ_K be the characteristic function of K . By the assumption of topological mixing, there exists $f \in L^\Phi(G)$ such that

$$\|f - \chi_K\|_\Phi < \varepsilon^2 \quad \text{and} \quad \|T_{a,w}^n f + \chi_K\|_\Phi < \varepsilon^2.$$

Applying a similar argument as in the proof of Theorem 2.1, one can obtain condition (ii). □

3 Disjoint topological transitivity

In this section, we turn our attention to give sufficient conditions and necessary conditions for powers of weighted translations on $L^\Phi(G)$ to be disjoint topologically transitive and disjoint topologically mixing.

Theorem 3.1 *Let G be a locally compact group, and let $a \in G$. Let Φ be a Young function. Given some $N \geq 2$, let $T_l = T_{a,w_l}$ be a weighted translation on $L^\Phi(G)$, generated by a weight w_l on G for $1 \leq l \leq N$. Let $r_l \in \mathbb{N}$ for $1 \leq l \leq N$. For $1 \leq r_1 < r_2 < \dots < r_N$, (ii) implies (i).*

- (i) $T_1^{r_1}, T_2^{r_2}, \dots, T_N^{r_N}$ are disjoint topologically transitive on $L^\Phi(G)$.
- (ii) For each compact subset K of G with $\lambda(K) > 0$, there exist a sequence of Borel sets (E_k) in K and a strictly increasing sequence $(n_k) \subset \mathbb{N}$ such that

$$\limsup_{k \rightarrow \infty} \int_{K \setminus E_k} |v(x)| d\lambda(x) = 0,$$

and the two sequences

$$\varphi_{l,r_l n} := \prod_{j=1}^{r_l n} w_l * \delta_{a^{-1}}^j \quad \text{and} \quad \tilde{\varphi}_{l,r_l n} := \left(\prod_{j=0}^{r_l n-1} w_l * \delta_a^j \right)^{-1}$$

satisfy (for $1 \leq l \leq N$)

$$\limsup_{k \rightarrow \infty} \int_{v \in \Omega} \int_{E_k} \varphi_{l,r_1 n_k}(x) |v(xa^{r_1 n_k})| d\lambda(x) = 0,$$

$$\limsup_{k \rightarrow \infty} \int_{v \in \Omega} \int_{E_k} \tilde{\varphi}_{l,r_1 n_k}(x) |v(xa^{-r_1 n_k})| d\lambda(x) = 0,$$

and (for $1 \leq s < l \leq L$)

$$\limsup_{k \rightarrow \infty} \int_{v \in \Omega} \int_{E_k} \frac{\tilde{\varphi}_{s,(r_l-r_s)n_k} \cdot \tilde{\varphi}_{l,r_1 n_k}(x)}{\tilde{\varphi}_{s,r_1 n_k}} |v(xa^{r_s n_k - r_1 n_k})| d\lambda(x) = 0,$$

$$\limsup_{k \rightarrow \infty} \int_{v \in \Omega} \int_{E_k} \frac{\varphi_{l,(r_l-r_s)n_k} \cdot \tilde{\varphi}_{s,r_s n_k}(x)}{\tilde{\varphi}_{l,r_s n_k}} |v(xa^{r_1 n_k - r_s n_k})| d\lambda(x) = 0,$$

where Ω is the set of all Borel functions v on G satisfying $\int_G \Psi(|v|) d\lambda \leq 1$.

Proof We show that $T_1^{r_1}, T_2^{r_2}, \dots, T_N^{r_N}$ are disjoint topologically transitive. For $1 \leq l \leq N$, let U and V_l be non-empty open subsets of $L^\Phi(G)$. Since the space $C_c(G)$ of continuous functions on G with compact support is dense in $L^\Phi(G)$, we can pick $f, g_l \in C_c(G)$ with $f \in U$ and $g_l \in V_l$ for $l = 1, 2, \dots, N$. Let K be the union of the compact supports of f and all g_l . Let $E_k \subset K$ and the sequences $(\varphi_{l,r_1 n_k}), (\tilde{\varphi}_{l,r_1 n_k})$ satisfy condition (ii).

First, for $1 \leq l \leq N$, we have

$$\begin{aligned} & \|T_l^{r_1 n_k}(f \chi_{E_k})\|_\Phi \\ &= \sup_{v \in \Omega} \int_G |T_l^{r_1 n_k}(f \chi_{E_k})(x)v(x)| d\lambda(x) \\ &= \sup_{v \in \Omega} \int_G |w_l(x)w_l(xa^{-1}) \cdots w_l(xa^{-r_1 n_k + 1})f(xa^{-r_1 n_k})\chi_{E_k}(xa^{-r_1 n_k})v(x)| d\lambda(x) \\ &= \sup_{v \in \Omega} \int_G |w_l(xa^{r_1 n_k})w_l(xa^{r_1 n_k - 1}) \cdots w_l(xa)f(x)\chi_{E_k}(x)v(xa^{r_1 n_k})| d\lambda(x) \\ &\leq \|f\|_\infty \cdot \sup_{v \in \Omega} \int_{E_k} \varphi_{l,r_1 n_k}(x) |v(xa^{r_1 n_k})| d\lambda(x) \rightarrow 0 \end{aligned}$$

as $k \rightarrow \infty$. Here we denote S_{a,w_l} by S_l . Applying similar arguments to the iterates $S_l^{r_1 n_k}$, and using the sequence $(\tilde{\varphi}_{l,r_1 n_k})$, for $1 \leq l \leq N$, we have

$$\begin{aligned} & \|S_l^{r_1 n_k}(g_l \chi_{E_k})\|_\Phi \\ &= \sup_{v \in \Omega} \int_G |S_l^{r_1 n_k}(g_l \chi_{E_k})(x)v(x)| d\lambda(x) \\ &= \sup_{v \in \Omega} \int_G \frac{1}{|w_l(xa)w_l(xa^2) \cdots w_l(xa^{r_1 n_k})|} |g_l(xa^{r_1 n_k})\chi_{E_k}(xa^{r_1 n_k})v(x)| d\lambda(x) \\ &= \sup_{v \in \Omega} \int_G \frac{1}{|w_l(xa^{-r_1 n_k + 1})w_l(xa^{-r_1 n_k + 2}) \cdots w_l(x)|} |g_l(x)\chi_{E_k}(x)v(xa^{-r_1 n_k})| d\lambda(x) \\ &\leq \|g_l\|_\infty \cdot \sup_{v \in \Omega} \int_{E_k} \tilde{\varphi}_{l,r_1 n_k}(x) |v(xa^{-r_1 n_k})| d\lambda(x) \rightarrow 0 \end{aligned}$$

as $k \rightarrow \infty$. Moreover, for $1 \leq s < l \leq N$, we have

$$\begin{aligned} & \|T_l^{r_l n_k} (S_s^{r_s n_k} g_s \chi_{E_k})\|_{\Phi} \\ &= \sup_{v \in \Omega} \int_G |w_l(x)w_l(xa^{-1}) \dots w_l(xa^{-(r_l n_k - 1)})| |S_s^{r_s n_k} g_s \chi_{E_k}(xa^{-r_l n_k})v(x)| d\lambda(x) \\ &= \sup_{v \in \Omega} \int_G \frac{|w_l(x)w_l(xa^{-1}) \dots w_l(xa^{-(r_l n_k - 1)})|}{|w_s(xa^{-r_l n_k + 1})w_s(xa^{-r_l n_k + 2}) \dots w_s(xa^{-r_l n_k + r_s n_k})|} \\ &\quad \times |g_s \chi_{E_k}(xa^{-r_l n_k + r_s n_k})v(x)| d\lambda(x) \\ &= \sup_{v \in \Omega} \int_{E_k} \frac{|w_l(xa^{-(r_s - r_l)n_k})w_l(xa^{-(r_s - r_l)n_k - 1}) \dots w_l(xa^{-(r_s n_k - 1)})|}{|w_s(xa^{-(r_s n_k - 1)})w_s(xa^{-(r_s n_k - 2)}) \dots w_s(x)|^p} \\ &\quad \times |g_s(x)v(xa^{r_l n_k - r_s n_k})| d\lambda(x) \\ &= \sup_{v \in \Omega} \int_{E_k} \frac{\varphi_{l,(r_l - r_s)n_k}(x) \cdot \tilde{\varphi}_{s,r_s n_k}(x)}{\tilde{\varphi}_{l,r_s n_k}(x)} |g_s(x)v(xa^{r_l n_k - r_s n_k})| d\lambda(x) \\ &\leq \|g_s\|_{\infty} \cdot \sup_{v \in \Omega} \int_{E_k} \frac{\varphi_{l,(r_l - r_s)n_k}(x) \cdot \tilde{\varphi}_{s,r_s n_k}(x)}{\tilde{\varphi}_{l,r_s n_k}(x)} |v(xa^{r_l n_k - r_s n_k})| d\lambda(x) \rightarrow 0 \end{aligned}$$

as $k \rightarrow \infty$. Similarly, we have

$$\begin{aligned} & \|T_s^{r_s n_k} (S_l^{r_l n_k} g_l \chi_{E_k})\|_{\Phi} \\ &= \sup_{v \in \Omega} \int_G |w_s(x)w_s(xa^{-1}) \dots w_s(xa^{-(r_s n_k - 1)})| |S_l^{r_l n_k} g_l \chi_{E_k}(xa^{-r_s n_k})v(x)| d\lambda(x) \\ &= \sup_{v \in \Omega} \int_G \frac{|w_s(x)w_s(xa^{-1}) \dots w_s(xa^{-(r_s n_k - 1)})|}{|w_l(xa^{-r_s n_k + 1})w_l(xa^{-r_s n_k + 2}) \dots w_l(xa^{-r_s n_k + r_l n_k})|} \\ &\quad \times |g_l \chi_{E_k}(xa^{-r_s n_k + r_l n_k})v(x)| d\lambda(x) \\ &= \sup_{v \in \Omega} \int_{E_k} \frac{|w_s(xa^{-(r_l - r_s)n_k})w_s(xa^{-(r_l - r_s)n_k - 1}) \dots w_s(xa^{-(r_l n_k - 1)})|}{|w_l(xa^{-(r_l n_k - 1)})w_l(xa^{-(r_l n_k - 2)}) \dots w_l(x)|} \\ &\quad \times |g_l(x)v(xa^{-(r_l n_k - r_s n_k)})| d\lambda(x) \\ &= \sup_{v \in \Omega} \int_{E_k} \frac{\tilde{\varphi}_{l,(r_l - r_s)n_k}(x) \cdot \tilde{\varphi}_{s,r_l n_k}(x)}{\tilde{\varphi}_{s,r_l n_k}(x)} |g_l(x)v(xa^{-(r_l n_k - r_s n_k)})| d\lambda(x) \\ &\leq \|g_l\|_{\infty} \cdot \sup_{v \in \Omega} \int_{E_k} \frac{\tilde{\varphi}_{l,(r_l - r_s)n_k}(x) \cdot \tilde{\varphi}_{s,r_l n_k}(x)}{\tilde{\varphi}_{s,r_l n_k}(x)} |v(xa^{-(r_l n_k - r_s n_k)})| d\lambda(x) \rightarrow 0 \end{aligned}$$

as $k \rightarrow \infty$. Also, as in the proof of Theorem 2.1, we have

$$\lim_{k \rightarrow \infty} \|f \chi_{E_k} - f\|_{\Phi} = \lim_{k \rightarrow \infty} \|g_l \chi_{E_k} - g_l\|_{\Phi} = 0$$

by the condition

$$\lim_{k \rightarrow \infty} \sup_{v \in \Omega} \int_{K \setminus E_k} |v(x)| d\lambda(x) = 0.$$

Now for each $k \in \mathbb{N}$, we let

$$v_k = f \chi_{E_k} + S_1^{r_1 n k} (g_1 \chi_{E_k}) + S_2^{r_2 n k} (g_2 \chi_{E_k}) + \dots + S_N^{r_N n k} (g_N \chi_{E_k}).$$

Then, by using the Minkowski inequality, we arrive at

$$\|v_k - f\|_\Phi \leq \|f \chi_{E_k} - f\|_\Phi + \sum_{l=1}^N \|S_l^{r_l n k} (g_l \chi_{E_k})\|_\Phi$$

and

$$\begin{aligned} & \|T_l^{r_l n k} v_k - g_l\|_\Phi \\ & \leq \|T_l^{r_l n k} (f \chi_{E_k})\|_\Phi + \|T_l^{r_l n k} S_1^{r_1 n k} (g_1 \chi_{E_k})\|_\Phi + \dots + \|T_l^{r_l n k} S_{l-1}^{r_{l-1} n k} (g_{l-1} \chi_{E_k})\|_\Phi \\ & \quad + \|g_l \chi_{E_k} - g_l\|_\Phi + \|T_l^{r_l n k} S_{l+1}^{r_{l+1} n k} (g_{l+1} \chi_{E_k})\|_\Phi + \dots + \|T_l^{r_l n k} S_N^{r_N n k} (g_N \chi_{E_k})\|_\Phi. \end{aligned}$$

Hence $\lim_{k \rightarrow \infty} v_k = f$ and $\lim_{k \rightarrow \infty} T_l^{r_l n k} v_k = g_l$ for $l = 1, 2, \dots, N$, which implies

$$\emptyset \neq U \cap T_1^{-r_1 n k} (V_1) \cap T_2^{-r_2 n k} (V_2) \cap \dots \cap T_N^{-r_N n k} (V_N). \quad \square$$

As in Sect. 2, one can strengthen the weight condition to characterize disjoint topological mixing.

Corollary 3.2 *Let G be a locally compact group, and let $a \in G$. Let Φ be a Young function. Given some $N \geq 2$, let $T_l = T_{a, w_l}$ be a weighted translation on $L^\Phi(G)$, generated by a weight w_l on G for $1 \leq l \leq N$. Let $r_l \in \mathbb{N}$ for $1 \leq l \leq N$. For $1 \leq r_1 < r_2 < \dots < r_N$, we have (ii) implies (i).*

- (i) $T_1^{r_1}, T_2^{r_2}, \dots, T_N^{r_N}$ are disjoint topologically mixing on $L^\Phi(G)$.
- (ii) For each compact subset K of G with $\lambda(K) > 0$, there exists a sequence of Borel sets (E_n) in K such that

$$\lim_{n \rightarrow \infty} \sup_{v \in \Omega} \int_{K \setminus E_n} |v(x)| d\lambda(x) = 0,$$

and the two sequences

$$\varphi_{l, r_l n} := \prod_{j=1}^{r_l n} w_l * \delta_{a^{-1}}^j \quad \text{and} \quad \tilde{\varphi}_{l, r_l n} := \left(\prod_{j=0}^{r_l n - 1} w_l * \delta_a^j \right)^{-1}$$

satisfy (for $1 \leq l \leq N$)

$$\begin{aligned} & \lim_{n \rightarrow \infty} \sup_{v \in \Omega} \int_{E_n} \varphi_{l, r_l n}(x) |v(x a^{r_l n})| d\lambda(x) = 0, \\ & \lim_{n \rightarrow \infty} \sup_{v \in \Omega} \int_{E_n} \tilde{\varphi}_{l, r_l n}(x) |v(x a^{-r_l n})| d\lambda(x) = 0, \end{aligned}$$

and (for $1 \leq s < l \leq N$)

$$\lim_{n \rightarrow \infty} \sup_{v \in \Omega} \int_{E_n} \frac{\tilde{\varphi}_{s, (r_l - r_s)n} \cdot \tilde{\varphi}_{l, r_l n}}{\tilde{\varphi}_{s, r_l n}}(x) |v(x a^{r_s n - r_l n})| d\lambda(x) = 0,$$

$$\limsup_{n \rightarrow \infty} \sup_{v \in \Omega} \int_{E_n} \frac{\varphi_{l,(r_l-r_s)n} \cdot \tilde{\varphi}_{s,r_s n}(x)}{\tilde{\varphi}_{l,r_s n}} |v(xa^{r_l n - r_s n})| d\lambda(x) = 0,$$

where Ω is the set of all Borel functions v on G satisfying $\int_G \Psi(|v|) d\lambda \leq 1$.

Apart from the above sufficient condition for disjoint topological transitivity, one can deduce a necessary condition below.

Theorem 3.3 *Let G be a locally compact group, and let $a \in G$. Let Φ be a Young function. Given some $N \geq 2$, let $T_l = T_{a,w_l}$ be a weighted translation on $L^\Phi(G)$, generated by a weight w_l on G for $1 \leq l \leq N$. Let $r_l \in \mathbb{N}$ for $1 \leq l \leq N$. For $1 \leq r_1 < r_2 < \dots < r_N$, we have (i) implies (ii).*

- (i) $T_1^{r_1}, T_2^{r_2}, \dots, T_N^{r_N}$ are disjoint topologically transitive on $L^\Phi(G)$.
- (ii) For each compact subset K of G with $\lambda(K) > 0$, there exist a sequence of Borel sets (E_k) in K and a strictly increasing sequence $(n_k) \subset \mathbb{N}$ such that

$$\limsup_{k \rightarrow \infty} \sup_{v \in \Omega} \int_{K \setminus E_k} |v(x)| d\lambda(x) = 0,$$

and the two sequences

$$\varphi_{l,r_l n} := \prod_{j=1}^{r_l n} w_l * \delta_{a^{-1}}^j \quad \text{and} \quad \tilde{\varphi}_{l,r_l n} := \left(\prod_{j=0}^{r_l n - 1} w_l * \delta_a^j \right)^{-1}$$

satisfy (for $1 \leq l \leq N$)

$$\begin{aligned} \limsup_{k \rightarrow \infty} \sup_{v \in \Omega} \int_{E_k} \varphi_{l,r_l n_k}(x) |v(xa^{r_l n_k})| d\lambda(x) &= 0, \\ \limsup_{k \rightarrow \infty} \sup_{v \in \Omega} \int_{E_k} \tilde{\varphi}_{l,r_l n_k}(x) |v(xa^{-r_l n_k})| d\lambda(x) &= 0, \end{aligned}$$

where Ω is the set of all Borel functions v on G satisfying $\int_G \Psi(|v|) d\lambda \leq 1$.

Proof Let $T_1^{r_1}, T_2^{r_2}, \dots, T_N^{r_N}$ be disjoint transitive. Let $\varepsilon \in (0, 1)$, and let $K \subset G$ be a compact set with $\lambda(K) > 0$. Then there exist $f \in L^\Phi(G)$ and some $m \in \mathbb{N}$ such that

$$\|f - \chi_K\|_\Phi < \varepsilon^2 \quad \text{and} \quad \|T_l^{r_l m} f + \chi_K\|_\Phi < \varepsilon^2.$$

Let

$$A = \{x \in K : |f(x) - 1| \geq \varepsilon\} \quad \text{and} \quad B_{l,m} = \{x \in K : |T_l^{r_l m} f(x) + 1| \geq \varepsilon\}.$$

Let

$$E_k = (K \setminus A) \setminus \bigcup_{1 \leq l \leq N} B_{l,m}.$$

Then as in the proof of Theorem 2.1, one can obtain

$$\sup_{v \in \Omega} \int_{E_k} \varphi_{l,r_l m}(x) |v(xa^{r_l m})| d\lambda(x) < \frac{\varepsilon^2}{1 - \varepsilon}$$

and

$$\sup_{v \in \Omega} \int_{E_k} \tilde{\varphi}_{l,r_l m}(x) |v(xa^{-r_l m})| d\lambda(x) < \frac{\varepsilon^2}{1 - \varepsilon}.$$

Also, as in the proof of Theorem 2.1,

$$\sup_{v \in \Omega} \int_A |v(x)| d\lambda(x) < \varepsilon \quad \text{and} \quad \sup_{v \in \Omega} \int_{B_{l,m}} |v(x)| d\lambda(x) < \varepsilon.$$

Hence

$$\begin{aligned} & \sup_{v \in \Omega} \int_{K \setminus E_m} |v(x)| d\lambda(x) \\ &= \sup_{v \in \Omega} \int_{A \cup B_{1,m} \cup \dots \cup B_{N,m}} |v(x)| d\lambda(x) \\ &\leq \sup_{v \in \Omega} \int_A |v(x)| d\lambda(x) + \sum_{l=1}^N \sup_{v \in \Omega} \int_{B_{l,m}} |v(x)| d\lambda(x) \\ &< \varepsilon + N\varepsilon = (N + 1)\varepsilon. \end{aligned}$$

Combining all these, condition (ii) follows. □

Similarly, a necessary condition for disjoint topological mixing follows.

Corollary 3.4 *Let G be a locally compact group, and let $a \in G$. Let Φ be a Young function. Given some $N \geq 2$, let $T_l = T_{a,w_l}$ be a weighted translation on $L^\Phi(G)$, generated by a weight w_l on G for $1 \leq l \leq N$. Let $r_l \in \mathbb{N}$ for $1 \leq l \leq N$. For $1 \leq r_1 < r_2 < \dots < r_N$, we have (i) implies (ii).*

- (i) $T_1^{r_1}, T_2^{r_2}, \dots, T_N^{r_N}$ are disjoint topologically mixing on $L^\Phi(G)$.
- (ii) For each compact subset K of G with $\lambda(K) > 0$, there exists a sequence of Borel sets (E_n) in K such that

$$\lim_{n \rightarrow \infty} \sup_{v \in \Omega} \int_{K \setminus E_n} |v(x)| d\lambda(x) = 0,$$

and the two sequences

$$\varphi_{l,r_l n} := \prod_{j=1}^{r_l n} w_l * \delta_{a^{-1}}^j \quad \text{and} \quad \tilde{\varphi}_{l,r_l n} := \left(\prod_{j=0}^{r_l n-1} w_l * \delta_a^j \right)^{-1}$$

satisfy (for $1 \leq l \leq N$)

$$\begin{aligned} & \lim_{n \rightarrow \infty} \sup_{v \in \Omega} \int_{E_n} \varphi_{l,r_l n}(x) |v(xa^{r_l n})| d\lambda(x) = 0, \\ & \lim_{n \rightarrow \infty} \sup_{v \in \Omega} \int_{E_n} \tilde{\varphi}_{l,r_l n}(x) |v(xa^{-r_l n})| d\lambda(x) = 0, \end{aligned}$$

where Ω is the set of all Borel functions v on G satisfying $\int_G \Psi(|v|) d\lambda \leq 1$.

Acknowledgements

The authors deeply thank the reviewers and the editor for the careful reading and numerous helpful suggestions to improve this paper. The first author was supported by grant no. MOST 106-2115-M-142-002 of the Ministry of Science and Technology of the Republic of China; the second author was supported by grant no. MOST 106-2115-M-017-002 of the Ministry of Science and Technology of the Republic of China.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

Both authors contributed equally to this work. Both authors read and approved the final manuscript.

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Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 30 January 2018 Accepted: 6 April 2018 Published online: 16 April 2018

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