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Hybrid CQ projection algorithm with line-search process for the split feasibility problem

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Abstract

In this paper, we propose a hybrid CQ projection algorithm with two projection steps and one Armijo-type line-search step for the split feasibility problem. The line-search technique is intended to construct a hyperplane that strictly separates the current point from the solution set. The next iteration is obtained by the projection of the initial point on a regress region (the intersection of three sets). Hence, algorithm converges faster than some other algorithms. Under some mild conditions, we show the convergence. Preliminary numerical experiments show that our algorithm is efficient.

MSC: 47H05; 47J05; 47J25

Keywords: split feasible problem; Armijo-type line-search technique; projection algorithm; convergence

1 Introduction

Split feasibility problem (SFP) is to find a point x satisfying

$$x \in C, \qquad Ax \in Q, \tag{1.1}$$

where C and Q are nonempty convex sets in $\Re^N$ and $\Re^M$, respectively, and A is an M by N real matrix. The SFP was originally introduced in [1] and has broad applications in many fields, such as image reconstruction problem, signal processing, and radiation therapy [2–4]. Various algorithms have been invented to solve it (see [5–16] and references therein). The well-known CQ algorithm presented in [1] is defined as follows: Denote by P_C the orthogonal projection onto C, that is, $P_C(x) = \arg\min_{y \in C} \|x - y\|$ for $x \in C$; then take an initial point x^0 arbitrarily and define the iterative step by

$$x^{k+1} = P_C\big(I - \gamma A^T(I - P_Q)A\big)\big(x^k\big), \tag{1.2}$$

where $0 < \gamma < 2/\rho(A^T A)$, and $\rho(A^T A)$ is the spectral radius of $A^T A$.

The algorithms mentioned use a fixed stepsize restricted by the Lipschitz constant L, which depends on the largest eigenvalue (spectral radius) of the matrix. We know that

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computing the largest eigenvalue may be very hard and conservative estimate of the constant L usually results in slow convergence. To overcome the difficulty in estimating the Lipschitz constant, He *et al.* [17] developed a selfadaptive method for solving variational problems. The numerical results reported in [17] have shown that the selfadaptive strategy is valid and robust for solving variational inequality problems. Subsequently, many self-adaptive projection methods were presented to solve the split feasibility problem [18, 19]. On the other hand, hybrid projection method was developed by Nakajo and Takahashi [20], Kamimure and Takahashi [21], and Martines-Yanes and Xu [22] to solve the problem of finding a common element of the set of fixed points of a nonexpansive mapping and the set of solutions of an equilibrium problem. Many modified hybrid projection methods were presented to solve different problems [23, 24].

In this paper, motivated by the selfadaptive method and hybrid projection method for solving variational inequality problem, based on the CQ algorithm for the SFP, we propose a hybrid CQ projection algorithm for the split feasibility problem, which uses different variable stepsizes in two projection steps. Algorithm performs a computationally inexpensive Armijo-type linear search along the search direction in order to generate separating hyperplane, which is different from the general selfadaptive Armijo-type procedure [18, 19]. For the second projection step, we select the projection onto the intersection of the set C with the halfspaces, which makes an optimal stepsize available at each iteration and hence guarantees that the next iteration is the 'closest' to the solution set. Therefore, the iterative sequence generated by the algorithm converges more quickly. The algorithm is shown to be convergent to a point in the solution set under some assumptions.

The paper is organized as follows. In Section 2, we recall some preliminaries. In Section 3, we propose a hybrid CQ projection algorithm for the split feasibility problem and show its convergence. In Section 4, we give an example to test the efficiency. In Section 5, we give some concluding remarks.

2 Preliminaries

We denote by I the identity operator and by $\text{Fix}(T)$ the set of fixed points of an operator T, that is, $\text{Fix}(T) := \{x | x = Tx\}$.

Recall that a mapping $T : \Re^n \to \Re$ is said to be monotone if

$$\langle T(x) - T(y), x - y \rangle \geq 0, \quad \forall x, y \in \Re^n.$$

For a monotone mapping T, if $\langle T(x) - T(y), x - y \rangle = 0$ iff $x = y$, then it is said to be strictly monotone.

A mapping $T : \Re^n \to \Re^n$ is called nonexpansive if

$$\| T(x) - T(y) \| \leq \| x - y \|, \quad \forall x, y \in \Re^n.$$

Lemma 2.1 *Let Ω be a nonempty closed and convex subset in H. Then, for any $x, y \in H$ and $z \in \Omega$, it is well known that the following statements hold:*

(1) $\langle P_\Omega(x) - x, z - P_\Omega(x) \rangle \geq 0$.

(2) $\langle P_\Omega(x) - P_\Omega(y), x - y \rangle \geq 0$.

(3) $\|P_\Omega(x) - P_\Omega(y)\| \le \|x - y\|$, $\forall x, y \in \Re^n$, *or more precisely,*

$$\left\|P_\Omega(x) - P_\Omega(y)\right\|^2 \le \|x - y\|^2 - \left\|P_\Omega(x) - x + y - P_\Omega(y)\right\|^2.$$

(4) $\|P_\Omega(x) - z\| \le \|x - z\| - \|P_\Omega(x) - x\|$.

Remark 2.1 In fact, the projection property (1) also provides a sufficient and necessary condition for a vector $u \in K$ to be the projection of the vector x; that is, $u = P_K(x)$ if and only if

$$\langle u - x, z - u \rangle \ge 0, \quad \forall z \in K.$$

Throughout the paper, we by denote Γ the solution set of split feasibility problem, that is,

$$\Gamma := \{y \in C | Ay \in Q\}. \tag{2.1}$$

3 Algorithm and convergence analysis

Let

$$F(x) := \left(A^T(I - P_Q)A\right)(x).$$

From [10] we know that F is Lipschitz-continuous with constant $L = 1/\rho(A^T A)$ and is $\frac{1}{\|A\|}$-inverse strongly monotone. We first note that the solution set coincides with zeros of the following projected residual function:

$$e(x) := x - P_C\left(x - F(x)\right), \qquad e(x, \mu) := x - P_C\left(x - \mu F(x)\right);$$

with this definition, we have $e(x, 1) = e(x)$, and $x \in \Gamma$ if and only if $e(x, \mu) = 0$. For any $x \in \Re^N$ and $\alpha \ge 0$, define

$$x(\alpha) = P_C\left(x - \alpha F(x)\right), \qquad e(x, \alpha) = x - x(\alpha).$$

The following lemma is useful for the convergence analysis in the next section.

Lemma 3.1 [18] *Let F be a mapping from $\Re^N$ into $\Re^N$. For any $x \in \Re^N$ and $\alpha \ge 0$, we have*

$$\min\{1, \alpha\}\left\|e(x, 1)\right\| \le \left\|e(x, \alpha)\right\| \le \max\{1, \alpha\}\left\|e(x, 1)\right\|.$$

The detailed iterative processes are as follows:

Algorithm 3.1

Step 0. Choose an arbitrary initial point $x^0 \in C$, $\eta_0 > 0$, and three parameters $\gamma \in (0, 1)$, $\sigma \in (0, 1)$, and $\theta > 1$, and set $k = 0$.

Step 1. Given the current iterative point x^k, compute

$$z^k = P_C\left[x^k - \mu_k F\left(x^k\right)\right], \tag{3.1}$$

where $\mu_k := \min\{\theta \eta_{k-1}, 1\}$. Obviously, $e(x^k, \mu_k) = x^k - z^k$. If $e(x^k, \mu_k) = 0$, then stop;

Step 2. Compute

$$y^k = x^k - \eta_k e(x^k, \mu_k),$$

where $\eta_k = \gamma^{m_k} \mu_k$ with m_k being the smallest nonnegative integer m satisfying

$$\langle F(x^k - \gamma^k \mu_k e(x^k, \mu_k)), e(x^k, \mu_k) \rangle \geq \frac{\sigma}{\mu_k} \| e(x^k, \mu_k) \|^2. \tag{3.2}$$

Step 3. Compute

$$x^{k+1} = P_{C \cap H_k^1 \cap H_k^2}(x^0), \tag{3.3}$$

where

$$H_k^1 = \{ x \in \Re^N | \langle x - y^k, F(y^k) \rangle \leq 0 \},$$
$$H_k^2 = \{ x \in \Re^N | \langle x - x^k, x^0 - x^k \rangle \leq 0 \}.$$

Select $k = k + 1$ and go to Step 1.

Remark 3.1 (1) In the algorithm, a projection from $\Re^N$ onto the intersection $C \cap H_k^1 \cap H_k^2$ needs to be computed, that is, procedure (3.3) at each iteration. Surely, if the domain set C has a special structure such as a box or a ball, then the next iteration x^{k+1} can easily be computed. If the domain set C is defined by a set of linear (in)equalities, then computing the projection is equivalent to solving a strictly convex quadratic optimization problem.

(2) It can readily be verified that the hyperplane H_k^1 strictly separates the current point x^k from the solution set Γ if x^k is not a solution of the problem. That is, $\Gamma \subset H_k^-$, and the hyperplane H_k^2 strictly separates the initial point x^0 from the solution set Γ.

(3) Compared with the general hybrid projection method in [21, 22], besides the major modification made in the projection domain in the last projection step, the values of some parameters involved in the algorithm are also adjusted.

Before establishing the global convergence of Algorithm 3.1, we first give some lemmas.

Lemma 3.2 *For all $k \geq 0$, there exists a nonnegative number m satisfying* (3.2).

Proof Suppose that, for some k, (3.2) is not satisfied for any integer m, that is,

$$\langle F(x^k - \gamma^m \mu_k e(x^k, \mu_k)), e(x^k, \mu_k) \rangle \leq \frac{\sigma}{\mu_k} \| e(x^k, \mu_k) \|^2. \tag{3.4}$$

By the definition of $e(x^k, \mu_k)$ and Lemma 2.1 we know that

$$\langle P_C(x^k - \mu_k F(x^k)) - (x^k - \mu_k F(x^k)), x^k - P_C(x^k - \mu_k F(x^k)) \rangle \geq 0.$$

Then

$$\langle F(x^k), e(x^k, \mu_k) \rangle \geq \frac{1}{\mu_k} \| e(x^k, \mu_k) \|^2 > 0. \tag{3.5}$$

Since $\gamma \in (0, 1)$ and $\mu_k := \min\{\theta \eta_{k-1}, 1\}$, from (3.4) we get

$$\lim_{m \to \infty} \left(x^k - \gamma^m \mu_k e(x^k, \mu_k) \right) = x^k.$$

Hence,

$$\langle F(x^k), e(x^k, \mu_k) \rangle \leq \frac{\sigma}{\mu_k} \|e(x^k, \mu_k)\|^2. \tag{3.6}$$

But (3.6) contradicts (3.5) because $\sigma < 1$ and $\|e(x^k, \mu_k)\| \geq 0$. Hence, (3.2) is satisfied for some integer m. □

The following lemma shows that the halfspace H_k^1 in Algorithm 3.1 strictly separates x^k from the solution set Γ if Γ is nonempty.

Lemma 3.3 *If $\Gamma \neq \emptyset$, then the halfspace H_k^1 in Algorithm 3.1 separates the point x^k from the set Γ. Moreover,*

$$\Gamma \subset H_k^1 \cap C, \quad \forall k \geq 0.$$

Proof By the definition of $e(x^k, \mu_k)$ and Algorithm 3.1 we have

$$y^k = (1 - \eta_k)x^k + \eta_k z^k = x^k - \eta_k e(x^k, \mu_k),$$

which can be rewritten as

$$\eta_k e(x^k, \mu_k) = x^k - y^k.$$

Then, by this and by (3.2) we get

$$\langle F(y^k), x^k - y^k \rangle > 0. \tag{3.7}$$

Hence, by the definition of H_k^1 and by (3.7) we get $x^k \notin H_k^1$.

On the other hand, for any $x^* \in \Gamma$ and $x \in C$, by the monotonicity of F we have

$$\langle F(x), x - x^* \rangle \geq 0. \tag{3.8}$$

By the convexity of C it is easy to see that $y^k \in C$. Letting $x = y^k$ in (3.9), we have

$$\langle F(y^k), y^k - x^* \rangle \geq 0,$$

which implies $x^* \in H_k^1$. Moreover, it is easy to see that $\Gamma \subseteq H_k^1 \cap C, \forall k \geq 0$. □

The following lemma says that if the solution set is nonempty, then $\Gamma \subset H_k^1 \cap H_k^2 \cap C$ and thus $H_k^1 \cap H_k^2 \cap C$ is a nonempty set.

Lemma 3.4 *If the solution set $\Gamma \neq \emptyset$, then $\Gamma \subset H_k^1 \cap H_k^2 \cap C$ for all $k \geq 0$.*

Proof From the previous analysis it is sufficient to prove that $\Gamma \subset H_k^2$ for all $k \geq 0$. The proof will be given by induction. Obviously, if $k = 0$, then

$$\Gamma \subseteq H_0^2 = \Re^N.$$

Now, suppose that

$$\Gamma \subset H_k^2$$

for $k = l \geq 0$. Then

$$\Gamma \subset H_l^1 \cap H_l^2 \cap C.$$

For any $x^* \in \Gamma$, by Lemma 2.1 and the fact that

$$x^{l+1} = P_{H_l^1 \cap H_l^2 \cap C}(x^0)$$

we have that

$$\langle x^* - x^{l+1}, x^0 - x^{l+1} \rangle \leq 0.$$

Thus, $\Gamma \subset H_{l+1}^2$. This shows that $\Gamma \subset H_k^2$ for all $k \geq 0$, and the desired result follows.

For the case that the solution set is empty, we have that $H_l^1 \cap H_l^2 \cap C$ is also nonempty from the following lemma, which implies the feasibility of Algorithm 3.1. □

Lemma 3.5 *Suppose that $\Gamma = \emptyset$. Then $H_l^1 \cap H_l^2 \cap C \neq \emptyset$ for all $k \geq 0$.*

We next prove our main convergence result.

Theorem 3.1 *Suppose the solution set Γ is nonempty. Then the sequence $\{x^k\}$ generated by Algorithm 3.1 is bounded, and all its cluster points belong to the solution set. Moreover, the sequence $\{x^k\}$ globally converges to a solution x^* such that $x^* = P_\Gamma(x^0)$.*

Proof Take an arbitrary point $x^* \in \Gamma$. Then $x^* \in H_k^1 \cap H_k^2 \cap C$. Since

$$x^{k+1} = P_{H_k^1 \cap H_k^2 \cap C}(x^0),$$

by the definition of the projection we have that

$$\left\| x^{k+1} - x^0 \right\| \leq \left\| x^* - x^0 \right\|.$$

So, $\{x^k\}$ is a bounded sequence, and so is $\{y^k\}$.

Since $x^{k+1} \in H_k^2$, from the definition of the projection operator it is obvious that

$$P_{H_k^2}(x^{k+1}) = x^{k+1}.$$

For x^k, by the definition of H_k^2, for all $z \in H_k^2$, we have

$$\langle z - x^k, x^0 - x^k \rangle \leq 0.$$

Obviously, $x^k = P_{H_k^2}(x^0)$. Thus, using Lemma 2.1, we have

$$\left\| P_{H_k^2}(x^{k+1}) - P_{H_k^2}(x^0) \right\|^2 \leq \left\| x^{k+1} - x^0 \right\|^2 - \left\| P_{H_k^2}(x^{k+1}) - x^{k+1} + x^0 - P_{H_k^2}(x^0) \right\|,$$

that is,

$$\left\| x^{k+1} - x^k \right\|^2 \leq \left\| x^{k+1} - x^0 \right\|^2 - \left\| x^k - x^0 \right\|^2,$$

which can be written as

$$\left\| x^{k+1} - x^k \right\|^2 + \left\| x^k - x^0 \right\|^2 \leq \left\| x^{k+1} - x^0 \right\|^2.$$

Thus, the sequence $\{\|x^k - x^0\|\}$ is nondecreasing and bounded and hence convergent, which implies that

$$\lim_{k \to \infty} \left\| x^{k+1} - x^k \right\|^2 = 0. \tag{3.9}$$

On the other hand, by $x^{k+1} \in H_k^1$ we get

$$\langle x^{k+1} - y^k, F(y^k) \rangle \leq 0. \tag{3.10}$$

Since

$$y^k = (1 - \eta_k)x^k + \eta_k z^k = x^k - \eta_k e(x^k, \mu_k),$$

by (3.10) we have

$$\langle x^{k+1} - y^k, F(y^k) \rangle = \langle x^{k+1} - x^k + \eta_k e(x^k, \mu_k), F(y^k) \rangle \leq 0,$$

which implies

$$\eta_k \langle e(x^k, \mu_k), F(y^k) \rangle \leq \langle x^k - x^{k+1}, F(y^k) \rangle \leq 0.$$

Using the Cauchy-Schwarz inequality and (3.2), we obtain

$$\eta_k \frac{\sigma}{\mu_k} \left\| e(x^k, \mu_k) \right\|^2 \leq \eta_k \langle e(x^k, \mu_k), F(y^k) \rangle \leq \left\| x^k - x^{k+1} \right\| \cdot \left\| F(y^k) \right\|. \tag{3.11}$$

By Lemma 3.1,

$$\left\| e(x^k, \mu^k) \right\| \geq \min\{1, \mu_k\} \left\| e(x^k) \right\|.$$

Since $\mu_k = \min\{\theta\eta_{k-1}, 1\}$, we have $\mu_k \le 1$. Hence,

$$\left\|e(x^k, \mu^k)\right\| \ge \min\{1, \mu_k\}\left\|e(x^k)\right\| \ge \mu_k\left\|e(x^k)\right\|.$$

Therefore,

$$\eta_k\mu_k\sigma\left\|e(x^k)\right\|^2 \le \eta_k\frac{\sigma}{\mu_k}\left\|e(x^k, \mu_k)\right\|^2 \le \left\|x^k - x^{k+1}\right\| \cdot \left\|F(y^k)\right\|.$$

Taking into account that $\eta_k = \gamma^{m_k}\mu_k \le \mu_k$, we further obtain

$$\eta_k^2\sigma\left\|e(x^k)\right\|^2 \le \eta_k\frac{\sigma}{\mu_k}\left\|e(x^k, \mu_k)\right\|^2 \le \left\|x^k - x^{k+1}\right\| \cdot \left\|F(y^k)\right\|. \tag{3.12}$$

Since F is continuous, there exists a constant M such that $\|F(y^k)\| \le M$. By (3.9) and (3.12) it follows that

$$\lim_{k\to\infty} \eta_k\left\|e(x^k)\right\| = 0. \tag{3.13}$$

For any convergent subsequence $\{x^{k_j}\}$ of $\{x^k\}$, its limit is denoted by $\bar{x}$, that is,

$$\lim_{j\to\infty} x^{k_j} = \bar{x}.$$

Now, we consider the two possible cases for (3.13).
Suppose first that $\{\eta_{k_j}\}$ has a limit. Then

$$\lim_{j\to\infty} \eta_{k_j} = 0.$$

By the choice of η_{k_j} in Algorithm 3.1 we know that

$$\left\langle F\left(x^{k_j} - \frac{\eta_{k_j}}{\gamma}e(x^{k_j}, \mu_{k_j})\right), e(x^{k_j}, \mu_{k_j})\right\rangle \le \frac{\sigma}{\mu_k}\left\|e(x^{k_j}, \mu_{k_j})\right\|^2. \tag{3.14}$$

Since

$$\lim_{j\to\infty}\left(x^{k_j} - \frac{\eta_{k_j}}{\gamma}e(x^{k_j}, \mu_{k_j})\right) = \bar{x}.$$

Furthermore,

$$\lim_{j\to\infty} F\left(x^{k_j} - \frac{\eta_{k_j}}{\gamma}e(x^{k_j}, \mu_{k_j})\right) = F(\bar{x}).$$

So, by (3.14) we obtain

$$\lim_{j\to\infty}\left\langle F\left(x^{k_j} - \frac{\eta_{k_j}}{\gamma}e(x^{k_j}, \mu_{k_j})\right), e(x^{k_j}, \mu_{k_j})\right\rangle$$

$$= \left\langle F(\bar{x}), e(\bar{x}, \mu_{k_j})\right\rangle$$

$$\le \lim_{j\to\infty}\frac{\sigma}{\mu_k}\left\|e(x^{k_j}, \mu_{k_j})\right\|^2 = \frac{\sigma}{\mu_k}\left\|e(\bar{x}, \mu_k)\right\|^2. \tag{3.15}$$

Using the similar arguments of (3.2), we have

$$\langle F(\bar{x}), e(\bar{x}, \mu_{k_j}) \rangle \geq \frac{1}{\mu_k} \| e(\bar{x}, \mu_k) \|^2.$$

Since

$$\| e(x^k, \mu^k) \| \geq \min\{1, \mu_k\} \| e(x^k) \|,$$

from (3.15) we get that $e(\bar{x}) = 0$, and thus $\bar{x}$ is a solution of problem (1.1).

Suppose now that $\limsup_{k \to \infty} \eta_k > 0$. Because of (3.13), it must be that $\liminf_{k \to \infty} \| e(x^k) \| = 0$. Since $e(\cdot)$ is continuous, we get $e(\bar{x}) = 0$, and thus $\bar{x}$ is a solution of problem (1.1).

Now, we prove that the sequence $\{x^k\}$ converges to a point contained in Γ.

Let $x^* = P_\Gamma(x^0)$. Since $x^* \in \Gamma$, by Lemma 3.4 we have

$$x^* \in H^1_{k_j - 1} \cap H^2_{k_j - 1} \cap C$$

for all j. So, by the iterative sequence of Algorithm 3.1 we have

$$\| x^{k_j} - x^0 \| \leq \| x^* - x^0 \|.$$

Thus,

$$\begin{aligned}
\| x^{k_j} - x^* \|^2 &= \| x^{k_j} - x^0 + x^0 - x^* \|^2 \\
&= \| x^{k_j} - x^0 \|^2 + \| x^0 - x^* \|^2 + 2\langle x^{k_j} - x^0, x^0 - x^* \rangle \\
&\leq \| x^* - x^0 \|^2 + \| x^0 - x^* \|^2 + 2\langle x^{k_j} - x^0, x^0 - x^* \rangle.
\end{aligned}$$

Letting $j \to \infty$, we have

$$\begin{aligned}
\| \bar{x} - x^* \|^2 &\leq 2\| x^0 - x^* \|^2 + 2\langle x^{k_j} - x^0, x^0 - x^* \rangle \\
&= 2\langle \bar{x} - x^*, x^0 - x^* \rangle \leq 0,
\end{aligned}$$

where the last inequality is due to Lemma 2.1 and the fact that $x^* = P_\Gamma(x^0)$ and $\bar{x} \in \Gamma$. So,

$$\bar{x} = x^* = P_\Gamma(x^0).$$

Thus, the sequence $\{x^k\}$ has a unique cluster point $P_\Gamma(x^0)$, which shows the global convergence of $\{x^k\}$. □

4 Numerical experiments

To test the effectiveness of our algorithm in this paper, we implemented it in MATLAB to solve the following example. We use $\| e(x^k, \mu_k) \| \leq \varepsilon = 10^{-5}$ as the stopping criterion. We denote Algorithm 3.1 in [14] as Algorithm 3.1*. Throughout the computational experiment, the parameters used in Algorithm 3.1 were set as $\gamma = 0.6$, $\sigma = 0.8$, $\theta = 1.5$, $\eta_0 = 0.3$, $\beta = 1$. The numerical results of the example are given in Table 1, *Iter* denotes the number of iterations, *CPU* denotes the computing time, and x^* denotes the approximate solution.

Table 1 Results for Example

Initiative point	Algorithm 3.1* with $\beta = 1$	Algorithm 3.1
$x^0 = (3, 2, 0)$	*Iter* = 78; *CPU* (**s**) = 0.113 $x^* = (0.0238, 0.0581, -0.1203)$	*Iter* = 43; *CPU* (**s**) = 0.098 $x^* = (1.0512; -2.3679; 1.0613)$
$x^0 = (0, -4, 0)$	*Iter* = 39; *CPU* (**s**) = 0.096 $x^* = (-0.1608, -0.5033, 0.8663)$	*Iter* = 20; *CPU* (**s**) = 0.053 $x^* = (-2.0711, 1.2634, 2.1036)$

Example Let $C = \{x \in \Re^3 | x_1^2 + x_2^2 \leq 40\}$ and $Q = \{x \in \Re^3 | x_2 - x_3^2 \leq 1\}$, $A = I$. Find $x \in C$ with $Ax \in Q$.

From the numerical experiments for the simple example we can see that our proposed method has good convergence properties.

5 Some concluding remarks

This paper presented a hybrid CQ projection algorithm with two projection steps and one Armijo linear-search step for solving the split feasibility problem (SFP). Different from the self-adaptive projection methods proposed by Zhang *et al.* [18], we use a new liner-search rule, which ensures that the hyperplane H_k separates the current x^k from the solution set Γ. The next iteration is generated by the projection of the starting point on a shrinking projection region (the intersection of three sets). Preliminary numerical experiments demonstrate a good behavior. However, whether the idea can be used to solve multiple-set SFP deserves further research.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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