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Some fixed point results on quasi-b-metric-like spaces

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Abstract

In this paper, we investigate the existence and uniqueness of a fixed point of certain operators in the setting of complete quasi-b-metric-like spaces via admissible mappings. Our results improve, extend, and unify several well-known existence results.

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1 Introduction and preliminaries

Throughout this paper, we denote $\mathbb{R}_0^+ = [0, +\infty)$ and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, where $\mathbb{N}$ is the set of all positive integers. First, we recall some basic concepts and notation.

The concept of b-metric was introduced by Czerwik [1] as a generalization of metric (see also Bakhtin [2, 3]) to extend the celebrated Banach contraction mapping principle. Following the initial paper of Czerwik [1], a number of researchers in nonlinear analysis investigated the topology of the paper and proved several fixed point theorems in the context of complete b-metric spaces (see [4–8] and references therein).

Definition 1.1 [1] Let X be a nonempty set, and $s \geq 1$ be a given real number. A mapping $d : X \times X \to [0, +\infty)$ is said to be a b-metric if for all $x, y, z \in X$, the following conditions are satisfied:

(b_1) $d(x, y) = 0$ if and only if $x = y$;
(b_2) $d(x, y) = d(y, x)$;
(b_3) $d(x, z) \leq s[d(x, y) + d(y, z)]$.

In this case, the pair (X, d) is called a b-metric space (with constant s).

Definition 1.2 [9] Let X be a nonempty set, and $s \geq 1$ be a given real number. A mapping $d : X \times X \to [0, +\infty)$ is said to be a quasi-b-metric if for all $x, y, z \in X$, the following conditions are satisfied:

(bm_1) $d(x, y) = 0$ if and only if $x = y$;
(bm_2) $d(x, z) \leq s[d(x, y) + d(y, z)]$.

In this case, the pair (X, d) is called a quasi-b-metric space (with constant s).

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Definition 1.3 [10] Let X be a nonempty set, and $s \geq 1$ be a given real number. A mapping $d : X \times X \to [0, +\infty)$ is said to be a quasi-b-metric-like if for all $x, y, z \in X$, the following conditions are satisfied:

(bM_1) $d(x, y) = 0$ implies $x = y$;
(bM_2) $d(x, z) \leq s[d(x, y) + d(y, z)]$.

In this case, the pair (X, d) is called a quasi-b-metric-like space (with constant s).

Example 1.4 Let $X = \{0, \frac{1}{2}, \frac{1}{3}\} \cup [1, \infty)$, and let $d : X \times X \to [0, +\infty)$ be defined as

$$d(x,y) = \begin{cases} 6 & \text{if } x = y = 0, \\ 3 & \text{if } x = y = \frac{1}{3}, \\ 2 & \text{if } x = 0, y = \frac{1}{2}, \\ \frac{1}{2} & \text{if } x = 0, y = \frac{1}{3}, \\ \frac{3}{2} & \text{if } x = \frac{1}{3}, y = 0, \\ |x - y| & \text{otherwise.} \end{cases}$$

It is clear that (X, d) is a quasi-b-metric-like space with constant $s = 9$.

Definition 1.5 (see *e.g.* [10]) Let (X, d) be a quasi-b-metric-like space. Then:

$(i)_a$ a sequence $\{x_n\}$ in X is called a left-Cauchy sequence if and only if for every $\varepsilon > 0$, there exists a positive integer $N = N(\varepsilon)$ such that $d(x_n, x_m) < \varepsilon$ for all $n > m > N$;
$(ii)_b$ a sequence $\{x_n\}$ in X is called a right-Cauchy sequence if and only if for every $\varepsilon > 0$, there exists a positive integer $N = N(\varepsilon)$ such that $d(x_n, x_m) < \varepsilon$ for all $m > n > N$;
$(iii)_a$ a quasi-partial metric space is said to be left-complete if every left-Cauchy sequence $\{x_n\}$ in X converges with respect to d to a point $u \in X$ such that

$$\lim_{n\to\infty} d(x_n, u) = d(u, u) = \lim_{n,m\to\infty} d(x_m, x_n) = 0, \quad \text{where } m \geq n;$$

$(iii)_b$ a quasi-partial metric space is said to be right-complete if every left-Cauchy sequence $\{x_n\}$ in X converges with respect to d to a point $u \in X$ such that

$$\lim_{n\to\infty} d(u, x_n) = d(u, u) = \lim_{n,m\to\infty} d(x_n, x_m) = 0, \quad \text{where } m \geq n.$$

Let (X, d) and (Y, α) be quasi-b-metric-like spaces, and let $f : X \to Y$ be a continuous mapping. Then

$$\lim_{n\to\infty} x_n = u \quad \Rightarrow \quad \lim_{n\to\infty} fx_n = fu.$$

In 2012, Samet *et al.* [11] introduced the concept of α-admissible mappings, and in 2013, Karapınar *et al.* [12] improved this notion as triangular α-admissible mappings.

Definition 1.6 [11, 12] Let $\alpha : X \times X \to [0, +\infty)$ be a function. A self-mapping f is called an α-admissible mapping if

$$\alpha(x, y) \geq 1 \quad \Rightarrow \quad \alpha(fx, fy) \geq 1$$

for all $x, y \in X$. If, further, f satisfies the condition

$$\alpha(x,z) \geq 1 \quad \text{and} \quad \alpha(z,y) \geq 1 \quad \Rightarrow \quad \alpha(x,y) \geq 1$$

for all $x, y, z \in X$, then it is called triangular α-admissible mapping.

Very recently, Popescu [13] improved these notions as follows.

Definition 1.7 [13] Let $\alpha : X \times X \to [0,\infty)$ be a function. If $f : X \to X$ satisfies the condition

$$\text{(T1)}' \quad \alpha(x,fx) \geq 1 \quad \Rightarrow \quad \alpha\bigl(fx,f^2x\bigr) \geq 1$$

for all $x \in X$, then it is called a right-α-orbital admissible mapping. If f satisfies the condition

$$\text{(T1)}'' \quad \alpha(fx,x) \geq 1 \quad \Rightarrow \quad \alpha\bigl(f^2x,fx\bigr) \geq 1$$

for all $x \in X$, then it is called a left-α-orbital admissible mapping. Furthermore, if f is both right-α-orbital admissible and left-α-orbital admissible, then f is called an α-orbital admissible mapping.

Triangular α-admissible mappings defined by Popescu [13] impose the following definitions.

Definition 1.8 [13] Let $f : X \to X$ be a self-mapping, and $\alpha : X \times X \to [0,\infty)$ be a function. Then f is said to be triangular right-α-orbital admissible if f is right-α-orbital admissible and

$$\text{(T2)}' \quad \alpha(x,y) \geq 1 \quad \text{and} \quad \alpha(y,fy) \geq 1 \quad \Rightarrow \quad \alpha(x,fy) \geq 1$$

and is said to be triangular left-α-orbital admissible if f is α-orbital admissible and

$$\text{(T2)}'' \quad \alpha(fx,x) \geq 1 \quad \text{and} \quad \alpha(x,y) \geq 1 \quad \Rightarrow \quad \alpha(fx,y) \geq 1.$$

If T satisfies both (T2)$'$ and (T2)$''$, then it is called triangular α-orbital admissible.

It is easy to conclude that each α-admissible mapping is an α-orbital admissible mapping and each triangular α-admissible mapping is a triangular α-orbital admissible mapping. However, the converses of the statements are false. In the following example, we see that a mapping that is triangular α-orbital admissible need not be triangular α-admissible.

Example 1.9 Let $X = \{x_i : i = 1,\dots,n\}$ for some $n \geq 4$, and $d : X \times X \to \mathbb{R}_0^+$ with $d(x,y) = |x-y|$. We define a self-mapping $f : X \to X$ such that $fx_i = x_i$ for $i = 1,2$, $fx_i = x_j$ for $i,j \in \{3,4\}$, $i \neq j$, $fx_i = x_{i+1}$ for $i \in \{5,\dots,n-1\}$, and $fx_n = fx_5$. Moreover, let $\alpha : X \times X \to \mathbb{R}_0^+$ be such that

$$\alpha(x,y) = \begin{cases} 1 & \text{if } (x,y) \in \{(x_1,x_3),(x_1,x_4),(x_3,x_3),(x_4,x_4), \\ & \qquad (x_3,x_4),(x_4,x_3),(x_3,x_2),(x_4,x_2)\}, \\ 0 & \text{otherwise.} \end{cases}$$

Note that f is α-orbital admissible since $\alpha(x_3, fx_3) = \alpha(x_3, x_4) = 1$ and $\alpha(x_4, fx_4) = \alpha(x_4, x_3) = 1$. On the other hand, we have $\alpha(x_1, x_3) = \alpha(x_3, x_2) = 1$, but $\alpha(x_1, x_2) = 0$. Hence, T is not triangular α-admissible.

Definition 1.10 [13] Let (X, d) be a quasi-b-metric-like space. Then X is said to be α-regular if for every sequence $\{x_n\}$ in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x \in X$ as $n \to \infty$, there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha(x_{n(k)}, x) \geq 1$ for all k.

2 Main result

The notion of (b)-comparison was introduced by Berinde [14] in order to extend the notion of (c)-comparison.

Definition 2.1 [14] Let $s \geq 1$ be a real number. A mapping $\psi : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is called a (b)-comparison function if the following conditions are fulfilled:

(1) ψ is monotone increasing;
(2) there exist $k_0 \in \mathbb{N}$, $a \in (0,1)$, and a convergent series of nonnegative terms $\sum_{k=1}^{\infty} v_k$ such that $s^{k+1}\psi^{k+1}(t) \leq as^k\psi^k(t) + v_k$ for all $k \geq k_0$ and $t \in [0, \infty)$.

The class of (b)-comparison functions will be denoted by Ψ_b. Notice that the notion of a (b)-comparison function reduces to the concept of a (c)-comparison function if $s = 1$.

The following lemma will be used in the proof of our main result.

Lemma 2.2 [15, 16] *Let $s \geq 1$ be a real number. If $\psi : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is a (b)-comparison function, then*:

(1) *the series $\sum_{k=0}^{\infty} s^k\psi^k(t)$ converges for any $t \in \mathbb{R}_0^+$*;
(2) *the function $p_s : [0, \infty) \to [0, \infty)$ defined by*

$$p_s(t) = \sum_{k=0}^{\infty} s^k\psi^k(t) \quad \text{for all } t \in [0, \infty)$$

is increasing and continuous at 0.

Remark 2.3 It is easy to see that if $\psi(t) \in \Psi_b$, then $\psi(t) < t$ for all $t > 0$. In fact, if there is a $t^* > 0$ such that $\psi(t^*) \geq t^*$, then we have $\psi^2(t^*) \geq \psi(t^*) \geq t^*$ (since ψ is increasing). Continuing in the same manner, we get $\psi^n(t^*) \geq t^* > 0$, $n \in \mathbb{N}$. This contradicts Lemma 2.2.

Definition 2.4 Let (X, d) be a complete quasi-b-metric-like space with a constant $s \geq 1$. A self-mapping $f : X \to X$ is called (α, ψ)-contractive mapping if there exist two functions $\psi \in \Psi_b$ and $\alpha : X \times X \to [0, \infty)$ satisfying the following condition:

$$\alpha(x, y)d(fx, fy) \leq \psi\bigl(d(x, y)\bigr) \tag{2.1}$$

for all $x, y \in X$.

Theorem 2.5 *Let (X, d) be a complete quasi-b-metric-like space, and let $f : X \to X$ be an (α, ψ)-contractive mapping. Suppose also that*

(i) *f is α-orbital admissible*;

(ii) *there exists* $x_0 \in X$ *such that* $\alpha(x_0, fx_0) \geq 1$ *and* $\alpha(fx_0, x_0) \geq 1$;
(iii) f *is continuous*.

Then f *has a fixed point* u *in* X, *and* $d(u,u) = 0$.

Proof By (ii) there exists $x_0 \in X$ such that $\alpha(x_0, fx_0) \geq 1$ and $\alpha(fx_0, x_0) \geq 1$. Define the iterative sequence $\{x_n\}$ in X by $x_{n+1} = fx_n$ for all $n \in \mathbb{N}_0$. Note that if there exists $n_0 \in \mathbb{N}_0$ such that $x_{n_0} = x_{n_0+1}$, then x_{n_0} becomes a fixed point, which completes the proof. Hence, throughout the proof, we suppose that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}_0$. Regarding the fact that f is α-orbital admissible, from (ii) we derive that

$$\alpha(x_0, x_1) = \alpha(x_0, fx_0) \geq 1 \quad \Rightarrow \quad \alpha(fx_0, fx_1) = \alpha(x_1, x_2) \geq 1.$$

Inductively, we get that

$$\alpha(x_n, x_{n+1}) \geq 1 \quad \text{for all } n \in \mathbb{N}_0. \tag{2.2}$$

Analogously, again by (ii) and the fact that f is α-orbital admissible we find that

$$\alpha(x_1, x_0) = \alpha(fx_0, x_0) \geq 1 \quad \Rightarrow \quad \alpha(fx_1, fx_0) = \alpha(x_2, x_1) \geq 1.$$

Consequently, we observe that

$$\alpha(x_{n+1}, x_n) \geq 1 \quad \text{for all } n \in \mathbb{N}_0. \tag{2.3}$$

From (2.1), by taking $x = x_n$ and $y = x_{n-1}$, we find that

$$\begin{aligned} d(x_{n+1}, x_n) &= d(fx_n, fx_{n-1}) \\ &\leq \alpha(x_n, x_{n-1}) d(fx_n, fx_{n-1}) \\ &\leq \psi\bigl(d(x_n, x_{n-1})\bigr). \end{aligned}$$

In view of Remark 2.3, we get that

$$d(x_{n+1}, x_n) \leq \psi\bigl(d(x_n, x_{n-1})\bigr) < d(x_n, x_{n-1}) \quad \text{for all } n \in \mathbb{N}. \tag{2.4}$$

By analogy, again by (2.1) and by substituting $x = x_{n-1}$ and $y = x_n$, we have

$$\begin{aligned} d(x_n, x_{n+1}) &= d(fx_{n-1}, fx_n) \\ &\leq \alpha(x_{n-1}, x_n) d(fx_{n-1}, fx_n) \\ &\leq \psi\bigl(d(x_{n-1}, x_n)\bigr). \end{aligned}$$

Consequently,

$$d(x_n, x_{n+1}) \leq \psi\bigl(d(x_{n-1}, x_n)\bigr) < d(x_{n-1}, x_n) \quad \text{for all } n \in \mathbb{N}. \tag{2.5}$$

From (2.4) and (2.5) we derive that

$$d(x_n,x_{n+1}) \le \psi^n\bigl(d(x_0,x_1)\bigr) \quad \text{and} \quad d(x_{n+1},x_n) \le \psi^n\bigl(d(x_1,x_0)\bigr) \quad \text{for all } n\in\mathbb{N}. \tag{2.6}$$

By Lemma 2.2(1) and letting $n\to\infty$ in (2.6), we have $\lim_{n\to\infty} d(x_n,x_{n+1}) = \lim_{n\to\infty} d(x_{n+1}, x_n) = 0$.

We further prove that the sequence $\{x_n\}$ is right-Cauchy and left-Cauchy. For all $n,p\in\mathbb{N}$, we have

$$\begin{aligned} d(x_n,x_{n+p}) &\le \sum_{i=1}^{p-1} s^i d(x_{n+i-1},x_{n+i}) + s^{p-1} d(x_{n+p-1},x_{n+p}) \\ &< \sum_{i=1}^{p} s^i \psi^{n+i-1}\bigl(d(x_0,x_1)\bigr) \\ &= \frac{1}{s^{n-1}} \sum_{k=n}^{n+p-1} s^k \psi^k\bigl(d(x_0,x_1)\bigr). \end{aligned}$$

By letting $n,p\to\infty$ we get that

$$\lim_{n,p\to\infty} d(x_n,x_{n+p}) = 0,$$

that is, the sequence $\{x_n\}$ is right-Cauchy.

Analogously,

$$\lim_{n,p\to\infty} d(x_{n+p},x_n) = 0,$$

that is, the sequence $\{x_n\}$ is left-Cauchy. As a result, the sequence $\{x_n\}$ is a Cauchy sequence. Since (X,d) is complete, there exists a point $u\in X$ such that

$$\lim_{n\to\infty} d(u,x_n) = \lim_{n\to\infty} d(x_n,u) = d(u,u) = \lim_{n,m\to\infty} d(x_n,x_m) = \lim_{n,m\to\infty} d(x_m,x_n) = 0. \tag{2.7}$$

Since f is continuous, we have

$$u = \lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} fx_n = fu. \qquad \square$$

Example 2.6 Let (X,d) be a quasi b-metric like space defined in Example 1.4, and let the mapping $f : X \mapsto X$ be defined as

$$fx = \begin{cases} \frac{1}{3} & \text{if } x = 0, \\ \frac{1}{2} & \text{if } x = \frac{1}{3}, \\ \frac{1}{2} & \text{if } x = \frac{1}{2}, \\ x+1 & \text{if } x \ge 1. \end{cases}$$

Let $\psi(t) = \frac{t}{2}$, $t\ge 0$, and let $\alpha : X\times X \to [0,\infty)$ be defined as

$$\alpha(x,y) = \begin{cases} 1 & \text{if } x,y \in \{0,\frac{1}{3},\frac{1}{2}\}, \\ 0 & \text{otherwise.} \end{cases}$$

Then $\psi \in \Psi_b$, and f is an (α,ψ)-contractive mapping. Since the conditions of Theorem 2.5 are satisfied, it follows that f has a fixed point in X.

It is possible to remove the heavy condition of continuity of the self-mapping f in Theorem 2.5. For this purpose, we need the following result, which is inspired from the results in [17].

Lemma 2.7 *Let (X,d) be a quasi-b-metric-like space with constant s and assume that $\{x_n\}$ and $\{y_n\}$ are sequences in X converging to x and y, respectively. Then*

$$\begin{aligned}\frac{1}{s^2}d(x,y)-\frac{1}{s}d(x,x)-d(y,y) &\le \liminf_{n\to\infty} d(x_n,y_n) \le \limsup_{n\to\infty} d(x_n,y_n)\\ &\le sd(x,x)+s^2d(y,y)+s^2d(x,y).\end{aligned}$$

In particular, if $d(x,y)=0$, then $\lim_{n\to\infty} d(x_n,y_n)=0$.

Moreover, for each $z\in X$, we have

$$\frac{1}{s}d(x,z)-d(x,x)\le \liminf_{n\to\infty} d(x_n,z)\le \limsup_{n\to\infty} d(x_n,z)\le sd(x,z)+sd(x,x). \tag{2.8}$$

If $d(x,x)=0$, then

$$\frac{1}{s}d(x,z)\le \liminf_{n\to\infty} d(x_n,z)\le \limsup_{n\to\infty} d(x_n,z)\le sd(x,z).$$

Theorem 2.8 *Let (X,d) be a complete quasi-b-metric-like space, and let $f:X\to X$ be an (α,ψ)-contractive mapping. Suppose also that*

(i) *f is α-orbital admissible*;
(ii) *there exists $x_0\in X$ such that $\alpha(x_0,fx_0)\ge 1$ and $\alpha(fx_0,x_0)\ge 1$*;
(iii) *X is α-regular.*

Then f has a fixed point u in X, and $d(u,u)=0$.

Proof By verbatim of the proof of Theorem 2.5 we find an iterative sequence $\{x_n\}$ that converges to a point $u\in X$ such that (2.7) holds.

Since $d(u,u)=0$, by Lemma 2.7 we have

$$\begin{aligned}\frac{1}{s}d(u,fu) &\le \liminf_{n\to\infty} d(x_{n+1},fu)\\ &\le \limsup_{n\to\infty} d(x_{n+1},fu)\\ &= \limsup_{n\to\infty} d(fx_n,fu)\\ &\le \limsup_{n\to\infty} \alpha(x_n,u)d(fx_n,fu)\\ &\le \limsup_{n\to\infty} \psi\bigl(d(x_n,u)\bigr).\end{aligned}$$

By letting $n\to\infty$ in these inequalities we derive that $\frac{1}{s}d(u,fu)=0$ and hence $fu=u$. □

It is natural to consider the uniqueness of a fixed point of an (α, ψ)-contractive mapping. We notice that we need to add an additional condition to guarantee the uniqueness.

(U) For all $x, y \in \mathrm{Fix}(f)$, either $\alpha(x,y) \geq 1$ or $\alpha(y,x) \geq 1$.

Here, $\mathrm{Fix}(f)$ denotes the set of all fixed points of f.

Theorem 2.9 *Adding condition* (U) *to hypotheses of Theorem* 2.5 (*or Theorem* 2.8), *we obtain the uniqueness of a fixed point of* f.

Proof Suppose that x^* and y^* are two distinct fixed points of f, so that $d(x^*, y^*) > 0$.

If, for example, $\alpha(x^*, y^*) \geq 1$, then

$$\begin{aligned} d(x^*, y^*) &= d(fx^*, fy^*) \\ &\leq \alpha(x^*, y^*) d(fx^*, fy^*) \\ &\leq \psi\bigl(d(x^*, y^*)\bigr) \\ &< d(x^*, y^*), \end{aligned}$$

which is a contradiction. □

Definition 2.10 Let (X, d) be a complete quasi-b-metric-like space with a constant $s \geq 1$. A self-mapping $f : X \to X$ is called a generalized (α, ψ)-contractive mapping of type (A) if there exist two functions $\psi \in \Psi_b$ and $\alpha : X \times X \to [0, \infty)$ satisfying the following condition:

$$\alpha(x,y) d(fx, fy) \leq \psi\bigl(M(x,y)\bigr) \tag{2.9}$$

for all $x, y \in X$, where

$$M(x,y) = \max\bigl\{d(x,y), d(x,fx), d(y,fy)\bigr\}. \tag{2.10}$$

Theorem 2.11 *Let* (X, d) *be a complete quasi-b-metric-like space, and let* $f : X \to X$ *be a generalized* (α, ψ)*-contractive mapping of type* (A). *Assume that*

(i) f *is* α*-orbital admissible*;

(ii) *there exists* $x_0 \in X$ *such that* $\alpha(x_0, fx_0) \geq 1$ *and* $\alpha(fx_0, x_0) \geq 1$;

(iii) f *is continuous*.

Then f *has a fixed point* u *in* X, *and* $d(u,u) = 0$.

Proof As in the proof of Theorem 2.5, we construct an iterative sequence $x_{n+1} = fx_n$, $n \in \mathbb{N}_0$, where the existence of $x_0 \in X$ is guaranteed by (ii). By the same reason as in the proof of Theorem 2.5, we may assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}_0$, and we can conclude that

$$\alpha(x_n, x_{n+1}) \geq 1 \quad \text{and} \quad \alpha(x_{n+1}, x_n) \geq 1 \quad \text{for all } n \in \mathbb{N}_0. \tag{2.11}$$

From (2.9) we have

$$\begin{aligned} d(x_n, x_{n+1}) &= d(fx_{n-1}, fx_n) \\ &\leq \alpha(x_{n-1}, x_n) d(fx_{n-1}, fx_n) \\ &\leq \psi\bigl(M(x_{n-1}, x_n)\bigr) \end{aligned}$$

for all $n \in \mathbb{N}$, where

$$\begin{aligned} M(x_{n-1}, x_n) &= \max\{d(x_{n-1}, x_n), d(x_n, fx_n), d(x_{n-1}, fx_{n-1})\} \\ &= \max\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\}. \end{aligned}$$

If $M(x_{n-1}, x_n) = d(x_n, x_{n+1})$, then since we assumed that $x_n \neq x_{n+1}$,

$$d(x_n, x_{n+1}) \le \psi\big(d(x_n, x_{n+1})\big) < d(x_n, x_{n+1}),$$

which is a contradiction. It allows us to conclude that $M(x_{n-1}, x_n) = d(x_{n-1}, x_n)$, $n \in \mathbb{N}$.

Thus,

$$d(x_n, x_{n+1}) \le \psi\big(d(x_{n-1}, x_n)\big) < d(x_{n-1}, x_n) \quad \text{for all } n \in \mathbb{N}$$

and

$$d(x_n, x_{n+1}) \le \psi^n\big(d(x_0, x_1)\big) \quad \text{for all } n \in \mathbb{N}. \tag{2.12}$$

Analogously, letting $x = x_n$ and $y = x_{n-1}$ in (2.9), we get

$$\begin{aligned} d(x_{n+1}, x_n) &= d(fx_n, fx_{n-1}) \\ &\le \alpha(x_n, x_{n-1}) d(fx_n, fx_{n-1}) \\ &\le \psi\big(M(x_n, x_{n-1})\big) \end{aligned} \tag{2.13}$$

for all $n \in \mathbb{N}$, where

$$\begin{aligned} M(x_n, x_{n-1}) &= \max\{d(x_n, x_{n-1}), d(x_n, fx_n), d(x_{n-1}, fx_{n-1})\} \\ &= \max\{d(x_n, x_{n-1}), d(x_n, x_{n+1}), d(x_{n-1}, x_n)\}. \end{aligned}$$

For the estimation of $d(x_{n+1}, x_n)$, we will consider three different cases.

Case 1. If $M(x_n, x_{n-1}) = d(x_{n-1}, x_n)$, then, by (2.13),

$$d(x_{n+1}, x_n) \le \psi\big(d(x_{n-1}, x_n)\big). \tag{2.14}$$

Case 2. If $M(x_n, x_{n-1}) = d(x_n, x_{n+1})$, then

$$d(x_{n+1}, x_n) \le \psi\big(d(x_n, x_{n+1})\big).$$

By Remark 2.3 we find that

$$d(x_{n+1}, x_n) \le \psi\big(d(x_n, x_{n+1})\big) < \psi^{n+1}\big(d(x_0, x_1)\big).$$

Case 3. Otherwise, $M(x_n, x_{n-1}) = d(x_n, x_{n-1})$ and

$$d(x_{n+1}, x_n) \le \psi\big(d(x_n, x_{n-1})\big). \tag{2.15}$$

Observing (2.14) and (2.15), it follows that, for any $n \in \mathbb{N}$,

$$d(x_{n+1}, x_n) \le \max\{\psi^n(d(x_0, x_1)), \psi^n(d(x_1, x_0))\}. \tag{2.16}$$

Obviously, in all considered cases, we deduce that

$$\lim_{n\to\infty} d(x_{n+1}, x_n) = \lim_{n\to\infty} d(x_n, x_{n+1}) = 0.$$

Since ψ is an increasing function, let

$$\nu = \max\{d(x_0, x_1), d(x_1, x_0)\}.$$

Consequently, we have that $d(x_{n+1}, x_n) \le \psi^n(\nu)$ and $d(x_n, x_{n+1}) \le \psi^n(\nu)$. By applying ($\mathrm{bM}_2$) for any $n, p \in \mathbb{N}$ it follows that

$$\begin{aligned} d(x_n, x_{n+p}) &\le \sum_{i=1}^{p-1} s^i d(x_{n+i-1}, x_{n+i}) + s^{p-1} d(x_{n+p-1}, x_{n+p}) \\ &\le \sum_{i=1}^{p} s^i d(x_{n+i-1}, x_{n+i}) \\ &\le \sum_{i=1}^{p} s^i \psi^{n+i-1}(\nu) \\ &= \frac{1}{s^{n-1}} \sum_{i=1}^{p} s^{n+i-1} \psi^{n+i-1}(\nu). \end{aligned}$$

Therefore, $\lim_{n,p\to\infty} d(x_n, x_{n+p}) = 0$ and, likewise, $\lim_{n,p\to\infty} d(x_{n+p}, x_n) = 0$. Since, X is complete, there exists $u \in X$ such that $\lim_{n\to\infty} x_n = u$ and

$$\lim_{n\to\infty} d(u, x_n) = \lim_{n\to\infty} d(x_n, u) = d(u, u) = 0. \tag{2.17}$$

Furthermore, f is a continuous mapping, and hence $u = \lim_{n\to\infty} x_n = \lim_{n\to\infty} fx_{n-1} = fu$. □

Theorem 2.12 *Adding condition* (U) *to hypotheses of Theorem* 2.11, *we obtain the uniqueness of a fixed point of* T.

Proof Suppose that $fx^* = x^*$ and $fy^* = y^*$. Then

$$\begin{aligned} d(x^*, y^*) &= d(fx^*, fy^*) \\ &\le \alpha(x^*, y^*) \psi(d(fx^*, fy^*)) \\ &\le \psi(M(x^*, y^*)) \\ &= \psi(d(x^*, y^*)), \end{aligned}$$

so that $d(x^*, y^*) = 0 \Rightarrow x^* = y^*$. □

In the following example, we show the existence of a function satisfying conditions of Theorem 2.11 but not satisfying conditions of Theorem 2.5.

Example 2.13 Let (X,d) be a quasi-b-metric-like space described in Example 1.4, and $f : X \mapsto X$ the mapping

$$fx = \begin{cases} \frac{1}{2} & \text{if } x = 0, \\ 0, & \text{if } x = \frac{1}{3}, \\ \frac{1}{2} & \text{if } x = \frac{1}{2}, \\ x+1 & \text{if } x \geq 1. \end{cases}$$

Let $\psi(t) = \frac{t}{2}$, $t \geq 0$, and let $\alpha : X \times X \to [0,\infty)$ be defined as

$$\alpha(x,y) = \begin{cases} 1 & \text{if } (x,y) \in \{(0,\frac{1}{3}),(0,\frac{1}{2}),(\frac{1}{2},0),(\frac{1}{2},\frac{1}{3}),(\frac{1}{2},\frac{1}{2})\}, \\ 0 & \text{otherwise.} \end{cases}$$

Then (2.1) does not hold, for example, for $x = 0$ and $y = \frac{1}{3}$, but (2.9) holds, f has a unique fixed point $u = \frac{1}{2}$, and $d(u,u) = 0$.

Definition 2.14 Let (X,d) be a complete quasi-b-metric-like space with a constant $s \geq 1$. A self-mapping $f : X \to X$ is called a generalized (α,ψ)-contractive mapping of type (B) if there exist two functions $\psi \in \Psi_b$ and $\alpha : X \times X \to [0,\infty)$ satisfying the following condition:

$$\alpha(x,y)d(fx,fy) \leq \psi\big(N(x,y)\big) \tag{2.18}$$

for all $x,y \in X$, where

$$N(x,y) = \max\left\{d(x,y), \frac{d(x,fx)+d(y,fy)}{2}\right\}. \tag{2.19}$$

The following theorem can be deduced from the inequality $N(x,y) \leq M(x,y)$ for all x, y, together with the monotonicity of ψ.

Theorem 2.15 *Let (X,d) be a complete quasi-b-metric-like space, and let $f : X \to X$ be a generalized (α,ψ)-contractive mapping of type (B). Assume that*

(i) *f is α-orbital admissible*;
(ii) *there exists $x_0 \in X$ such that $\alpha(x_0,fx_0) \geq 1$ and $\alpha(fx_0,x_0) \geq 1$*;
(iii) *f is continuous.*

Then f has a fixed point u in X, and $d(u,u) = 0$.

Definition 2.16 Let (X,d) be a complete quasi-b-metric-like space with a constant $s \geq 1$. A self-mapping $f : X \to X$ is called a generalized (α,ψ)-contractive mapping of type (C) if there exist two functions $\psi \in \Psi_b$ and $\alpha : X \times X \to [0,\infty)$ satisfying the following condition:

$$s\alpha(x,y)d(fx,fy) \leq \psi\big(M(x,y)\big) \tag{2.20}$$

for all $x, y \in X$, where

$$M(x,y) = \max\bigl\{d(x,y), d(x,fx), d(y,fy)\bigr\}. \tag{2.21}$$

The following theorem is easily observed from Theorem 2.11 since inequality (2.9) can be easily derived from inequality (2.20).

Theorem 2.17 *Let (X,d) be a complete quasi-b-metric-like space, and let $f : X \to X$ be a generalized (α,ψ)-contractive mapping of type (C). Assume that*
(i) *f is α-orbital admissible;*
(ii) *there exists $x_0 \in X$ such that $\alpha(x_0,fx_0) \geq 1$ and $\alpha(fx_0,x_0) \geq 1$;*
(iii) *f is continuous.*
Then f has a fixed point u in X, and $d(u,u) = 0$.

In the next theorems, we establish a fixed point result for a generalized (α,ψ)-contractive mapping of type (C) without any continuity assumption on the mapping f.

Theorem 2.18 *Let (X,d) be a complete quasi-b-metric-like space, and let $f : X \to X$ be a generalized (α,ψ)-contractive mapping of type (C). Suppose that*
(i) *f is α-orbital admissible;*
(ii) *there exists $x_0 \in X$ such that $\alpha(x_0,fx_0) \geq 1$ and $\alpha(fx_0,x_0) \geq 1$;*
(iii) *X is α-regular.*
Then f has a fixed point u in X, and $d(u,u) = 0$.

Proof As in the proof of Theorem 2.5, we consider an iterative sequence $\{x_n\}$, and we obtain the existence of $u \in X$ such that (2.17) holds. By Lemma 2.7 we get

$$\begin{aligned} d(u,fu) &\leq s \liminf_{n\to\infty} d(x_{n+1},fu) \\ &\leq s \limsup_{n\to\infty} d(x_{n+1},fu) \\ &\leq s \limsup_{n\to\infty} \alpha(x_n,u) d(fx_n,fu) \\ &\leq \limsup_{n\to\infty} \psi\bigl(M(x_n,u)\bigr), \end{aligned}$$

where

$$M(x_n,u) = \max\bigl\{d(x_{n-1},u), d(x_{n-1},x_n), d(u,fu)\bigr\}.$$

According to (2.17) and the fact that $\lim_{n\to\infty} d(x_{n-1},x_n) = 0$, it remains to discuss only the case $M(x_n,u) = d(u,fu)$ because otherwise it follows $d(u,fu) = 0 \Rightarrow u = fu$.

Notice that, under this assumption, $d(u,fu) \leq \psi(d(u,fu))$ also implies $d(u,fu) = 0$ since $\psi(t) < t$ for any $t > 0$. Hence, u is a fixed point of the mapping f. □

Theorem 2.19 *Adding condition* (U) *to hypotheses of Theorem* 2.17 (*or Theorem* 2.18), *we obtain the uniqueness of a fixed point of T.*

Example 2.20 Let (X,d) be a quasi b-metric like space defined in Example 1.4, and let the mapping $f : X \mapsto X$ be defined as

$$fx = \begin{cases} 1 & \text{if } x = 0, \\ \frac{1}{2} & \text{if } x = \frac{1}{3}, \\ \frac{1}{2} & \text{if } x = \frac{1}{2}, \\ x+1 & \text{if } x \geq 1. \end{cases}$$

Let $\psi(t) = \frac{t}{2}$, $t \geq 0$, and $\alpha : X \times X \to [0,\infty)$ be defined as

$$\alpha(x,y) = \begin{cases} 1 & \text{if } (x,y) \in \{(\frac{1}{3},\frac{1}{2}),(\frac{1}{2},\frac{1}{2})\}, \\ \frac{1}{9} & \text{if } (x,y) \in \{0,\frac{1}{3},\frac{1}{2}\} \times \{0,\frac{1}{3},\frac{1}{2}\} \setminus \{(\frac{1}{3},\frac{1}{2}),(\frac{1}{2},\frac{1}{2})\}, \\ 0 & \text{otherwise.} \end{cases}$$

Then $\psi \in \Psi_b$, and f is a generalized (α,ψ)-contractive mapping of type (C). Since the conditions of Theorem 2.18 are satisfied, it follows that f has a fixed point in X.

Definition 2.21 Let (X,d) be a complete quasi-b-metric-like space with a constant $s \geq 1$. A self-mapping $f : X \to X$ is called a generalized (α,ψ)-contractive mapping of type (D) if there exist two functions $\psi \in \Psi_b$ and $\alpha : X \times X \to [0,\infty)$ satisfying the following condition:

$$\alpha(x,y)d(fx,fy) \leq \psi\bigl(L(x,y)\bigr) \tag{2.22}$$

for all $x,y \in X$, where

$$L(x,y) = \max\left\{d(x,y), \frac{d(x,fx)+d(y,fy)}{2s}\right\}. \tag{2.23}$$

Theorem 2.22 *Let (X,d) be a complete quasi-b-metric-like space, and let $f : X \to X$ be a generalized (α,ψ)-contractive mapping of type (D). Suppose that*

(i) *f is triangular α-orbital admissible;*
(ii) *there exists $x_0 \in X$ such that $\alpha(x_0,fx_0) \geq 1$ and $\alpha(fx_0,x_0) \geq 1$;*
(iii) *X is α-regular.*

Then f has a fixed point in X, that is, there exists $u \in X$ such that $fu = u$ and $d(u,u) = 0$.

Proof As in the proof of Theorem 2.11, we consider an iterative sequence $\{x_n\}$ and obtain the existence of $u \in X$ such that (2.17) holds. By Lemma 2.7 we get

$$\begin{aligned} \frac{1}{s}d(u,fu) &\leq \liminf_{n\to\infty} d(x_{n+1},fu) \\ &\leq \limsup_{n\to\infty} d(x_{n+1},fu) \\ &\leq \limsup_{n\to\infty} \alpha(x_n,u)d(fx_n,fu) \\ &\leq \limsup_{n\to\infty} \psi\bigl(N(x_n,u)\bigr), \end{aligned}$$

where

$$N(x_n,u) = \max\left\{d(x_{n-1},u), \frac{d(x_{n-1},x_n)+d(u,fu)}{2s}\right\}.$$

If $N(x_n,u) = d(x_{n-1},u)$, then we conclude the result due to (2.17). Taking $\lim_{n\to\infty} d(x_{n-1}, x_n) = 0$ into account, we deduce that $\lim_{n\to\infty} N(x_n,u) = \frac{d(u,fu)}{2s}$. Notice that, under this assumption, $\frac{1}{s}d(u,fu) \leq \psi(\frac{d(u,fu)}{2s})$ also implies $d(u,fu) = 0$ since $\psi(t) < t$ for any $t > 0$. Hence, u is a fixed point of the mapping f. □

Theorem 2.23 *Adding condition* (U) *to hypotheses of Theorem* 2.15 (*and respectively, Theorem* 2.22), *we obtain the uniqueness of a fixed point of* T.

3 Consequences

In this section, we will list some consequences of our main results.

3.1 For standard quasi-*b*-metric-like

Corollary 3.1 *Let* (X,d) *be a complete quasi-b-metric-like space, and let* $f : X \to X$ *be a mapping such that*

$$d(fx,fy) \leq \psi\bigl(\max\bigl\{d(x,y), d(x,fx), d(y,fy)\bigr\}\bigr) \tag{3.1}$$

for all $x,y \in X$, *where* $\psi \in \Psi_b$. *If* f *is continuous, then* f *has a fixed point* u *in* X, *and* $d(u,u) = 0$.

Proof The proof of Corollary 3.1 follows from Theorem 2.12 by taking $\alpha(x,y) = 1$ for all $x,y \in X$, so (ii) is satisfied for any $x_0 \in X$, f is obviously an α-orbital admissible, and (U) holds. Inequality (3.1) allows us to conclude that f is a generalized (α,ψ)-contractive mapping of type (A). □

Corollary 3.2 *Let* (X,d) *be a complete quasi-b-metric-like space, and let* $f : X \to X$ *be a continuous mapping such that*

$$d(fx,fy) \leq \psi\left(\max\left\{d(x,y), \frac{d(x,fx) + d(y,fy)}{2}\right\}\right) \tag{3.2}$$

for all $x,y \in X$, *where* $\psi \in \Psi_b$. *Then* f *has a fixed point* u *in* X, *and* $d(u,u) = 0$.

Proof The proof of Corollary 3.2 follows from Theorem 2.15 by taking $\alpha(x,y) = 1$ for all $x,y \in X$ since then (2.18) follows from (3.2). □

Notice that the continuity condition of f in Corollary 3.1 can be removed by adding an extra term s.

Corollary 3.3 *Let* (X,d) *be a complete quasi-b-metric-like space, and let* $f : X \to X$ *be a mapping such that*

$$d(fx,fy) \leq s\psi\bigl(\max\bigl\{d(x,y), d(x,fx), d(y,fy)\bigr\}\bigr) \tag{3.3}$$

for all $x,y \in X$, *where* $\psi \in \Psi_b$. *Then* f *has a fixed point* u *in* X *such that* $d(u,u) = 0$.

Proof The proof of Corollary 3.3 follows from Theorem 2.18 by taking $\alpha(x,y) = 1$ for all $x,y \in X$. Then f is an α-orbital admissible mapping, and both inequalities in (ii) hold for any $x_0 \in X$. Notice that since $\alpha(x,y) = 1$, any constructive sequence turns to be regular, and thus X is α-regular. □

Corollary 3.4 *Let (X,d) be a complete quasi-b-metric-like space, and let $f : X \to X$ be a mapping such that*

$$d(fx,fy) \leq \psi\bigl(d(x,y)\bigr) \tag{3.4}$$

for all $x,y \in X$, where $\psi \in \Psi_b$. Then f has a fixed point u in X such that $d(u,u) = 0$.

Proof The proof of Corollary 3.4 follows from Theorem 2.8 by taking $\alpha(x,y) = 1$ for all $x,y \in X$ and observing that X is α-regular and that (i) and (ii) hold. □

3.2 For standard quasi-*b*-metric-like spaces with a partial order

In this section, we deduce various fixed point results on a quasi-b-metric-like space endowed with a partial order. We, first, recollect some basic notions and notation.

Definition 3.5 Let $(X,\preceq)$ be a partially ordered set, and $f : X \to X$ be a given mapping. We say that f is nondecreasing with respect to $\preceq$ if for all $x,y \in X$,

$$x \preceq y \quad \Rightarrow \quad fx \preceq fy.$$

Definition 3.6 Let $(X,\preceq)$ be a partially ordered set. A sequence $\{x_n\} \subseteq X$ is said to be nondecreasing (respectively, nonincreasing) with respect to $\preceq$ if $x_n \preceq x_{n+1}$, $n \in \mathbb{N}$ (respectively, $x_{n+1} \preceq x_n$, $n \in \mathbb{N}$).

Definition 3.7 Let $(X,\preceq)$ be a partially ordered set, and d be a b-metric-like on X. We say that $(X,\preceq,d)$ is regular if for every nondecreasing (respectively, nonincreasing) sequence $\{x_n\} \subseteq X$ such that $x_n \to x \in X$ as $n \to \infty$, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $x_{n_k} \preceq x$ (respectively, $x \preceq x_{n_k}$) for all k.

We have the following result.

Corollary 3.8 *Let $(X,\preceq)$ be a partially ordered set (which does not contain an infinite totally unordered subset), and d be a b-metric-like on X with constant $s \geq 1$ such that (X,d) is complete. Let $f : X \to X$ be a nondecreasing mapping with respect to $\preceq$. Suppose that there exists $\psi \in \Psi_b$ such that*

$$d(fx,fy) \leq \psi\bigl(M(x,y)\bigr) \tag{3.5}$$

for all $x,y \in X$ with $x \preceq y$ or $y \preceq x$, where $M(x,y)$ is defined as in (2.10). *Suppose also that the following conditions hold*:

(i) *there exists $x_0 \in X$ such that $x_0 \preceq fx_0$ and $fx_0 \preceq x_0$*;
(ii) *f is continuous or*
(ii)′ *$(X,\preceq,d)$ is regular, and d is continuous.*

Then f has a fixed point $u \in X$ with $d(u,u) = 0$. Moreover, if for all $x,y \in X$, there exists $z \in X$ such that $x \preceq z$ and $y \preceq z$, then f has a unique fixed point.

Proof Define the mapping $\alpha : X \times X \to [0,\infty)$ by

$$\alpha(x,y) = \begin{cases} 1 & \text{if } x \preceq y \text{ or } x \succeq y, \\ 0 & \text{otherwise.} \end{cases}$$

Clearly, f satisfies (2.20), that is,

$$\alpha(x,y)sd(fx,fy) \le \psi\bigl(M(x,y)\bigr)$$

for all $x, y \in X$. From condition (i) we have $x_0 \preceq fx_0$ and $fx_0 \preceq x_0$. Moreover, for all $x, y \in X$, from the monotone property of f we have

$$\begin{aligned} &\alpha(x,y) \ge 1 \quad \Rightarrow \quad x \succeq y \quad \text{or} \\ &x \preceq y \quad \Rightarrow \quad fx \succeq fy \quad \text{or} \\ &fx \preceq fy \quad \Rightarrow \quad \alpha(fx,fy) \ge 1. \end{aligned}$$

Hence, the self-mapping f is α-admissible. Similarly, we can prove that f is triangular α-admissible and so triangular α-orbital admissible. Now, if f is continuous, then the existence of a fixed point follows from Theorem 2.17.

Suppose that $(X, \preceq, d)$ is regular. Let $\{x_n\}$ be a sequence in X such that $\alpha(x_n, x_{n+1}) \ge 1$ for all n and $x_n \to x \in X$ as $n \to \infty$. By the regularity hypothesis, since X does not contain an infinite totally unordered subset, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $x_{n_k} \preceq x$ or $x \preceq x_{n_k}$ for all k.

This implies from the definition of α that $\alpha(x_{n_k}, x) \ge 1$ for all k. In this case, the existence of a fixed point follows again from Theorem 2.18. □

Corollary 3.9 *Let* $(X, \preceq)$ *be a partially ordered set* (*which does not contain an infinite totally unordered subset*), *and* d *be a b-metric-like on* X *with constant* $s \ge 1$ *such that* (X, d) *is complete. Let* $f : X \to X$ *be a nondecreasing mapping with respect to* $\preceq$. *Suppose that there exists* $\psi \in \Psi_b$ *such that*

$$d(fx,fy) \le \psi\bigl(d(x,y)\bigr) \tag{3.6}$$

for all $x, y \in X$ *with* $x \succeq y$ *or* $y \succeq x$. *Suppose also that the following conditions hold*:

(i) *there exists* $x_0 \in X$ *such that* $x_0 \preceq fx_0$ *and* $fx_0 \preceq x_0$;

(ii) f *is continuous*.

Then T *has a fixed point* $u \in X$ *with* $d(u,u) = 0$. *Moreover, if for all* $x, y \in X$, *there exists* $z \in X$ *such that* $x \preceq z$ *and* $y \preceq z$, *we have the uniqueness of a fixed point*.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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