Shukla et al. *Journal of Inequalities and Applications* (2015) 2015:89
DOI 10.1186/s13660-015-0605-8

Journal of Inequalities and Applications
a SpringerOpen Journal

ERRATUM **Open Access**

Erratum: On σ-type zero of Sheffer polynomials

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After publication of our work [1], we realized that there are some mathematical errors in Theorem 2 and Theorem 4. Our aim is to correct and modify Theorems 2 and 4.

Brown [2] discussed that $\{B_n(x)\}$ is a polynomial sequence which is simple and of degree precisely n. $\{B_n(x)\}$ is a binomial sequence if

$$B_n(x+y) = \sum_{k=0}^{n} \binom{n}{k} B_{n-k}(x) B_k(y), \quad n = 0, 1, 2, \ldots,$$

and a simple polynomial sequence $\{P_n(x)\}$ is a Sheffer sequence if there is a binomial sequence $\{B_n(x)\}$ such that

$$P_n(x+y) = \sum_{k=0}^{n} \binom{n}{k} B_{n-k}(x) P_k(y), \quad n = 0, 1, 2, \ldots.$$

The correct theorem is given as follows.

Theorem 2 *A necessary and sufficient condition that $p_n(x)$ be of σ-type zero and there exists a sequence h_k independent of x and n such that*

$$\sum_{k=0}^{n-1} \sum_{i=1}^{r} \left(\varepsilon_i^{k+1} h_k\right) p_{n-k-1}(x) = \sigma p_n(x), \tag{3}$$

where $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_r$ are roots of unity and r is a fixed positive integer.

Proof If $p_n(x)$ is of σ-type zero, then it follows from Theorem 1 (see [1]) that

$$\sum_{n=0}^{\infty} p_n(x) t^n = \sum_{i=1}^{r} A_i(t) {}_0F_q\bigl(-; b_1, b_2, \ldots, b_q; xH(\varepsilon_i t)\bigr).$$

This can be written as

$$\begin{aligned}\sum_{n=0}^{\infty} \sigma p_n(x) t^n &= \sum_{i=1}^{r} A_i(t) \sigma {}_0F_q\bigl(-; b_1, b_2, \ldots, b_q; xH(\varepsilon_i t)\bigr) \\ &= \sum_{n=0}^{\infty} \sum_{k=0}^{n} \sum_{i=1}^{r} \left(\varepsilon_i^{k+1} h_k\right) p_{n-k}(x) t^{n+1} = \sum_{n=1}^{\infty} \sum_{k=0}^{n-1} \sum_{i=1}^{r} \left(\varepsilon_i^{k+1} h_k\right) p_{n-k-1}(x) t^n.\end{aligned}$$

Thus

$$\sigma p_n(x) = \sum_{k=0}^{n-1}\sum_{i=1}^{r}\left(\varepsilon_i^{k+1} h_k\right) p_{n-k-1}(x).$$

This gives the proof of the statement. □

The correct theorem is given as follows.

Theorem 4 *A necessary and sufficient condition that $p_n(x,y)$ be symmetric, a class of polynomials in two variables and σ-type zero, there exists a sequence g_k and h_k, independent of x, y and n such that*

$$\sigma p_n(x,y) = \sum_{k=0}^{n-1}\sum_{i=1}^{r}\varepsilon_i^{k+1}(g_k + h_k) p_{n-k-1}(x,y), \tag{6}$$

where $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_r$ are roots of unity and r is a fixed positive integer.

Proof If $p_n(x,y)$ is of σ-type zero, then it follows from Theorem 3 (see [1]) that

$$\sum_{n=0}^{\infty} p_n(x,y)t^n = \sum_{i=1}^{r} A_i(t)\, {}_0F_p\left(-; b_1, b_2, \ldots, b_p; xG(\varepsilon_i t)\right) {}_0F_q\left(-; c_1, c_2, \ldots, c_q; yH(\varepsilon_i t)\right).$$

This can be written as

$$\begin{aligned}
\sum_{n=0}^{\infty} \sigma p_n(x,y)t^n &= \sum_{i=1}^{r} A_i(t)\sigma\, {}_0F_p\left(-; b_1, b_2, \ldots, b_p; xG(\varepsilon_i t)\right) {}_0F_q\left(-; c_1, c_2, \ldots, c_q; yH(\varepsilon_i t)\right) \\
&= \sum_{n=0}^{\infty}\sum_{k=0}^{n}\sum_{i=1}^{r} \varepsilon_i^{k+1}(g_k + h_k) p_{n-k}(x,y) t^{n+1} \\
&= \sum_{n=1}^{\infty}\sum_{k=0}^{n-1}\sum_{i=1}^{r} \varepsilon_i^{k+1}(g_k + h_k) p_{n-k-1}(x,y) t^{n}.
\end{aligned}$$

Thus

$$\sigma p_n(x,y) = \sum_{k=0}^{n-1}\sum_{i=1}^{r}\varepsilon_i^{k+1}(g_k + h_k) p_{n-k-1}(x,y).$$

This is the proof of Theorem 4. □

Acknowledgements

Authors are grateful to Prof. MEH Ismail for his comments and suggestions. The second author is thankful to SVNIT, Surat, India for awarding JRF and SRF.

Received: 3 February 2015 Accepted: 3 February 2015 Published online: 06 March 2015

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