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Blow-up of the weakly dissipative Novikov equation

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Abstract

In this paper, we investigate the Novikov equation with a weakly dissipative term. A new blow-up criterion independent of the initial energy is established.

MSC: 37L05; 35Q58; 26A12

Keywords: dissipative; Novikov equation; blow-up

1 Introduction

Recently, Vladimir Novikov [1] derived the following integrable partial differential equation

$$u_t - u_{xxt} + 4u^2 u_x - 3uu_x u_{xx} - u^2 u_{xxx} = 0, \quad t > 0, x \in \mathbb{R}. \tag{1.1}$$

In [2], Hone and Wang gave a matrix Lax pair for the Novikov equation and showed how it was related by a reciprocal transformation to a negative flow in the Sawada-Kotera hierarchy. Infinite conserved quantities were found as well as a bi-Hamiltonian structure. Then in [3], Hone, Lundmark and Szmigielski calculated the explicit formulas for multipeakon solutions of (1.1), using the matrix Lax pair found by Hone and Wang.

A detailed description of the corresponding strong solutions to (1.1) with initial data u_0 was given by Ni and her collaborators in [4, 5]. In [5], they proved that the Cauchy problem of the Novikov equation is locally well posed in the Besov spaces $B^s_{2,r}$ with the critical index $s = 3/2$. Then, well-posedness in H^s with $s > 3/2$ was also established by applying Kato's semigroup theory. In [4], they found sufficient conditions on the initial data to guarantee the formulation of singularities in finite time. A global existence result was also established in [4].

In this paper, we consider the following weakly dissipative Novikov equation:

$$u_t - u_{xxt} + 4u^2 u_x - 3uu_x u_{xx} - u^2 u_{xxx} + \lambda(u - u_{xx}) = 0, \quad t > 0, x \in \mathbb{R}, \tag{1.2}$$

where $u(x)$ denotes the velocity field and $y(x,t) = u - u_{xx}$.

In [6], local well-posedness for weakly dissipative Novikov equation (1.2) by Kato's theorem and some blow-up results were proved. The global existence of strong solutions to the weakly dissipative equation was also presented.

Now, we recall some elementary results which will be used in the paper.

Theorem 1.1 [6] *Given $u_0 \in H^s(\mathbb{R})$, $s > \frac{3}{2}$, there exist T and a unique solution u to* (1.1) *such that*

$$u(x,t) \in C\big([0,T); H^s(\mathbb{R})\big) \cap C^1\big([0,T); H^{s-1}(\mathbb{R})\big).$$

Theorem 1.2 [6] *Let $u_0 \in H^s$ with $s > \frac{3}{2}$, and let T be the maximal existence time of the solution $u(x,t)$ to* (1.2) *with the initial data $u_0(x)$. Then the corresponding strong solution to* (1.2) *blows up if and only if*

$$\lim_{t\to T} \sup_{0\le\tau\le t} \big\|u_x(x)\big\|_{L^\infty} = +\infty.$$

Theorem 1.3 [6] *Let $u_0 \in H^s$ with $s > \frac{3}{2}$, and let T be the maximal existence time of the solution $u(x,t)$ to* (1.2) *with u_0 as the initial datum. Assume that there exists $x_0 \in \mathbb{R}$ such that $y_0 = (1-\partial_x^2)u_0(x_0)$,*

$$y_0(x) \ge 0 \quad \text{for } x \in (-\infty, x_0) \quad \text{and} \quad y_0(x) \le 0 \quad \text{for } x \in (x_0, \infty)$$

and

$$u_0(x_0)u_0'(x_0) < -2\lambda - \sqrt{\frac{1}{4}\|u\|_{H^1}^4 + 4\lambda^2}. \tag{1.3}$$

Then the corresponding solution to (1.2) *with $u_0(x)$ as initial data blows up in finite time.*

For studies on related dissipative equations, we can refer to [7–11].

2 Blow-up phenomenon

Before going to the main results, we introduce some notations and do some preliminaries. Letting $\Lambda = (1-\partial_x^2)^{\frac{1}{2}}$, the operator Λ^{-2} can be expressed by its associated Green's function $G = \frac{1}{2}e^{-|x|}$ as

$$\Lambda^{-2}f(x) = G * f(x) = \frac{1}{2}\int_{\mathbb{R}} e^{-|x-y|}f(y)\,dy. \tag{2.1}$$

Due to (2.1), equation (1.2) is equivalent to the following one

$$u_t + u^2u_x + G * \big(3uu_xu_{xx} + 2u_x^3 + 3u^2u_x\big) + \lambda u = 0. \tag{2.2}$$

Motivated by Mckean's observation for the Camassa-Holm equation [12], we can do the similar particle trajectory as

$$\begin{cases} \frac{dq(x,t)}{dt} = u^2(q(x,t),t), & 0 < t < T, x \in \mathbb{R}, \\ q(x,t=0) = x, & x \in \mathbb{R}, \end{cases} \tag{2.3}$$

where T is the life span of the solution. Differentiating the first equation in (2.3) with respect to x, one has

$$\frac{dq_t}{dx} = q_{xt} = 2uu_x(q,t)q_x, \quad t \in (0,T).$$

Hence

$$q_x(x,t) = \exp\left\{\int_0^t 2uu_x(q,s)\,ds\right\} \quad \text{and} \quad q_x(x,0) = 1.$$

Then $q(x,t)$ is a diffeomorphism of the line before blow-up. Since

$$\frac{d}{dt}\left(y(q)q_x^{\frac{3}{2}}\right) = \left[y_t(q) + u^2(q,t)y_x(q) + 3uu_x(q,t)y(q)\right]q_x^{\frac{3}{2}} = -\lambda y q_x^{\frac{3}{2}},$$

it follows that

$$y\bigl(q(x,t),t\bigr)q_x^{\frac{3}{2}}(x,t) = y_0(x)e^{-\lambda t}. \tag{2.4}$$

The first result reads as follows.

Theorem 2.1 *Suppose that* $u_0 \in H^2(\mathbb{R})$ *and there exists* $x_0 \in \mathbb{R}$ *such that* $u_0(x_0) > 0$, $y_0(x_0) = (1-\partial_x^2)u_0(x_0) = 0$,

$$y_0(x) \geq 0 \ (\not\equiv 0) \quad \text{for } x \in (-\infty, x_0) \quad \text{and} \quad y_0(x) \leq 0 \ (\not\equiv 0) \quad \text{for } x \in (x_0, \infty) \tag{2.5}$$

as well as

$$u_0(u_0 - u_{0x})(x_0) > 4\lambda \quad \text{and} \quad u_0(u_0 + u_{0x})(x_0) < -4\lambda. \tag{2.6}$$

Then the corresponding solution $u(x,t)$ *to equation* (1.2) *with* u_0 *as the initial datum blows up in finite time.*

Remark 2.1 Due to the effect of the weakly dissipative term, we add condition (2.6) by comparing it with the blow-up result of the Novikov equation [4]. But unlike condition (1.3), here it does not depend on the initial energy at all.

Proof Due to equation (2.4) and initial condition (2.5), we have

$$\begin{cases} y(q(x_0,t),t) = 0, \\ y(\xi,t) \geq 0 \ (\not\equiv 0), & \text{for } \xi \in (-\infty, (q(x_0,t),t)), \\ y(\xi,t) \leq 0 \ (\not\equiv 0), & \text{for } \xi \in ((q(x_0,t),t), \infty) \end{cases}$$

for all $t \geq 0$. Since $u(x,t) = G * y(x,t)$, one can write $u(x,t)$ and $u_x(x,t)$ as

$$u(x,t) = \frac{1}{2}e^{-x}\int_{-\infty}^x e^{\xi}y(\xi,t)\,d\xi + \frac{1}{2}e^x\int_x^\infty e^{-\xi}y(\xi,t)\,d\xi,$$
$$u_x(x,t) = -\frac{1}{2}e^{-x}\int_{-\infty}^x e^{\xi}y(\xi,t)\,d\xi + \frac{1}{2}e^x\int_x^\infty e^{-\xi}y(\xi,t)\,d\xi.$$

Consequently, we can obtain

$$u_x^2(x,t) - u^2(x,t) = -\int_{-\infty}^x e^{\xi}y(\xi,t)\,d\xi \int_x^\infty e^{-\xi}y(\xi,t)\,d\xi$$

for all $t > 0$. Rewrite (2.2) as

$$u_t + u^2 u_x + \partial_x G * \left(u^3 + \frac{3}{2}uu_x^2\right) + G * \left(\frac{1}{2}u_x^3\right) + \lambda u = 0.$$

By differentiating the above equation, we get

$$u_{tx} + u^2 u_{xx} + \frac{1}{2}uu_x^2 - u^3 + G * \left(u^3 + \frac{3}{2}uu_x^2\right) + \partial_x G * \left(\frac{1}{2}u_x^3\right) + \lambda u_x = 0.$$

Use the above equation and differentiate $uu_x(q(x_0,t),t)$ with respect to t:

$$\begin{aligned}
\frac{d}{dt}uu_x &= \left(u_t + u_x u^2\right)u_x + u\left(u_{xt} + u_{xx}u^2\right) \\
&= -\left[\partial_x G * \left(u^3 + \frac{3}{2}uu_x^2\right) + G * \left(\frac{1}{2}u_x^3\right) + \lambda u\right]u_x \\
&\quad - u\left[\frac{1}{2}uu_x^2 - u^3 + G * \left(u^3 + \frac{3}{2}uu_x^2\right) + \partial_x G * \left(\frac{1}{2}u_x^3\right) + \lambda u_x\right] \\
&= u^4 - \frac{1}{2}u^2u_x^2 - 2\lambda u_x u - \left[\partial_x G * \left(u^3 + \frac{3}{2}uu_x^2\right) + G * \left(\frac{1}{2}u_x^3\right)\right]u_x \\
&\quad - u\left[G * \left(u^3 + \frac{3}{2}uu_x^2\right) + \partial_x G * \left(\frac{1}{2}u_x^3\right)\right] \\
&= u^4 - \frac{1}{2}u^2u_x^2 - 2\lambda u_x u - (u - u_x)\frac{1}{2}e^{-q}\int_{-\infty}^{q} e^{\xi}\left(u^3 + \frac{3}{2}uu_x^2 - \frac{1}{2}u_x^3\right)d\xi \\
&\quad - (u + u_x)\frac{1}{2}e^{q}\int_{q}^{\infty} e^{-\xi}\left(u^3 + \frac{3}{2}uu_x^2 + \frac{1}{2}u_x^3\right)d\xi \\
&\le u^4 - \frac{1}{2}u^2u_x^2 - 2\lambda u_x u - \frac{1}{4}(u - u_x)u^3 - \frac{1}{4}(u + u_x)u^3 \\
&= \frac{1}{2}u^4 - \frac{1}{2}u^2u_x^2 - 2\lambda uu_x, \qquad (2.7)
\end{aligned}$$

where q is the diffeomorphism defined in (2.3) and we also apply the following inequalities in [4]:

$$\int_{-\infty}^{q(x_0,t)} e^{\xi}\left(u^3 + \frac{3}{2}uu_x^2 - \frac{1}{2}u_x^3\right)(\xi,t)\,d\xi \ge e^{q(x_0,t)}u^3\left(q(x_0,t),t\right)$$

and

$$\int_{q(x_0,t)}^{\infty} e^{-\xi}\left(u^3 + \frac{3}{2}uu_x^2 + \frac{1}{2}u_x^3\right)(\xi,t)\,d\xi \le e^{-q(x_0,t)}u^3\left(q(x_0,t),t\right).$$

Owing to condition (2.6), we can derive

$$\left(u_0u_{0x}(x_0) + 2\lambda\right)^2 - \left(u_0^2(x_0) + 2\lambda\right)^2 = \left[u_0(u_{0x} - u_0)\right]\left[u_0(u_{0x} + u_0) + 4\lambda\right] > 0$$

and

$$\left(u_0u_{0x}(x_0) + 2\lambda\right)^2 - \left(u_0^2(x_0) - 2\lambda\right)^2 = \left[u_0(u_{0x} - u_0) + 4\lambda\right]\left[u_0(u_{0x} + u_0)\right] > 0.$$

Claim $uu_x(q(x_0,t),t) < 0$ *is decreasing and*

$$\begin{cases} (u^2(q(x_0,t),t) + 2\lambda)^2 < (uu_x(q(x_0,t),t) + 2\lambda)^2, \\ (u^2(q(x_0,t),t) - 2\lambda)^2 < (uu_x(q(x_0,t),t) + 2\lambda)^2 \end{cases}$$

for all $t \geq 0$.

Proof Suppose not, there exists t_0 such that $(u^2(q(x_0,t),t) + 2\lambda)^2 < (uu_x(q(x_0,t),t) + 2\lambda)^2$ and $(u^2(q(x_0,t),t) - 2\lambda)^2 < (uu_x(q(x_0,t),t) + 2\lambda)^2$ on $[0,t)$. Then we have $(u^2(q(x_0,t),t) + 2\lambda)^2 = (uu_x(q(x_0,t),t) + 2\lambda)^2$ or $(u^2(q(x_0,t),t) - 2\lambda)^2 = (uu_x(q(x_0,t),t) + 2\lambda)^2$. Let

$$I(t) := u(u - u_x)\big(q(x_0,t),t\big)$$

and

$$II(t) := u(u + u_x)\big(q(x_0,t),t\big).$$

Firstly, differentiating $I(t)$, we have

$$\begin{aligned} \frac{dI(t)}{dt} &= -u^2\big(q(x_0,t),t\big)\left(e^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi}y(\xi,t)\,d\xi\right)^2 \\ &\quad + \left(e^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi}y(\xi,t)\,d\xi\right)\left(e^{-q(x_0,t)}\int_{-\infty}^{q(x_0,t)} e^{\xi}y_t(\xi,t)\,d\xi\right) \\ &\quad + \frac{1}{2}\int_{-\infty}^{q(x_0,t)} e^{\xi}y_t(\xi,t)\,d\xi \int_{q(x_0,t)}^{\infty} e^{-\xi}y(\xi,t)\,d\xi \\ &\quad + \frac{1}{2}\int_{-\infty}^{q(x_0,t)} e^{\xi}y(\xi,t)\,d\xi \int_{q(x_0,t)}^{\infty} e^{-\xi}y_t(\xi,t)\,d\xi \\ &\leq \frac{1}{2}u^2\big(u_x^2 - u^2\big) - 2\lambda u^2 + 2\lambda uu_x \\ &= \frac{1}{2}(uu_x + 2\lambda)^2 - \frac{1}{2}\big(u^2 + 2\lambda\big)^2 > 0. \end{aligned} \tag{2.8}$$

Secondly, by the same argument, we get

$$\begin{aligned} \frac{dII(t)}{dt} &= u^2\big(q(x_0,t),t\big)\left(e^{q(x_0,t)}\int_{q(x_0,t)}^{\infty} e^{-\xi}y(\xi,t)\,d\xi\right)^2 \\ &\quad + \left(e^{q(x_0,t)}\int_{q(x_0,t)}^{\infty} e^{-\xi}y(\xi,t)\,d\xi\right)\left(e^{q(x_0,t)}\int_{q(x_0,t)}^{\infty} e^{-\xi}y_t(\xi,t)\,d\xi\right) \\ &\quad + \frac{1}{2}\int_{-\infty}^{q(x_0,t)} e^{\xi}y_t(\xi,t)\,d\xi \int_{q(x_0,t)}^{\infty} e^{-\xi}y(\xi,t)\,d\xi \\ &\quad + \frac{1}{2}\int_{-\infty}^{q(x_0,t)} e^{\xi}y(\xi,t)\,d\xi \int_{q(x_0,t)}^{\infty} e^{-\xi}y_t(\xi,t)\,d\xi \\ &\leq -\frac{1}{2}u^2\big(u_x^2 - u^2\big) - 2\lambda u^2 - 2\lambda uu_x \\ &= -\frac{1}{2}(uu_x + 2\lambda)^2 + \frac{1}{2}\big(u^2 - 2\lambda\big)^2 < 0. \end{aligned} \tag{2.9}$$

Hence, following from (2.8), (2.9) and the continuity property of ODEs, we deduce

$$\begin{aligned}\bigl(uu_x\bigl(q(x_0,t),t\bigr)+2\lambda\bigr)^2-\bigl(u^2\bigl(q(x_0,t),t\bigr)+2\lambda\bigr)^2&=-I(t)\bigl(II(t)+4\lambda\bigr)\\&>-I(0)\bigl(II(0)+4\lambda\bigr)>0\end{aligned}$$

and

$$\begin{aligned}\bigl(uu_x\bigl(q(x_0,t),t\bigr)+2\lambda\bigr)^2-\bigl(u^2\bigl(q(x_0,t),t\bigr)-2\lambda\bigr)^2&=-\bigl(I(t)-4\lambda\bigr)II(t)\\&>-\bigl(I(0)-4\lambda\bigr)II(0)>0,\end{aligned}$$

for all $t>0$, which implies that t_0 can be extended to the infinity. This is a contradiction. Thus the claim is true. □

Using (2.8) and (2.9) again and $2uu_x(q(x_0,t),t)=-I(t)+II(t)$, we have the following inequality for $[2(uu_x+2\lambda)^2-(u^2+2\lambda)^2-(u^2-2\lambda)^2](q(x_0,t),t)$:

$$\begin{aligned}&\frac{d}{dt}\bigl[2(uu_x+2\lambda)^2-\bigl(u^2+2\lambda\bigr)^2-\bigl(u^2-2\lambda\bigr)^2\bigr]\bigl(q(x_0,t),t\bigr)\\&\quad=-\frac{d}{dt}\bigl[I(t)\bigl(II(t)+4\lambda\bigr)\bigr]-\frac{d}{dt}\bigl[\bigl(I(t)-4\lambda\bigr)II(t)\bigr]\\&\quad\geq-\left[\frac{1}{2}(uu_x+2\lambda)^2-\frac{1}{2}\bigl(u^2+2\lambda\bigr)^2\right]\bigl(q(x_0,t),t\bigr)\bigl(II(t)+4\lambda\bigr)\\&\qquad+\left[\frac{1}{2}(uu_x+2\lambda)^2-\frac{1}{2}\bigl(u^2-2\lambda\bigr)^2\right]\bigl(q(x_0,t),t\bigr)I(t)\\&\qquad-\left[\frac{1}{2}(uu_x+2\lambda)^2-\frac{1}{2}\bigl(u^2+2\lambda\bigr)^2\right]\bigl(q(x_0,t),t\bigr)II(t)\\&\qquad+\left[\frac{1}{2}(uu_x+2\lambda)^2-\frac{1}{2}\bigl(u^2-2\lambda\bigr)^2\right]\bigl(q(x_0,t),t\bigr)\bigl(I(t)-4\lambda\bigr)\\&\quad\geq-\frac{1}{2}(uu_x+4\lambda)\bigl[2(uu_x+2\lambda)^2-\bigl(u^2+2\lambda\bigr)^2-\bigl(u^2-2\lambda\bigr)^2\bigr]\bigl(q(x_0,t),t\bigr).\end{aligned}\tag{2.10}$$

Recalling (2.7), we get

$$\begin{aligned}\partial_t\bigl[uu_x\bigl(q(x_0,t),t\bigr)\bigr]&\leq\left[\frac{1}{2}u^4-\frac{1}{2}u^2u_x^2-2\lambda uu_x\right]\bigl(q(x_0,t),t\bigr)\\&=-\frac{1}{4}\bigl[2(uu_x+2\lambda)^2-\bigl(u^2+2\lambda\bigr)^2-\bigl(u^2-2\lambda\bigr)^2\bigr]\bigl(q(x_0,t),t\bigr).\end{aligned}\tag{2.11}$$

Substituting (2.11) into (2.10) yields

$$\begin{aligned}&\frac{d}{dt}\bigl[2(uu_x+2\lambda)^2-\bigl(u^2+2\lambda\bigr)^2-\bigl(u^2-2\lambda\bigr)^2\bigr]\bigl(q(x_0,t),t\bigr)\\&\quad\geq\frac{1}{4}\bigl[2(uu_x+2\lambda)^2-\bigl(u^2+2\lambda\bigr)^2-\bigl(u^2-2\lambda\bigr)^2\bigr]\bigl(q(x_0,t),t\bigr)\\&\qquad\times\int_0^t\bigl[2(uu_x+2\lambda)^2-\bigl(u^2+2\lambda\bigr)^2-\bigl(u^2-2\lambda\bigr)^2\bigr]\bigl(q(x_0,\tau),\tau\bigr)\,d\tau\\&\qquad-\bigl[2(uu_x+2\lambda)^2-\bigl(u^2+2\lambda\bigr)^2-\bigl(u^2-2\lambda\bigr)^2\bigr]\bigl(q(x_0,t),t\bigr)\bigl(u_0u_{0x}(x_0)+2\lambda\bigr).\end{aligned}\tag{2.12}$$

Before completing the proof, we need the following technical lemma.

Lemma 2.2 [13] *Suppose that $\Psi(t)$ is twice continuously differential satisfying*

$$\begin{cases} \Psi''(t) \geq C_0\Psi'(t)\Psi(t), & t > 0, C_0 > 0, \\ \Psi(t) > 0, & \Psi'(t) > 0. \end{cases} \tag{2.13}$$

Then $\psi(t)$ blows up in finite time. Moreover, the blow-up time can be estimated in terms of the initial datum as

$$T \leq \max\left\{\frac{2}{C_0\Psi(0)}, \frac{\Psi(0)}{\Psi'(0)}\right\}.$$

Letting $\Psi(t) := \int_0^t [2(uu_x + 2\lambda)^2 - (u^2 + 2\lambda)^2 - (u^2 - 2\lambda)^2](q(x_0,\tau),\tau)\,d\tau - 4u_0u_{0x}(x_0) - 8\lambda$, then (2.12) is an equation of type (2.13) with $C_0 = \frac{1}{4}$. The proof is complete by applying Lemma 2.2. □

Similarly, if we change the signs of $u_0(x_0)$ and $y_0(x)$ in Theorem 2.1, it still holds. More precisely, we have the following blow-up criterion.

Theorem 2.3 *Suppose that $u_0 \in H^2(\mathbb{R})$ and there exists $x_0 \in \mathbb{R}$ such that $u_0(x_0) < 0$, $y_0(x_0) = (1 - \partial_x^2)u_0(x_0) = 0$,*

$$y_0(x) \leq 0\ (\not\equiv 0) \quad \textit{for } x \in (-\infty, x_0) \quad \textit{and} \quad y_0(x) \geq 0\ (\not\equiv 0) \quad \textit{for } x \in (x_0, \infty)$$

as well as

$$u_0(u_0 - u_{0x})(x_0) > 4\lambda \quad \textit{and} \quad u_0(u_0 + u_{0x})(x_0) < -4\lambda.$$

Then the corresponding solution $u(x,t)$ to equation (1.2) with u_0 as the initial datum blows up in finite time.

Remark 2.2 We know that $\|u\|_{L^\infty} \leq \frac{1}{2}\|u\|_{H^1} = \frac{1}{2}e^{-\lambda t}\|u_0\|_{H^1}$, which implies $u_x(q(x_0,t),t)$ goes to $-\infty$ in finite time for the case in Theorem 2.1. However, for the case in Theorem 2.3, $u_x(q(x_0,t),t)$ goes to $+\infty$ in finite time.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

CM proposed the problem. The authors proved Theorems together. All authors read and approved the final manuscript.

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