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Fixed point results for implicit contractions on spaces with two metrics

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Abstract
We establish new fixed-point results involving implicit contractions on a metric space endowed with two metrics. The main results in this paper extend and generalize several existing fixed-point theorems in the literature.

Keywords: fixed point; implicit contraction; two metrics

1 Introduction

Fixed-point theory is a major branch of nonlinear analysis because of its wide applicability. The existence problem of fixed points of mappings satisfying a given metrical contractive condition has attracted many researchers in past few decades. The Banach contraction principle [1] is one of the most important theorems in this direction. Many generalizations of this famous principle exist in the literature, see, for examples, [2–6] and references therein. On the other hand, several classical fixed-point theorems have been unified by considering general contractive conditions expressed by an implicit condition, see for examples, Turinici [7], Popa [8, 9], Berinde [10], and references therein.

This paper presents fixed-point theorems for implicit contractions on a metric space endowed with two metrics. This paper will be divided into two main sections. Section 2 presents local and global fixed-point results for implicit contractions involving α-admissible mappings, a recent concept introduced in [11]. Section 3 presents some interesting consequences that can be obtained from the results established in the previous section.

2 Main results

Let $\mathcal{F}$ be the set of functions $F : [0,+\infty)^6 \to \mathbb{R}$ satisfying the following conditions:

(i) F is continuous;
(ii) F is non-decreasing in the first variable;
(iii) F is non-increasing in the fifth variable;
(iv) $\exists h \in (0,1) \mid F(u,v,v,u,u+v,0) \le 0 \Longrightarrow u \le hv$.

Example Let $F : [0,+\infty)^6 \to \mathbb{R}$ be the function defined by

$$F(t_1,t_2,t_3,t_4,t_5,t_6) := t_1 - q \max\left\{t_2,t_3,t_4,\frac{t_5+t_6}{2}\right\},$$

where $q \in (0,1)$. We can check easily that $F \in \mathcal{F}$.

Let X be a nonempty set endowed with two metrics d and d'. If $x_0 \in X$ and $r > 0$, let

$$B(x_0, r) := \{x \in X : d(x_0, x) < r\}.$$

We denote by $\overline{B(x_0,r)}^{d'}$ the d'-closure of $B(x_0, r)$.

Let $T : \overline{B(x_0,r)}^{d'} \to X$ and $\alpha : X \times X \to [0,\infty)$. We say that T is α-admissible (see [11]) if the following condition holds: for all $x, y \in B(x_0, r)$, we have

$$\alpha(x,y) \geq 1 \quad \Longrightarrow \quad \alpha(Tx, Ty) \geq 1.$$

We say that X satisfies the property (H) with respect to the metric d if the following condition holds:

If $\lim_{n\to\infty} d(x_n, x) = 0$ for some $x \in X$ and $\alpha(x_n, x_{n+1}) \geq 1$ for all n, then there exist a positive integer κ and a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha(x_{n(k)}, x) \geq 1$ for all $k \geq \kappa$.

Our first result is the following.

Theorem 2.1 *Let (X, d') be a complete metric space, d another metric on X, $x_0 \in X$, $r > 0$, $T : \overline{B(x_0,r)}^{d'} \to X$, and $\alpha : X \times X \to [0,\infty)$. Suppose there exists $F \in \mathcal{F}$ such that for $x, y \in \overline{B(x_0,r)}^{d'}$, we have*

$$F\big(\alpha(x,y)d(Tx,Ty), d(x,y), d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\big) \leq 0. \tag{1}$$

In addition, assume the following properties hold:

(I) *$d(x_0, Tx_0) < (1-h)r$ and $\alpha(x_0, Tx_0) \geq 1$*;

(II) *T is α-admissible*;

(III) *if $d \not\geq d'$, assume T is uniformly continuous from $(B(x_0,r), d)$ into (X, d')*;

(IV) *if $d = d'$, assume X satisfies the property* (H) *with respect to the metric d*;

(V) *if $d \neq d'$, assume T is continuous from $(\overline{B(x_0,r)}^{d'}, d')$ into (X, d').*

Then T has a fixed point.

Proof Let $x_1 = Tx_0$. From (I), we have

$$d(x_0, x_1) = d(x_0, Tx_0) \leq (1-h)r < r,$$

which implies that $x_1 \in B(x_0, r)$. Let $x_2 = Tx_1$. From (1), we have

$$F\big(\alpha(x_0,x_1)d(Tx_0,Tx_1), d(x_0,x_1), d(x_0,x_1), d(x_1,x_2), d(x_0,x_2), 0\big) \leq 0.$$

From (I), we have

$$d(Tx_0, Tx_1) \leq \alpha(x_0,x_1)d(Tx_0,Tx_1).$$

Since F is non-decreasing in the first variable (property (i)), we obtain

$$F\big(d(x_1,x_2), d(x_0,x_1), d(x_0,x_1), d(x_1,x_2), d(x_0,x_2), 0\big) \leq 0.$$

Since $d(x_0,x_2) \leq d(x_0,x_1) + d(x_1,x_2)$, using (iii), we obtain

$$F\bigl(d(x_1,x_2), d(x_0,x_1), d(x_0,x_1), d(x_1,x_2), d(x_0,x_1) + d(x_1,x_2), 0\bigr) \leq 0,$$

which implies from (iv) that

$$d(x_1,x_2) \leq hd(x_0,x_1) \leq h(1-h)r < r.$$

Now, we have

$$d(x_0,x_2) \leq d(x_0,x_1) + hd(x_0,x_1) = (1+h)d(x_0,x_1) \leq (1+h)(1-h)r < r.$$

This implies that $x_2 \in B(x_0,r)$. Again, let $x_3 = Tx_2$. Since T is α-admissible and $\alpha(x_0,x_1) \geq 1$, we have

$$d(x_2,x_3) \leq \alpha(x_1,x_2)d(Tx_1,Tx_2).$$

Then, from (1), we obtain

$$F\bigl(d(x_2,x_3), d(x_1,x_2), d(x_1,x_2), d(x_2,x_3), d(x_1,x_3), 0\bigr) \leq 0.$$

Using (iii), we obtain

$$F\bigl(d(x_2,x_3), d(x_1,x_2), d(x_1,x_2), d(x_2,x_3), d(x_1,x_2) + d(x_2,x_3), 0\bigr) \leq 0,$$

which implies from (iv) that

$$d(x_2,x_3) \leq hd(x_1,x_2) \leq h^2(1-h)r < r.$$

Now, we have

$$d(x_0,x_3) \leq d(x_0,x_2) + d(x_2,x_3) \leq (1+h)(1-h)r + h^2(1-h)r = \bigl(1-h^3\bigr)r < r.$$

This implies that $x_3 \in B(x_0,r)$. Continuing this process, by induction, we can define the sequence $\{x_n\}$ by

$$x_{n+1} = Tx_n, \quad \forall n \in \mathbb{N}.$$

Such sequence satisfies the following property:

$$x_n \in B(x_0,r), \quad \alpha(x_n,x_{n+1}) \geq 1 \quad \text{and} \quad d(x_n,x_{n+1}) \leq h^n(1-h)r, \quad \forall n \in \mathbb{N}. \tag{2}$$

Since $h \in (0,1)$, it follows from (2) that $\{x_n\}$ is a Cauchy sequence with respect to the metric d.

Now, we shall prove that $\{x_n\}$ is also a Cauchy sequence with respect to d'. If $d' \leq d$, the result follows immediately from (2). If $d \ngeq d'$, from (III), given $\varepsilon > 0$, there exists $\delta >$ such that

$$x, y \in B(x_0, r), \quad d(x, y) < \delta \quad \Longrightarrow \quad d'(Tx, Ty) < \varepsilon. \tag{3}$$

On the other hand, since $\{x_n\}$ is Cauchy with respect to d, there exists a positive integer N such that

$$d(x_n, x_m) < \delta, \quad \forall n, m \geq N.$$

Using (3), we have

$$d'(x_{n+1}, x_{m+1}) < \varepsilon, \quad \forall n, m \geq N.$$

Thus we proved that $\{x_n\}$ is Cauchy with respect to d'.

Since (X, d') is complete, there exists $z \in \overline{B(x_0, r)}^{d'}$ such that

$$\lim_{n\to\infty} d'(x_n, z) = 0. \tag{4}$$

We shall prove that z is a fixed point of T. We consider two cases.

Case 1. If $d = d'$.

From (IV), there exist a positive integer κ and a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that

$$\alpha(x_{n(k)}, z) \geq 1, \quad \forall k \geq \kappa. \tag{5}$$

Using (1), for all $k \geq \kappa$, we obtain

$$F\bigl(\alpha(x_{n(k)}, z)d(Tx_{n(k)}, Tz), d(x_{n(k)}, z), d(x_{n(k)}, x_{n(k)+1}), d(z, Tz), d(x_{n(k)}, Tz), d(z, x_{n(k)+1})\bigr) \\ \leq 0.$$

Using (5) and condition (ii), for all $k \geq \kappa$, we obtain

$$F\bigl(d(x_{n(k)+1}, Tz), d(x_{n(k)}, z), d(x_{n(k)}, x_{n(k)+1}), d(z, Tz), d(x_{n(k)}, Tz), d(z, x_{n(k)+1})\bigr) \leq 0.$$

Letting $k \to \infty$, using (4) and the continuity of F, we obtain

$$F\bigl(d(z, Tz), 0, 0, d(z, Tz), d(z, Tz), 0\bigr) \leq 0,$$

which implies from (iv) that $d(z, Tz) = 0$.

Case 2. If $d \neq d'$.

In this case, using (V) and (4), we obtain

$$\lim_{n\to\infty} d'(Tx_n, Tz) = \lim_{n\to\infty} d'(x_{n+1}, Tz) = 0.$$

The uniqueness of the limit gives $z = Tz$. □

Taking $d = d'$ in Theorem 2.1, we obtain the following result.

Theorem 2.2 *Let* (X,d) *be a complete metric space,* $x_0 \in X$, $r > 0$, $T : \overline{B(x_0,r)}^d \to X$, *and* $\alpha : X \times X \to [0,\infty)$. *Suppose there exists* $F \in \mathcal{F}$ *such that for* $x, y \in \overline{B(x_0,r)}^d$, *we have*

$$F\big(\alpha(x,y)d(Tx,Ty), d(x,y), d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\big) \leq 0.$$

In addition, assume the following properties hold:
(I) $d(x_0,Tx_0) < (1-h)r$ *and* $\alpha(x_0,Tx_0) \geq 1$;
(II) T *is* α*-admissible*;
(III) X *satisfies the property* (H) *with respect to the metric* d.
Then T *has a fixed point.*

From Theorem 2.1, we can deduce the following global result.

Theorem 2.3 *Let* (X,d') *be a complete metric space,* d *another metric on* X, $T : X \to X$, *and* $\alpha : X \times X \to [0,\infty)$. *Suppose there exists* $F \in \mathcal{F}$ *such that for* $x, y \in X$, *we have*

$$F\big(\alpha(x,y)d(Tx,Ty), d(x,y), d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\big) \leq 0.$$

In addition, assume the following properties hold:
(I) *there exists* $x_0 \in X$ *such that* $\alpha(x_0,Tx_0) \geq 1$;
(II) T *is* α*-admissible* $(x, y \in X, \alpha(x,y) \geq 1 \Longrightarrow \alpha(Tx,Ty) \geq 1)$;
(III) *if* $d \not\geq d'$, *assume* T *is uniformly continuous from* (X,d) *into* (X,d');
(IV) *if* $d = d'$, *assume* X *satisfies the property* (H) *with respect to the metric* d;
(V) *if* $d \neq d'$, *assume* T *is continuous from* (X,d') *into* (X,d').
Then T *has a fixed point.*

Proof We take $r > 0$ such that $d(x_0,Tx_0) < (1-h)r$. From Theorem 2.1, T has a fixed point in $\overline{B(x_0,r)}^{d'}$. □

Taking $d = d'$ in Theorem 2.3, we obtain the following result.

Theorem 2.4 *Let* (X,d) *be a complete metric space,* $T : X \to X$, *and* $\alpha : X \times X \to [0,\infty)$. *Suppose there exists* $F \in \mathcal{F}$ *such that for* $x, y \in X$, *we have*

$$F\big(\alpha(x,y)d(Tx,Ty), d(x,y), d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\big) \leq 0.$$

In addition, assume the following properties hold:
(I) *there exists* $x_0 \in X$ *such that* $\alpha(x_0,Tx_0) \geq 1$;
(II) T *is* α*-admissible* $(x, y \in X, \alpha(x,y) \geq 1 \Longrightarrow \alpha(Tx,Ty) \geq 1)$;
(III) X *satisfies the property* (H) *with respect to the metric* d.
Then T *has a fixed point.*

3 Consequences

We present here some interesting consequences that can be obtained from our main results.

3.1 The case $\alpha(x,y) = 1$

Taking $\alpha(x,y) := 1$ for all $x,y \in X$, from Theorems 2.1, 2.2, 2.3, and 2.4, we obtain the following results that are generalizations of the fixed-point results in [2, 3, 5, 8, 10, 12, 13].

Corollary 3.1 *Let (X,d') be a complete metric space, d another metric on X, $x_0 \in X$, $r > 0$, and $T : \overline{B(x_0,r)}^{d'} \to X$. Suppose there exists $F \in \mathcal{F}$ such that for $x,y \in \overline{B(x_0,r)}^{d'}$, we have*

$$F\big(d(Tx,Ty),d(x,y),d(x,Tx),d(y,Ty),d(x,Ty),d(y,Tx)\big) \le 0.$$

In addition, assume the following properties hold:

(I) $d(x_0,Tx_0) < (1-h)r$;

(II) *if $d \ngeq d'$, assume T is uniformly continuous from $(B(x_0,r),d)$ into (X,d')*;

(III) *if $d \neq d'$, assume T is continuous from $(\overline{B(x_0,r)}^{d'},d')$ into (X,d').*

Then T has a fixed point.

Corollary 3.2 *Let (X,d) be a complete metric space, $x_0 \in X$, $r > 0$, and $T : \overline{B(x_0,r)}^{d} \to X$. Suppose there exists $F \in \mathcal{F}$ such that for $x,y \in \overline{B(x_0,r)}^{d}$, we have*

$$F\big(d(Tx,Ty),d(x,y),d(x,Tx),d(y,Ty),d(x,Ty),d(y,Tx)\big) \le 0.$$

In addition, assume that $d(x_0,Tx_0) < (1-h)r$. Then T has a fixed point.

Corollary 3.3 *Let (X,d') be a complete metric space, d another metric on X, and $T : X \to X$. Suppose there exists $F \in \mathcal{F}$ such that for $x,y \in X$, we have*

$$F\big(d(Tx,Ty),d(x,y),d(x,Tx),d(y,Ty),d(x,Ty),d(y,Tx)\big) \le 0.$$

In addition, assume the following properties hold:

(I) *if $d \ngeq d'$, assume T is uniformly continuous from (X,d) into (X,d')*;

(II) *if $d \neq d'$, assume T is continuous from (X,d') into (X,d').*

Then T has a fixed point.

Corollary 3.4 *Let (X,d) be a complete metric space and $T : X \to X$. Suppose there exists $F \in \mathcal{F}$ such that for $x,y \in X$, we have*

$$F\big(d(Tx,Ty),d(x,y),d(x,Tx),d(y,Ty),d(x,Ty),d(y,Tx)\big) \le 0.$$

Then T has a fixed point.

Corollary 3.4 is an enriched version of Popa [8] that unifies the most important metrical fixed-point theorems for contractive mappings in Rhoades' classification [6].

3.2 The case of a partial ordered set

Let $\preceq$ be a partial order on X. Let $\lhd$ be the binary relation on X defined by

$$(x,y) \in X \times X, \quad x \lhd y \quad \Longleftrightarrow \quad x \preceq y \quad \text{or} \quad y \preceq x.$$

We say that $(X, \lhd)$ satisfies the property (H) with respect to the metric d if the following condition holds:

If $\lim_{n\to\infty} d(x_n, x) = 0$ for some $x \in X$ and $x_n \lhd x_{n+1}$ for all n, then there exist a positive integer κ and a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $x_{n(k)} \lhd x$ for all $k \geq \kappa$.

From Theorems 2.1, 2.2, 2.3, and 2.4, we obtain the following results that are extensions and generalizations of the fixed-point results in [14, 15].

At first, we denote by $\widetilde{\mathcal{F}}$ the set of functions $F : [0, +\infty)^6 \to \mathbb{R}$ satisfying the following conditions:

(j) $F \in \mathcal{F}$;

(jj) $F(0, t_2, t_3, t_4, t_5, t_6) \leq 0$ for all $t_i \geq 0$, $i = 2, \ldots, 6$.

We start with the following fixed-point result.

Corollary 3.5 *Let (X, d') be a complete metric space, d another metric on X, $x_0 \in X$, $r > 0$, and $T : \overline{B(x_0, r)}^{d'} \to X$. Suppose there exists $F \in \widetilde{\mathcal{F}}$ such that for $x, y \in \overline{B(x_0, r)}^{d'}$ with $x \lhd y$, we have*

$$F\big(d(Tx, Ty), d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\big) \leq 0.$$

In addition, assume the following properties hold:

(I) *$d(x_0, Tx_0) < (1 - h)r$ and $x_0 \lhd Tx_0$;*

(II) *$x, y \in \overline{B(x_0, r)}^{d'}$, $x \lhd y \Longrightarrow Tx \lhd Ty$;*

(III) *if $d \ngeq d'$, assume T is uniformly continuous from $(B(x_0, r), d)$ into (X, d');*

(IV) *if $d = d'$, assume $(X, \lhd)$ satisfies the property* (H) *with respect to the metric d;*

(V) *if $d \neq d'$, assume T is continuous from $(\overline{B(x_0, r)}^{d'}, d')$ into (X, d').*

Then T has a fixed point.

Proof It follows from Theorem 2.1 by taking

$$\alpha(x, y) := \begin{cases} 1 & \text{if } x \lhd y; \\ 0 & \text{if } x \ntriangleleft y. \end{cases}$$ □

Similarly, from Theorem 2.2, we obtain the following result.

Corollary 3.6 *Let (X, d) be a complete metric space, $x_0 \in X$, $r > 0$, and $T : \overline{B(x_0, r)}^{d} \to X$. Suppose there exists $F \in \widetilde{\mathcal{F}}$ such that for $x, y \in \overline{B(x_0, r)}^{d}$ with $x \lhd y$, we have*

$$F\big(d(Tx, Ty), d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\big) \leq 0.$$

In addition, assume the following properties hold:

(I) *$d(x_0, Tx_0) < (1 - h)r$ and $x_0 \lhd Tx_0$;*

(II) *$x, y \in \overline{B(x_0, r)}^{d'}$, $x \lhd y \Longrightarrow Tx \lhd Ty$;*

(III) *$(X, \lhd)$ satisfies the property* (H) *with respect to the metric d;*

Then T has a fixed point.

From Theorem 2.3, we obtain the following global result.

Corollary 3.7 *Let (X, d') be a complete metric space, d another metric on X, and $T : X \to X$. Suppose there exists $F \in \widetilde{\mathcal{F}}$ such that for $x, y \in X$ with $x \lhd y$, we have*

$$F\big(d(Tx, Ty), d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\big) \leq 0.$$

In addition, assume the following properties hold:

(I) *there exists $x_0 \in X$ such that $x_0 \lhd Tx_0$*;
(II) $x, y \in X$, $x \lhd y \Longrightarrow Tx \lhd Ty$;
(III) *if $d \ngeq d'$, assume T is uniformly continuous from* (X, d) *into* (X, d');
(IV) *if $d = d'$, assume* $(X, \lhd)$ *satisfies the property* (H) *with respect to the metric d*;
(V) *if $d \neq d'$, assume T is continuous from* (X, d') *into* (X, d').

Then T has a fixed point.

Finally, from Theorem 2.4, we obtain the following fixed-point result.

Corollary 3.8 *Let (X, d) be a complete metric space and $T : X \to X$. Suppose there exists $F \in \widetilde{\mathcal{F}}$ such that for $x, y \in X$ with $x \lhd y$, we have*

$$F\big(d(Tx, Ty), d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\big) \leq 0.$$

In addition, assume the following properties hold:

(I) *there exists $x_0 \in X$ such that $x_0 \lhd Tx_0$*;
(II) $x, y \in X$, $x \lhd y \Longrightarrow Tx \lhd Ty$;
(III) $(X, \lhd)$ *satisfies the property* (H) *with respect to the metric d.*

Then T has a fixed point.

3.3 The case of cyclic mappings

From Theorem 2.4, we obtain the following fixed-point result that is a generalization of Theorem 1.1 in [16].

Corollary 3.9 *Let (Y, d) be a complete metric space, $\{A, B\}$ a pair of nonempty closed subsets of Y, and $T : A \cup B \to A \cup B$. Suppose there exists $F \in \widetilde{\mathcal{F}}$ such that for $x \in A$, $y \in B$, we have*

$$F\big(d(Tx, Ty), d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\big) \leq 0.$$

In addition, assume that $T(A) \subseteq B$ and $T(B) \subseteq A$.

Then T has a fixed point in $A \cap B$.

Proof Let $X := A \cup B$. Clearly (since A and B are closed), (X, d) is a complete metric space. Define $\alpha : X \times X \to [0, \infty)$ by

$$\alpha(x, y) := \begin{cases} 1 & \text{if } (x, y) \in (A \times B) \cup (B \times A); \\ 0 & \text{if } (x, y) \notin (A \times B) \cup (B \times A). \end{cases}$$

Clearly (since $F \in \widetilde{\mathcal{F}}$), for all $x, y \in X$, we have

$$F\big(\alpha(x, y) d(Tx, Ty), d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\big) \leq 0.$$

Taking any point $x_0 \in A$, since $T(A) \subseteq B$, we have $Tx_0 \in B$, which implies that $\alpha(x_0, Tx_0) \geq 1$.

Now, let $(x, y) \in X \times X$ such that $\alpha(x, y) \geq 1$. We have two cases.

Case 1. $(x, y) \in A \times B$.

Since $T(A) \subseteq B$ and $T(B) \subseteq A$, we have $(Tx, Ty) \in B \times A$, which implies that $\alpha(Tx, Ty) \geq 1$.

Case 2. $(x, y) \in B \times A$.

In this case, we have $(Tx, Ty) \in A \times B$, which implies that $\alpha(Tx, Ty) \geq 1$.

Then T is α-admissible.

Finally, we shall prove that X satisfies the property (H) with respect to the metric d.

Let $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} d(x_n, x) = 0$ for some $x \in X$, and $\alpha(x_n, x_{n+1}) \geq 1$ for all n. From the definition of α, this implies that $(x_n, x_{n+1}) \in (A \times B) \cup (B \times A)$ for all n. Since A and B are closed, we get $x \in A \cap B$. Then we have $\alpha(x_n, x) = 1$ for all n. Thus, we proved that X satisfies the property (H) with respect to the metric d.

Now, from Theorem 2.4, T has a fixed point in X, that is, there exists $z \in A \cup B$ such that $Tz = z$. Since $T(A) \subseteq B$ and $T(B) \subseteq A$, obviously, we have $z \in A \cap B$. □

Competing interests
The author declares that they have no competing interests.

Author's contributions
The author read and approved the final manuscript.

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