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Investigation of the spectrum and the Jost solutions of discrete Dirac system on the whole axis

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Abstract

We consider the boundary value problem (BVP) for the discrete Dirac equations

$$\begin{cases} y_{n+1}^{(2)} - y_n^{(2)} + p_n y_n^{(1)} = \lambda y_n^{(1)}, \\ y_{n-1}^{(1)} - y_n^{(1)} + q_n y_n^{(2)} = \lambda y_n^{(2)}, \end{cases} \qquad n \in \mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\}; \qquad y_0^{(1)} = 0,$$

where (p_n) and (q_n), $n \in \mathbb{Z}$ are real sequences, and λ is an eigenparameter. We find a polynomial type Jost solution of this BVP. Then we investigate the analytical properties and asymptotic behavior of the Jost solution. Using the Weyl compact perturbation theorem, we prove that a self-adjoint discrete Dirac system has a continuous spectrum filling the segment $[-2, 2]$. We also prove that the Dirac system has a finite number of real eigenvalues.

1 Introduction

Let us consider the BVP generated by the Sturm-Liouville equation,

$$-y'' + q(x)y = \lambda^2 y, \quad x \in \mathbb{R}_+ = [0, \infty), \tag{1}$$

and the boundary condition

$$y(0) = 0, \tag{2}$$

where q is a real valued function and λ is a spectral parameter. The bounded solution of (1) satisfying the condition

$$\lim_{x\to\infty} y(x,\lambda)e^{-i\lambda x} = 1, \quad \lambda \in \overline{\mathbb{C}}_+ := \{\lambda \in \mathbb{C} : \operatorname{Im}\lambda \geq 0\} \tag{3}$$

will be denoted by $e(x,\lambda)$. The solution $e(x,\lambda)$ satisfies the integral equation

$$e(x,\lambda) = e^{i\lambda x} + \int_x^\infty \frac{\sin\lambda(t-x)}{\lambda} q(t)e(t,\lambda)\,dt. \tag{4}$$

It has been shown that, under the condition

$$\int_0^\infty x|q(x)|\,dx < \infty, \tag{5}$$

the solution $e(x,\lambda)$ has the integral representation

$$e(x,\lambda) = e^{i\lambda x} + \int_x^\infty K(x,t)e^{i\lambda t}\,dt, \quad \lambda \in \overline{\mathbb{C}}_+, \tag{6}$$

where the function $K(x,t)$ is defined by q. The function $e(x,\lambda)$ is analytic with respect to λ in $\mathbb{C}_+ := \{\lambda \in \mathbb{C} : \operatorname{Im}\lambda > 0\}$, continuous in $\overline{\mathbb{C}}_+$, and

$$e(x,\lambda) = e^{i\lambda x}\bigl[1 + o(1)\bigr], \quad \lambda \in \overline{\mathbb{C}}_+, x \to \infty \tag{7}$$

holds [1].

The functions $e(x,\lambda)$ and $e(\lambda) := e(0,\lambda)$ are called the Jost solution and Jost function of the BVP (1) and (2), respectively. These functions play an important role in the solution of inverse problems of the quantum scattering theory [1–4]. In particular, the scattering data of the BVP (1) and (2) is defined in terms of Jost solution and Jost function.

Let us consider the system

$$\begin{array}{l} a_n y_{n+1}^{(2)} + b_n y_n^{(2)} + p_n y_n^{(1)} = \lambda y_n^{(1)}, \\ a_{n-1} y_{n-1}^{(1)} + b_n y_n^{(1)} + q_n y_n^{(2)} = \lambda y_n^{(2)}, \end{array} \quad n \in \mathbb{Z}, \tag{8}$$

where $\{\binom{y_n^{(1)}}{y_n^{(2)}}\}$ are vector sequences, $\{a_n\}_{n\in\mathbb{Z}}$, $\{b_n\}_{n\in\mathbb{Z}}$, $\{p_n\}_{n\in\mathbb{Z}}$, and $\{q_n\}_{n\in\mathbb{Z}}$ are real sequences, $a_n \neq 0$, $b_n \neq 0$ for all $n \in \mathbb{Z}$, and λ is a spectral parameter.

If for all $n \in \mathbb{Z}$, $a_n \equiv 1$ and $b_n \equiv -1$, then the system (8) reduces to

$$\begin{cases} \Delta y_{n+1}^{(2)} + p_n y_n^{(1)} = \lambda y_n^{(1)}, \\ -\Delta y_{n-1}^{(1)} + q_n y_n^{(2)} = \lambda y_n^{(2)}, \end{cases} \quad n \in \mathbb{Z}, \tag{9}$$

where Δ is the forward difference operator, *i.e.*,

$$\Delta u_n = u_{n+1} - u_n.$$

The system (9) is the discrete analogue of the well-known Dirac system

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} + \begin{pmatrix} p(x) & 0 \\ 0 & q(x) \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \lambda \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

([5], Chapter 2). Therefore the system (9) is called a discrete Dirac system.

Various problems of spectral analysis of self-adjoint difference equations have been investigated in detail [6, 7]. But all of them give an exponential type Jost solution of the difference equations. In this paper, we find a polynomial type Jost solution of (9) with the boundary condition

$$y_0^{(1)} = 0, \tag{10}$$

which is analytic in $D := \{z : |z| < 1\} \backslash \{0\}$.

2 Jost solutions of (9)

We will assume that the real sequences $\{p_n\}_{n\in\mathbb{Z}}$, $\{q_n\}_{n\in\mathbb{Z}}$ satisfy

$$\sum_{n\in\mathbb{Z}} |n|\big(|p_n| + |q_n|\big) < \infty. \tag{11}$$

If $p_n = q_n = 0$ for all $n \in \mathbb{Z}$ and

$$\lambda = -iz - (iz)^{-1},$$

from (9), we get

$$\begin{aligned} y_{n+1}^{(2)} - y_n^{(2)} &= \big[-iz - (iz)^{-1}\big] y_n^{(1)}, \\ y_{n-1}^{(1)} - y_n^{(1)} &= \big[-iz - (iz)^{-1}\big] y_n^{(2)}. \end{aligned} \tag{12}$$

It is clear that

$$e_n(z) = \begin{pmatrix} e_n^{(1)}(z) \\ e_n^{(2)}(z) \end{pmatrix} = \begin{pmatrix} z \\ -i \end{pmatrix} z^{2n}, \quad n \in \mathbb{Z}, \tag{13}$$

and

$$h_n(z) = \begin{pmatrix} h_n^{(1)}(z) \\ h_n^{(2)}(z) \end{pmatrix} = \begin{pmatrix} -i \\ z \end{pmatrix} z^{-2n}, \quad n \in \mathbb{Z}, \tag{14}$$

are the solutions of (9).

Now we find the solutions $f_n(z) = \binom{f_n^{(1)}}{f_n^{(2)}}$, $n \in \mathbb{Z}$, and $g_n(z) = \binom{g_n^{(1)}}{g_n^{(2)}}$, $n \in \mathbb{Z}$, of (9) for $\lambda = -iz - (iz)^{-1}$, satisfying the condition

$$f_n(z) = \big[I + o(1)\big] e_n(z), \quad |z| = 1, n \to \infty, \tag{15}$$

and

$$g_n(z) = \big[I + o(1)\big] h_n(z), \quad |z| = 1, n \to -\infty, \tag{16}$$

respectively, where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Theorem 1 *Under the condition* (11) *for* $\lambda = -iz - (iz)^{-1}$ *and* $|z| = 1$, (9) *has the solutions* $f_n(z) = \{\binom{f_n^{(1)}}{f_n^{(2)}}\}_{n\in\mathbb{Z}}$ *and* $g_n(z) = \{\binom{g_n^{(1)}}{g_n^{(2)}}\}_{n\in\mathbb{Z}}$ *having the representations*

$$f_n(z) = \begin{pmatrix} f_n^{(1)} \\ f_n^{(2)} \end{pmatrix} = \left[I + \sum_{m=1}^{\infty} K_{nm} z^{2m} \right] \begin{pmatrix} z \\ -i \end{pmatrix} z^{2n}, \quad n \in \mathbb{Z}, \tag{17}$$

$$g_n(z) = \begin{pmatrix} g_n^{(1)} \\ g_n^{(2)} \end{pmatrix} = \left[I + \sum_{m=-1}^{-\infty} B_{nm} z^{-2m} \right] \begin{pmatrix} -i \\ z \end{pmatrix} z^{-2n}, \quad n \in \mathbb{Z}, \tag{18}$$

$$f_0^{(1)}(z) = z + \sum_{m=1}^{\infty} \left(K_{0m}^{11} z^{2m+1} - i K_{0m}^{12} z^{2m} \right), \tag{19}$$

where

$$K_{nm} = \begin{pmatrix} K_{nm}^{11} & K_{nm}^{12} \\ K_{nm}^{21} & K_{nm}^{22} \end{pmatrix},$$

and

$$B_{nm} = \begin{pmatrix} B_{nm}^{11} & B_{nm}^{12} \\ B_{nm}^{21} & B_{nm}^{22} \end{pmatrix}.$$

Proof Substituting the vector-valued functions f and g defined by (17) and (18) in (9), taking $\lambda = -iz - (iz)^{-1}$, $|z| = 1$, we get

$$K_{n1}^{12} = -\sum_{k=n+1}^{\infty} (p_k + q_k),$$

$$K_{n1}^{11} = \sum_{k=n+1}^{\infty} p_k K_{k1}^{12},$$

$$K_{n1}^{22} = K_{n-1,1}^{11} = \sum_{k=n}^{\infty} p_k K_{k1}^{12},$$

$$K_{n1}^{21} = K_{n1}^{12} + p_n K_{n1}^{11} + \sum_{k=n+1}^{\infty} \left[q_k K_{k1}^{22} + p_k K_{k1}^{11} \right],$$

$$K_{n2}^{12} = -\sum_{k=n+1}^{\infty} \left[p_k K_{k1}^{11} + q_k K_{k1}^{22} \right],$$

$$K_{n2}^{11} = -K_{n+1,1}^{22} + \sum_{k=n+1}^{\infty} \left[p_k K_{k2}^{12} - q_k K_{k1}^{21} \right],$$

$$K_{n2}^{22} = -K_{n1}^{11} + \sum_{k=n}^{\infty} \left[p_k K_{k2}^{12} - q_{k+1} K_{k+1,1}^{21} \right],$$

$$K_{n2}^{21} = K_{n2}^{12} + \sum_{k=n}^{\infty} \left[p_k K_{k2}^{11} + q_{k+1} K_{k+1,2}^{22} \right],$$

and

$$B_{n,-1}^{21} = -\sum_{k=n-1}^{-\infty} (p_k + q_k),$$

$$B_{n,-1}^{22} = \sum_{k=n-1}^{-\infty} q_k B_{k,-1}^{21},$$

$$B^{11}_{n,-1} = \sum_{k=n}^{-\infty} q_k B^{21}_{k,-1},$$

$$B^{12}_{n,-1} = -\sum_{k=n-1}^{-\infty} (p_k + q_k) + \sum_{k=n}^{-\infty} \left[p_{k-1} B^{11}_{k-1,-1} + q_k B^{22}_{k,-1}\right],$$

$$B^{21}_{n,-2} = -\sum_{k=n-1}^{-\infty} \left(p_k B^{11}_{k,-1} + q_k B^{22}_{k,-1}\right),$$

$$B^{22}_{n,-2} = -B^{11}_{n-1,-1} - \sum_{k=n-1}^{-\infty} \left[q_k B^{21}_{k,-2} - p_k B^{12}_{k,-1}\right],$$

$$B^{11}_{n,-2} = -B^{22}_{n,-1} + \sum_{k=n}^{-\infty} \left[q_k B^{21}_{k,-2} - p_{k-1} B^{12}_{k-1,-1}\right],$$

$$B^{12}_{n,-2} = B^{21}_{n,-2} + \sum_{k=n-1}^{-\infty} \left[p_k B^{11}_{k,-2} + q_{k+1} B^{22}_{k+1,-2}\right],$$

where $n \in \mathbb{Z}$. For $m \geq 3$ and $n \in \mathbb{Z}$, we obtain

$$K^{12}_{nm} = K^{21}_{n+1,m-2} - \sum_{k=n+1}^{\infty} \left[q_k K^{22}_{k,m-1} + p_k K^{11}_{k,m-1}\right],$$

$$K^{11}_{nm} = -K^{22}_{n+1,m-1} + \sum_{k=n+1}^{\infty} \left[p_k K^{12}_{km} - q_k K^{21}_{k,m-1}\right],$$

$$K^{22}_{nm} = -K^{11}_{n,m-1} + \sum_{k=n}^{\infty} \left[p_k K^{12}_{km} - q_{k+1} K^{21}_{k+1,m-1}\right],$$

$$K^{21}_{nm} = K^{12}_{nm} + p_n K^{11}_{nm} + \sum_{k=n+1}^{\infty} \left[q_k K^{22}_{km} + p_k K^{11}_{km}\right].$$

Also for $m \leq -3$ and $n \in \mathbb{Z}$, we get

$$B^{21}_{nm} = -\sum_{k=n-1}^{-\infty} \left[q_k B^{21}_{k,m+1} + p_k B^{11}_{k,m+1}\right] + B^{12}_{n-1,m+2},$$

$$B^{22}_{nm} = -B^{11}_{n-1,m+1} + \sum_{k=n}^{-\infty} \left[q_k B^{21}_{km} - p_k B^{12}_{k,m+1}\right],$$

$$B^{11}_{nm} = -B^{22}_{n,m+1} + \sum_{k=n}^{-\infty} \left[q_k B^{21}_{km} - p_{k-1} B^{12}_{k-1,m+1}\right],$$

$$B^{12}_{nm} = B^{21}_{n+1,m-2} + \sum_{k=n}^{-\infty} \left[p_k B^{11}_{k,m-1} + q_k B^{21}_{k,m-1}\right].$$

Due to the condition (11), the series in the definition of K^{ij}_{nm} and B^{ij}_{nm} $(i,j = 1,2)$ are absolutely convergent. Therefore, K^{ij}_{nm} and B^{ij}_{nm} $(i,j = 1,2)$ can uniquely be defined by p_n and q_n $(n \in \mathbb{Z})$, *i.e.*, the system (9) for $\lambda = -iz - (iz)^{-1}$, $|z| = 1$, have the solutions $f_n(z)$ given by (17) and $g_n(z)$ given by (18). □

The solutions f and g are called Jost solutions of (9). Using the equalities for K_{nm}^{ij} and B_{nm}^{ij} $(i,j=1,2)$ given in Theorem 1, we find

$$|K_{nm}^{ij}| \le C \sum_{k=n+\lfloor \frac{m}{2} \rfloor}^{\infty} \bigl(|p_k|+|q_k|\bigr), \quad i,j=1,2, \tag{20}$$

and

$$|B_{nm}^{ij}| \le C \sum_{k=n+\lfloor \frac{m}{2} \rfloor+1}^{-\infty} \bigl(|p_k|+|q_k|\bigr), \quad i,j=1,2, \tag{21}$$

by induction, where $\lfloor \frac{m}{2} \rfloor$ is the integer part of $\frac{m}{2}$ and $C>0$ is a constant.

Using (20), (21), and the definitions of f and g, we obtain (15) and (16). Also the Jost solutions have an analytic continuation from $\{z:|z|=1\}$ to $D:=\{z:|z|<1\}\backslash\{0\}$. Because of (11) and (20), we see that the series $\sum_{m=1}^{\infty} K_{nm}z^{2m}$ and $\sum_{m=1}^{\infty} mK_{nm}z^{2m-1}$ are uniformly convergent in D. Similarly from (11) and (21), we see that the series $\sum_{m=-1}^{-\infty} B_{nm}z^{2m}$ and $\sum_{m=-1}^{-\infty} mB_{nm}z^{-2m-1}$ are uniformly convergent in D.

Theorem 2 *The following asymptotics hold*:

$$\begin{pmatrix} f_n^{(1)} \\ f_n^{(2)} \end{pmatrix} = \bigl[I+o(1)\bigr]\begin{pmatrix} z \\ -i \end{pmatrix} z^{2n}, \quad z\in \overline{D}:=\bigl\{z:|z|\le 1\bigr\}\backslash\{0\}, n\to\infty, \tag{22}$$

$$\begin{pmatrix} g_n^{(1)} \\ g_n^{(2)} \end{pmatrix} = \bigl[I+o(1)\bigr]\begin{pmatrix} -i \\ z \end{pmatrix} z^{-2n}, \quad z\in \overline{D}, n\to -\infty. \tag{23}$$

Proof From (17), we obtain

$$f_n^{(1)}(z)z^{-2n-1} = 1+\sum_{m=1}^{\infty} K_{nm}^{11}z^{2m} - i\sum_{m=1}^{\infty} K_{nm}^{12}z^{2m-1}, \quad z\in D. \tag{24}$$

Using (20) and (24), we get

$$\begin{aligned} \bigl|f_n^{(1)}(z)z^{-2n-1}\bigr| &\le 1+\sum_{m=1}^{\infty}\bigl|K_{nm}^{11}\bigr| + \sum_{m=1}^{\infty}\bigl|K_{nm}^{12}\bigr| \\ &\le 1+2C\sum_{m=1}^{\infty}\sum_{k=n+\lfloor \frac{m}{2} \rfloor}^{\infty}\bigl(|p_k|+|q_k|\bigr) \\ &\le 1+2C\sum_{k=n+1}^{\infty}(k-n)\bigl(|p_k|+|q_k|\bigr) \\ &\le 1+2C\sum_{k=n+1}^{\infty}k\bigl(|p_k|+|q_k|\bigr), \end{aligned} \tag{25}$$

where C is constant. So we have by (25)

$$f_n^{(1)}(z) = z^{2n+1}\bigl(1+o(1)\bigr), \quad z\in \overline{D}, n\to\infty. \tag{26}$$

In a manner similar to (26), we obtain

$$f_n^{(2)}(z) = -iz^{2n}\big(1 + o(1)\big), \quad z \in \overline{D}, n \to \infty. \tag{27}$$

From (26) and (27), we get (22). Also from (18), we obtain

$$ig_n^{(1)}(z)z^{2n} = 1 + \sum_{m=-1}^{-\infty} B_{nm}^{11} z^{-2m} + i \sum_{m=-1}^{-\infty} B_{nm}^{12} z^{-2m+1}, \quad z \in D. \tag{28}$$

Using (21) and (28), we have

$$\begin{aligned} \big|ig_n^{(1)}(z)z^{2n}\big| &\le 1 + \sum_{m=-1}^{-\infty} \big|B_{nm}^{11}\big| + \sum_{m=-1}^{-\infty} \big|B_{nm}^{12}\big| \\ &\le 1 + 2C \sum_{m=-1}^{-\infty} \sum_{k=n+\lfloor \frac{m}{2} \rfloor+1}^{-\infty} \big(|p_k| + |q_k|\big) \\ &\le 1 + 2C \sum_{k=n}^{-\infty} -k\big(|p_k| + |q_k|\big), \end{aligned} \tag{29}$$

where C is constant. So, we get by (29)

$$g_n^{(1)}(z) = -iz^{-2n}\big(1 + o(1)\big), \quad z \in \overline{D}, n \to -\infty. \tag{30}$$

Similarly, we can obtain

$$g_n^{(2)}(z) = z^{-2n+1}\big(1 + o(1)\big), \quad z \in \overline{D}, n \to -\infty. \tag{31}$$

From (30) and (31), we obtain (23). □

3 Continuous and discrete spectrum of the BVP (9)

Let $l_2(\mathbb{Z}, \mathbb{C}^2)$ denote the Hilbert space of all complex vector sequences

$$y = \begin{Bmatrix} y_n^{(1)} \\ y_n^{(2)} \end{Bmatrix}_{n \in \mathbb{Z}}$$

with the norm

$$\|y\|^2 := \sum_{n \in \mathbb{Z}} \big|y_n^{(1)}\big|^2 + \big|y_n^{(2)}\big|^2.$$

We also define the operator L generated in $\ell_2(\mathbb{Z}, \mathbb{C}^2)$ by (9). The operator L is self-adjoint.

Theorem 3 *If the condition* (11) *holds, then* $\sigma_c(L) = [-2, 2]$, *where* $\sigma_c(L)$ *denotes the continuous spectrum of* L.

Proof Let L_0 denote the operator generated in $\ell_2(\mathbb{Z},\mathbb{C}^2)$ by the BVP

$$\begin{cases} \Delta y_n^{(2)} = \lambda y_n^{(1)}, \\ -\nabla y_n^{(1)} = \lambda y_n^{(2)}, \end{cases}$$

$$y_0^{(1)} = 0.$$

We also define the operator L_1 in $\ell_2(\mathbb{Z},\mathbb{C}^2)$ by the following:

$$L_1 \begin{pmatrix} y_n^{(1)} \\ y_n^{(2)} \end{pmatrix} := \begin{pmatrix} p_n & 0 \\ 0 & q_n \end{pmatrix} \begin{pmatrix} y_n^{(1)} \\ y_n^{(2)} \end{pmatrix} = \begin{pmatrix} p_n y_n^{(1)} \\ q_n y_n^{(2)} \end{pmatrix}.$$

It is clear that $L_0 = L_0^*$, $L = L_0 + L_1$ and we can easily prove that

$$\sigma(L_0) = \sigma_c(L_0) = [-2, 2].$$

It follows from (11) that the operator L_1 is compact in $\ell_2(\mathbb{Z},\mathbb{C}^2)$ [8].

By the Weyl Theorem [9] of a compact perturbation, we get

$$\sigma_c(L) = \sigma_c(L_0) = [-2, 2].$$ □

The Wronskian of the solutions

$$U_n := \left\{ \begin{pmatrix} U_n^{(1)} \\ U_n^{(2)} \end{pmatrix} \right\}_{n\in\mathbb{Z}}, \qquad V_n := \left\{ \begin{pmatrix} V_n^{(1)} \\ V_n^{(2)} \end{pmatrix} \right\}_{n\in\mathbb{Z}}$$

of (8) is defined by

$$W[U_n, V_n] = a_n \big[U_n^{(1)} V_{n+1}^{(2)} - U_{n+1}^{(2)} V_n^{(1)} \big].$$

If we define $F(z) = W[f_n(z), g_n(z)]$, then F is analytic in D. Since the operator L is self-adjoint, the eigenvalues of L is real. From the definition of the eigenvalues we obtain

$$\sigma_d(L) = \big\{ \lambda : \lambda = -iz - (iz)^{-1}, iz \in (-1, 0) \cup (0, 1), F(z) = 0 \big\}, \tag{32}$$

where $\sigma_d(L)$ denotes the set of all eigenvalues of L.

Definition 1 The multiplicity of a zero of the function $F(z)$ is called the multiplicity of the corresponding eigenvalue of L.

Theorem 4 *Under the condition* (11) *the operator L has a finite number of real eigenvalues in D.*

Proof To prove the theorem, we have to show that the function $F(z)$ has a finite number of real zeros in D. The cluster points of the zeros of the analytic function F could be $-i$, 0 and i. Since L is a self-adjoint bounded operator its eigenvalues should be different from infinity and as z is '0', the eigenvalue λ is infinity, we cannot consider '0' as a zero of the function F. Also, for z is $\pm i$, the eigenvalue λ is ± 2 and D is bounded. But, as we know, ± 2 are elements of the continuous spectrum of the operator L. On the other hand from the operator theory, the eigenvalues of the self-adjoint operator are not the elements of the continuous spectrum of that operator. Therefore, from the Bolzano Weierstrass Theorem the set of zeros of the function F in D are finite *i.e.*, the operator L has a finite number of eigenvalues. □

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

Both authors completed the paper together. Both authors read and approved the final manuscript.

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