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# Forced oscillation of second-order differential equations with mixed nonlinearities

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Dedicated to Professor Ravi P Agarwal

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**Abstract**
We study oscillatory behavior of a class of second-order forced differential equations with mixed nonlinearities. Some new oscillation theorems are presented that improve and complement those related results given in the literature. An example is provided to illustrate the main results.
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## 1 Introduction

This paper is concerned with the oscillation of solutions to a class of second-order forced differential equations with mixed nonlinearities

$$\bigl(rx'\bigr)'(t)+q_0(t)x\bigl(\tau_0(t)\bigr)+\sum_{i=1}^{n}q_i(t)\bigl|x\bigl(\tau_i(t)\bigr)\bigr|^{\beta_i-1}x\bigl(\tau_i(t)\bigr)=e(t)\operatorname{sgn}\bigl(x(t)\bigr), \tag{1.1}$$

where $t\geq t_0>0$, $n\geq 1$ is a natural number, $\beta_i\geq 1$ $(i=1,2,\ldots,n)$ are constants, $r\in \mathrm{C}^1([t_0,\infty),\mathbb{R})$, $q_j,\tau_j,e\in \mathrm{C}([t_0,\infty),\mathbb{R})$, $r(t)>0$, $r'(t)\geq 0$, $q_j(t)\geq 0$ $(j=0,1,2,\ldots,n)$, $e(t)\leq 0$. We also assume that there exists a function $\tau\in \mathrm{C}^1([t_0,\infty),\mathbb{R})$ such that $\tau(t)\leq\tau_j(t)$ $(j=0,1,2,\ldots,n)$, $\tau(t)\leq t$, $\lim_{t\to\infty}\tau(t)=\infty$, and $\tau'(t)>0$.

We consider only those solutions $x$ of equation (1.1) which satisfy condition $\sup\{|x(t)|: t\geq T\}>0$ for all $T\geq t_0$. We assume that (1.1) possesses such solutions. As usual, a solution of (1.1) is called oscillatory if it has arbitrarily large zeros on the interval $[t_0,\infty)$; otherwise, it is termed nonoscillatory. Equation (1.1) is said to be oscillatory if all its solutions are oscillatory.

Functional differential equations arise in many applied problems in natural sciences, technology, and automatic control; see, for instance, Hale [1]. In mechanical and engineering problems, questions related to the existence of oscillatory and nonoscillatory solutions play an important role. As a result, many theoretical studies have been undertaken during the past few years. We refer the reader to [2–12] and the references cited therein.

In what follows, we briefly comment on the related results that motivate our study. Li and Cheng [9] studied a differential equation

$$\left(r\left|x'\right|^{\alpha-1}x'\right)'(t) + q(t)\left|x(t)\right|^{\alpha-1}x(t) = e(t).$$

Zheng *et al.* [11] considered the equation

$$\left(r\left|x'\right|^{\alpha-1}x'\right)'(t) + q_0(t)\left|x(t)\right|^{\alpha-1}x(t) + \sum_{i=1}^{n} q_i(t)\left|x(t)\right|^{\beta_i-1}x(t) = e(t).$$

Equation (1.1) was studied by Zhong *et al.* [12] who established the following oscillation theorem.

**Theorem 1.1** (see [12, Theorem 3.1]) *Assume that*

$$\int_{t_0}^{\infty} r^{-1}(t)\,\mathrm{d}t = \infty \tag{1.2}$$

*and there exists a function* $\rho \in \mathrm{C}^1([t_0,\infty),(0,\infty))$ *such that*

$$\int_{t_0}^{\infty} \left(\rho(t)Q(t) - \frac{r(\tau(t))(\rho_+'(t))^2}{4\rho(t)\tau'(t)}\right)\mathrm{d}t = \infty, \tag{1.3}$$

*where*

$$Q(t) := q_0(t) + \sum_{i=1}^{n} \beta_i\left[n(\beta_i - 1)\right]^{(1-\beta_i)/\beta_i}\left(q_i(t)\right)^{1/\beta_i}\left|e(t)\right|^{(\beta_i-1)/\beta_i}$$

*and*

$$\rho_+'(t) := \max\left\{0, \rho'(t)\right\}. \tag{1.4}$$

*Then equation* (1.1) *is oscillatory.*

The purpose of this paper is to refine Theorem 1.1 in some cases and analyze the oscillatory behavior of solutions to (1.1) in the case when the integral in (1.2) is finite. This paper proceeds as follows: in Section 2, we present our main results; in Section 3, an example is provided to illustrate the results obtained.

## 2 Oscillation criteria

In what follows, all functional inequalities are tacitly assumed to hold eventually, that is, for all $t$ large enough. Before stating the main results, we begin with the following lemma.

**Lemma 2.1** (Bernoulli's inequality) *For* $y \geq -1$ *and* $\gamma \geq 1$,

$$(1 + y)^{\gamma} \geq 1 + \gamma y.$$

**Theorem 2.2** *Assume that condition* (1.2) *is satisfied, and let*

$$\sum_{i=1}^{n}(1-\beta_i)q_i(t)-e(t)\geq 0. \tag{2.1}$$

*If there exists a function* $\rho \in C^1([t_0,\infty),(0,\infty))$ *such that, for all constants* $M>0$,

$$\limsup_{t\to\infty}\int_{t_0}^{t}\left[\rho(s)\left(q_0(s)+\sum_{i=1}^{n}\beta_i q_i(s)+\frac{\sum_{i=1}^{n}(1-\beta_i)q_i(s)-e(s)}{M\tau(s)}\right)-\frac{r(\tau(s))(\rho'_+(s))^2}{4\rho(s)\tau'(s)}\right]\mathrm{d}s=\infty, \tag{2.2}$$

*where* $\rho'_+$ *is defined as in* (1.4), *then equation* (1.1) *is oscillatory.*

*Proof* Assume that (1.1) has a nonoscillatory solution $x$. Without loss of generality, we can assume that $x$ is an eventually positive solution, *i.e.*, there exists a $t_1\geq t_0$ such that $x(t)>0$ for $t\geq t_1$. Equation (1.1) yields

$$\left(rx'\right)'(t)=-q_0(t)x\left(\tau_0(t)\right)-\sum_{i=1}^{n}q_i(t)x^{\beta_i}\left(\tau_i(t)\right)+e(t)\leq 0. \tag{2.3}$$

With a proof similar to that of [12, Theorem 3.1], we conclude that

$$x(t)>0,\qquad x'(t)>0,\qquad x''(t)\leq 0,\qquad \left(rx'\right)'(t)\leq 0. \tag{2.4}$$

For $t\geq t_1$, define a function

$$u(t):=\rho(t)\frac{r(t)x'(t)}{x(\tau(t))}. \tag{2.5}$$

Then $u(t)>0$ for $t\geq t_1$. Differentiating (2.5), by virtue of (2.3) and (2.4), we have $x'(\tau(t))\geq r(t)x'(t)/r(\tau(t))$, and so

$$\begin{aligned}u'(t) &= \frac{\rho'(t)}{\rho(t)}u(t)-\rho(t)\frac{r(t)x'(t)}{x^2(\tau(t))}x'\left(\tau(t)\right)\tau'(t)-\rho(t)\frac{q_0(t)x(\tau_0(t))}{x(\tau(t))}\\ &\quad-\rho(t)\frac{\sum_{i=1}^{n}q_i(t)x^{\beta_i}(\tau_i(t))-e(t)}{x(\tau(t))}\\ &\leq \frac{\rho'_+(t)}{\rho(t)}u(t)-\frac{\tau'(t)u^2(t)}{\rho(t)r(\tau(t))}-\rho(t)\left[q_0(t)+\frac{\sum_{i=1}^{n}q_i(t)x^{\beta_i}(\tau_i(t))-e(t)}{x(\tau(t))}\right].\end{aligned} \tag{2.6}$$

Let $y:=x(\tau_i(t))-1$. It follows from Lemma 2.1 that

$$x^{\beta_i}\left(\tau_i(t)\right)\geq \beta_i x\left(\tau_i(t)\right)+1-\beta_i. \tag{2.7}$$

Hence, we deduce that

$$\begin{aligned}\frac{\sum_{i=1}^n q_i(t)x^{\beta_i}(\tau_i(t)) - e(t)}{x(\tau(t))} &\geq \frac{\sum_{i=1}^n q_i(t)[\beta_i x(\tau_i(t)) + (1-\beta_i)] - e(t)}{x(\tau(t))} \\ &\geq \sum_{i=1}^n \beta_i q_i(t) + \frac{\sum_{i=1}^n (1-\beta_i)q_i(t) - e(t)}{x(\tau(t))}. \end{aligned} \tag{2.8}$$

By virtue of (2.4), there exists a constant $M > 0$ such that

$$x(t) \leq Mt.$$

Thus, by (2.8), we obtain

$$\frac{\sum_{i=1}^n q_i(t)x^{\beta_i}(\tau_i(t)) - e(t)}{x(\tau(t))} \geq \sum_{i=1}^n \beta_i q_i(t) + \frac{\sum_{i=1}^n (1-\beta_i)q_i(t) - e(t)}{M\tau(t)}. \tag{2.9}$$

Substitution of (2.9) into (2.6) implies that

$$\begin{aligned} u'(t) &\leq \frac{\rho'_+(t)}{\rho(t)}u(t) - \frac{\tau'(t)u^2(t)}{\rho(t)r(\tau(t))} - \rho(t)\left(q_0(t) + \sum_{i=1}^n \beta_i q_i(t) + \frac{\sum_{i=1}^n (1-\beta_i)q_i(t) - e(t)}{M\tau(t)}\right) \\ &\leq \frac{r(\tau(t))(\rho'_+(t))^2}{4\rho(t)\tau'(t)} - \rho(t)\left(q_0(t) + \sum_{i=1}^n \beta_i q_i(t) + \frac{\sum_{i=1}^n (1-\beta_i)q_i(t) - e(t)}{M\tau(t)}\right). \end{aligned}$$

Integrating the latter inequality from $t_1$ to $t$, we conclude that

$$\int_{t_1}^t \left[\rho(s)\left(q_0(s) + \sum_{i=1}^n \beta_i q_i(s) + \frac{\sum_{i=1}^n (1-\beta_i)q_i(s) - e(s)}{M\tau(s)}\right) - \frac{r(\tau(s))(\rho'_+(s))^2}{4\rho(s)\tau'(s)}\right] \mathrm{d}s \leq u(t_1),$$

which contradicts (2.2). This completes the proof. □

On the basis of Theorem 2.2, we can obtain the following results due to condition (2.1).

**Corollary 2.3** *Assume that conditions* (1.2) *and* (2.1) *are satisfied. If there exists a function* $\rho \in \mathrm{C}^1([t_0,\infty),(0,\infty))$ *such that*

$$\limsup_{t\to\infty} \int_{t_0}^t \left[\rho(s)\left(q_0(s) + \sum_{i=1}^n \beta_i q_i(s)\right) - \frac{r(\tau(s))(\rho'_+(s))^2}{4\rho(s)\tau'(s)}\right] \mathrm{d}s = \infty,$$

*where* $\rho'_+$ *is defined as in* (1.4), *then equation* (1.1) *is oscillatory.*

Using $\rho(t) = t$ in Corollary 2.3, we can get the following criterion.

**Corollary 2.4** *Assume that conditions* (1.2) *and* (2.1) *are satisfied. If*

$$\limsup_{t\to\infty} \int_{t_0}^t \left[s\left(q_0(s) + \sum_{i=1}^n \beta_i q_i(s)\right) - \frac{r(\tau(s))}{4s\tau'(s)}\right] \mathrm{d}s = \infty,$$

*then equation* (1.1) *is oscillatory.*

In what follows, we derive some oscillation criteria for (1.1) in the case where

$$\int_{t_0}^{\infty} r^{-1}(t)\,\mathrm{d}t < \infty. \tag{2.10}$$

**Theorem 2.5** *Assume that conditions* (2.1) *and* (2.10) *are satisfied, and let* $\tau_j(t) \le t$ $(j = 0, 1, 2, \dots, n)$. *Suppose also that there exists a function* $\rho \in \mathrm{C}^1([t_0,\infty),(0,\infty))$ *such that* (2.2) *holds. If*

$$\limsup_{t\to\infty} \int_{t_0}^{t} \left[\delta(s)\left(q_0(s) + \sum_{i=1}^{n} \beta_i q_i(s) + \frac{\sum_{i=1}^{n}(1-\beta_i)q_i(s) - e(s)}{K}\right) - \frac{1}{4r(s)\delta(s)}\right]\mathrm{d}s = \infty \tag{2.11}$$

*holds for all constants* $K > 0$, *where*

$$\delta(t) := \int_{t}^{\infty} r^{-1}(s)\,\mathrm{d}s, \tag{2.12}$$

*then equation* (1.1) *is oscillatory.*

*Proof* Assume that (1.1) has a nonoscillatory solution $x$. As above, we may assume that there is a $t_1 \ge t_0$ such that $x(t) > 0$ for $t \ge t_1$. By virtue of (1.1), we have (2.3). Then there exist two possible cases, *i.e.*, $x'(t) > 0$ or

$$x'(t) < 0. \tag{2.13}$$

Assume first that $x'(t) > 0$. Then we obtain (2.4). Proceeding as in the proof of Theorem 2.2, we can obtain a contradiction to (2.2). Suppose now that (2.13) holds. Define a new function $\omega$ by

$$\omega(t) := \frac{r(t)x'(t)}{x(t)}, \quad t \ge t_1. \tag{2.14}$$

Then $\omega(t) < 0$ for $t \ge t_1$ and

$$\begin{aligned} \omega'(t) &= \frac{(rx')'(t)x(t) - r(t)x'(t)x'(t)}{x^2(t)} \\ &= \frac{-q_0(t)x(\tau_0(t)) - \sum_{i=1}^{n} q_i(t)x^{\beta_i}(\tau_i(t)) + e(t)}{x(t)} - \frac{\omega^2(t)}{r(t)}. \end{aligned} \tag{2.15}$$

On the other hand, we have (2.7), and so

$$\begin{aligned} \frac{\sum_{i=1}^{n} q_i(t)x^{\beta_i}(\tau_i(t)) - e(t)}{x(t)} &\ge \frac{\sum_{i=1}^{n} q_i(t)[\beta_i x(\tau_i(t)) + (1-\beta_i)] - e(t)}{x(t)} \\ &\ge \sum_{i=1}^{n} \beta_i q_i(t) + \frac{\sum_{i=1}^{n}(1-\beta_i)q_i(t) - e(t)}{x(t)} \end{aligned} \tag{2.16}$$

due to $\tau_i(t) \le t$ $(i = 1, 2, \ldots, n)$. By (2.13), there exists a constant $K > 0$ such that $x(t) \le K$. Hence, by virtue of (2.16), we conclude that

$$\frac{\sum_{i=1}^n q_i(t)x^{\beta_i}(\tau_i(t)) - e(t)}{x(t)} \ge \sum_{i=1}^n \beta_i q_i(t) + \frac{\sum_{i=1}^n (1-\beta_i)q_i(t) - e(t)}{K}. \tag{2.17}$$

It follows now from (2.15), (2.17), and $\tau_0(t) \le t$ that

$$\omega'(t) \le -q_0(t) - \sum_{i=1}^n \beta_i q_i(t) - \frac{\sum_{i=1}^n (1-\beta_i)q_i(t) - e(t)}{K} - \frac{\omega^2(t)}{r(t)}. \tag{2.18}$$

Using the condition $(rx')'(t) \le 0$, we have, for $s \ge t$,

$$x'(s) \le \frac{r(t)x'(t)}{r(s)}.$$

Integrating the latter inequality from $t$ to $l$, we deduce that

$$x(l) - x(t) \le r(t)x'(t) \int_t^l r^{-1}(s)\,\mathrm{d}s.$$

Passing to the limit as $l \to \infty$, we have

$$-x(t) \le r(t)x'(t)\delta(t),$$

which yields

$$\frac{r(t)x'(t)}{x(t)}\delta(t) \ge -1,$$

*i.e.*,

$$\omega(t)\delta(t) \ge -1. \tag{2.19}$$

Multiplying (2.18) by $\delta(t)$ and integrating the resulting inequality from $t_1$ to $t$, we obtain

$$\begin{aligned}&\omega(t)\delta(t) - \omega(t_1)\delta(t_1) + \int_{t_1}^t \delta(s)\left(q_0(s) + \sum_{i=1}^n \beta_i q_i(s) + \frac{\sum_{i=1}^n (1-\beta_i)q_i(s) - e(s)}{K}\right)\mathrm{d}s \\ &\quad + \int_{t_1}^t \frac{\omega(s)}{r(s)}\,\mathrm{d}s + \int_{t_1}^t \frac{\omega^2(s)\delta(s)}{r(s)}\,\mathrm{d}s \le 0.\end{aligned}$$

Hence, we derive from (2.19) that

$$\begin{aligned}&\int_{t_1}^t \left[\delta(s)\left(q_0(s) + \sum_{i=1}^n \beta_i q_i(s) + \frac{\sum_{i=1}^n (1-\beta_i)q_i(s) - e(s)}{K}\right) - \frac{1}{4r(s)\delta(s)}\right]\mathrm{d}s \\ &\quad \le 1 + \omega(t_1)\delta(t_1),\end{aligned}$$

which contradicts (2.11). The proof is complete. □

**Theorem 2.6** *Assume that conditions* (2.1) *and* (2.10) *are satisfied, and let* $\tau_j(t) \geq t$ $(j = 0, 1, 2, \ldots, n)$. *Suppose further that there exists a function* $\rho \in C^1([t_0, \infty), (0, \infty))$ *such that* (2.2) *holds*. *If, for all constants* $K > 0$,

$$\limsup_{t\to\infty} \int_{t_0}^{t} \Bigg[ \delta(s) \Bigg( q_0(s) \frac{\delta(\tau_0(s))}{\delta(s)} + \sum_{i=1}^{n} \beta_i q_i(s) \frac{\delta(\tau_i(s))}{\delta(s)} \Bigg) + \delta(s) \frac{\sum_{i=1}^{n} (1-\beta_i) q_i(s) - e(s)}{K} - \frac{1}{4r(s)\delta(s)} \Bigg] \mathrm{d}s = \infty, \tag{2.20}$$

*where* $\delta$ *is as in* (2.12), *then equation* (1.1) *is oscillatory.*

*Proof* Assume again that there exists a $t_1 \geq t_0$ such that $x(t) > 0$ for $t \geq t_1$. From (1.1), we have (2.3). Then there exist two possible cases, *i.e.*, $x'(t) > 0$ or (2.13). Suppose that $x'(t) > 0$. Following the same lines as in Theorem 2.2, we can obtain a contradiction to (2.2). Assume now that (2.13) is satisfied. Define the function $\omega$ by (2.14). We have $\omega(t) < 0$ for $t \geq t_1$ and (2.15). On the other hand, it has been established in Theorems 2.2 and 2.5 that (2.7) and (2.19) hold. By virtue of (2.19),

$$\left(\frac{x}{\delta}\right)'(t) \geq 0.$$

It follows from the latter inequality, $\tau_j(t) \geq t$ $(j = 0, 1, 2, \ldots, n)$, and (2.7) that

$$\frac{x(\tau_0(t))}{x(t)} \geq \frac{\delta(\tau_0(t))}{\delta(t)} \tag{2.21}$$

and

$$\begin{aligned} \frac{\sum_{i=1}^{n} q_i(t) x^{\beta_i}(\tau_i(t)) - e(t)}{x(t)} &\geq \frac{\sum_{i=1}^{n} q_i(t)[\beta_i x(\tau_i(t)) + (1-\beta_i)] - e(t)}{x(t)} \\ &\geq \sum_{i=1}^{n} \beta_i q_i(t) \frac{\delta(\tau_i(t))}{\delta(t)} + \frac{\sum_{i=1}^{n} (1-\beta_i) q_i(t) - e(t)}{x(t)}. \end{aligned} \tag{2.22}$$

Since $x'(t) < 0$, there exists a constant $K > 0$ such that $x(t) \leq K$. Hence, by (2.22), we conclude that

$$\frac{\sum_{i=1}^{n} q_i(t) x^{\beta_i}(\tau_i(t)) - e(t)}{x(t)} \geq \sum_{i=1}^{n} \beta_i q_i(t) \frac{\delta(\tau_i(t))}{\delta(t)} + \frac{\sum_{i=1}^{n} (1-\beta_i) q_i(t) - e(t)}{K}. \tag{2.23}$$

Using (2.15), (2.21), and (2.23), we obtain

$$\omega'(t) \leq -q_0(t) \frac{\delta(\tau_0(t))}{\delta(t)} - \sum_{i=1}^{n} \beta_i q_i(t) \frac{\delta(\tau_i(t))}{\delta(t)} - \frac{\sum_{i=1}^{n} (1-\beta_i) q_i(t) - e(t)}{K} - \frac{\omega^2(t)}{r(t)}. \tag{2.24}$$

The remainder of the proof is similar to that of Theorem 2.5 and hence is omitted. This completes the proof. □

**Remark 2.7** From the proof of Theorems 2.5 and 2.6, one can obtain oscillation results for equation (1.1) with delayed and advanced arguments. The details are left to the reader.

## 3 Example

The following example illustrates possible applications of the theoretical results presented in this paper.

**Example 3.1** For $t \geq 1$, consider a second-order differential equation

$$x''(t) + \frac{\gamma}{t^2} x\left(\frac{t}{2}\right) + \frac{1}{t^2}\left|x\left(\frac{t}{5}\right)\right| x\left(\frac{t}{5}\right) + \frac{1}{t^2} x^3\left(\frac{t}{40}\right) = -\frac{3}{t^2}\operatorname{sgn}\bigl(x(t)\bigr), \tag{3.1}$$

where $\gamma > 0$ is a constant. Let $n = 2$, $r(t) = 1$, $q_0(t) = \gamma/t^2$, $q_1(t) = q_2(t) = 1/t^2$, $\tau_0(t) = t/2$, $\tau_1(t) = t/5$, $\tau(t) = \tau_2(t) = t/40$, $e(t) = -3/t^2$, $\beta_1 = 2$, and $\beta_2 = 3$. Then $\sum_{i=1}^{n}(1-\beta_i)q_i(t) - e(t) = 0$ and

$$\begin{aligned}
&\limsup_{t\to\infty} \int_{t_0}^{t} \left[ s\left( q_0(s) + \sum_{i=1}^{n} \beta_i q_i(s) \right) - \frac{r(\tau(s))}{4s\tau'(s)} \right] \mathrm{d}s \\
&\quad = (\gamma - 5) \limsup_{t\to\infty} \int_{1}^{t} \frac{\mathrm{d}s}{s} = \infty, \quad \text{provided that } \gamma > 5.
\end{aligned}$$

Hence, by Corollary 2.4, equation (3.1) is oscillatory for any $\gamma > 5$.

Let $Q$ be defined as in Theorem 1.1. Then

$$\begin{aligned}
Q(t) &= q_0(t) + \sum_{i=1}^{n} \beta_i \bigl[n(\beta_i - 1)\bigr]^{(1-\beta_i)/\beta_i} \bigl(q_i(t)\bigr)^{1/\beta_i} \bigl|e(t)\bigr|^{(\beta_i-1)/\beta_i} \\
&= \frac{1}{t^2}\left[ \gamma + \sum_{i=1}^{2} \beta_i \left( \frac{2(\beta_i - 1)}{3} \right)^{(1-\beta_i)/\beta_i} \right] \\
&= \frac{1}{t^2}\left[ \gamma + 2\left(\frac{3}{2}\right)^{1/2} + 3\left(\frac{3}{4}\right)^{2/3} \right] \\
&< \frac{1}{t^2}(\gamma + 2.45 + 2.477) = \frac{1}{t^2}(\gamma + 4.927).
\end{aligned}$$

Using the latter inequality and $\rho(t) = t$ in (1.3), we observe that Theorem 1.1 cannot ensure oscillation of (3.1) on the interval $(5, 5.073]$. Therefore, Corollary 2.4 improves Theorem 1.1.

**Remark 3.2** In this paper, several new oscillation criteria for equation (1.1) are obtained by using the Riccati substitution and Bernoulli's inequality. Employing inequalities different from those exploited in [12], we improve Theorem 1.1; see Example 3.1. Furthermore, Theorems 2.5 and 2.6 complement those by Zhong *et al.* [12] since our results can be applied to the case where (2.10) holds.

**Competing interests**
The authors declare that they have no competing interests.

**Authors' contributions**
All three authors contributed equally to this work. They all read and approved the final version of the manuscript.

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