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Asymptotic behavior of a third-order nonlinear neutral delay differential equation

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Abstract
The objective of this paper is to study asymptotic nature of a class of third-order neutral delay differential equations. By using a generalized Riccati substitution and the integral averaging technique, a new Philos-type criterion is obtained which ensures that every solution of the studied equation is either oscillatory or converges to zero. An illustrative example is included.

MSC: 34K11

Keywords: asymptotic behavior; third-order neutral differential equation; oscillation; generalized Riccati substitution

1 Introduction

In this work, we study the oscillation and asymptotic behavior of a third-order nonlinear neutral differential equation with variable delay arguments

$$\left(r(t)\left[x(t)+P(t)x\left(t-\tau(t)\right)\right]''\right)'+\sum_{i=1}^{m}Q_i(t)f_i\left(x\left(t-\sigma_i(t)\right)\right)=0,\quad t\geq t_0,\tag{1}$$

where $m\geq 1$ is an integer and $t_0>0$. We assume that the following hypotheses are satisfied.

(A$_1$) $r\in \mathrm{C}^1([t_0,\infty),(0,\infty))$, $P,\tau,Q_i,\sigma_i\in \mathrm{C}([t_0,\infty),[0,\infty))$, $f_i\in \mathrm{C}(\mathbb{R},\mathbb{R})$, and $uf_i(u)>0$ for $u\neq 0$, $i=1,2,\dots,m$;
(A$_2$) $r'(t)\geq 0$, $\int_{t_0}^{\infty}r^{-1}(t)\,\mathrm{d}t=\infty$, and $0\leq P(t)\leq p_0<1$;
(A$_3$) $\lim_{t\to\infty}(t-\tau(t))=\lim_{t\to\infty}(t-\sigma_i(t))=\infty$, $i=1,2,\dots,m$;
(A$_4$) there exist constants $\alpha_i>0$ such that $f_i(u)/u\geq\alpha_i$ for $u\neq 0$ and $i=1,2,\dots,m$.

Throughout, we define

$$z(t):=x(t)+P(t)x\left(t-\tau(t)\right).\tag{2}$$

By a solution of equation (1), we mean a function $x\in \mathrm{C}([T_x,\infty),\mathbb{R})$, $T_x\geq t_0$, which has the properties $z\in \mathrm{C}^2([T_x,\infty),\mathbb{R})$, $rz''\in \mathrm{C}^1([T_x,\infty),\mathbb{R})$, and satisfies (1) on $[T_x,\infty)$. We consider only those solutions x of (1) which satisfy assumption $\sup\{|x(t)|:t\geq T\}>0$ for all $T\geq T_x$. We assume that (1) possesses such solutions. A solution of (1) is called oscillatory if it has arbitrarily large zeros on $[T_x,\infty)$; otherwise, it is termed nonoscillatory.

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As is well known, the third-order differential equations are derived from many different areas of applied mathematics and physics, for instance, deflection of buckling beam with a fixed or variable cross-section, three-layer beam, electromagnetic waves, gravity-driven flows, *etc.* In recent years, the oscillation theory of third-order differential equations has received a great deal of attention since it has been widely applied in research of physical sciences, mechanics, radio technology, lossless high-speed computer network, control system, life sciences, and population growth.

Numerous research activities are concerned with the oscillation of solutions to different functional differential equations, for some related contributions, we refer the reader to [1–16] and the references cited therein. In the following, we provide some background details regarding the study of various classes of neutral differential equations. Baculíková and Džurina [3] studied a second-order neutral differential equation

$$\left(r(t)\left[x(t)+p(t)x\left(\tau(t)\right)\right]'\right)'+q(t)x\left(\sigma(t)\right)=0.$$

Agarwal *et al.* [1], Grace *et al.* [7], and Zhang *et al.* [16] considered a third-order nonlinear differential equation

$$\left(a(t)\left(b(t)x'(t)\right)'\right)'+q(t)x^{\gamma}\left(\sigma(t)\right)=0.$$

Baculíková and Džurina [4], Candan [5, 6], Karpuz [8], Li and Rogovchenko [9], Li and Thandapani [10], and Li *et al.* [11, 12] investigated a class of third-order neutral differential equations

$$\left(r(t)\left[x(t)+p(t)x\left(\tau(t)\right)\right]''\right)'+q(t)x\left(\sigma(t)\right)=0. \tag{3}$$

Define $\tilde{\tau}(t) := t-\tau(t)$ and $\tilde{\sigma}_i(t) := t-\sigma_i(t)$, $i=1,2,\ldots,m$. It follows from conditions (A_1) and (A_3) that $\tilde{\tau}(t)\le t$, $\tilde{\sigma}_i(t)\le t$, and $\lim_{t\to\infty}\tilde{\tau}(t)=\lim_{t\to\infty}\tilde{\sigma}_i(t)=\infty$, $i=1,2,\ldots,m$. Hence, equation (3) is a special case of (1). As a matter of fact, equation (1) reduces to the form of (3) if $m=1$ and $f_1(u)=u$.

There are two techniques in the study of oscillation of third-order neutral differential equations. One of them is comparison method which is used to reduce the third-order neutral differential equations to the first-order differential equations or inequalities; see, *e.g.*, [8–10]. Another technique is the Riccati technique; see, *e.g.*, [4–6, 10–12]. In this paper, using a *generalized* Riccati substitution which differs from those reported in [4–6, 10–12], a new asymptotic criterion for (1) is presented. In what follows, all functional inequalities are tacitly supposed to hold for all sufficiently large t.

2 Some lemmas

Lemma 1 *Assume that conditions* (A_1)-(A_4) *hold and* x *is a positive solution of* (1). *Then there are only the following two possible cases for* z *defined by* (2):

(I) $z(t)>0$, $z'(t)>0$, $z''(t)>0$, $z'''(t)\le 0$, *and* $(r(t)z''(t))'\le 0$;

(II) $z(t)>0$, $z'(t)<0$, $z''(t)>0$, $z'''(t)\le 0$, *and* $(r(t)z''(t))'\le 0$,

for $t\ge T$, *where* $T\ge t_0$ *is sufficiently large.*

Proof The proof is similar to that of Baculíková and Džurina [4, Lemma 1], and hence is omitted. □

Lemma 2 *Assume that conditions* (A_1)-(A_4) *hold and let* x *be a positive solution of* (1) *and corresponding* z *satisfy case* (II) *in Lemma* 1. *If*

$$\int_{t_0}^{\infty}\int_{v}^{\infty}\frac{1}{r(u)}\left(\sum_{i=1}^{m}\int_{u}^{\infty}Q_i(s)\,\mathrm{d}s\right)\mathrm{d}u\,\mathrm{d}v=\infty, \tag{4}$$

then $\lim_{t\to\infty}x(t)=\lim_{t\to\infty}z(t)=0$.

Proof Suppose that x is a positive solution of (1). Since $z(t)>0$ and $z'(t)<0$, there exists a finite constant $l\geq 0$ such that $\lim_{t\to\infty}z(t)=l\geq 0$. We shall prove that $l=0$. Assume now that $l>0$. Then for any $\varepsilon>0$, there exists a $t_1\geq T$ such that $l+\varepsilon>z(t)>l$ for $t\geq t_1$. Choose $0<\varepsilon<l(1-p_0)/p_0$. It is not hard to find that

$$\begin{aligned}x(t)&=z(t)-P(t)x\bigl(t-\tau(t)\bigr)>l-P(t)x\bigl(t-\tau(t)\bigr)>l-p_0z\bigl(t-\tau(t)\bigr)\\&>l-p_0(l+\varepsilon):=N(l+\varepsilon)>Nz(t),\end{aligned}\tag{5}$$

where $N:=(l-p_0(l+\varepsilon))/(l+\varepsilon)>0$. Using (1) and (5), we conclude that

$$\begin{aligned}0&=\bigl(r(t)z''(t)\bigr)'+\sum_{i=1}^{m}Q_i(t)f_i\bigl(x\bigl(t-\sigma_i(t)\bigr)\bigr)\\&\geq\bigl(r(t)z''(t)\bigr)'+\sum_{i=1}^{m}\alpha_iQ_i(t)x\bigl(t-\sigma_i(t)\bigr)\\&\geq\bigl(r(t)z''(t)\bigr)'+N\sum_{i=1}^{m}\alpha_iQ_i(t)z\bigl(t-\sigma_i(t)\bigr)\\&\geq\bigl(r(t)z''(t)\bigr)'+N\sum_{i=1}^{m}\alpha_iQ_i(t)z(t).\end{aligned}\tag{6}$$

Integrating (6) from t to ∞, we obtain

$$0\geq -r(t)z''(t)+N\sum_{i=1}^{m}\alpha_i\int_{t}^{\infty}Q_i(s)z(s)\,\mathrm{d}s.$$

Noting that $z(t)>l$, we get

$$0\geq -z''(t)+\frac{lN}{r(t)}\sum_{i=1}^{m}\alpha_i\int_{t}^{\infty}Q_i(s)\,\mathrm{d}s. \tag{7}$$

Integrating (7) from t to ∞, we have

$$0\geq z'(t)+lN\int_{t}^{\infty}\frac{1}{r(u)}\left(\sum_{i=1}^{m}\alpha_i\int_{u}^{\infty}Q_i(s)\,\mathrm{d}s\right)\mathrm{d}u. \tag{8}$$

Integrating (8) from t_1 to ∞, we deduce that

$$\int_{t_1}^{\infty}\int_{v}^{\infty}\frac{1}{r(u)}\left(\sum_{i=1}^{m}\alpha_i\int_{u}^{\infty}Q_i(s)\,\mathrm{d}s\right)\mathrm{d}u\,\mathrm{d}v\leq\frac{z(t_1)}{lN},$$

which contradicts (4). Hence, $l = 0$ and $\lim_{t\to\infty} z(t) = 0$. Then it follows from $0 \leq x(t) \leq z(t)$ that $\lim_{t\to\infty} x(t) = 0$. The proof is complete. □

Lemma 3 (See [4, Lemma 3]) *Assume that $u(t) > 0$, $u'(t) > 0$, and $u''(t) \leq 0$ for $t \geq t_0$. If $\sigma \in C([t_0,\infty),[0,\infty))$, $\sigma(t) \leq t$, and $\lim_{t\to\infty} \sigma(t) = \infty$, then for every $\alpha \in (0,1)$, there exists a $T_\alpha \geq t_0$ such that $u(\sigma(t))/\sigma(t) \geq \alpha u(t)/t$ for $t \geq T_\alpha$.*

Remark 1 If u satisfies conditions of Lemma 3, then $u(t-\sigma_i(t))/u(t) \geq \alpha(t-\sigma_i(t))/t$ for $i = 1,2,\dots,m$ when using conditions (A_1) and (A_3).

Lemma 4 (See [4, Lemma 4]) *Assume that $u(t) > 0$, $u'(t) > 0$, $u''(t) > 0$, and $u'''(t) \leq 0$ for $t \geq t_0$. Then for each $\beta \in (0,1)$, there exists a $T_\beta \geq t_0$ such that $u(t) \geq \beta t u'(t)/2$ for $t \geq T_\beta$.*

Remark 2 If u satisfies conditions of Lemma 4, then $u(t-\sigma_i(t))/u'(t-\sigma_i(t)) \geq \beta(t-\sigma_i(t))/2$ for $i = 1,2,\dots,m$ when using condition (A_3).

3 Main results

We use the integral averaging technique to establish a Philos-type (see Philos [13]) criterion for (1). Let

$$\mathbb{D} := \{(t,s) : t \geq s \geq t_0\} \quad \text{and} \quad \mathbb{D}_0 := \{(t,s) : t > s \geq t_0\}.$$

We say that a function $H \in C(\mathbb{D},\mathbb{R})$ belongs to the class X if

(i) $H(t,t) = 0$, $t \geq t_0$, $H(t,s) > 0$, $(t,s) \in \mathbb{D}_0$;

(ii) H has a nonpositive continuous partial derivative $\partial H/\partial s$ on $\mathbb{D}_0$ with respect to the second variable, and there exist functions $\rho \in C^1([t_0,\infty),(0,\infty))$, $\delta \in C^1([t_0,\infty),\mathbb{R})$, and $h \in C(\mathbb{D}_0,\mathbb{R})$ such that

$$\frac{\partial H(t,s)}{\partial s} + \left(2\delta(s) + \frac{\rho'(s)}{\rho(s)}\right)H(t,s) = -h(t,s)\sqrt{H(t,s)}. \tag{9}$$

Theorem 1 *Assume that conditions (A_1)-(A_4) and (4) are satisfied. If*

$$\limsup_{t\to\infty} \frac{1}{H(t,t_0)} \int_{t_0}^{t} \left[H(t,s)G(s) - \frac{1}{4}\rho(s)r(s)h^2(t,s)\right] ds = \infty \tag{10}$$

holds for some $\alpha \in (0,1)$, $\beta \in (0,1)$, and for some $H \in X$, where

$$G(t) := \rho(t)\left[\frac{\alpha\beta(1-p_0)}{2}\sum_{i=1}^{m}\alpha_i Q_i(t)\frac{(t-\sigma_i(t))^2}{t} + r(t)\delta^2(t) - \bigl(r(t)\delta(t)\bigr)'\right], \tag{11}$$

then every solution x of (1) is either oscillatory or satisfies $\lim_{t\to\infty} x(t) = 0$.

Proof Suppose to the contrary and assume that (1) has a nonoscillatory solution x. Without loss of generality, we can assume that there exists a $t_1 \geq t_0$ such that $x(t) > 0$, $x(t-\tau(t)) > 0$, and $x(t-\sigma_i(t)) > 0$ for $t \geq t_1$ and $i = 1,2,\dots,m$. By Lemma 1, we observe that z satisfies either (I) or (II) for $t \geq T$, where $T \geq t_1$ is large enough. We consider each of the two cases separately.

Assume first that case (I) holds. It follows from $z'(t) > 0$ that

$$\begin{aligned} x(t) &= z(t) - P(t)x\bigl(t-\tau(t)\bigr) \ge z(t) - p_0 x\bigl(t-\tau(t)\bigr) \\ &\ge z(t) - p_0 z\bigl(t-\tau(t)\bigr) \ge (1-p_0)z(t). \end{aligned} \tag{12}$$

Using (1) and (12), we deduce that

$$\begin{aligned} \bigl(r(t)z''(t)\bigr)' &= -\sum_{i=1}^{m} Q_i(t) f_i\bigl(x\bigl(t-\sigma_i(t)\bigr)\bigr) \\ &\le -\sum_{i=1}^{m} \alpha_i Q_i(t) x\bigl(t-\sigma_i(t)\bigr) \\ &\le -(1-p_0)\sum_{i=1}^{m} \alpha_i Q_i(t) z\bigl(t-\sigma_i(t)\bigr). \end{aligned} \tag{13}$$

Define a generalized Riccati substitution by

$$\omega(t) := \rho(t)\left[\frac{r(t)z''(t)}{z'(t)} + r(t)\delta(t)\right]. \tag{14}$$

Then we have

$$\begin{aligned} \omega' &= \rho'\left[\frac{rz''}{z'} + r\delta\right] + \rho\left[\frac{rz''}{z'} + r\delta\right]' \\ &= \frac{\rho'}{\rho}\omega + \rho(r\delta)' + \rho\left(\frac{rz''}{z'}\right)' \\ &= \frac{\rho'}{\rho}\omega + \rho(r\delta)' + \rho\frac{(rz'')'}{z'} - \rho r\left(\frac{z''}{z'}\right)^2. \end{aligned} \tag{15}$$

By virtue of (14), we conclude that

$$\left(\frac{z''}{z'}\right)^2 = \left[\frac{\omega}{\rho r} - \delta\right]^2 = \left(\frac{\omega}{\rho r}\right)^2 + \delta^2 - 2\frac{\omega\delta}{\rho r}. \tag{16}$$

Substituting (13) and (16) into (15), we obtain

$$\begin{aligned} \omega' &= \rho\frac{(rz'')'}{z'} + \frac{\rho'}{\rho}\omega + \rho(r\delta)' - \rho r\left[\frac{\omega^2}{\rho^2 r^2} + \delta^2 - 2\frac{\omega\delta}{\rho r}\right] \\ &= \rho\frac{(rz'')'}{z'} - \rho\bigl[r\delta^2 - (r\delta)'\bigr] + \left(\frac{\rho'}{\rho} + 2\delta\right)\omega - \frac{\omega^2}{r\rho} \\ &\le -(1-p_0)\rho\sum_{i=1}^{m}\alpha_i Q_i \frac{z(t-\sigma_i(t))}{z'(t)} - \rho\bigl[r\delta^2 - (r\delta)'\bigr] + \left(\frac{\rho'}{\rho} + 2\delta\right)\omega - \frac{\omega^2}{r\rho}. \end{aligned} \tag{17}$$

It follows from Remarks 1 and 2 that, for any $\alpha \in (0,1)$ and $\beta \in (0,1)$,

$$\frac{z(t-\sigma_i(t))}{z'(t)} = \frac{z(t-\sigma_i(t))}{z'(t-\sigma_i(t))}\frac{z'(t-\sigma_i(t))}{z'(t)} \ge \frac{\alpha\beta}{2}\frac{(t-\sigma_i(t))^2}{t}, \tag{18}$$

$i = 1, 2, \ldots, m$. Combining (17) and (18), we get

$$\begin{aligned}\omega'(t) &\le -\frac{\alpha\beta(1-p_0)}{2}\rho(t)\sum_{i=1}^{m}\alpha_i Q_i(t)\frac{(t-\sigma_i(t))^2}{t} \\ &\quad - \rho(t)\left[r(t)\delta^2(t) - \left(r(t)\delta(t)\right)'\right] + \left(\frac{\rho'(t)}{\rho(t)} + 2\delta(t)\right)\omega(t) - \frac{\omega^2(t)}{r(t)\rho(t)} \\ &= -G(t) + A(t)\omega(t) - B(t)\omega^2(t),\end{aligned}$$

where G is defined as in (11), $A(t) := (\rho'(t)/\rho(t)) + 2\delta(t)$, and $B(t) := 1/(r(t)\rho(t))$. Replacing in the latter inequality t with s, multiplying both sides by $H(t,s)$ and integrating with respect to s from some T_1 ($T_1 \ge T$) to t, we derive from $H(t,t) = 0$ and (9) that

$$\begin{aligned}&\int_{T_1}^{t} H(t,s)G(s)\,\mathrm{d}s \\ &\quad\le \int_{T_1}^{t} H(t,s)\left[-\omega'(s) + A(s)\omega(s) - B(s)\omega^2(s)\right]\mathrm{d}s \\ &\quad= H(t,T_1)\omega(T_1) + \int_{T_1}^{t}\left[\left(\frac{\partial H(t,s)}{\partial s} + A(s)H(t,s)\right)\omega(s) - H(t,s)B(s)\omega^2(s)\right]\mathrm{d}s \\ &\quad= H(t,T_1)\omega(T_1) - \int_{T_1}^{t}\left[h(t,s)\sqrt{H(t,s)}\omega(s) + H(t,s)B(s)\omega^2(s)\right]\mathrm{d}s \\ &\quad= H(t,T_1)\omega(T_1) - \int_{T_1}^{t}\left(\sqrt{H(t,s)B(s)}\omega(s) + \frac{h(t,s)}{2\sqrt{B(s)}}\right)^2\mathrm{d}s + \int_{T_1}^{t}\frac{h^2(t,s)}{4B(s)}\,\mathrm{d}s \\ &\quad\le H(t,T_1)\omega(T_1) + \int_{T_1}^{t}\frac{h^2(t,s)}{4B(s)}\,\mathrm{d}s,\end{aligned}$$

and hence

$$\limsup_{t\to\infty}\frac{1}{H(t,T_1)}\int_{T_1}^{t}\left[H(t,s)G(s) - \frac{1}{4}\rho(s)r(s)h^2(t,s)\right]\mathrm{d}s \le \omega(T_1),$$

which contradicts condition (10).

Assume now that case (II) holds. By virtue of Lemma 2, $\lim_{t\to\infty} x(t) = 0$. This completes the proof. □

Corollary 1 *The conclusion of Theorem* 1 *remains intact if condition* (10) *is replaced by the assumptions*

$$\limsup_{t\to\infty}\frac{1}{H(t,t_0)}\int_{t_0}^{t} H(t,s)G(s)\,\mathrm{d}s = \infty$$

and

$$\limsup_{t\to\infty}\frac{1}{H(t,t_0)}\int_{t_0}^{t} \rho(s)r(s)h^2(t,s)\,\mathrm{d}s < \infty.$$

As an application of Theorem 1, we provide the following example.

Example 1 For $t \geq 1$, consider a third-order neutral delay differential equation

$$\left(x(t) + \frac{1}{3}x\left(\frac{t}{3}\right)\right)''' + t^{-3}x\left(\frac{t}{4}\right) + 4t^{-3}x\left(\frac{t}{2}\right) = 0. \tag{19}$$

Let $\rho(t) = t$, $\delta(t) = 0$, and $H(t,s) = (t-s)^2$. It is not difficult to verify that all assumptions of Theorem 1 are satisfied. Hence, every solution x of (19) is either oscillatory or satisfies $\lim_{t\to\infty} x(t) = 0$. As a matter of fact, one such solution is $x(t) = t^{-1}$.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
Both authors contributed equally to this work. They both read and approved the final version of the manuscript.

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