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Positive solution for q-fractional four-point boundary value problems with p-Laplacian operator

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Abstract

This paper investigates a class of four-point boundary value problems of fractional q-difference equations with p-Laplacian operator $D_q^\beta(\varphi_p(D_q^\alpha u(t))) = f(t,u(t))$, $t \in (0,1)$, $u(0)=0$, $u(1)=au(\xi)$, $D_q^\alpha u(0)=0$, and $D_q^\alpha u(1) = bD_q^\alpha u(\eta)$, where D_q^α and D_q^β are the fractional q-derivative of the Riemann-Liouville type, p-Laplacian operator is defined as $\varphi_p(s) = |s|^{p-2}s$, $p>1$, and $f(t,u)$ may be singular at $t=0,1$ or $u=0$. By applying the upper and lower solutions method associated with the Schauder fixed point theorem, some sufficient conditions for the existence of at least one positive solution are established. Furthermore, two examples are presented to illustrate the main results.

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Keywords: fractional q-difference equations; four-point boundary conditions; p-Laplacian operator; positive solution; upper and lower solutions method

1 Introduction

Recently, fractional differential equations with p-Laplacian operator have gained its popularity and importance due to its distinguished applications in numerous diverse fields of science and engineering, such as viscoelasticity mechanics, non-Newtonian mechanics, electrochemistry, fluid mechanics, combustion theory, and material science. There have appeared some results for the existence of solutions or positive solutions of boundary value problems for fractional differential equations with p-Laplacian operator; see [1–7] and the references therein. For example, under different conditions, Wang *et al.* [8] and Ren and Chen [9] established the existence of positive solutions to four-point boundary value problems for nonlinear fractional differential equations with p-Laplacian operator by using the upper and lower solutions method and fixed point theorems, respectively.

Since Al-Salam [10] and Agarwal [11] proposed the fractional q-difference calculus, new developments in this theory of fractional q-difference calculus have been made due to the explosion in research within the fractional differential calculus setting. For example, some researcher obtained q-analogs of the integral and differential fractional operators properties such as the q-Laplace transform, the q-Taylor formula, the Mittag-Leffler function [12–15], and so on.

Recently, the theory of boundary value problems for nonlinear fractional q-difference equations has been addressed extensively by several researchers. There have been some papers dealing with the existence and multiplicity of solutions or positive solutions for boundary value problems involving nonlinear fractional q-difference equations by the use of some well-known fixed point theorems and the upper and lower solutions method. For some recent developments on the subject, see [16–25] and the references therein. El-Shahed and Al-Askar [26] studied the existence of multiple positive solutions to the nonlinear q-fractional boundary value problems by using the Guo-Krasnoselskii fixed point theorem in a cone. Under different conditions, Graef and Kong [27, 28] investigated the existence of positive solutions for the boundary value problem with fractional q-derivatives in terms of different ranges of λ, respectively. Zhao *et al.* [29] showed some existence results of positive solutions to nonlocal q-integral boundary value problem of nonlinear fractional q-derivatives equation using the generalized Banach contraction principle, the monotone iterative method, and the Krasnoselskii fixed point theorem. Ahmad *et al.* [30] considered the existence of solutions for the nonlinear fractional q-difference equation with nonlocal boundary conditions by applying some well-known tools of fixed point theory such as the Banach contraction principle, the Krasnoselskii fixed point theorem, and the Leray-Schauder nonlinear alternative. By applying the nonlinear alternative of Leray-Schauder type and Krasnoselskii fixed point theorems, the author [31] established sufficient conditions for the existence of positive solutions for nonlinear semipositone fractional q-difference system with coupled integral boundary conditions. Relying on the standard tools of fixed point theory, Agarwal *et al.* [32] and Ahmad *et al.* [33] discussed the existence and uniqueness of solutions for a new class of sequential q-fractional integrodifferential equations with q-antiperiodic boundary conditions and nonlocal four-point boundary conditions, respectively.

In [34], Aktuğlu and Özarslan dealt with the following Caputo q-fractional boundary value problem involving the p-Laplacian operator:

$$\begin{aligned}
&D_q\bigl(\varphi_p\bigl({}^cD_q^\alpha x(t)\bigr)\bigr) = f\bigl(t,x(t)\bigr), \quad t \in (0,1), \\
&D_q^k x(0) = 0, \quad \text{for } k = 2,3,\ldots,n-1, \qquad x(0) = a_0 x(1), \qquad D_q x(0) = a_1 D_q x(1),
\end{aligned}$$

where $a_0, a_1 \neq 0$, $1 < \alpha \in \mathbb{R}$, and $f \in C([0,1] \times \mathbb{R}, \mathbb{R})$. Under some conditions, the authors obtained the existence and uniqueness of the solution for the above boundary value problem by using the Banach contraction mapping principle.

In [35], Miao and Liang studied the following three-point boundary value problem with p-Laplacian operator:

$$\begin{aligned}
&D_q^\gamma\bigl(\phi_p\bigl(D_q^\alpha u(t)\bigr)\bigr) + f\bigl(t,u(t)\bigr) = 0, \quad 0 < t < 1, 2 < \alpha < 3, \\
&u(0) = (D_q u)(0) = 0, \qquad (D_q u)(1) = 0, \qquad D_{0+}^\gamma u(t)|_{t=0} = 0,
\end{aligned}$$

where $0 < \beta\eta^{\alpha-2} < 1$, $0 < q < 1$. The authors proved the existence and uniqueness of a positive and nondecreasing solution for the boundary value problems by using a fixed point theorem in partially ordered sets.

In [36], the author investigated the following fractional q-difference boundary value problem with p-Laplacian operator:

$$\begin{aligned}
&D_q^\beta\bigl(\varphi_p\bigl(D_q^\alpha u(t)\bigr)\bigr) = f\bigl(t,u(t)\bigr), \quad 0 < t < 1, \\
&u(0) = u(1) = 0, \qquad D_q^\alpha u(0) = D_q^\alpha u(1) = 0,
\end{aligned}$$

where $1 < \alpha, \beta \leq 2$. The existence results for the above boundary value problem were obtained by using the upper and lower solutions method associated with the Schauder fixed point theorem.

In this paper, motivated greatly by the above mentioned works, we consider the following fractional q-difference boundary value problem with p-Laplacian operator:

$$\begin{aligned} &D_q^\beta\big(\varphi_p\big(D_q^\alpha u(t)\big)\big) = f\big(t,u(t)\big), \quad t \in (0,1), \\ &u(0) = 0, \qquad u(1) = au(\xi), \qquad D_q^\alpha u(0) = 0, \qquad D_q^\alpha u(1) = bD_q^\alpha u(\eta), \end{aligned} \tag{1.1}$$

where D_q^α, D_q^β are the fractional q-derivative of the Riemann-Liouville type with $1 < \alpha, \beta \leq 2$, $0 \leq a, b \leq 1$, $0 < \xi, \eta < 1$, $\varphi_p(s) = |s|^{p-2}s$, $p > 1$, $(\phi_p)^{-1} = \phi_r$, $(1/p) + (1/r) = 1$, and $f(t,u) : (0,1) \times (0,+\infty) \to [0,\infty)$ is continuous and may be singular at $t = 0,1$ or $u = 0$. By applying the upper and lower solutions method associated with the Schauder fixed point theorem, the existence results of at least one positive solution for the above fractional q-difference boundary value problem with p-Laplacian operator are established. This work improves essentially the results of [36]. At the end of this paper, we will give two examples to show the effectiveness of the main results.

2 Preliminaries

For the convenience of the reader, we present some necessary definitions and lemmas of fractional q-calculus theory to facilitate the analysis of problem (1.1). These details can be found in the recent literature; see [37] and references therein.

Let $q \in (0,1)$ and define

$$[a]_q = \frac{q^a - 1}{q - 1}, \quad a \in \mathbb{R}.$$

The q-analog of the Pochhammer symbol (the q-shifted factorial) is defined by

$$(a;q)_0 = 1, \qquad (a;q)_n = \prod_{k=0}^{n-1}\big(a - bq^k\big), \quad n \in \mathbb{N} \cup \{\infty\}.$$

The q-analog of the power $(a-b)^n$ with $n \in \mathbb{N}_0 = \{0,1,2,\ldots\}$ is

$$(a-b)^{(0)} = 1, \qquad (a-b)^{(n)} = \prod_{k=0}^{n-1}\big(a - bq^k\big), \quad n \in \mathbb{N}_0, a, b \in \mathbb{R}.$$

The following relation between them holds:

$$(a-b)^{(n)} = a^n(b/a;q)_n, \quad a \neq 0.$$

Their natural extensions to the reals are

$$(a;q)_\gamma = \frac{(a;q)_\infty}{(aq^\gamma;q)_\infty} \quad \text{and} \quad (a-b)^{(\gamma)} = a^\gamma \frac{(b/a;q)_\infty}{(q^\gamma b/a;q)_\infty}, \quad \gamma \in \mathbb{R}.$$

Clearly, $(a-b)^{(\gamma)} = a^{\gamma}(b/a;q)_{\gamma}$, $a \neq 0$. Note that, if $b = 0$ then $a^{(\alpha)} = a^{\alpha}$. The q-gamma function is defined by

$$\Gamma_q(x) = (1-q)^{(x-1)}(1-q)^{1-x}, \quad x \in \mathbb{R} \setminus \{0,-1,-2,\ldots\},$$

and satisfies $\Gamma_q(x+1) = [x]_q \Gamma_q(x)$.

The q-derivative of a function f is here defined by

$$(D_q f)(x) = \frac{f(x) - f(qx)}{(1-q)x}, \qquad (D_q f)(0) = \lim_{x \to 0} (D_q f)(x),$$

and q-derivatives of higher order by

$$\bigl(D_q^0 f\bigr)(x) = f(x) \quad \text{and} \quad \bigl(D_q^n f\bigr)(x) = D_q\bigl(D_q^{n-1} f\bigr)(x), \quad n \in \mathbb{N}.$$

The q-integral of a function f defined in the interval $[0,b]$ is given by

$$(I_q f)(x) = \int_0^x f(t)\,d_q t = x(1-q)\sum_{n=0}^{\infty} f\bigl(xq^n\bigr)q^n, \quad x \in [0,b].$$

If $a \in [0,b]$ and f is defined in the interval $[0,b]$, its integral from a to b is defined by

$$\int_a^b f(t)\,d_q t = \int_0^b f(t)\,d_q t - \int_0^a f(t)\,d_q t.$$

Similarly as done for derivatives, an operator I_q^n can be defined, namely,

$$\bigl(I_q^0 f\bigr)(x) = f(x) \quad \text{and} \quad \bigl(I_q^n f\bigr)(x) = I_q\bigl(I_q^{n-1} f\bigr)(x), \quad n \in \mathbb{N}.$$

The fundamental theorem of calculus applies to these operators I_q and D_q, *i.e.*,

$$(D_q I_q f)(x) = f(x),$$

and if f is continuous at $x = 0$, then

$$(I_q D_q f)(x) = f(x) - f(0).$$

Basic properties of the two operators can be found in the book [37]. We now point out three formulas that will be used later (${}_iD_q$ denotes the derivative with respect to variable i):

$$\bigl[a(t-s)\bigr]^{(\alpha)} = a^{\alpha}(t-s)^{(\alpha)}, \qquad {}_tD_q(t-s)^{(\alpha)} = [\alpha]_q(t-s)^{(\alpha-1)},$$
$$\left({}_xD_q \int_0^x f(x,t)\,d_q t\right)(x) = \int_0^x {}_xD_q f(x,t)\,d_q t + f(qx,x).$$

Denote that if $\alpha > 0$ and $a \leq b \leq t$, then $(t-a)^{(\alpha)} \geq (t-b)^{(\alpha)}$ [18].

Definition 2.1 ([11]) Let $\alpha \geq 0$ and f be function defined on $[0,1]$. The fractional q-integral of the Riemann-Liouville type is $I_q^0 f(x) = f(x)$ and

$$\left(I_q^\alpha f\right)(x) = \frac{1}{\Gamma_q(\alpha)} \int_0^x (x-qt)^{(\alpha-1)} f(t)\, d_q t, \quad \alpha > 0, x \in [0,1].$$

Definition 2.2 ([13]) The fractional q-derivative of the Riemann-Liouville type of order $\alpha \geq 0$ is defined by $D_q^0 f(x) = f(x)$ and

$$\left(D_q^\alpha f\right)(x) = \left(D_q^m I_q^{m-\alpha} f\right)(x), \quad \alpha > 0,$$

where m is the smallest integer greater than or equal to α.

Lemma 2.3 ([14]) *Let $\alpha, \beta \geq 0$ and f be a function defined on $[0,1]$. Then the next formulas hold*:

(1) $(I_q^\beta I_q^\alpha f)(x) = I_q^{\alpha+\beta} f(x)$,
(2) $(D_q^\alpha I_q^\alpha f)(x) = f(x)$.

Lemma 2.4 ([18]) *Let $\alpha > 0$ and p be a positive integer. Then the following equality holds*:

$$\left(I_q^\alpha D_q^p f\right)(x) = \left(D_q^p I_q^\alpha f\right)(x) - \sum_{k=0}^{p-1} \frac{x^{\alpha-p+k}}{\Gamma_q(\alpha+k-p+1)} \left(D_q^k f\right)(0).$$

Lemma 2.5 *Let $y \in C[0,1]$, $1 < \alpha \leq 2$, $0 < \xi < 1$, and $0 \leq a \leq 1$. Then the unique solution of the following linear fractional q-difference boundary value problem*:

$$\begin{aligned} &D_q^\alpha u(t) + y(t) = 0, \quad t \in (0,1), \\ &u(0) = 0, \qquad u(1) = au(\xi), \end{aligned} \tag{2.1}$$

is given by

$$u(t) = \int_0^1 G(t,qs) y(s)\, d_q s, \tag{2.2}$$

where

$$G(t,s) = g(t,s) + \frac{a g(\xi,s) t^{\alpha-1}}{1 - a\xi^{\alpha-1}}, \tag{2.3}$$

$$g(t,s) = \frac{1}{\Gamma_q(\alpha)} \begin{cases} (t(1-s))^{(\alpha-1)} - (t-s)^{(\alpha-1)}, & 0 \leq s \leq t \leq 1, \\ (t(1-s))^{(\alpha-1)}, & 0 \leq t \leq s \leq 1. \end{cases}$$

Proof By applying Lemma 2.4, we may reduce (2.1) to an equivalent integral equation

$$u(t) = -I_q^\alpha y(t) + c_1 t^{\alpha-1} + c_2 t^{\alpha-2}, \quad c_1, c_2 \in \mathbb{R}. \tag{2.4}$$

From $u(0) = 0$ and (2.4), we have $c_2 = 0$. Consequently the general solution of (2.1) is

$$u(t) = -I_q^\alpha y(t) + c_1 t^{\alpha-1} = -\int_0^t \frac{(t-qs)^{(\alpha-1)}}{\Gamma_q(\alpha)} y(s)\, d_q s + c_1 t^{\alpha-1}. \tag{2.5}$$

By (2.5), one has

$$u(1) = -\int_0^1 \frac{(1-qs)^{(\alpha-1)}}{\Gamma_q(\alpha)} y(s)\,d_qs + c_1, \qquad u(\xi) = -\int_0^\xi \frac{(\xi-qs)^{(\alpha-1)}}{\Gamma_q(\alpha)} y(s)\,d_qs + c_1\xi^{\alpha-1}.$$

And from $u(1) = au(\xi)$, then we have

$$c_1 = \int_0^1 \frac{(1-qs)^{(\alpha-1)}}{(1-a\xi^{\alpha-1})\Gamma_q(\alpha)} y(s)\,d_qs - \int_0^\xi \frac{a(\xi-qs)^{(\alpha-1)}}{(1-a\xi^{\alpha-1})\Gamma_q(\alpha)} y(s)\,d_qs.$$

So, the unique solution of problem (2.1) is

$$\begin{aligned} u(t) &= -\int_0^t \frac{(t-qs)^{(\alpha-1)}}{\Gamma_q(\alpha)} y(s)\,d_qs + \int_0^1 \frac{t^{\alpha-1}(1-qs)^{(\alpha-1)}}{(1-a\xi^{\alpha-1})\Gamma_q(\alpha)} y(s)\,d_qs \\ &\quad - \int_0^\xi \frac{at^{\alpha-1}(\xi-qs)^{(\alpha-1)}}{(1-a\xi^{\alpha-1})\Gamma_q(\alpha)} y(s)\,d_qs \\ &= -\int_0^t \frac{(t-qs)^{(\alpha-1)}}{\Gamma_q(\alpha)} y(s)\,d_qs + \int_0^1 \frac{t^{\alpha-1}(1-qs)^{(\alpha-1)}}{\Gamma_q(\alpha)} y(s)\,d_qs \\ &\quad + \int_0^1 \frac{a\xi^{\alpha-1}t^{\alpha-1}(1-qs)^{(\alpha-1)}}{(1-a\xi^{\alpha-1})\Gamma_q(\alpha)} y(s)\,d_qs - \int_0^\xi \frac{at^{\alpha-1}(\xi-qs)^{(\alpha-1)}}{(1-a\xi^{\alpha-1})\Gamma_q(\alpha)} y(s)\,d_qs \\ &= \int_0^1 G(t,qs)y(s)\,d_qs, \end{aligned} \tag{2.6}$$

where $G(t,s)$ is defined in (2.3). The proof is completed. □

Lemma 2.6 *Let $y \in C[0,1]$, $1 < \alpha, \beta \le 2$, $0 < \xi, \eta < 1$, and $0 \le a, b \le 1$. Then the following fractional q-difference boundary value problem with p-Laplacian operator*:

$$\begin{aligned} &D_q^\beta\bigl(\varphi_p\bigl(D_q^\alpha u(t)\bigr)\bigr) = y(t), \quad t \in (0,1), \\ &u(0) = 0, \qquad u(1) = au(\xi), \qquad D_q^\alpha u(0) = 0, \qquad D_q^\alpha u(1) = bD_q^\alpha u(\eta), \end{aligned} \tag{2.7}$$

has unique solution given by

$$u(t) = \int_0^1 G(t,qs)\varphi_r\left(\int_0^1 H(s,q\tau)y(\tau)\,d_q\tau\right) d_qs, \tag{2.8}$$

where $b_1 = b^{p-1}$, $G(t,s)$ is defined by (2.3) *and*

$$H(t,s) = h(t,s) + \frac{b_1 h(\xi,s)t^{\alpha-1}}{1-b_1\xi^{\alpha-1}}, \tag{2.9}$$

$$h(t,s) = \frac{1}{\Gamma_q(\beta)} \begin{cases} (t(1-s))^{(\beta-1)} - (t-s)^{(\beta-1)}, & 0 \le s \le t \le 1, \\ (t(1-s))^{(\beta-1)}, & 0 \le t \le s \le 1. \end{cases}$$

Proof By applying Lemma 2.4, we may reduce (2.7) to an equivalent integral equation,

$$\varphi_p\bigl(D_q^\alpha u(t)\bigr) = I_q^\beta y(t) + c_3t^{\beta-1} + c_4t^{\beta-2}, \quad c_3, c_4 \in \mathbb{R}. \tag{2.10}$$

From $D_q^\alpha u(0) = 0$ and (2.10), we have $c_4 = 0$. Consequently the general solution of (2.7) is

$$\varphi_p\big(D_q^\alpha u(t)\big) = I_q^\beta y(t) + c_3 t^{\beta-1} = \int_0^t \frac{(t-qs)^{(\beta-1)}}{\Gamma_q(\beta)} y(s)\,d_qs + c_3 t^{\beta-1}. \tag{2.11}$$

By (2.11), one has

$$\begin{aligned}
\varphi_p\big(D_q^\alpha u(1)\big) &= \int_0^1 \frac{(1-qs)^{(\beta-1)}}{\Gamma_q(\beta)} y(s)\,d_qs + c_3, \\
\varphi_p\big(D_q^\alpha u(\eta)\big) &= \int_0^\eta \frac{(\eta-qs)^{(\beta-1)}}{\Gamma_q(\beta)} y(s)\,d_qs + c_3\eta^{\beta-1}.
\end{aligned}$$

From $D_q^\alpha u(1) = bD_q^\alpha u(\eta)$, we have

$$c_3 = \int_0^1 \frac{(1-qs)^{(\beta-1)}}{(1-b_1\eta^{\beta-1})\Gamma_q(\beta)} y(s)\,d_qs - \int_0^\eta \frac{b_1(\eta-qs)^{(\beta-1)}}{(1-b_1\eta^{\beta-1})\Gamma_q(\beta)} y(s)\,d_qs,$$

where $b_1 = b^{p-1}$. Similar to Lemma 2.6, we have

$$\varphi_p\big(D_q^\alpha u(t)\big) = -\int_0^1 H(t,qs)y(s)\,d_qs.$$

Consequently, the fractional boundary value problem (2.7) is equivalent to the following problem:

$$\begin{aligned}
&D_q^\alpha u(t) + \varphi_r\left(\int_0^1 H(t,qs)y(s)\,d_qs\right) = 0, \quad t\in(0,1), \\
&u(0) = 0, \qquad u(1) = au(\xi).
\end{aligned}$$

Lemma 2.6 implies that the fractional boundary value problem (2.7) has a unique solution

$$u(t) = \int_0^1 G(t,qs)\varphi_r\left(\int_0^1 H(s,q\tau)y(\tau)\,d_q\tau\right) d_qs.$$

The proof is completed. □

Lemma 2.7 *Let* $1 < \alpha, \beta \le 2$, $0 < \xi, \eta < 1$, *and* $0 \le a, b \le 1$. *Then functions* $G(t,s)$ *and* $H(t,s)$ *defined by* (2.3) *and* (2.9), *respectively, are continuous on* $[0,1]\times[0,1]$ *satisfying*

(a) $G(t,qs) \ge 0$, $H(t,qs) \ge 0$, *for all* $t,s \in [0,1]$;

(b) *for all* $t,s\in[0,1]$, $\sigma_1(qs)t^{\alpha-1} \le G(t,qs) \le \sigma_2(qs)t^{\alpha-1}$, *where*

$$\sigma_1(s) = \frac{ag(\xi,s)}{1-a\xi^{\alpha-1}}, \qquad \sigma_2(s) = \frac{(1-s)^{(\alpha-1)}}{\Gamma_q(\alpha)} + \frac{ag(\xi,s)}{1-a\xi^{\alpha-1}}.$$

Proof The proof is obvious, so we omit the proof. □

From Lemmas 2.5 and 2.7, it is easy to obtain the following lemma.

Lemma 2.8 *Let $u(t) \in C([0,1],\mathbb{R})$ satisfies $u(0) = 0$, $u(1) = \varphi_p(b)u(\eta)$, and $D_q^\beta u(t) \geq 0$ for any $t \in (0,1)$, then $u(t) \leq 0$, for $t \in [0,1]$.*

Let $E = \{u : u, \varphi_p(D_q^\alpha u) \in C^2[0,1]\}$. Now we introduce the following definitions about the upper and lower solutions of the fractional q-difference boundary value problem (1.1).

Definition 2.9 A function $\phi(t)$ is called a lower solution of the fractional q-difference boundary value problem (1.1), if $\phi(t) \in E$ and $\phi(t)$ satisfies

$$\begin{aligned}
&D_q^\beta\big(\varphi_p\big(D_q^\alpha\phi(t)\big)\big) \leq f\big(t,\phi(t)\big), \quad t \in (0,1),\\
&\phi(0) \leq 0, \qquad \phi(1) \leq a\phi(\xi), \qquad D_q^\alpha\phi(0) \geq 0, \qquad D_q^\alpha\phi(1) \geq bD_q^\alpha\phi(\eta).
\end{aligned}$$

Definition 2.10 A function $\psi(t)$ is called an upper solution of the fractional q-difference boundary value problem (1.1), if $\psi(t) \in E$ and $\psi(t)$ satisfies

$$\begin{aligned}
&D_q^\beta\big(\varphi_p\big(D_q^\alpha\psi(t)\big)\big) \geq f\big(t,\psi(t)\big), \quad t \in (0,1),\\
&\psi(0) \leq 0, \qquad \psi(1) \leq a\psi(\xi), \qquad D_q^\alpha\psi(0) \geq 0, \qquad D_q^\alpha\psi(1) \geq bD_q^\alpha\psi(\eta).
\end{aligned}$$

3 Main results

For the sake of simplicity, we make the following assumptions throughout this paper.

(H_1) $f(t,u) \in C[(0,1) \times (0,+\infty),[0,+\infty)]$ and $f(t,u)$ is decreasing in u.
(H_2) Set $e(t) = t^{\alpha-1}$. For any constant $\rho > 0, f(t,\rho) \not\equiv 0$, and

$$0 < \int_0^1 \sigma_2(qs)\varphi_r\left(\int_0^1 H(s,q\tau)f\big(\tau,\rho e(\tau)\big)\,d_q\tau\right)d_qs < +\infty.$$

We define $P = \{u \in C[0,1]$: there exist two positive constants $0 < l_u < L_u$ such that $l_u e(t) \leq u(t) \leq L_u e(t), t \in [0,1]\}$. Obviously, $e(t) \in P$. Therefore, P is not empty. For any $u \in P$, define an operator T by

$$(Tu)(t) = \int_0^1 G(t,qs)\varphi_r\left(\int_0^1 H(s,q\tau)f\big(\tau,u(\tau)\big)\,d_q\tau\right)d_qs, \quad t \in [0,1].$$

Theorem 3.1 *Suppose that conditions* (H_1)-(H_2) *are satisfied, then the boundary value problem* (1.1) *has at least one positive solution u, and there exist two positive constants $0 < \lambda_1 < 1 < \lambda_2$ such that $\lambda_1 e(t) \leq u(t) \leq \lambda_2 e(t)$, $t \in [0,1]$.*

Proof We will divide our proof into four steps.

Step 1. We show that T is well defined on P and $T(P) \subset P$, and T is decreasing in u.

In fact, for any $u \in P$, by the definition of P, there exist two positive constants $0 < l_u < 1 < L_u$ such that $l_u e(t) \leq u(t) \leq L_u e(t)$ for any $t \in [0,1]$. It follows from Lemma 2.7 and conditions (H_1)-(H_2) that

$$\begin{aligned}
(Tu)(t) &= \int_0^1 G(t,qs)\varphi_r\left(\int_0^1 H(s,q\tau)f\big(\tau,u(\tau)\big)\,d_q\tau\right)d_qs\\
&\leq e(t)\int_0^1 \sigma_2(qs)\varphi_r\left(\int_0^1 H(s,q\tau)f\big(\tau,l_u e(\tau)\big)\,d_q\tau\right)d_qs < +\infty. \qquad (3.1)
\end{aligned}$$

On the other hand, it follows from Lemma 2.7 that

$$\begin{aligned}(Tu)(t) &= \int_0^1 G(t,qs)\varphi_r\left(\int_0^1 H(s,q\tau)f\big(\tau,u(\tau)\big)\,d_q\tau\right)d_qs\\ &\geq e(t)\int_0^1 \sigma_1(qs)\varphi_r\left(\int_0^1 H(s,q\tau)f\big(\tau,L_ue(\tau)\big)\,d_q\tau\right)d_qs.\end{aligned}\tag{3.2}$$

Take

$$\begin{aligned}l'_u &= \min\left\{1,\int_0^1 \sigma_1(qs)\varphi_r\left(\int_0^1 H(s,q\tau)f\big(\tau,L_ue(\tau)\big)\,d_q\tau\right)d_qs\right\},\\ L'_u &= \max\left\{1,\int_0^1 \sigma_2(qs)\varphi_r\left(\int_0^1 H(s,q\tau)f\big(\tau,l_ue(\tau)\big)\,d_q\tau\right)d_qs\right\},\end{aligned}$$

then by (3.1) and (3.2), $l'_ue(t) \leq (Tu)(t) \leq L'_ue(t)$, which implies that T is well defined and $T(P) \subset P$. It follows from (H_1) that the operator T is decreasing in u. By direct computations, we can state that

$$\begin{aligned}&D_q^\beta\big(\varphi_p\big(D_q^\alpha(Tu)(t)\big)\big) = f\big(t,Tu(t)\big),\quad 0<t<1,\\ &(Tu)(0)=0,\qquad (Tu)(1)=a(Tu)(\xi),\\ &D_q^\alpha(Tu)(0)=0,\qquad D_q^\alpha(Tu)(1)=bD_q^\alpha(Tu)(\eta).\end{aligned}\tag{3.3}$$

Step 2. We focus on lower and upper solutions of the fractional q-difference boundary value problem (1.1). Let

$$m(t)=\min\big\{e(t),(Te)(t)\big\},\qquad n(t)=\max\big\{e(t),(Te)(t)\big\},\tag{3.4}$$

then, if $e(t) = (Te)(t)$, the conclusion of Theorem 3.1 holds. If $e(t) \not\equiv (Te)(t)$, clearly, $m(t), n(t) \in P$, and

$$m(t) \leq e(t) \leq n(t).\tag{3.5}$$

We will prove that the functions $\phi(t) = Tn(t)$, $\psi(t) = Tm(t)$ are a couple of lower and upper solutions of the fractional q-difference boundary value problem (1.1), respectively.

From (H_1), we know that T is nonincreasing relative to u. Thus it follows from (3.4) and (3.5) that

$$\begin{aligned}&\phi(t)=Tn(t)\leq Tm(t)=\psi(t),\\ &Tn(t)\leq Te(t)\leq n(t),\qquad Tm(t)\geq Te(t)\geq m(t),\end{aligned}\tag{3.6}$$

and $\phi(t), \psi(t) \in P$. It follows from (3.3)-(3.6) that

$$\begin{aligned}&D_q^\beta\big(\varphi_p\big(D_q^\alpha\phi(t)\big)\big)-f\big(t,\phi(t)\big)\leq D_q^\beta\big(\varphi_p\big(D_q^\alpha(Tn)(t)\big)\big)-f\big(t,n(t)\big)=0,\\ &\phi(0)=0,\qquad \phi(1)=a\phi(\xi),\qquad D_q^\alpha\phi(0)=0,\qquad D_q^\alpha\phi(1)=bD_q^\alpha\phi(\eta),\\ &D_q^\beta\big(\varphi_p\big(D_q^\alpha\psi(t)\big)\big)-f\big(t,\psi(t)\big)\geq D_q^\beta\big(\varphi_p\big(D_q^\alpha(Tm)(t)\big)\big)-f\big(t,m(t)\big)=0,\\ &\psi(0)=0,\qquad \psi(1)=a\psi(\xi),\qquad D_q^\alpha\psi(0)=0,\qquad D_q^\alpha\psi(1)=bD_q^\alpha\psi(\eta),\end{aligned}\tag{3.7}$$

that is, $\phi(t)$ and $\psi(t)$ are a couple of lower and upper solutions of the fractional q-difference boundary value problem (1.1), respectively.

Step 3. We will show that the fractional q-difference boundary value problem

$$\begin{aligned} &D_q^\beta\big(\varphi_p\big(D_q^\alpha u(t)\big)\big) = g\big(t,u(t)\big), \quad 0<t<1, \\ &u(0)=0, \qquad u(1)=au(\xi), \qquad D_q^\alpha u(0)=0, \qquad D_q^\alpha u(1) = bD_q^\alpha u(\eta), \end{aligned} \tag{3.8}$$

has at least one positive solution, where

$$g\big(t,u(t)\big) = \begin{cases} f(t,\phi(t)), & \text{if } u(t)<\phi(t), \\ f(t,u(t)), & \text{if } \phi(t)\le u(t)\le \psi(t), \\ f(t,\psi(t)), & \text{if } u(t)>\psi(t). \end{cases} \tag{3.9}$$

It follows from (H_1) and (3.9) that $g(t,u):[0,1]\times\mathbb{R}\to\mathbb{R}$ is continuous. To see this, we consider the operator $A: C[0,1]\to C[0,1]$ defined as follows:

$$Au(t) = \int_0^1 G(t,qs)\varphi_r\left(\int_0^1 H(s,q\tau)g\big(\tau,u(\tau)\big)\,d_q\tau\right)d_qs,$$

where $G(t,s)$ is defined as (2.3), $H(t,s)$ is defined as (2.9). It is clear that $Au\ge 0$, for all $u\in P$, and a fixed point of the operator A is a solution of the boundary value problem (3.8). Noting that $\phi(t)\in P$, there exists a positive constant $0<l_\phi<1$ such that $\phi(t)\ge l_\phi e(t)$, $t\in[0,1]$. It follows from Lemma 2.7, (3.9), and (H_2) that

$$\begin{aligned} Au(t) &= \int_0^1 G(t,qs)\varphi_r\left(\int_0^1 H(s,q\tau)g\big(\tau,u(\tau)\big)\,d_q\tau\right)d_qs \\ &\le e(t)\int_0^1 \sigma_2(qs)\varphi_r\left(\int_0^1 H(s,q\tau)g\big(\tau,u(\tau)\big)\,d_q\tau\right)d_qs \\ &\le e(t)\int_0^1 \sigma_2(qs)\varphi_r\left(\int_0^1 H(s,q\tau)g\big(\tau,l_\phi e(\tau)\big)\,d_q\tau\right)d_qs < +\infty, \end{aligned}$$

which implies that the operator A is uniformly bounded.

On the other hand, since $G(t,s)$ is continuous on $[0,1]\times[0,1]$, it is uniformly continuous on $[0,1]\times[0,1]$. So, for fixed $s\in[0,1]$ and for any $\varepsilon>0$, there exists a constant $\delta>0$, such that any $t_1,t_2\in[0,1]$ and $|t_1-t_2|<\delta$,

$$\big|G(t_1,qs)-G(t_2,qs)\big| < \frac{\varepsilon}{\int_0^1\varphi_r(\int_0^1 H(s,q\tau)g(\tau,l_\phi e(\tau))\,d_q\tau)\,d_qs}.$$

Then, for all $u(t)\in C[0,1]$, we have

$$\begin{aligned} &\big|Au(t_1)-Au(t_2)\big| \\ &\quad= \int_0^1 \big|G(t_1,qs)-G(t_2,qs)\big|\varphi_r\left(\int_0^1 H(s,q\tau)g\big(\tau,u(\tau)\big)\,d_q\tau\right)d_qs \\ &\quad< \int_0^1 \frac{\varepsilon}{\int_0^1\varphi_r(\int_0^1 H(s,q\tau)g(\tau,l_\phi e(\tau))\,d_q\tau)\,d_qs}\varphi_r\left(\int_0^1 H(s,q\tau)f\big(\tau,\varphi(\tau)\big)\,d_q\tau\right)d_qs \\ &\quad= \frac{\varepsilon}{\int_0^1\varphi_r(\int_0^1 H(s,q\tau)g(\tau,l_\phi e(\tau))\,d_q\tau)\,d_qs}\int_0^1 \varphi_r\left(\int_0^1 H(s,q\tau)f\big(\tau,\varphi(\tau)\big)\,d_q\tau\right)d_qs = \varepsilon, \end{aligned}$$

that is to say, A is equicontinuous. Thus, from the Arzela-Ascoli theorem, we know that A is a compact operator, by using the Schauder fixed point theorem, the operator A has a fixed point u such that $u = Au$; *i.e.*, the fractional q-difference boundary value problem (3.8) has a positive solution.

Step 4. We will prove that the boundary value problem (1.1) has at least one positive solution. Suppose that $u(t)$ is a solution of (3.3), we only need to prove that $\phi(t) \leq u(t) \leq \psi(t)$, $t \in [0,1]$. Now we claim that $\phi(t) \leq u(t) \leq \psi(t)$, $t \in [0,1]$. In fact, since u is fixed point of A and (3.7), we get

$$\begin{aligned} &u(0) = 0, \qquad u(1) = au(\xi), \qquad D_q^\alpha u(0) = 0, \qquad D_q^\alpha u(1) = bD_q^\alpha u(\eta), \\ &\psi(0) = 0, \qquad \psi(1) = a\psi(\xi), \qquad D_q^\alpha \psi(0) = 0, \qquad D_q^\alpha \psi(1) = bD_q^\alpha \psi(\eta). \end{aligned} \tag{3.10}$$

Suppose by contradiction that $u(t) \geq \psi(t)$. According to the definition of g, one verifies that

$$D_q^\beta\big(\varphi_p\big(D_q^\alpha u(t)\big)\big) = g\big(t, u(t)\big) = f\big(t, \psi(t)\big), \quad 0 < t < 1. \tag{3.11}$$

On the other hand, since ψ is an upper solution to (1.1), we obviously have

$$D_q^\beta\big(\varphi_p\big(D_q^\alpha \psi(t)\big)\big) \geq f\big(t, \psi(t)\big), \quad 0 < t < 1. \tag{3.12}$$

Let $z(t) = \varphi_p(D_q^\alpha \psi(t)) - \varphi_p(D_q^\alpha u(t))$, $0 < t < 1$. From (3.11) and (3.12), we can get

$$\begin{aligned} &D_q^\beta z(t) = D_q^\beta\big(\varphi_p\big(D_q^\alpha \psi(t)\big)\big) - D_q^\beta\big(\varphi_p\big(D_q^\alpha u(t)\big)\big) \geq f\big(t, \psi(t)\big) - f\big(t, \psi(t)\big) = 0, \\ &z(0) = 0, \qquad z(1) = \varphi_p(b)z(\eta). \end{aligned}$$

Thus, by Lemma 2.8, we have $z(t) \leq 0$, $t \in [0,1]$, which implies that

$$\varphi_p\big(D_q^\alpha \psi(t)\big) \leq \varphi_p\big(D_q^\alpha u(t)\big), \quad t \in [0,1].$$

Since φ_p is monotone increasing, we obtain $D_q^\alpha \psi(t) \leq D_q^\alpha u(t)$, *i.e.*, $D_q^\alpha(\psi - u)(t) \leq 0$. Combining Lemma 2.8, we have $(\psi - u)(t) \geq 0$. Therefore, $\psi(t) \geq u(t)$, $t \in [0,1]$, a contradiction to the assumption that $u(t) > \psi(t)$. Hence, $u(t) > \psi(t)$ is impossible.

Similarly, suppose by contradiction that $u(t) \leq \phi(t)$. According to the definition of g, one verifies that

$$g\big(t, u(t)\big) = f\big(t, \phi(t)\big), \quad 0 < t < 1.$$

Consequently, we obtain

$$D_q^\beta\big(\varphi_p\big(D_q^\alpha u(t)\big)\big) = f\big(t, \phi(t)\big), \quad 0 < t < 1. \tag{3.13}$$

On the other hand, since ϕ is an upper solution to (1.1), we obviously have

$$D_q^\beta\big(\varphi_p\big(D_q^\alpha \phi(t)\big)\big) \leq f\big(t, \phi(t)\big), \quad 0 < t < 1. \tag{3.14}$$

Let $z(t) = \varphi_p(D_q^\alpha u(t)) - \varphi_p(D_q^\alpha \phi(t))$, $0 < t < 1$. From (3.13) and (3.14), we get

$$D_q^\beta z(t) = D_q^\beta\bigl(\varphi_p\bigl(D_q^\alpha u(t)\bigr)\bigr) - D_q^\beta\bigl(\varphi_p\bigl(D_q^\alpha \varphi(t)\bigr)\bigr) \geq f\bigl(t, \phi(t)\bigr) - f\bigl(t, \phi(t)\bigr) = 0,$$
$$z(0) = 0, \qquad z(1) = \varphi_p(b)z(\eta).$$

Thus, by Lemma 2.5, we have $z(t) \leq 0$, $t \in [0,1]$, which implies that

$$\varphi_p\bigl(D_q^\alpha u(t)\bigr) \leq \varphi_p\bigl(D_q^\alpha \varphi(t)\bigr), \quad t \in [0,1].$$

Since φ_p is monotone increasing, we obtain $D_q^\alpha u(t) \leq D_q^\alpha \phi(t)$, *i.e.*, $D_q^\alpha (u - \phi)(t) \leq 0$. Combining Lemma 2.5, we have $(u - \phi)(t) \geq 0$. Therefore, $u(t) \geq \phi(t)$, $t \in [0,1]$, a contradiction to the assumption that $u(t) < \phi(t)$. Hence, $u(t) < \phi(t)$ is impossible.

Consequently, we have $\phi(t) \leq u(t) \leq \psi(t)$, $t \in [0,1]$, that is, $u(t)$ is a positive solution of the boundary value problem (1.1). Furthermore, $\phi(t), \psi(t) \in P$ implies that there exist two positive constants $0 < \lambda_1 < 1 < \lambda_2$ such that $\lambda_1 e(t) \leq u(t) \leq \lambda_2 e(t)$, $t \in [0,1]$. Thus, we have finished the proof of Theorem 3.1. □

Theorem 3.2 *If $f(t,u) \in C([0,1] \times [0,+\infty), [0,+\infty))$ is decreasing in u and $f(t,\rho) \not\equiv 0$ for any $\rho > 0$, then the boundary value problem* (1.1) *has at least one positive solution u, and there exist two positive constants $0 < \lambda_1 < 1 < \lambda_2$ such that $\lambda_1 e(t) \leq u(t) \leq \lambda_2 e(t)$, $t \in [0,1]$.*

Proof The proof is similar to Theorem 3.1, we omit it here. □

4 Two examples

Example 4.1 Consider the p-Laplacian fractional q-difference boundary value problem

$$\begin{aligned} &D_{1/2}^{4/3}\bigl(\varphi_2\bigl(D_{1/2}^{3/2}u(t)\bigr)\bigr) = \frac{2(1+\sqrt[3]{t})}{\sqrt{tu(t)}}, \quad 0 < t < 1, \\ &u(0) = 0, \qquad u(1) = \frac{1}{2}u\left(\frac{1}{3}\right), \\ &D_{1/2}^{3/2}u(0) = 0, \qquad D_{1/2}^{3/2}u(1) = \frac{1}{2}D_{1/2}^{3/2}u\left(\frac{1}{2}\right). \end{aligned} \tag{4.1}$$

It is easy to check that (H_1) holds. For any $\rho > 0, f(t,\rho) \not\equiv 0$, we have

$$\begin{aligned} 0 &< \int_0^1 \sigma_2(qs)\varphi_2\left(\int_0^1 H(s,q\tau)f\bigl(\tau,\rho e(\tau)\bigr)\,d_q\tau\right)d_qs \\ &\leq \int_0^1 \sigma_2(qs)\varphi_2\left(\int_0^1 H(1,q\tau)f\bigl(\tau,\rho e(\tau)\bigr)\,d_q\tau\right)d_qs \\ &= \frac{1}{\sqrt{\rho}}\int_0^1 \sigma_2(qs)\,d_qs\int_0^1 H(1,q\tau)\frac{2(1+\sqrt[3]{\tau})}{\tau^{3/4}}\,d_q\tau < +\infty, \end{aligned}$$

which implies that (H_2) holds. Theorem 3.1 implies that the boundary value problem (4.1) has at least one positive solution.

Example 4.2 Consider the p-Laplacian fractional q-difference boundary value problem

$$\begin{aligned} &D_{1/2}^{4/3}\bigl(\varphi_p\bigl(D_{1/2}^{3/2}u(t)\bigr)\bigr) = t^2 + \frac{1}{\sqrt{u(t)+4}}, \quad 0 < t < 1, \\ &u(0) = 0, \qquad u(1) = \frac{1}{2}u\left(\frac{1}{3}\right), \\ &D_{1/2}^{3/2}u(0) = 0, \qquad D_{1/2}^{3/2}u(1) = \frac{1}{2}D_{1/2}^{3/2}u\left(\frac{1}{2}\right). \end{aligned} \tag{4.2}$$

It is not difficult to check that $f(t,u) : [0,1] \times [0,+\infty) \to [0,+\infty)$ is continuous and decreasing in u and $f(t,\rho) \not\equiv 0$ for any $\rho > 0$. Theorem 3.2 implies that the boundary value problem (4.2) has at least one positive solution.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors contributed to each part of this work equally and read and approved the final version of the manuscript.

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