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Robust stability of probabilistic delays fuzzy stochastic p-Laplace dynamic equations

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Abstract

In this paper, the stability of a class of time-delay Takagi-Sugeno (T-S) fuzzy Markovian jumping partial differential equations (PDEs) with p-Laplace and probabilistic time-varying delays is investigated, and the robust exponential stability criterion is obtained by way of some variational methods in Sobolev space $W^{1,p}(\Omega)$, the Lyapunov functional method and the linear matrix inequalities technique. Moreover, a numerical example shows the effectiveness of the proposed methods due to the large allowable variation range of time delay.

Keywords: probabilistic time-varying delays; Markovian jumping; Takagi-Sugeno fuzzy mathematical model

1 Introduction and preparation

Given a complete probability space $(\boldsymbol{\Omega}, \mathcal{F}, \mathbb{P})$ with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$, where $\boldsymbol{\Omega}$ is a sample space, $\mathcal{F}$ is σ-algebra of a subset of the sample space, and $\mathbb{P}$ is the probability measure defined on $\mathcal{F}$. Let $S = \{1, 2, \ldots, N\}$ and the random form process $\{r(t), t \in [0, +\infty)\}$ be a homogeneous, finite-state Markovian process with right continuous trajectories with generator $\Pi = (\pi_{ij})_{N\times N}$ and transition probability from mode i at time t to mode j at time $t + \Delta t$, $i, j \in S$, $\mathbb{P}(r(t + \delta) = j \mid r(t) = i) = \pi_{ij}\delta + o(\delta)$ if $j \neq i$, and $\mathbb{P}(r(t + \delta) = j \mid r(t) = i) = 1 + \pi_{ij}\delta + o(\delta)$ if $j = i$, where $\pi_{ij} \geq 0$ is transition probability rate from i to j ($j \neq i$) and $\pi_{ii} = -\sum_{j=1,j\neq i}^{s} \pi_{ij}$, $\delta > 0$ and $\lim_{\delta\to 0} o(\delta)/\delta = 0$.

Let us consider the following delayed Markovian jumping PDEs:

$$\begin{aligned} du(t,x) &= \bigl[\nabla \cdot \bigl(\mathcal{D}(t,x,u) \circ \nabla_p u(t,x)\bigr) - B\bigl(u(t,x)\bigr) + C\bigl(r(t),t\bigr) f\bigl(u(t,x)\bigr) + D\bigl(r(t),t\bigr) \\ &\quad \times g\bigl(u\bigl(t-\tau\bigl(r(t),t\bigr),x\bigr)\bigr)\bigr]\,dt + \sigma\bigl(u(t,x), u\bigl(t-\tau\bigl(r(t),t\bigr),x\bigr)\bigr)\,dw(t), \\ &\quad t \geq 0, x \in \Omega \end{aligned} \tag{1.1}$$

equipped with the initial condition $u(\theta, x) = \phi(\theta, x)$, $(\theta, x) \in [-\tau, 0] \times \Omega$ and zero-boundary condition

$$\mathfrak{B}\bigl[u_i(t,x)\bigr] = 0, \quad (t,x) \in [-\tau, +\infty) \times \partial\Omega, i = 1, 2, \ldots, n, \tag{1.1a}$$

where $w(t)$ is a standard one-dimensional Brownian motion defined on the probability space. $p > 1$ is a positive scalar, $\Omega \in R^m$ is a bounded domain with a smooth boundary

$\partial\Omega$ of class $\mathcal{C}^2$ by Ω, $u(t,x) = (u_1(t,x), u_2(t,x), \dots, u_n(t,x))^T \in R^n$. In what follows, $u(t,x)$ is always denoted by u for convenience sake. $\mathcal{D}(t,x,u) \circ \nabla_p u(t,x)$ denotes the Hadamard product of matrix $\mathcal{D}(t,x,u)$ and $\nabla_p u$ (see [1] or [2]), and $\mathcal{D}(t,x,v) = (\mathcal{D}_{jk}(t,x,u))_{n\times m}$ satisfies $\mathcal{D}_{jk}(t,x,u) \geq 0$ for all j, k, (t,x,u). In mode $r(t) = i \in S = \{1,2,\dots,N\}$, we denote $C_i(t) = C(r(t),t)$ and $D_i(t) = D(r(t),t)$. Denote by $\tau_i(t)$ the time delay $\tau(r(t),t)$ which satisfies $0 \leq \tau_i(t) \leq \tau$ for any mode $i \in S$. Functions $B(u) = (B_1(u_1), B_2(u_2), \dots, B_n(u_n))^T \in R^n$, $f(u) = (f_1(u_1), f_2(u_2), \dots, f_n(u_n))^T \in R^n$, $g(u) = (g_1(u_1), g_2(u_2), \dots, g_n(u_n))^T \in R^n$. The boundary condition (1.1a) is called Dirichlet boundary condition if $\mathfrak{B}[u_i(t,x)] = u_i(t,x)$ and Neumann boundary condition if $\mathfrak{B}[u_i(t,x)] = \frac{\partial u_i(t,x)}{\partial \nu}$. Here, $\frac{\partial u_i(t,x)}{\partial \nu} = (\frac{\partial u_i(t,x)}{\partial x_1}, \frac{\partial u_i(t,x)}{\partial x_2}, \dots, \frac{\partial u_i(t,x)}{\partial x_m})^T$ denotes the outward normal derivative on $\partial\Omega$.

For mode $i \in S$, PDEs (1.1) is simply denoted as

$$\begin{aligned} du = {} & \bigl[\nabla \cdot \bigl(\mathcal{D}(t,x,u) \circ \nabla_p u\bigr) - B(u) + C_i(t) f(u) + D_i(t) g\bigl(u\bigl(t - \tau_i(t), x\bigr)\bigr)\bigr]\, dt \\ & + \sigma\bigl(u, u\bigl(t - \tau_i(t), x\bigr)\bigr)\, dw(t), \quad t \geq 0, x \in \Omega. \end{aligned} \tag{1.2}$$

The T-S fuzzy mathematical model with time delay is described as follows.

Fuzzy rule *j*:

IF $\omega_1(t)$ is μ_{j1} and $\dots \omega_s(t)$ is μ_{js} **THEN**

$$\begin{aligned} du = {} & \bigl[\nabla \cdot \bigl(\mathcal{D}(t,x,u) \circ \nabla_p u\bigr) - B(u) + C_{ij}(t) f(u) + D_{ij}(t) g\bigl(u\bigl(t - \tau_i(t), x\bigr)\bigr)\bigr]\, dt \\ & + \sigma\bigl(u, u\bigl(t - \tau_i(t), x\bigr)\bigr)\, dw(t), \end{aligned} \tag{1.3}$$

where $\omega_k(t)$ $(k = 1,2,\dots,s)$ is the premise variable, μ_{jk} $(j = 1,2,\dots,r; k = 1,2,\dots,s)$ is the fuzzy set that is characterized by membership function, r is the number of the **IF-THEN** rules, and s is the number of the premise variables.

For any mode $r(t) = i \in S$, we assume that C_{ij}, D_{ij} are real constant matrices of appropriate dimensions, and ΔC_{ij}, ΔD_{ij} are real-valued matrix functions which stand for time-varying parameter uncertainties, satisfying

$$C_{ij}(t) = C_{ij} + \Delta C_{ij}(t), \qquad D_{ij}(t) = D_{ij} + \Delta D_{ij}(t). \tag{1.4}$$

By way of a standard fuzzy inference method, system (1.3) is inferred as follows:

$$\begin{aligned} du = {} & \Biggl\{ \nabla \cdot \bigl(\mathcal{D}(t,x,u) \circ \nabla_p u\bigr) - B(u) \\ & + \sum_{j=1}^{r} h_j\bigl(\omega(t)\bigr)\bigl[C_{ij}(t) f(u) + D_{ij}(t) g\bigl(u\bigl(t - \tau_i(t), x\bigr)\bigr)\bigr] \Biggr\}\, dt \\ & + \sigma\bigl(u, u\bigl(t - \tau_i(t), x\bigr)\bigr)\, dw(t), \end{aligned} \tag{1.5}$$

where $\omega(t) = [\omega_1(t), \omega_2(t), \dots, \omega_s(t)]$, $h_j(\omega(t)) = \frac{w_j(\omega(t))}{\sum_{k=1}^{r} w_k(\omega(t))}$, $w_j(\omega(t)) : R^s \to [0,1]$ $(j = 1,2,\dots,r)$ is the membership function of the system with respect to the fuzzy rule j. h_j can be regarded as the normalized weight of each **IF-THEN** rule, satisfying $h_j(\omega(t)) \geq 0$ and $\sum_{j=1}^{r} h_j(\omega(t)) = 1$.

Next, we consider the following information for probability distribution of time delays $\tau_i(t)$ for all $i \in S$:

$$\mathbb{P}\big(0 \le \tau_i(t) \le \tau_{1i}\big) = c_0, \qquad \mathbb{P}\big(\tau_{1i} < \tau_i(t) \le \tau_{2i}\big) = 1 - c_0.$$

Here the nonnegative scalar $c_0 \le 1$. Define a random variable as follows:

$$\mathscr{C}(t) = \begin{cases} 1, & 0 \le \tau_i(t) \le \tau_{1i}; \\ 0, & \tau_{1i} < \tau_i(t) \le \tau_{2i}. \end{cases}$$

So, in this paper, we consider the following delayed Takagi-Sugeno (T-S) fuzzy Markovian jumping p-Laplace partial differential equations (PDEs) with probabilistic time-varying delays:

$$\begin{aligned} du &= \nabla \cdot \big(\mathcal{D}(t,x,u) \circ \nabla_p u\big)\, dt \\ &\quad - A(u)\Bigg\{ B(u) - \sum_{j=1}^{r} h_j\big(\omega(t)\big)\big[C_{ij}(t)f(u) + c_0 D_{ij}(t) g\big(u\big(t-\tau_{1i}(t),x\big)\big) \\ &\quad + (1-c_0) D_{ij}(t) g\big(u\big(t-\tau_{2i}(t),x\big)\big) + \big(\mathscr{C}(t) - c_0\big)\big(D_{ij}(t) g\big(u\big(t-\tau_{1i}(t),x\big)\big) \\ &\quad - D_{ij}(t) g\big(u\big(t-\tau_{2i}(t),x\big)\big)\big)\big]\Bigg\}\, dt \\ &\quad + \sigma\big(u, u\big(t-\tau_{1i}(t),x\big), u\big(t-\tau_{2i}(t),x\big), i\big)\, dw(t), \end{aligned} \tag{1.6}$$

where $0 \le \tau_{1i}(t) \le \tau_{1i}$, $\tau_{1i} < \tau_{2i}(t) \le \tau_{2i}$, and $\mathscr{C}(t)$ is the Bernoulli distributed sequence, satisfying $\mathbb{P}(\mathscr{C}(t) = 1) = \mathbb{P}(0 \le \tau_{1i}(t) \le \tau_{1i}) = \mathbb{E}(\mathscr{C}(t)) = c_0$, and $\mathbb{P}(\mathscr{C}(t) = 0) = \mathbb{P}(\tau_{1i} < \tau_{2i}(t) \le \tau_{2i}) = 1 - \mathbb{E}(\mathscr{C}(t)) = 1 - c_0$. Here, $\mathbb{E}(\mathscr{C}(t))$ denotes the mathematical expectation of $\mathscr{C}(t)$. Note that the global existence of the solution of system (1.6) was investigated in [3]. To study the stability of (1.6), we need to assume

(A1) Let $A(u) = \mathrm{diag}(a_1(u_1(t,x)), a_2(u_2(t,x)), \ldots, a_n(u_n(t,x)))$, $\underline{A} = \mathrm{diag}(\underline{a}_1, \underline{a}_2, \ldots, \underline{a}_n)$, and $\overline{A} = \mathrm{diag}(\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_n)$ such that $0 < \underline{a}_j \le a_j(r) \le \overline{a}_j$, $j = 1, 2, \ldots, n$;

(A2) Let $B(u) = (B_1(u_1), B_2(u_2), \ldots, B_n(u_n))^T \in R^n$, there exists a positive definite diagonal matrix $\mathbb{B} = \mathrm{diag}(b_1, b_2, \ldots, b_n)$ such that $\frac{B_j(r)}{r} \ge b_j$, $\forall j = 1, 2, \ldots, n$, and $0 \ne r \in R$;

(A3) There exist constant diagonal matrices $G_k = \mathrm{diag}(G_1^{(k)}, G_2^{(k)}, \ldots, G_n^{(k)})$, $F_k = \mathrm{diag}(F_1^{(k)}, F_2^{(k)}, \ldots, F_n^{(k)})$, $k = 1, 2$ with $|F_j^{(1)}| \le F_j^{(2)}$, $|G_j^{(1)}| \le G_j^{(2)}$, $j = 1, 2, \ldots, n$, such that $F_j^{(1)} \le \frac{f_j(r)}{r} \le F_j^{(2)}$, $G_j^{(1)} \le \frac{g_j(r)}{r} \le G_j^{(2)}$, $\forall j = 1, 2, \ldots, n$, and $r \in R$.

(A4) There exist positive define symmetric matrices Γ_{1i}, Γ_{2i}, Γ_{3i} such that $\mathrm{Trace}[\sigma^T(t)\sigma(t)] \le u^T \Gamma_{1i} u + u^T(t-\tau_{1i},x)\Gamma_{2i} u(t-\tau_{1i},x) + u^T(t-\tau_{2i},x)\Gamma_{3i} u(t-\tau_{2i},x)$, $i \in S$.

(A5) $\dot{\tau}_{ki}(t) + \sum_{l \in S} \pi_{il}\tau_{kl}(t) \le a_0 < 1$ for any mode $i \in S$, and $k = 1, 2$.

In addition, one can assume that $u = 0$ is a trivial solution of PDEs (1.6) provided that $B(0) = f(0) = g(0) = 0$. For any mode $i \in S$, the parameter uncertainties considered here are norm-bounded and of the following forms:

$$\begin{pmatrix} \Delta C_{ij}(t) & \Delta D_{ij}(t) \end{pmatrix} = E_{ij}\mathfrak{F}(t)\begin{pmatrix} H_{ij} & M_{ij} \end{pmatrix}, \quad \forall i \in S.$$

Here $\mathfrak{F}(t)$ is an unknown matrix function satisfying $|\mathfrak{F}^T(t)||\mathfrak{F}(t)| \leq I$, and E_{ij}, G_{ij}, H_{ij} are known real constant matrices. Throughout this paper, for a matrix $C = (c_{ij})_{n\times n}$, we denote the matrix $|C| = (|c_{ij}|)_{n\times n}$. In addition, we denote by I the identity matrix with compatible dimension, and denote $\|u\|_2^2 = \int_\Omega u(t,x)\,dx$.

Lemma 1.1 *Let $\varepsilon > 0$ be any given scalar, and $\mathcal{M}$, $\mathfrak{E}$ and $\mathcal{K}$ be matrices with appropriate dimensions. If $\mathcal{K}^T\mathcal{K} \leq I$, then we have $\mathcal{M}\mathcal{K}\mathfrak{E} + \mathfrak{E}^T\mathcal{K}^T\mathcal{M}^T \leq \varepsilon^{-1}\mathcal{M}\mathcal{M}^T + \varepsilon\mathfrak{E}^T\mathfrak{E}$.*

Lemma 1.2 ([2, Lemma 6]) *Let $P = \operatorname{diag}(p_1, p_2, \dots, p_n)$ be a positive definite matrix, and v be a solution of system* (1.6) *with boundary condition* (1.1a). *Then we have*

$$\begin{aligned}\int_\Omega v^T P\big(\nabla\cdot\big(\mathcal{D}(t,x,v)\circ\nabla_p v\big)\big)\,dx &= -\sum_{k=1}^m\sum_{j=1}^n\int_\Omega p_j\mathcal{D}_{jk}(t,x,v)|\nabla v_j|^{p-2}\left(\frac{\partial v_j}{\partial x_k}\right)^2 dx\\ &= \int_\Omega\big(\nabla\cdot\big(\mathcal{D}(t,x,v)\circ\nabla_p v\big)\big)^T P v\,dx.\end{aligned}$$

2 Main result

Theorem 2.1 *Assume $p > 1$. PDEs* (1.6) *is global stochastic exponential robust stability in the mean square if there exist a positive scalar $\beta > 0$ and positive definite diagonal matrices P_i ($i \in S$), L_1, L_2 and Q_1, Q_2 such that for each $i \in S$, $j = 1, 2, \dots, r$, the following LMI conditions hold*:

$$\begin{pmatrix} \frac{1}{r}a_{i1} & 0 & 0 & \frac{1}{r}a_{i4} & \frac{1}{r}a_{i5} & \frac{1}{r}a_{i6} & P_i\overline{A}|E_{ij}| & 0\\ * & \frac{1}{r}a_{i2} & 0 & 0 & \frac{1}{r}(G_1+G_2)L_2 & 0 & 0 & 0\\ * & * & \frac{1}{r}a_{i3} & 0 & 0 & \frac{1}{r}(G_1+G_2)L_2 & 0 & 0\\ * & * & * & -\frac{2}{r}L_1 & 0 & 0 & 0 & |H_{ij}^T|\\ * & * & * & * & -\frac{2}{r}L_2 & 0 & 0 & c_0|M_{ij}^T|\\ * & * & * & * & * & -\frac{2}{r}L_2 & 0 & (1-c_0)|M_{ij}^T|\\ * & * & * & * & * & * & -I & 0\\ * & * & * & * & * & * & * & -I \end{pmatrix} < 0, \tag{2.1}$$

where $a_{i1} = -2P_i\underline{A}\mathbb{B} - 2F_1L_1F_2 + \beta P_i + \sum_{l\in S}\pi_{il}P_l + P_i\Gamma_{1i} + Q_1 + Q_2$; $a_{i2} = P_i\Gamma_{2i} - (1-a_0)e^{-\tau_{1i}\beta}Q_1 - 2G_1L_2G_2$; $a_{i3} = P_i\Gamma_{3i} - (1-a_0)e^{-\tau_{2i}\beta}Q_2 - 2G_1L_2G_2$; $a_{i4} = \sum_{j=1}^r P_i\overline{A}|C_{ij}| + (F_1 + F_2)L_1$; $a_{i5} = c_0\sum_{j=1}^r P_i\overline{A}|D_{ij}|$; $a_{i6} = (1-c_0)\sum_{j=1}^r P_i\overline{A}|D_{ij}|$.

Proof Consider the Lyapunov-Krasovskii functional $V(t,i) = V_{1i} + V_{2i}$, $\forall i \in S$, where $V_{1i} = e^{\beta t}\int_\Omega u^T(t,x)P_i u(t,x)\,dx$, and $V_{2i} = e^{\beta t}[\int_\Omega\int_{-\tau_{1i}(t)}^0 e^{\beta\theta}u^T(t+\theta,x)Q_1u(t+\theta,x)\,d\theta\,dx + \int_\Omega\int_{-\tau_{2i}(t)}^0 e^{\beta\theta}u^T(t+\theta,x)Q_2u(t+\theta,x)\,d\theta\,dx]$.

It follows immediately by Lemma 1.2 that $\int_\Omega u^T P_i(\nabla\cdot(\mathcal{D}(t,x,u)\circ\nabla_p u))\,dx \leq 0$.

Let $\mathscr{L}$ be the weak infinitesimal operator such that $\mathscr{L}V(t,u(t,x),i) = \mathscr{L}V_{1i} + \mathscr{L}V_{2i}$ for any given $i \in S$. Then we have

$$\begin{aligned}
\mathscr{L}V_{1i} \le{}& -e^{\beta t}\Biggl\{2\int_\Omega u^T P_i\underline{A}\mathbb{B}u\,dx - 2\int_\Omega \sum_{j=1}^r h_j\bigl(\omega(t)\bigr)\bigl[u^T P_i\overline{A}C_{ij}(t)f(u) \\
&+ c_0 u^T P_i\overline{A}D_{ij}(t)g\bigl(u\bigl(t-\tau_{1i}(t),x\bigr)\bigr) \\
&+ (1-c_0)u^T P_i\overline{A}D_{ij}(t)g\bigl(u\bigl(t-\tau_{2i}(t),x\bigr)\bigr)\bigr]\,dx + \int_\Omega u^T\Biggl(\sum_{l\in S}\pi_{il}P_l\Biggr)u\,dx\Biggr\} \\
&+ e^{\beta t}\int_\Omega \bigl[u^T P_i\Gamma_{1i}u + u^T\bigl(t-\tau_{1i}(t)\bigr)P_i\Gamma_{2i}u\bigl(t-\tau_{1i}(t)\bigr) \\
&+ u^T\bigl(t-\tau_{2i}(t)\bigr)P_i\Gamma_{3i}u\bigl(t-\tau_{2i}(t)\bigr)\bigr]\,dx + \beta e^{\beta t}\int_\Omega u^T P_i u\,dx, \\
\mathscr{L}V_{2i} \le{}& e^{\beta t}\biggl[\int_\Omega u^T(Q_1+Q_2)u\,dx \\
&- (1-a_0)e^{-\tau_{1i}\beta}\int_\Omega u^T\bigl(t-\tau_{1i}(t),x\bigr)Q_1u\bigl(t-\tau_{1i}(t),x\bigr)\,dx \\
&- (1-a_0)e^{-\tau_{2i}\beta}\int_\Omega u^T\bigl(t-\tau_{2i}(t),x\bigr)Q_2u\bigl(t-\tau_{2i}(t),x\bigr)\,dx\biggr].
\end{aligned}$$

From (A3), we have $2|f^T(u)|L_1|f(u)| - 2|u^T|(F_1+F_2)L_1|f(u)| + 2|u^T|F_1L_1F_2|u| \le 0$, $2|g^T(u(t-\tau_{1i}(t),x))|L_2|g(u(t-\tau_{1i}(t),x))| + 2|u^T(t-\tau_{1i}(t),x)|G_1L_2G_2|u(t-\tau_{1i}(t),x)| \le 2|u^T(t-\tau_{1i}(t),x)|(G_1+G_2)L_2|g(u(t-\tau_{1i}(t),x))|$, and $2|g^T(u(t-\tau_{2i}(t),x))|L_2|g(u(t-\tau_{2i}(t),x))| + 2|u^T(t-\tau_{2i}(t),x)|G_1L_2G_2|u(t-\tau_{2i}(t),x)| \le 2|u^T(t-\tau_{2i}(t),x)|(G_1+G_2)L_2|g(u(t-\tau_{2i}(t),x))|$.

Combining the above inequalities results in $\mathscr{L}V(t,i) \le e^{\beta t}\int_\Omega \zeta^T(t,x)\mathfrak{A}_i\zeta(t,x)\,dx$, where $\zeta(t,x) = (|u^T(t,x)|, |u^T(t-\tau_{1i}(t),x)|, |u^T(t-\tau_{2i}(t),x)|, |f^T(u(t,x))|, |g^T(u(t-\tau_{1i}(t),x))|, |g^T(u(t-\tau_{2i}(t),x))|)^T$,

$$\mathfrak{A}_i = \begin{pmatrix}
a_{i1} & 0 & 0 & \widetilde{a}_{i4} & \widetilde{a}_{i5} & \widetilde{a}_{i6} \\
 & a_{i2} & 0 & 0 & (G_1+G_2)L_2 & 0 \\
 & * & a_{i3} & 0 & 0 & (G_1+G_2)L_2 \\
 & * & * & -2L_1 & 0 & 0 \\
 & * & * & * & -2L_2 & 0 \\
 & * & * & * & * & -2L_2
\end{pmatrix},$$

and $\widetilde{a}_{i4} = \sum_{j=1}^r h_j(\omega(t))P_i\overline{A}|C_{ij}(t)| + (F_1+F_2)L_1$, $\widetilde{a}_{i5} = c_0\sum_{j=1}^r h_j(\omega(t))P_i\overline{A}|D_{ij}(t)|$, $\widetilde{a}_{i6} = (1-c_0)\sum_{j=1}^r h_j(\omega(t))P_i\overline{A}|D_{ij}(t)|$.

Further, we can apply the Schur complement [4] to (2.1), and derive $\mathfrak{A}_i < 0$ by Lemma 1.1. Hence, $\mathscr{L}V(t,i) \le 0$. Define $\mathcal{V}(t,i) = \int_\Omega u^T(t,x)P_iu(t,x)\,dx + \int_\Omega\int_{-\tau_i(t)}^0 e^{\beta\theta}u^T(t+\theta,x) \times Qu(t+\theta,x)\,d\theta\,dx$. From the Dynkin formula, we can derive that $e^{\beta t}\mathbb{E}\mathcal{V}(t) - \mathbb{E}\mathcal{V}(0) = \mathbb{E}\int_0^t \mathscr{L}(e^{\beta s}\mathcal{V}(s))\,ds \le 0$. Now, for any $\phi(\theta,x) \in L^2_{\mathcal{F}_0}([-\tau,0]\times\Omega;R^n)$ and any system mode $i \in S$, the solution $u(t,x,\phi,i_0)$ of system (1.6) with the initial value ϕ satisfies $\min_{i\in S}\{\underline{\alpha}_i\}e^{\beta t}\mathbb{E}(\|u(t,x,\phi,i_0)\|_2^2) \le (\max_{i\in S}\{\overline{\alpha}_i\} + \lambda_{\max}Q)\sup_{-\tau\le\theta\le 0}\mathbb{E}(\|\phi(\theta)\|_2^2)$, $\forall t \ge 0$, or $\mathbb{E}(\|u(t,x;\phi,i_0)\|_2^2) \le \gamma e^{-\beta t}\sup_{-\tau\le\theta\le 0}\mathbb{E}(\|\phi(\theta,x)\|_2^2)$, $\forall t \ge 0$, where positive scalars $\underline{\alpha}_i, \overline{\alpha}_i$ satisfy $\underline{\alpha}_i I \le P_i$ and $\overline{\alpha}I \ge P_i$ for any mode $i \in S$, scalars $\gamma = \frac{1}{\min_{i\in S}\{\underline{\alpha}_i\}}(\max_{i\in S}\{\overline{\alpha}_i\} + \lambda_{\max}Q) > 0$,

$\beta > 0$. Therefore, PDEs (1.6) is global stochastic exponential robust stability in the mean square. □

Remark 2.1 As pointed out in [1], diffusion effect exists really in the neural networks when electrons are moving in asymmetric electromagnetic fields [5]. Strictly speaking, reaction-diffusion terms should be considered in any neural networks model [6–8]. Usually, the diffusion behaviors were simulated by linear Laplace diffusion items [9–16]. But not all diffusion behaviors can be simply considered as the linear reaction-diffusion. Indeed, there are various works related to the nonlinear reaction-diffusion [17–21], and even the nonlinear p-Laplace diffusion [17, 20]. So, in this paper, the stability of p-Laplace PDEs was investigated.

Example 2.1 Consider PDEs (1.6) with the following parameters: $\underline{A} = I_2 = \overline{A}$, $B = 1.5I_2$, $F_1 = G_1 = 0$, $F_2 = G_2 = 0.1I_2$, $\beta = 0.01$, $r = 2$, $S = \{1,2\}$, $\pi_{11} = -0.8$, $\pi_{12} = 0.8$, $\pi_{21} = 0.5$, $\pi_{22} = -0.5$, $c_0 = 0.9$, $a_0 = 0.1$. Let $i = 1,2$; $j = 1,2$; $k = 1,2,3$, and $C_{ij} = D_{ij} = 0.1I_2$, $\tau_{2i} = 380$, $\tau_{1i} = 10$, $\Gamma_{ki} \equiv 0.01I_2$, $E_{ij} = H_{ij} = M_{ij} = 0.01I_2$. By using Matlab LMI toolbox, we solve LMI condition (2.1) and obtain $t_{\min} = -0.0149 < 0$, which implies feasible (see [2, Remark 29(3)] for details). Further, one can extract data as follows:

$$P_1 = \begin{pmatrix} 15.8743 & 0 \\ 0 & 15.8743 \end{pmatrix}, \qquad P_2 = \begin{pmatrix} 15.6796 & 0 \\ 0 & 15.6796 \end{pmatrix},$$
$$Q_1 = \begin{pmatrix} 4.9980 & 0 \\ 0 & 4.9980 \end{pmatrix}, \qquad Q_2 = \begin{pmatrix} 25.9273 & 0 \\ 0 & 25.9273 \end{pmatrix},$$
$$L_1 = \begin{pmatrix} 4.8023 & 0 \\ 0 & 4.8023 \end{pmatrix}, \qquad L_2 = \begin{pmatrix} 0.5294 & 0 \\ 0 & 0.5294 \end{pmatrix}.$$

Then Theorem 2.1 derives that PDEs (1.6) is global stochastic exponential robust stability in the mean square with a large allowable variation range of time delay $[0, 380]$.

Remark 2.2 To the best of our knowledge, it is the first attempt to investigate the robust stability of T-S fuzzy Markovian jumping Itô-type stochastic dynamic equations with p-Laplace and probabilistic time-varying delays (see [1, 2, 20, 22–25]). Example 2.1 shows the effectiveness of the proposed methods due to the large allowable variation range of time delay.

Remark 2.3 As pointed out in [26], almost all the above related literature did not point out the role that the nonlinear p-Laplace items play, except [1] and [20]. In fact, when $p = 2$, 2-Laplace is the linear Laplace, and there are many papers (see, *e.g.*, [10–13]) in which the Laplace diffusion item plays its role in their stability criteria, for the linear Laplace PDEs can be considered in the special Hilbert space $H^1(\Omega)$ that can be orthogonally decomposed into the direct sum of infinitely many eigenfunction spaces. However, the nonlinear p-Laplace ($p > 1$, $p \neq 2$) brings great difficulties for the nonlinear p-Laplace PDEs should be considered in the frame of the Sobolev space $W^{1,p}(\Omega)$ that is only a reflexive Banach space. Indeed, owing to the great difficulties, the authors only provide in [1] and [20] the stability criterion in which the nonlinear p-Laplace items play roles in the case of $1 < p < 2$ and $p > 1$ under the Dirichlet boundary condition. So, a further profound study is very interesting, which may call for some new mathematical methods, and even new mathematical theories. Under the Neumann boundary condition, the problem of the role of the nonlinear p-Laplace ($p > 1$) item in the stability criteria for fuzzy stochastic p-Laplace PDEs with probabilistic delays still remains open and challenging.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
XW, the first author of this manuscript, carried out the main part of this manuscript. RR is the corresponding author of this manuscript. All authors typed, read and approved the final manuscript.

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