Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH Open Access

Fixed point theory in partial metric spaces via φ-fixed point's concept in metric spaces

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Abstract
Let X be a non-empty set. We say that an element $x \in X$ is a φ-fixed point of T, where $\varphi : X \to [0,\infty)$ and $T : X \to X$, if x is a fixed point of T and $\varphi(x) = 0$. In this paper, we establish some existence results of φ-fixed points for various classes of operators in the case, where X is endowed with a metric d. The obtained results are used to deduce some fixed point theorems in the case where X is endowed with a partial metric p.

MSC: 54H25; 47H10

Keywords: φ-fixed point; fixed point; metric space; partial metric space

1 Introduction and preliminaries

In 1994, Matthews [1] introduced the concept of partial metric spaces as a part of the study of denotational semantics of dataflow networks and showed that the Banach contraction principle can be generalized to the partial metric context for applications in program verification.

We start by recalling some basic definitions and properties of partial metric spaces (see [1, 2] for more details).

A partial metric on a non-empty set X is a function $p : X \to X \to [0,\infty)$ such that for all $x, y, z \in X$, we have

(P1) $p(x,x) = p(y,y) = p(x,y) \Leftrightarrow x = y$;

(P2) $p(x,x) \leq p(x,y)$;

(P3) $p(x,y) = p(y,x)$;

(P4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$.

A partial metric space is a pair (X,p) such that X is a non-empty set and p is a partial metric on X. It is clear that, if $p(x,y) = 0$, then from (P1) and (P2), $x = y$; but if $x = y$, $p(x,y)$ may not be 0. A basic example of a partial metric space is the pair $([0,\infty), p)$, where $p(x,y) = \max\{x,y\}$. Other examples of partial metric spaces which are interesting from a computational point of view may be found in [1].

Each partial metric p on X generates a T_0 topology τ_p on X which has as a base the family of open p-balls $\{B_p(x,\varepsilon) : x \in X, \varepsilon > 0\}$, where

$$B_p(x,\varepsilon) := \big\{y \in X : p(x,y) < p(x,x) + \varepsilon\big\}.$$

Let (X,p) be a partial metric space. A sequence $\{x_n\} \subset X$ converges to some $x \in X$ with respect to p if and only if

$$\lim_{n\to\infty} p(x_n,x) = p(x,x).$$

A sequence $\{x_n\} \subset X$ is said to be a Cauchy sequence if and only if $\lim_{m,n\to\infty} p(x_n,x_m)$ exists and is finite. The partial metric space (X,p) is said to be complete if and only if every Cauchy sequence $\{x_n\}$ in X converges to some $x \in X$ such that $\lim_{n,m\to\infty} p(x_n,x_m) = p(x,x)$.

If p is a partial metric on X, then the function $d_p : X \to X \to [0,\infty)$ defined by

$$d_p(x,y) = 2p(x,y) - p(x,x) - p(y,y), \quad (x,y) \in X^2 \tag{1}$$

is a metric on X.

Lemma 1.1 *Let (X,p) be a partial metric space. Then*
- (i) *$\{x_n\}$ is a Cauchy sequence in (X,p) if and only if $\{x_n\}$ is a Cauchy sequence in the metric space (X,d_p);*
- (ii) *the partial metric space (X,p) is complete if and only if the metric space (X,d_p) is complete.*

Recently, many works on fixed point theory in the partial metric context have been published. For more details, we refer to [2–22]. On the other hand, Haghi *et al.* [10] observed that some fixed point theorems for certain classes of operators can be deuced easily from the same theorems in metric spaces. The idea presented in [10] is interesting, however it cannot be applied for a large class of operators, as, for example, in the case of an implicit contraction.

In [23], Rus presented three interesting open problems in the context of a complete partial metric space (X,p).

Problem 1. If $T : (X,p) \to (X,p)$ is an operator satisfying a certain contractive condition with respect to p, which condition satisfies T with respect to the metric d_p defined by (1)?

Problem 2. The problem is to give fixed point theorems for these new classes of operators on a metric space.

Problem 3. Use the results for the above problems to give fixed point theorems in a partial metric space.

In [18], Samet answered to the above problems by considering Boyd-Wong contraction mappings. Other types of contractions were considered in [11, 19].

In this paper, we introduce the concept of a φ-fixed point, and we establish some φ-fixed point results for various classes of operators defined on a metric space (X,d). The obtained results are then used to obtain some fixed point theorems, in the case where X is endowed with a partial metric p.

2 φ-Fixed point results

Let (X,d) be a metric space, $\varphi : X \to [0,\infty)$ be a given function, and $T : X \to X$ be an operator.

We denote by

$$T^0 := 1_X, \qquad T^1 := T, \qquad T^{n+1} := T \circ T^n, \quad n \in \mathbb{N}$$

the iterate operators of T. The set of all fixed points of the operator T will be denoted by

$$F_T := \{x \in X : Tx = x\}.$$

The set of all zeros of the function φ will be denoted by

$$Z_\varphi := \{x \in X : \varphi(x) = 0\}.$$

We introduce the notion of φ-fixed point as follows.

Definition 2.1 An element $z \in X$ is said to be a φ-fixed point of the operator T if and only if $z \in F_T \cap Z_\varphi$.

Definition 2.2 We say that the operator T is a φ-Picard operator if and only if
(i) $F_T \cap Z_\varphi = \{z\}$;
(ii) $T^n x \to z$ as $n \to \infty$, for each $x \in X$.

Definition 2.3 We say that the operator T is a weakly φ-Picard operator if and only if
(i) $F_T \cap Z_\varphi \neq \emptyset$;
(ii) the sequence $\{T^n x\}$ converges for each $x \in X$, and the limit is a φ-fixed point of T.

We denote by $\mathcal{F}$ the set of functions $F : [0,\infty)^3 \to [0,\infty)$ satisfying the following conditions:
(F1) $\max\{a,b\} \le F(a,b,c)$, for all $a,b,c \in [0,\infty)$;
(F2) $F(0,0,0) = 0$;
(F3) F is continuous.
As examples, the following functions belong to $\mathcal{F}$:
(i) $F(a,b,c) = a + b + c$;
(ii) $F(a,b,c) = \max\{a,b\} + c$;
(iii) $F(a,b,c) = a + a^2 + b + c$.
In this section, we study the existence and uniqueness of φ-fixed points for various classes of operators.

2.1 (F,φ)-Contraction mappings

Definition 2.4 Let (X,d) be a metric space, $\varphi : X \to [0,\infty)$ be a given function, and $F \in \mathcal{F}$. We say that the operator $T : X \to X$ is an (F,φ)-contraction with respect to the metric d if and only if

$$F\big(d(Tx,Ty),\varphi(Tx),\varphi(Ty)\big) \le kF\big(d(x,y),\varphi(x),\varphi(y)\big), \quad (x,y) \in X^2, \tag{2}$$

for some constant $k \in (0,1)$.

Our first main result is the following.

Theorem 2.1 *Let (X,d) be a complete metric space, $\varphi : X \to [0,\infty)$ be a given function, and $F \in \mathcal{F}$. Suppose that the following conditions hold*:
(H1) *φ is lower semi-continuous*;
(H2) *$T : X \to X$ is an (F,φ)-contraction with respect to the metric d.*

Then

(i) $F_T \subseteq Z_\varphi$;

(ii) *T is a φ-Picard operator*;

(iii) *for all $x \in X$, for all $n \in \mathbb{N}$, we have*

$$d(T^n x, z) \leq \frac{k^n}{1-k} F\big(d(Tx, x), \varphi(Tx), \varphi(x)\big),$$

where $\{z\} = F_T \cap Z_\varphi = F_T$.

Proof Suppose that $\xi \in X$ is a fixed point of T. Applying (2) with $x = y = \xi$, we obtain

$$F\big(0, \varphi(\xi), \varphi(\xi)\big) \leq kF\big(0, \varphi(\xi), \varphi(\xi)\big),$$

which implies (since $k \in (0, 1)$) that

$$F\big(0, \varphi(\xi), \varphi(\xi)\big) = 0. \tag{3}$$

On the other hand, from (F1), we have

$$\varphi(\xi) \leq F\big(0, \varphi(\xi), \varphi(\xi)\big). \tag{4}$$

Using (3) and (4), we obtain $\varphi(\xi) = 0$, which proves (i).

Let $x \in X$ be an arbitrary point. Using (2), we have

$$\begin{aligned} &F\big(d(T^{n+1}x, T^n x), \varphi(T^{n+1}x), \varphi(T^n x)\big) \\ &\quad \leq kF\big(d(T^n x, T^{n-1}x), \varphi(T^n x), \varphi(T^{n-1}x)\big), \quad n \in \mathbb{N}. \end{aligned}$$

By induction, we obtain easily

$$F\big(d(T^{n+1}x, T^n x), \varphi(T^{n+1}x), \varphi(T^n x)\big) \leq k^n F\big(d(Tx, x), \varphi(Tx), \varphi(x)\big), \quad n \in \mathbb{N} \cup \{0\},$$

which implies by property (F1) that

$$\max\big\{d(T^{n+1}x, T^n x), \varphi(T^{n+1}x)\big\} \leq k^n F\big(d(Tx, x), \varphi(Tx), \varphi(x)\big), \quad n \in \mathbb{N} \cup \{0\}. \tag{5}$$

From (5), we have

$$d(T^{n+1}x, T^n x) \leq k^n F\big(d(Tx, x), \varphi(Tx), \varphi(x)\big), \quad n \in \mathbb{N} \cup \{0\},$$

which implies (since $k \in (0, 1)$) that $\{T^n x\}$ is a Cauchy sequence. Since (X, d) is complete, there is some $z \in X$ such that

$$\lim_{n \to \infty} d(T^n x, z) = 0. \tag{6}$$

Now, we shall prove that z is a φ-fixed point of T. Observe that from (5), we have

$$\lim_{n \to \infty} \varphi(T^{n+1}x) = 0. \tag{7}$$

Since φ is lower semi-continuous, from (6) and (7), we obtain

$$\varphi(z) = 0. \tag{8}$$

Using (2), we have

$$F\big(d\big(T^{n+1}x, Tz\big), \varphi\big(T^{n+1}x\big), \varphi(Tz)\big) \leq kF\big(d\big(T^n x, z\big), \varphi\big(T^n x\big), \varphi(z)\big), \quad n \in \mathbb{N} \cup \{0\}.$$

Letting $n \to \infty$ in the above inequality, using (6), (7), (8), (F2), and the continuity of F, we get

$$F\big(d(z, Tz), 0, \varphi(Tz)\big) \leq kF(0,0,0) = 0,$$

which implies from condition (F1) that

$$d(z, Tz) = 0. \tag{9}$$

It follows from (8) and (9) that z is a φ-fixed point of T.

Suppose now that $z' \in X$ is another φ-fixed point of T. Applying (2) with $x = z$ and $y = z'$, we obtain

$$F\big(d\big(z,z'\big), 0, 0\big) \leq kF\big(d\big(z,z'\big), 0, 0\big),$$

which implies that $d(z,z') = 0$, that is, $z = z'$. So we get (ii).

Finally, using (5) and the triangle inequality, we get

$$d\big(T^n x, T^{n+m}x\big) \leq \frac{k^n(1-k^m)}{1-k} F\big(d(Tx,x), \varphi(Tx), \varphi(x)\big), \quad n,m \in \mathbb{N} \cup \{0\}.$$

Letting $m \to \infty$ in the above inequality, from (6), we obtain

$$d\big(T^n x, z\big) \leq \frac{k^n}{1-k} F\big(d(Tx,x), \varphi(Tx), \varphi(x)\big), \quad n \in \mathbb{N} \cup \{0\},$$

which proves (iii). □

2.2 Graphic (F,φ)-contraction mappings

Definition 2.5 Let (X,d) be a metric space, $\varphi : X \to [0,\infty)$ be a given function, and $F \in \mathcal{F}$. We say that the operator $T : X \to X$ is a graphic (F,φ)-contraction with respect to the metric d if and only if

$$F\big(d\big(T^2x, Tx\big), \varphi\big(T^2x\big), \varphi(Tx)\big) \leq kF\big(d(Tx,x), \varphi(Tx), \varphi(x)\big), \quad x \in X, \tag{10}$$

for some constant $k \in (0,1)$.

Theorem 2.2 *Let (X,d) be a complete metric space, $\varphi : X \to [0,\infty)$ be a given function, and $F \in \mathcal{F}$. Suppose that the following conditions hold*:

(H1) *φ is lower semi-continuous*;

(H2) $T : X \to X$ *is a graphic* (F, φ)*-contraction with respect to the metric* d;
(H3) T *is continuous.*

Then

(i) $F_T \subseteq Z_\varphi$;
(ii) T *is a weakly* φ*-Picard operator*;
(iii) *for all* $x \in X$, *if* $T^n x \to z$ *as* $n \to \infty$, *then*

$$d(T^n x, z) \le \frac{k^n}{1-k} F\big(d(Tx, x), \varphi(Tx), \varphi(x)\big), \quad n \in \mathbb{N}.$$

Proof Suppose that $\xi \in X$ is a fixed point of T. Applying (10) with $x = \xi$, we obtain

$$F\big(0, \varphi(\xi), \varphi(\xi)\big) \le kF\big(0, \varphi(\xi), \varphi(\xi)\big),$$

which implies (since $k \in (0,1)$) that

$$F\big(0, \varphi(\xi), \varphi(\xi)\big) = 0. \tag{11}$$

On the other hand, from (F1), we have

$$\varphi(\xi) \le F\big(0, \varphi(\xi), \varphi(\xi)\big). \tag{12}$$

Using (11) and (12), we obtain $\varphi(\xi) = 0$, which proves (i).

Let $x \in X$ be an arbitrary point. Using (10), we have

$$\begin{aligned} &F\big(d(T^{n+1}x, T^n x), \varphi(T^{n+1}x), \varphi(T^n x)\big) \\ &\quad \le kF\big(d(T^n x, T^{n-1}x), \varphi(T^n x), \varphi(T^{n-1}x)\big), \quad n \in \mathbb{N}. \end{aligned}$$

By induction, we obtain easily

$$F\big(d(T^{n+1}x, T^n x), \varphi(T^{n+1}x), \varphi(T^n x)\big) \le k^n F\big(d(Tx, x), \varphi(Tx), \varphi(x)\big), \quad n \in \mathbb{N} \cup \{0\},$$

which implies by property (F1) that

$$\max\big\{d(T^{n+1}x, T^n x), \varphi(T^{n+1}x)\big\} \le k^n F\big(d(Tx, x), \varphi(Tx), \varphi(x)\big), \quad n \in \mathbb{N} \cup \{0\}. \tag{13}$$

From (13), we have

$$d(T^{n+1}x, T^n x) \le k^n F\big(d(Tx, x), \varphi(Tx), \varphi(x)\big), \quad n \in \mathbb{N} \cup \{0\},$$

which implies that $\{T^n x\}$ is a Cauchy sequence. Since (X, d) is complete, there is some $z \in X$ such that

$$\lim_{n\to\infty} d(T^n x, z) = 0. \tag{14}$$

Now, we shall prove that z is a φ-fixed point of T. Observe that from (13), we have

$$\lim_{n\to\infty} \varphi(T^{n+1}x) = 0. \tag{15}$$

Since φ is lower semi-continuous, from (14) and (15), we obtain $\varphi(z) = 0$. On the other hand, using the continuity of T and (14), we get $z = Tz$. Then z is a φ-fixed point of T. So T is a weakly φ-Picard operator.

Finally, the proof of (iii) follows by using similar arguments as in the proof of (iii), Theorem 2.1. □

2.3 (F, φ)-Weak contraction mappings

Definition 2.6 Let (X, d) be a metric space, $\varphi : X \to [0, \infty)$ be a given function, and $F \in \mathcal{F}$. We say that the operator $T : X \to X$ is an (F, φ)-weak contraction with respect to the metric d if and only if

$$\begin{aligned} F\big(d(Tx, Ty), \varphi(Tx), \varphi(Ty)\big) \le{}& kF\big(d(x, y), \varphi(x), \varphi(y)\big) \\ &+ L\big(F\big(d(y, Tx), \varphi(y), \varphi(Tx)\big) - F\big(0, \varphi(y), \varphi(Tx)\big)\big), \end{aligned} \tag{16}$$

for all $(x, y) \in X^2$, for some constants $k \in (0, 1)$ and $L \ge 0$.

For this class of operators, we have the following result.

Theorem 2.3 *Let (X, d) be a complete metric space, $\varphi : X \to [0, \infty)$ be a given function, and $F \in \mathcal{F}$. Suppose that the following conditions hold*:

(H1) *φ is lower semi-continuous*;

(H2) *$T : X \to X$ is an (F, φ)-weak contraction with respect to the metric d.*

Then

(i) $F_T \subseteq Z_\varphi$;

(ii) *T is a weakly φ-Picard operator*;

(iii) *for all $x \in X$, if $T^n x \to z$ as $n \to \infty$, then*

$$d\big(T^n x, z\big) \le \frac{k^n}{1 - k} F\big(d(x, Tx), \varphi(x), \varphi(Tx)\big), \quad n \in \mathbb{N}.$$

Proof Let $\xi \in X$ be a fixed point of T. Applying (16) with $x = y = \xi$, we get

$$\begin{aligned} F\big(0, \varphi(\xi), \varphi(\xi)\big) &\le kF\big(0, \varphi(\xi), \varphi(\xi)\big) \\ &\quad + L\big(F\big(0, \varphi(\xi), \varphi(\xi)\big) - F\big(0, \varphi(\xi), \varphi(\xi)\big)\big) \\ &= kF\big(0, \varphi(\xi), \varphi(\xi)\big), \end{aligned}$$

which implies that $F(0, \varphi(\xi), \varphi(\xi)) = 0$. Using property (F1), we obtain $\varphi(\xi) = 0$, that is, $\xi \in Z_\varphi$. Then (i) is proved.

Let $x \in X$ be an arbitrary point. Applying (16), we obtain

$$\begin{aligned} &F\big(d\big(T^n x, T^{n+1} x\big), \varphi\big(T^n x\big), \varphi\big(T^{n+1} x\big)\big) \\ &\quad \le kF\big(d\big(T^{n-1} x, T^n x\big), \varphi\big(T^{n-1} x\big), \varphi\big(T^n x\big)\big) \\ &\qquad + L\big(F\big(0, \varphi\big(T^n x\big), \varphi\big(T^n x\big)\big) - F\big(0, \varphi\big(T^n x\big), \varphi\big(T^n x\big)\big)\big) \\ &\quad = kF\big(d\big(T^{n-1} x, T^n x\big), \varphi\big(T^{n-1} x\big), \varphi\big(T^n x\big)\big), \quad n \in \mathbb{N}. \end{aligned}$$

By induction, we get

$$F\big(d\big(T^nx, T^{n+1}x\big), \varphi\big(T^nx\big), \varphi\big(T^{n+1}x\big)\big) \le k^n F\big(d(x, Tx), \varphi(x), \varphi(Tx)\big), \quad n \ge 0.$$

The rest of the proof follows using similar arguments to the proof of Theorem 2.2. □

3 Links with partial metric spaces

From the previous obtained results in metric spaces, we deduce in this section some fixed point theorems in partial metric spaces; see also [21].

We start by the Matthews fixed point theorem [1].

Corollary 3.1 *Let (X, p) be a complete partial metric space and let $T : X \to X$ be a mapping such that*

$$p(Tx, Ty) \le kp(x, y), \quad (x, y) \in X^2,$$

for some constant $k \in (0, 1)$. Then T has a unique fixed point $z \in X$. Moreover, we have $p(z, z) = 0$.

Proof Consider the metric d_p on X defined by (1) and the function $\varphi : X \to [0, \infty)$ defined by $\varphi(x) = p(x, x)$. Applying Theorem 2.1 with $F(a, b, c) = a + b + c$, and using Lemma 1.1, we obtain the desired result. □

Similarly, from Theorem 2.2, we obtain the following result.

Corollary 3.2 *Let (X, p) be a complete partial metric space and let $T : X \to X$ be a mapping such that*

$$p\big(T^2x, Tx\big) \le kp(Tx, x), \quad x \in X,$$

for some constant $k \in (0, 1)$. Then T has a fixed point $z \in X$. Moreover, we have $p(z, z) = 0$.

Finally, from Theorem 2.3, we obtain the following result.

Corollary 3.3 *Let (X, p) be a complete partial metric space and let $T : X \to X$ be a mapping such that*

$$p(Tx, Ty) \le kp(x, y) + L\left(p(y, Tx) - \frac{p(y, y) + p(Tx, Tx)}{2}\right), \quad (x, y) \in X^2,$$

for some constants $k \in (0, 1)$ and $L \ge 0$. Then T has a fixed point $z \in X$. Moreover, we have $p(z, z) = 0$.

Observe that if p is a metric on X, we obtain from Corollary 3.3 the Berinde fixed point theorem for (k, L)-weak contractions [24].

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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Acknowledgements
The authors would like to extend their sincere appreciation to the Deanship of Scientific Research at King Saud University for its funding of this research through the International Research Group Project No. IRG14-04.

Received: 29 May 2014 Accepted: 14 October 2014 Published: 30 Oct 2014

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10.1186/1029-242X-2014-426
Cite this article as: Jleli et al.: **Fixed point theory in partial metric spaces via φ-fixed point's concept in metric spaces.** *Journal of Inequalities and Applications* 2014, 2014:426