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Double sequence spaces over n-normed spaces defined by a sequence of Orlicz functions

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Abstract

In the present paper we introduce double sequence space $m^2(\mathcal{M}, A, \phi, p, \|\cdot, \ldots, \cdot\|)$ defined by a sequence of Orlicz functions over n-normed space. We examine some of its topological properties and establish some inclusion relations.

MSC: 40A05; 46A45

Keywords: double sequence spaces; paranormed space; Orlicz function; n-normed space

1 Introduction and preliminaries

The initial works on double sequences is found in Bromwich [1]. Later on, it was studied by Hardy [2], Moricz [3], Moricz and Rhoades [4], Başarır and Sonalcan [5] and many others. Hardy [2] introduced the notion of regular convergence for double sequences. Quite recently, Zeltser [6] in her PhD thesis has essentially studied both the theory of topological double sequence spaces and the theory of summability of double sequences. Mursaleen and Edely [7] have recently introduced the statistical convergence which was further studied in locally solid Riesz spaces [8]. Nextly, Mursaleen [9] and Mursaleen and Savas [10] have defined the almost regularity and almost strong regularity of matrices for double sequences and applied these matrices to establish core theorems and introduced the M-core for double sequences and determined those four dimensional matrices transforming every bounded double sequences $x = (x_{k,l})$ into one whose core is a subset of the M-core of x. More recently, Altay and Başar [11] have defined the spaces $\mathcal{BS}$, $\mathcal{BS}(t)$, $\mathcal{CS}_p$, $\mathcal{CS}_{bp}$, $\mathcal{CS}_r$ and $\mathcal{BV}$ of double sequences consisting of all double series whose sequence of partial sums are in the spaces $\mathcal{M}_u$, $\mathcal{M}_u(t)$, $\mathcal{C}_p$, $\mathcal{C}_{bp}$, $\mathcal{C}_r$ and $\mathcal{L}_u$, respectively and also examined some properties of these sequence spaces and determined the α-duals of the spaces $\mathcal{BS}$, $\mathcal{BV}$, $\mathcal{CS}_{bp}$ and the $\beta(v)$-duals of the spaces $\mathcal{CS}_{bp}$ and $\mathcal{CS}_r$ of double series. Recently Başar and Sever [12] have introduced the Banach space $\mathcal{L}_q$ of double sequences corresponding to the well known space ℓ_q of single sequences and examined some properties of the space $\mathcal{L}_q$. Now, recently Raj and Sharma [13] have introduced entire double sequence spaces. By the convergence of a double sequence we mean the convergence in the Pringsheim sense *i.e.* a double sequence $x = (x_{k,l})$ has Pringsheim limit L (denoted by P-$\lim x = L$) provided that given $\epsilon > 0$ there exists $n \in N$ such that $|x_{k,l} - L| < \epsilon$ whenever $k, l > n$, see [14]. The double sequence $x = (x_{k,l})$ is bounded if there exists a positive number M such that $|x_{k,l}| < M$ for all k and l.

Throughout this paper, $\mathbb{N}$ and $\mathbb{C}$ denote the set of positive integers and complex numbers, respectively. A complex double sequence is a function x from $N \times \mathbb{N}$ into $\mathbb{C}$ and briefly denoted by $\{x_{k,l}\}$. If for all $\epsilon > 0$, there is $n_\epsilon \in \mathbb{N}$ such that $|x_{k,l} - a| < \epsilon$ where $k > n_\epsilon$ and $l > n_\epsilon$, then a double sequence $\{x_{k,l}\}$ is said to be convergent to $a \in \mathbb{C}$. A real double sequence $\{x_{k,l}\}$ is non-decreasing, if $x_{k,l} \leq x_{p,q}$ for $(k,l) < (p,q)$. A double series is infinite sum $\sum_{k,l=1}^{\infty} x_{k,l}$ and its convergence implies the convergence of partial sums sequence $\{S_{n,m}\}$, where $S_{n,m} = \sum_{k=1}^{m} \sum_{l=1}^{n} x_{k,l}$ (see [15]). For recent development on double sequences, we refer to [16–20] and [21–23].

A double sequence space E is said to be solid if $\{x_{k,l}y_{k,l}\} \in E$ for all double sequences $\{y_{k,l}\}$ of scalars such that $|y_{k,l}| < 1$ for all $k, l \in N$ whenever $\{x_{k,l}\} \in E$.

Let $x = \{x_{k,l}\}$ be a double sequence. A set $S(x)$ is defined by

$$S(x) = \big\{\{X_{\pi_1(k),\pi_2(k)}\} : \pi_1 \text{ and } \pi_2 \text{ are permutation of } \mathbb{N}\big\}.$$

If $S(x) \subseteq E$ for all $x \in E$, then E is said to be symmetric. Now let $\mathcal{P}_s$ be a family of subsets σ having at most elements s in $\mathbb{N}$. Also $\mathcal{P}_{s,t}$ denotes the class of subsets $\sigma = \sigma_1 \times \sigma_2$ in $\mathbb{N} \times \mathbb{N}$ such that the element numbers of σ_1 and σ_2 are at most s and t, respectively. Besides $\{\phi_{k,l}\}$ is taken as a non-decreasing double sequence of the positive real numbers such that

$$k\phi_{k+1,l} \leq (k+1)\phi_{k,l},$$
$$l\phi_{k,l+1} \leq (l+1)\phi_{k,l}.$$

An Orlicz function $M : [0,\infty) \to [0,\infty)$ is a continuous, non-decreasing, and convex function such that $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$.

Lindenstrauss and Tzafriri [24] used the idea of Orlicz function to define the following sequence space:

$$\ell_M = \left\{x \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty\right\},$$

which is called an Orlicz sequence space. Also ℓ_M is a Banach space with the norm

$$\|x\| = \inf\left\{\rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1\right\}.$$

Also, it was shown that every Orlicz sequence space ℓ_M contains a subspace isomorphic to ℓ_p $(p \geq 1)$. The Δ_2-condition is equivalent to $M(Lx) \leq LM(x)$, for all L with $0 < L < 1$. An Orlicz function M can always be represented in the following integral form:

$$M(x) = \int_0^x \eta(t)\,dt,$$

where η, known as the kernel of M, is right differentiable for $t \geq 0$, $\eta(0) = 0$, $\eta(t) > 0$, η is non-decreasing and $\eta(t) \to \infty$ as $t \to \infty$.

For further reading on Orlicz spaces, we refer to [25–29].

Let X be a linear metric space. A function $p : X \to \mathbb{R}$ is called a paranorm if

(1) $p(x) \geq 0$ for all $x \in X$,
(2) $p(-x) = p(x)$ for all $x \in X$,
(3) $p(x+y) \leq p(x) + p(y)$ for all $x, y \in X$,
(4) if (λ_n) is a sequence of scalars with $\lambda_n \to \lambda$ as $n \to \infty$ and (x_n) is a sequence of vectors with $p(x_n - x) \to 0$ as $n \to \infty$, then $p(\lambda_n x_n - \lambda x) \to 0$ as $n \to \infty$.

A paranorm p for which $p(x) = 0$ implies $x = 0$ is called a total paranorm and the pair (X, p) is called a total paranormed space. It is well known that the metric of any linear metric space is given by some total paranorm (see [30], Theorem 10.4.2, p.183).

The concept of 2-normed spaces was initially developed by Gähler [31] in the mid-1960s, while that of n-normed spaces one can see in Misiak [32]. Since then, many others have studied this concept and obtained various results; see Gunawan [33, 34] and Gunawan and Mashadi [35] and references therein. Let $n \in \mathbb{N}$ and X be a linear space over the field $\mathbb{K}$, where $\mathbb{K}$ is the field of real or complex numbers of dimension d, where $d \geq n \geq 2$. A real valued function $\|\cdot, \dots, \cdot\|$ on X^n satisfying the following four conditions:

(1) $\|x_1, x_2, \dots, x_n\| = 0$ if and only if $x_1, x_2, \dots, x_n$ are linearly dependent in X;
(2) $\|x_1, x_2, \dots, x_n\|$ is invariant under permutation;
(3) $\|\alpha x_1, x_2, \dots, x_n\| = |\alpha| \|x_1, x_2, \dots, x_n\|$ for any $\alpha \in \mathbb{K}$, and
(4) $\|x + x', x_2, \dots, x_n\| \leq \|x, x_2, \dots, x_n\| + \|x', x_2, \dots, x_n\|$

is called a n-norm on X, and the pair $(X, \|\cdot, \dots, \cdot\|)$ is called a n-normed space over the field $\mathbb{K}$. For example, we may take $X = \mathbb{R}^n$ being equipped with the Euclidean n-norm $\|x_1, x_2, \dots, x_n\|_E$, the volume of the n-dimensional parallelepiped spanned by the vectors $x_1, x_2, \dots, x_n$ which may be given explicitly by the formula

$$\|x_1, x_2, \dots, x_n\|_E = \left|\det(x_{ij})\right|,$$

where $x_i = (x_{i1}, x_{i2}, \dots, x_{in}) \in \mathbb{R}^n$ for each $i = 1, 2, \dots, n$. Let $(X, \|\cdot, \dots, \cdot\|)$ be a n-normed space of dimension $d \geq n \geq 2$ and $\{a_1, a_2, \dots, a_n\}$ be linearly independent set in X. Then the function $\|\cdot, \dots, \cdot\|_\infty$ on X^{n-1} defined by

$$\|x_1, x_2, \dots, x_{n-1}\|_\infty = \max\left\{\|x_1, x_2, \dots, x_{n-1}, a_i\| : i = 1, 2, \dots, n\right\}$$

defines an $(n-1)$-norm on X with respect to $\{a_1, a_2, \dots, a_n\}$.

A sequence (x_k) in a n-normed space $(X, \|\cdot, \dots, \cdot\|)$ is said to converge to some $L \in X$ if

$$\lim_{k \to \infty} \|x_k - L, z_1, \dots, z_{n-1}\| = 0 \quad \text{for every } z_1, \dots, z_{n-1} \in X.$$

A sequence (x_k) in a n-normed space $(X, \|\cdot, \dots, \cdot\|)$ is said to be Cauchy if

$$\lim_{\substack{k \to \infty \\ p \to \infty}} \|x_k - x_p, z_1, \dots, z_{n-1}\| = 0 \quad \text{for every } z_1, \dots, z_{n-1} \in X.$$

If every Cauchy sequence in X converges to some $L \in X$, then X is said to be complete with respect to the n-norm. Any complete n-normed space is said to be n-Banach space.

The space $m(\phi)$ was introduced by Sargent [36]:

$$m(\phi) = \left\{ x = (x_k) \in w : \|x\|_{m(\phi)} = \sup_{s \geq 1, \sigma \in \mathcal{P}_s} \frac{1}{\phi_s} \sum_{k \in \sigma} |x_k| < \infty \right\},$$

which was further studied in [37, 38] and [39]. Recently, Duyar and Oğur [40] introduced the sequence space $m^2(M, A, \phi, p)$ and studied some of its properties.

Let $A = (a_{ijkl})$ be an infinite double matrix of complex numbers, $\mathcal{M} = (M_{k,l})$ be a sequence of Orlicz functions, and $p = (p_{k,l})$ be a bounded double sequence of positive real numbers. In the present paper we define the following sequence space:

$$\begin{aligned} & m^2(\mathcal{M}, A, \phi, p, \|\cdot, \ldots, \cdot\|) \\ & \quad = \left\{ x = (x_{k,l}) \in w^2(X) : \sup \left\{ \frac{1}{\phi_{s,t}} \sum_{i \in \sigma_1} \sum_{j \in \sigma_2} \sum_{k,l=1}^{\infty} M_{k,l} \left(\left\| \frac{A_{ij}(x)}{\rho}, z_1, \ldots, z_{n-1} \right\| \right)^{p_{ij}} : \right. \right. \\ & \qquad \left. \left. (s,t) \geq (1,1), \sigma_1 \times \sigma_2 \in \mathcal{P}_{s,t} \right\} < \infty \text{ for some } \rho > 0 \right\}, \end{aligned}$$

where $A(x) = (A_{ij}(x))$ if $A_{ij}(x) = \sum_{k,l=1}^{\infty} a_{ijkl} x_{k,l}$ converges for each $(i,j) \in \mathbb{N} \times \mathbb{N}$.

If $p = (p_{ij}) = 1$, we have

$$\begin{aligned} & m^2(\mathcal{M}, A, \phi, \|\cdot, \ldots, \cdot\|) \\ & \quad = \left\{ x = (x_{k,l}) \in w^2(X) : \sup \left\{ \frac{1}{\phi_{s,t}} \sum_{i \in \sigma_1} \sum_{j \in \sigma_2} \sum_{k,l=1}^{\infty} M_{k,l} \left(\left\| \frac{A_{ij}(x)}{\rho}, z_1, \ldots, z_{n-1} \right\| \right) : \right. \right. \\ & \qquad \left. \left. (s,t) \geq (1,1), \sigma_1 \times \sigma_2 \in \mathcal{P}_{s,t} \right\} < \infty \text{ for some } \rho > 0 \right\}. \end{aligned}$$

The following inequality will be used throughout the paper:

$$|a + b|^{p_{ij}} \leq \max(1, 2^{H-1}) (|a|^{p_{ij}} + |b|^{p_{ij}}), \tag{1.1}$$

where $a, b \in \mathbb{C}$ and $H = \sup\{p_{ij} : (i,j) \in \mathbb{N} \times \mathbb{N}\}$.

We examine some topological properties of $m^2(\mathcal{M}, A, \phi, p, \|\cdot, \ldots, \cdot\|)$ and establish some inclusion relations.

2 Main results

Theorem 2.1 *Let $\mathcal{M} = (M_{k,l})$ be a sequence of Orlicz functions and $p = (p_{k,l})$ be a bounded sequence of positive real numbers, then the space $m^2(\mathcal{M}, A, \phi, p, \|\cdot, \ldots, \cdot\|)$ is linear space over the field of complex number $\mathbb{C}$.*

Proof Let $x = (x_{k,l}), y = (y_{k,l}) \in m^2(\mathcal{M}, A, \phi, p, \|\cdot, \ldots, \cdot\|)$ and $\alpha, \beta \in \mathbb{C}$. Then there exist positive numbers ρ_1 and ρ_2 such that

$$\sup \left\{ \frac{1}{\phi_{s,t}} \sum_{i \in \sigma_1} \sum_{j \in \sigma_2} \sum_{k,l=1}^{\infty} M_{k,l} \left(\left\| \frac{A_{ij}(x)}{\rho_1}, z_1, \ldots, z_{n-1} \right\| \right)^{p_{ij}} : (s,t) \geq (1,1), \sigma_1 \times \sigma_2 \in \mathcal{P}_{s,t} \right\} < \infty$$

and

$$\sup\left\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M_{k,l}\left(\left\|\frac{A_{ij}(y)}{\rho_2},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}}:(s,t)\geq(1,1),\sigma_1\times\sigma_2\in\mathcal{P}_{s,t}\right\}<\infty.$$

Let $\rho_3 = \max(2|\alpha|\rho_1, 2|\beta|\rho_2)$. Since $\mathcal{M}$ is a non-decreasing and convex function, we have

$$\begin{aligned}
&\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M_{k,l}\left(\left\|\frac{A_{ij}(\alpha x+\beta y)}{\rho_3},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}}\\
&\quad\leq\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M_{k,l}\left(\left\|\frac{\alpha a_{ijkl}x_{k,l}+\beta a_{ijkl}y_{k,l}}{\rho_3},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}}\\
&\quad\leq\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M_{k,l}\left(\left\|\frac{\alpha a_{ijkl}x_{k,l}}{2|\alpha|\rho_1},z_1,\ldots,z_{n-1}\right\|+\left\|\frac{\beta a_{ijkl}y_{k,l}}{2|\beta|\rho_2},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}}\\
&\quad=\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M_{k,l}\left(\left\|\frac{a_{ijkl}x_{k,l}}{2\rho_1},z_1,\ldots,z_{n-1}\right\|+\left\|\frac{a_{ijkl}y_{k,l}}{2\rho_2},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}}\\
&\quad\leq\max\left(1,2^{H-1}\right)\left(\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M_{k,l}\left(\left\|\frac{a_{ijkl}x_{k,l}}{2\rho_1},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}}\right.\\
&\qquad\left.+\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M_{k,l}\left(\left\|\frac{a_{ijkl}y_{k,l}}{2\rho_2},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}}\right).
\end{aligned}$$

Thus, we have

$$\begin{aligned}
&\sup\left\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M_{k,l}\left(\left\|\frac{A_{ij}(\alpha x+\beta y)}{\rho_3},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}}:\right.\\
&\qquad\left.(s,t)\geq(1,1),\sigma_1\times\sigma_2\in\mathcal{P}_{s,t}\right\}\\
&\quad\leq\max\left(1,2^{H-1}\right)\left\{\sup\left\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M_{k,l}\left(\left\|\frac{a_{ijkl}x_{k,l}}{2\rho_1},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}}:\right.\right.\\
&\qquad\left.(s,t)\geq(1,1),\sigma_1\times\sigma_2\in\mathcal{P}_{s,t}\right\}\\
&\qquad+\sup\left\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}M_{k,l}\left(\left\|\frac{a_{ijkl}y_{k,l}}{2\rho_2},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}}:\right.\\
&\qquad\left.\left.(s,t)\geq(1,1),\sigma_1\times\sigma_2\in\mathcal{P}_{s,t}\right\}\right\}.
\end{aligned}$$

This proves that $\alpha x+\beta y\in m^2(\mathcal{M},A,\phi,p,\|\cdot,\ldots,\cdot\|)$. Hence $m^2(\mathcal{M},A,\phi,p,\|\cdot,\ldots,\cdot\|)$ is a linear space. This completes the proof of the theorem. □

Theorem 2.2 *$\mathcal{M}=(M_{k,l})$ be a sequence of Orlicz functions and $p=(p_{k,l})$ be a bounded sequence of positive real numbers, then the space $m^2(\mathcal{M},A,\phi,p,\|\cdot,\ldots,\cdot\|)$ is a paranormed* *space with the paranorm defined by*

$$g(x) = \inf\left\{\rho^{\frac{p_{qr}}{H}} : \left[\sup\left\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M_{k,l}\left(\left\|\frac{A_{ij}(x)}{\rho},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}} : (s,t)\geq(1,1),\sigma_1\times\sigma_2\in\mathcal{P}_{s,t}\right\}\leq 1\right]^{1/H}, q\in\mathbb{N}, r\in\mathbb{N}\right\}.$$

Proof It is clear that $g(x) = g(-x)$ and $g(x) = 0$ if $x = 0$. Then there exist positive numbers ρ_1 and ρ_2 such that

$$\sup\left\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M_{k,l}\left(\left\|\frac{A_{ij}(x)}{\rho_1},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}} : (s,t)\geq(1,1),\sigma_1\times\sigma_2\in\mathcal{P}_{s,t}\right\}<1$$

and

$$\sup\left\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M_{k,l}\left(\left\|\frac{A_{ij}(y)}{\rho_2},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}} : (s,t)\geq(1,1),\sigma_1\times\sigma_2\in\mathcal{P}_{s,t}\right\}<1.$$

Then, by using Minkowski's inequality, we have

$$\begin{aligned}
&\sup\left\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M_{k,l}\left(\left\|\frac{A_{ij}(x+y)}{\rho_1+\rho_2},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}} :\right.\\
&\qquad\left.(s,t)\geq(1,1),\sigma_1\times\sigma_2\in\mathcal{P}_{s,t}\right\}\\
&\quad\leq\sup\left\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M_{k,l}\left(\left\|\frac{A_{ij}(x)}{\rho_1+\rho_2},z_1,\ldots,z_{n-1}\right\|\right.\right.\\
&\qquad\left.\left.+\left\|\frac{A_{ij}(y)}{\rho_1+\rho_2},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}} : (s,t)\geq(1,1),\sigma_1\times\sigma_2\in\mathcal{P}_{s,t}\right\}\\
&\quad\leq\left(\frac{\rho_1}{\rho_1+\rho_2}\right)^{h}\left\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M_{k,l}\left(\left\|\frac{A_{ij}(x)}{\rho_1},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}} :\right.\\
&\qquad\left.(s,t)\geq(1,1),\sigma_1\times\sigma_2\in\mathcal{P}_{s,t}\right\}\\
&\qquad+\left(\frac{\rho_2}{\rho_1+\rho_2}\right)^{h}\left\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M_{k,l}\left(\left\|\frac{A_{ij}(y)}{\rho_2},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}} :\right.\\
&\qquad\left.(s,t)\geq(1,1),\sigma_1\times\sigma_2\in\mathcal{P}_{s,t}\right\},
\end{aligned}$$

where $h = \inf p_{ij}$. This shows that $g(x+y)\leq g(x)+g(y)$. Using this triangle inequality we can write

$$g(\lambda^n x^n - \lambda x)\leq g(\lambda^n x^n - \lambda^n x) + g(\lambda^n x - \lambda x).$$

Thus we have

$$\begin{aligned}
&g\big(\lambda^n x^n - \lambda^n x\big) \\
&= \inf\Biggl\{\rho_n^{\frac{p_{qr}}{H}} : \Biggl[\sup\Biggl\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty} M_{k,l}\Biggl(\Biggl\|\frac{A_{ij}(\lambda^n x^n - \lambda^n x)}{\rho_n}, z_1, \ldots, z_{n-1}\Biggr\|\Biggr)^{p_{ij}} : \\
&\quad (s,t) \geq (1,1), \sigma_1\sigma_2 \in \mathcal{P}_{s,t}\Biggr\} \leq 1\Biggr]^{1/H} \leq 1, q \in \mathbb{N}, r \in \mathbb{N}\Biggr\} \\
&= \inf\Biggl\{\rho_n^{p_{qr}/H} : \Biggl[\sup\Biggl\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty} M_{k,l}\Biggl(\Biggl\|\frac{A_{ij}(x^n - x)}{(\rho_n/|\lambda^n|)}, z_1, \ldots, z_{n-1}\Biggr\|\Biggr)^{p_{ij}} : \\
&\quad (s,t) \geq (1,1), \sigma_1 \times \sigma_2 \in \mathcal{P}_{s,t}\Biggr\} \leq 1\Biggr]^{1/H} \leq 1, q \in \mathbb{N}, r \in \mathbb{N}\Biggr\} \\
&= \inf\Biggl\{\big(\lambda^n \rho_n\big)^{p_{qr}/H} : \Biggl[\sup\Biggl\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty} M_{k,l}\Biggl(\Biggl\|\frac{A_{ij}(x^n - x)}{\rho_n}, z_1, \ldots, z_{n-1}\Biggr\|\Biggr)^{p_{ij}} : \\
&\quad (s,t) \geq (1,1), \sigma_1 \times \sigma_2 \in \mathcal{P}_{s,t}\Biggr\}\Biggr]^{1/H} \leq 1, q \in \mathbb{N}, r \in \mathbb{N}\Biggr\} \\
&\leq \max\bigl\{\big|\lambda^n\big|^{h/H}, \big|\lambda^n\big|\bigr\} \times \inf\Biggl\{\big(\big|\lambda^n\big|\rho_n\big)^{p_{qr}/H} : \\
&\quad \Biggl[\sup\Biggl\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty} M_{k,l}\Biggl(\Biggl\|\frac{A_{ij}(x^n - x)}{\rho_n}, z_1, \ldots, z_{n-1}\Biggr\|\Biggr)^{p_{ij}} : \\
&\quad (s,t) \geq (1,1), \sigma_1 \times \sigma_2 \in \mathcal{P}_{s,t}\Biggr\}\Biggr]^{1/H} \leq 1, q \in \mathbb{N}, r \in \mathbb{N}\Biggr\} \\
&= \max\bigl\{\big|\lambda^n\big|^{h/H}, \big|\lambda^n\big|\bigr\} \cdot g\big(x^n - x\big).
\end{aligned}$$

Thus

$$\begin{aligned}
&g\big(\lambda^n x - \lambda x\big) \\
&= \inf\Biggl\{\rho_n^{p_{qr}/H} : \Biggl[\sup\Biggl\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty} M_{k,l}\Biggl(\Biggl\|\frac{A_{ij}((\lambda^n - \lambda)x)}{\rho_n}, z_1, \ldots, z_{n-1}\Biggr\|\Biggr)^{p_{ij}} : \\
&\quad (s,t) \geq (1,1), \sigma_1 \times \sigma_2 \in \mathcal{P}_{s,t}\Biggr\}\Biggr]^{1/H} \leq 1, q \in \mathbb{N}, r \in \mathbb{N}\Biggr\} \\
&= \inf\Biggl\{\rho_n^{p_{qr}/H} : \Biggl[\sup\Biggl\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty} M_{k,l}\Biggl(\Biggl\|\frac{A_{ij}(x)}{\rho_n/|\lambda^n - \lambda|}, z_1, \ldots, z_{n-1}\Biggr\|\Biggr)^{p_{ij}} : \\
&\quad (s,t) \geq (1,1), \sigma_1 \times \sigma_2 \in \mathcal{P}_{s,t}\Biggr\}\Biggr]^{1/H} \leq 1, q \in \mathbb{N}, r \in \mathbb{N}\Biggr\} \\
&= \inf\Biggl\{\big(\big|\lambda^n - \lambda\big|\rho_n\big)^{p_{qr}/H} : \Biggl[\sup\Biggl\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty} M_{k,l}\Biggl(\Biggl\|\frac{A_{ij}(x)}{\rho_n}, z_1, \ldots, z_{n-1}\Biggr\|\Biggr)^{p_{ij}} :
\end{aligned}$$

$$(s,t) \geq (1,1), \sigma_1 \times \sigma_2 \in \mathcal{P}_{s,t}\biggr\}\biggr]^{1/H} \leq 1, q \in \mathbb{N}, r \in \mathbb{N}\biggr\}$$

$$\leq \max\{|\lambda^n - \lambda|^{h/H}, |\lambda^n - \lambda|\} g(x).$$

Hence $g(\lambda^n x^n - \lambda x) \to 0$ where $\lambda^n \to \lambda$ and $x^n \to x$ as $n \to \infty$. This proves that $m^2(\mathcal{M}, A, \phi, p, \|\cdot, \ldots, \cdot\|)$ is a paranormed space with the paranorm defined by g. This completes the proof of the theorem. □

Theorem 2.3 *Let* ϕ *and* ψ *be two double sequences then* $m^2(\mathcal{M}, A, \phi, p, \|\cdot, \ldots, \cdot\|) \subseteq m^2(\mathcal{M}, A, \psi, p, \|\cdot, \ldots, \cdot\|)$ *if and only if* $\sup_{(s,t)\geq(1,1)}(\phi_{s,t}/\psi_{s,t}) < \infty$.

Proof Let $K = \sup_{(s,t)\geq(1,1)}(\phi_{s,t}/\psi_{s,t}) < \infty$. Then $\phi_{s,t} \leq K \cdot \psi_{s,t}$ for all $(s,t) \geq (1,1)$. If $x = \{x_{k,l}\} \in m^2(\mathcal{M}, A, \phi, p, \|\cdot, \ldots, \cdot\|)$, then

$$\sup\left\{\frac{1}{\phi_{s,t}} \sum_{i\in\sigma_1} \sum_{j\in\sigma_2} \sum_{k,l=1}^{\infty} M_{k,l}\left(\left\|\frac{A_{ij}(x)}{\rho}, z_1, \ldots, z_{n-1}\right\|\right)^{p_{ij}} : (s,t) \geq (1,1), \sigma_1 \times \sigma_2 \in \mathcal{P}_{s,t}\right\}$$
$$< \infty \quad \text{for some } \rho > 0.$$

Thus

$$\sup\left\{\frac{1}{K\psi_{s,t}} \sum_{i\in\sigma_1} \sum_{j\in\sigma_2} \sum_{k,l=1}^{\infty} M_{k,l}\left(\left\|\frac{A_{ij}(x)}{\rho}, z_1, \ldots, z_{n-1}\right\|\right)^{p_{ij}} : (s,t) \geq (1,1), \sigma_1 \times \sigma_2 \in \mathcal{P}_{s,t}\right\}$$
$$< \infty \quad \text{for some } \rho > 0,$$

and hence $x = \{x_{k,l}\} \in m^2(\mathcal{M}, A, \psi, p, \|\cdot, \ldots, \cdot\|)$. This shows that

$$m^2(\mathcal{M}, A, \phi, p, \|\cdot, \ldots, \cdot\|) \subseteq m^2(\mathcal{M}, A, \psi, p, \|\cdot, \ldots, \cdot\|).$$

Conversely, let $m^2(\mathcal{M}, A, \phi, p, \|\cdot, \ldots, \cdot\|) \subseteq m^2(\mathcal{M}, A, \psi, p, \|\cdot, \ldots, \cdot\|)$ and $\alpha_{s,t} = \frac{\phi_{s,t}}{\psi_{s,t}}$ for all $(s,t) \geq (1,1)$, and suppose $\sup_{(s,t)\geq(1,1)} \alpha_{s,t} = \infty$. Then there exists a subsequence $\{\alpha_{s_i,t_i}\}$ of $\{\alpha_{s,t}\}$ such that $\lim_{i\to\infty} \alpha_{s_i,t_i} = \infty$. If $x = \{x_{k,l}\} \in m^2(\mathcal{M}, A, \phi, p, \|\cdot, \ldots, \cdot\|)$, then we have

$$\sup\left\{\frac{1}{\psi_{s,t}} \sum_{i\in\sigma_1} \sum_{j\in\sigma_2} \sum_{k,l=1}^{\infty} M_{k,l}^{\infty}\left(\left\|\frac{A_{ij}(x)}{\rho}, z_1, \ldots, z_{n-1}\right\|\right)^{p_{ij}} : (s,t) \geq (1,1), \sigma_1 \times \sigma_2 \in \mathcal{P}_{s,t}\right\}$$
$$= \sup\left\{\alpha_{s,t} \frac{1}{\phi_{s,t}} \sum_{i\in\sigma_1} \sum_{j\in\sigma_2} \sum_{k,l=1}^{\infty} M_{k,l}^{\infty}\left(\left\|\frac{A_{ij}(x)}{\rho}, z_1, \ldots, z_{n-1}\right\|\right)^{p_{ij}} :\right.$$
$$\left.(s,t) \geq (1,1), \sigma_1 \times \sigma_2 \in \mathcal{P}_{s,t}\right\}$$
$$\geq \left\{\sup_{m\geq1} \alpha_{s_m,t_m}\right\} \sup\left\{\frac{1}{\phi_{s_m,t_m}} \sum_{i\in\sigma_1} \sum_{j\in\sigma_2} \sum_{k,l=1}^{\infty} M_{k,l}^{\infty}\left(\left\|\frac{A_{ij}(x)}{\rho}, z_1, \ldots, z_{n-1}\right\|\right)^{p_{ij}} :\right.$$
$$\left.(s,t) \geq (1,1), \sigma_1 \times \sigma_2 \in \mathcal{P}_{s,t}\right\} = \infty.$$

This is a contradiction as $x = \{x_{k,l}\} \notin m^2(\mathcal{M}, A, \phi, p, \|\cdot, \ldots, \cdot\|)$. This completes the proof of the theorem. □

Corollary 2.4 *Let ϕ and ψ be two double sequences then $m^2(\mathcal{M}, A, \phi, p, \|\cdot, \ldots, \cdot\|) = m^2(\mathcal{M}, A, \psi, p, \|\cdot, \ldots, \cdot\|)$ if and only if $\sup_{(s,t)\geq(1,1)} \alpha_{s,t} < \infty$ and $\sup_{(s,t)\geq(1,1)} \alpha_{s,t}^{-1} < \infty$.*

Proof It is easy to prove so we omit the details. □

Theorem 2.5 *Let $\mathcal{M} = (M_{k,l})$, $\mathcal{M}' = (M'_{k,l})$ and $\mathcal{M}'' = (M''_{k,l})$ be sequences of Orlicz functions satisfying Δ_2-condition. Then*

(i) $m^2(\mathcal{M}, \phi, \|\cdot, \ldots, \cdot\|) \subseteq m^2(\mathcal{M} \circ \mathcal{M}', \phi, \|\cdot, \ldots, \cdot\|)$,

(ii) $m^2(\mathcal{M}', A, \phi, p, \|\cdot, \ldots, \cdot\|) \cap m^2(\mathcal{M}'', A, \phi, p, \|\cdot, \ldots, \cdot\|) \subseteq m^2(\mathcal{M}' + \mathcal{M}'', A, \phi, p, \|\cdot, \ldots, \cdot\|)$.

Proof (i) Let $x = \{x_{k,l}\} \in m^2(\mathcal{M}, A, \phi, p, \|\cdot, \ldots, \cdot\|)$. Then there exists $\rho > 0$ such that

$$\sup\left\{\frac{1}{\phi_{s,t}} \sum_{i\in\sigma_1} \sum_{j\in\sigma_2} \sum_{k,l=1}^{\infty} M_{k,l}\left(\left\|\frac{A_{ij}(x)}{\rho}, z_1, \ldots, z_{n-1}\right\|\right)^{p_{ij}} : (s,t) \geq (1,1), \sigma_1 \times \sigma_2 \in \mathcal{P}_{s,t}\right\} < \infty.$$

By the continuity of $\mathcal{M}$, we can take a number δ with $0 < \delta < 1$ such that $M_{k,l}(t) < \epsilon$, whenever $0 \leq t < \delta$, for arbitrary $0 < \epsilon < 1$. Now let

$$y_{i,j} = \left(\left\|\frac{A_{i,j}(x)}{\rho}, z_1, \ldots, z_{n-1}\right\|\right).$$

Thus we have

$$\sum_{i\in\sigma_1} \sum_{j\in\sigma_2} \sum_{k,l=1}^{\infty} M_{k,l}(y_{i,j})^{p_{i,j}} = \sum_{y_{i,j}\leq\delta} \sum_{k,l=1}^{\infty} M_{k,l}(y_{i,j})^{p_{i,j}} + \sum_{y_{i,j}>\delta} \sum_{k,l=1}^{\infty} M_{k,l}(y_{i,j})^{p_{i,j}}.$$

By the properties of the Orlicz function we have

$$\sum_{y_{i,j}\leq\infty} \sum_{k,l=1}^{\infty} M_{k,l}(y_{i,j})^{p_{i,j}} \leq \max\{1, M_{k,l}(1)^H\} \sum_{y_{i,j}\leq\infty} (y_{i,j})^{p_{i,j}}.$$

Again, we have

$$M_{k,l}(y_{i,j}) < M_{k,l}\left(1 + \frac{y_{i,j}}{\delta}\right) < \frac{1}{2} M_{k,l}(2) + \frac{1}{2} M_{k,l}\left(\frac{2y_{i,j}}{\delta}\right)$$

for $y_{i,j} > \delta$. If $\mathcal{M}$ satisfies the Δ_2-condition, then we have

$$M_{k,l}(y_{i,j}) < \frac{1}{2} T \frac{y_{i,j}}{\delta} M_{k,l}(2) + \frac{1}{2} T \frac{y_{i,j}}{\delta} M_{k,l}(2),$$

and so

$$\sum_{y_{i,j}>\delta} M_{k,l}(y_{i,j})^{p_{i,j}} \leq \max\left(1, \left(\frac{T}{\delta} M_{k,l}(2)\right)^H\right) \sum_{y_{i,j}>\delta} y_{i,j}.$$

Hence, we have

$$\sup\left\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M_{k,l}\left(M'_{k,l}\left(\left\|\frac{A_{ij}(x)}{\rho},z_1,\ldots,z_{n-1}\right\|\right)\right)^{p_{ij}}:\right.$$
$$\left.(s,t)\geq(1,1),\sigma_1\times\sigma_2\in\mathcal{P}_{s,t}\right\}$$
$$<\infty$$
$$\leq\max\{1,M_{k,l}(1)^H\}\sup\left\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M_{k,l}(y_{i,j})^{p_{ij}}:(s,t)\geq(1,1),\sigma_1\times\sigma_2\in\mathcal{P}_{s,t}\right\}$$
$$<\infty+\max\left(1,\left(\frac{T}{\delta}M_{k,l}(2)\right)^H\right)$$
$$\times\sup\left\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M_{k,l}(y_{i,j})^{p_{ij}}:(s,t)\geq(1,1),\sigma_1\times\sigma_2\in\mathcal{P}_{s,t}\right\}<\infty.$$

Thus, we have $x=\{x_{k,l}\}\in m^2(\mathcal{M}\circ\mathcal{M}',\phi,\|\cdot,\ldots,\cdot\|)$ and hence $m^2(\mathcal{M},\phi,\|\cdot,\ldots,\cdot\|)\subseteq m^2(\mathcal{M}\circ\mathcal{M}',\phi,\|\cdot,\ldots,\cdot\|)$.

(ii) Let $x=\{x_{k,l}\}\in m^2(\mathcal{M}',A,\phi,p,\|\cdot,\ldots,\cdot\|)\cap m^2(\mathcal{M}'',A,\phi,p,\|\cdot,\ldots,\cdot\|)$. Then there exists a $\rho>0$ such that

$$\sup\left\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M'_{k,l}\left(\left\|\frac{A_{i,j}(x)}{\rho},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}}:(s,t)\geq(1,1),\sigma_1\times\sigma_2\in\mathcal{P}_{s,t}\right\}<\infty$$

and

$$\sup\left\{\frac{1}{\phi_{s,t}}\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M''_{k,l}\left(\left\|\frac{A_{i,j}(x)}{\rho},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}}:(s,t)\geq(1,1),\sigma_1\times\sigma_2\in\mathcal{P}_{s,t}\right\}<\infty.$$

By the inequality, we have

$$\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}(M'_{k,l}+M''_{k,l})\left(\left\|\frac{A_{i,j}(x)}{\rho},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}}$$
$$\leq\max(1,2^{H-1})\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M'_{k,l}\left(\left\|\frac{A_{i,j}(x)}{\rho},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}}$$
$$+\max(1,2^{H-1})\sum_{i\in\sigma_1}\sum_{j\in\sigma_2}\sum_{k,l=1}^{\infty}M''_{k,l}\left(\left\|\frac{A_{i,j}(x)}{\rho},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}}.$$

Hence

$$m^2(\mathcal{M}',A,\phi,p,\|\cdot,\ldots,\cdot\|)\cap m^2(\mathcal{M}'',A,\phi,p,\|\cdot,\ldots,\cdot\|)$$
$$\subseteq m^2(\mathcal{M}'+\mathcal{M}'',A,\phi,p,\|\cdot,\ldots,\cdot\|).$$

This completes the proof of the theorem. □

Theorem 2.6 *The sequence space $m^2(\mathcal{M},\phi,p,\|\cdot,\ldots,\cdot\|)$ is solid.*

Proof Let $\alpha = \{\alpha_{k,l}\}$ be a double sequence of scalars such that $|\alpha_{k,l}| \leq 1$ and $y = \{y_{k,l}\} \in m^2(\mathcal{M},\phi,p,\|\cdot,\ldots,\cdot\|)$. Then we have

$$\sup\left\{\frac{1}{\phi_{s,t}}\sum_{k\in\sigma_1}\sum_{l\in\sigma_2}M_{k,l}\left(\left\|\frac{\alpha_{k,l}x_{k,l}}{\rho},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}} : (s,t)\geq(1,1),\sigma_1\times\sigma_2\in\mathcal{P}_{s,t}\right\}$$
$$\leq \sup\left\{\frac{1}{\phi_{s,t}}\sum_{k\in\sigma_1}\sum_{l\in\sigma_2}M_{k,l}\left(\left\|\frac{\alpha_{k,l}y_{k,l}}{\rho},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}} : (s,t)\geq(1,1),\sigma_1\times\sigma_2\in\mathcal{P}_{s,t}\right\}$$
$$\leq \sup\left\{\frac{1}{\phi_{s,t}}\sum_{k\in\sigma_1}\sum_{l\in\sigma_2}M_{k,l}\left(\left\|\frac{y_{k,l}}{\rho},z_1,\ldots,z_{n-1}\right\|\right)^{p_{ij}} : (s,t)\geq(1,1),\sigma_1\times\sigma_2\in\mathcal{P}_{s,t}\right\}.$$

This implies that $\{\alpha_{k,l}y_{k,l}\} \in m^2(\mathcal{M},\phi,p,\|\cdot,\ldots,\cdot\|)$. This proves that the space $m^2(\mathcal{M},\phi,p,\|\cdot,\ldots,\cdot\|)$ is a solid. □

Corollary 2.7 *The sequence space $m^2(\mathcal{M},\phi,p,\|\cdot,\ldots,\cdot\|)$ is monotone.*

Proof It is trivial so we omit the details. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

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