Journal of Inequalities and Applications
a SpringerOpen Journal

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On the strictly G-preinvex function

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Abstract

In this paper, we consider the strictly G-preinvex functions introduced by Antczak (J. Comput. Appl. Math. 217:212-226, 2008). The relationships between semistrictly G-preinvex functions and strictly G-preinvex functions, G-preinvex functions strictly G-preinvex functions are investigated under mild assumptions. Our results improve and extend the existing ones in the literature.

MSC: 90C26

Keywords: G-preinvex functions; semistrictly G-preinvex functions; strictly G-preinvex functions; optimization

1 Introduction

Convexity and some generalizations of convexity play a crucial role in mathematical economics, engineering, management science, and optimization theory. Therefore, it is important to consider wider classes of generalized convex functions and also to seek practical criteria for convexity or generalized convexity (see [1–11] and the references therein). A significant generalization of convex functions is the introduction of preinvex functions, which is due to Weir and Mond [3]. Yang and Li [1] presented some properties of preinvex functions; in [2] they introduced two new classes of generalized convex functions, called semistrictly preinvex functions and strictly preinvex functions. They established relationships between preinvex functions and semistrictly preinvex functions under a certain set of conditions. Recently, Antczak [4, 6] introduced the concept of G-preinvex function (strictly G-preinvex function), which includes the preinvex function (strictly preinvex function) [3] and r-preinvex function [5] as special cases. Relations of this G-preinvex function to preinvex functions and some properties of this class of functions were studied in [4]. Luo and Wu in [7] introduced the semistrictly G-preinvex functions and established relationships between G-preinvex functions and semistrictly G-preinvex functions under a certain set of conditions.

In this paper, we investigate the relationships between semistrictly G-preinvex functions and strictly G-preinvex functions, G-preinvex functions and strictly G-preinvex functions under mild assumptions. It is worth pointing out that the results obtained here improve and generalize the corresponding ones given in [12, 13].

The outline of the paper is as follows. In Section 2, we give some preliminaries. The main results of the paper are presented in Section 3. Section 4 gives some conclusions.

2 Preliminaries

In this section, we describe some definitions of generalized convexity.

Definition 2.1 ([3]) For a given set $K \subseteq R^n$ and a given function $\eta : R^n \times R^n \to R^n$, K is said to be an invex set with respect to η iff

$$\forall x, y \in K, \qquad \forall \lambda \in [0,1] \quad \Rightarrow \quad y + \lambda\eta(x,y) \in K.$$

Definition 2.2 ([3]) Let $K \subseteq R^n$ be an invex set with respect to $\eta : R^n \times R^n \to R^n$. The function $f : K \to R$ is said to be preinvex on K iff, $\forall x, y \in K$, $\forall \lambda \in [0,1]$,

$$f\bigl(y + \lambda\eta(x,y)\bigr) \le \lambda f(x) + (1-\lambda)f(y).$$

Definition 2.3 ([2]) Let $K \subseteq R^n$ be an invex set with respect to $\eta : K \times K \to R^n$. The function $f : K \to R$ is said to be semistrictly preinvex on K iff, $\forall x, y \in K, f(x) \neq f(y)$, $\forall \lambda \in (0,1)$,

$$f\bigl(y + \lambda\eta(x,y)\bigr) < \lambda f(x) + (1-\lambda)f(y).$$

Definition 2.4 ([2]) Let $K \subseteq R^n$ be an invex set with respect to $\eta : K \times K \to R^n$. The function $f : K \to R$ is said to be strictly preinvex on K iff, $\forall x, y \in K$, $x \neq y$, $\forall \lambda \in (0,1)$,

$$f\bigl(y + \lambda\eta(x,y)\bigr) < \lambda f(x) + (1-\lambda)f(y).$$

In [12], the relationships between preinvex functions and strictly preinvex functions, semistrictly preinvex functions and strictly preinvex functions were discussed under the following condition.

Condition C ([3]) Let $\eta : K \times K \to R^n$. We say that the function η satisfies Condition C iff, $\forall x, y \in K$, $\forall \lambda \in [0,1]$,

$$\eta\bigl(y, y + \lambda\eta(x,y)\bigr) = -\lambda\eta(x,y),$$
$$\eta\bigl(x, y + \lambda\eta(x,y)\bigr) = (1-\lambda)\eta(x,y).$$

Let $I_f(K)$ be the range of f, *i.e.*, the image of K under f, and let G^{-1} be the inverse of G.

Definition 2.5 ([4, 6]) Let $K \subseteq R^n$ be an invex set (with respect to η). The function $f : K \to R$ is said to be G-preinvex on K iff there exist a continuous real-valued increasing function $G : I_f(K) \to R$ and a vector-valued function $\eta : K \times K \to R^n$ such that, $\forall x, y \in K$, $\forall \lambda \in [0,1]$,

$$f\bigl(y + \lambda\eta(x,y)\bigr) \le G^{-1}\bigl(\lambda G\bigl(f(x)\bigr) + (1-\lambda)G\bigl(f(y)\bigr)\bigr).$$

We note that the G-preinvex function in Definition 2.5 reduces to the preinvex function in Definition 2.2 and the r-preinvex function in [5] when setting $G(t) = t$ and $G(t) = e^{rt}$, $r > 0$, respectively.

Definition 2.6 ([7]) Let $K \subseteq R^n$ be an invex set (with respect to η). The function $f : K \to R$ is said to be semistrictly G-preinvex on K iff there exist a continuous real-valued increasing function $G : I_f(K) \to R$ and a vector-valued function $\eta : K \times K \to R^n$ such that, $\forall x, y \in K$, $f(x) \neq f(y)$, $\forall\lambda \in (0,1)$,

$$f\big(y + \lambda\eta(x,y)\big) < G^{-1}\big(\lambda G\big(f(x)\big) + (1-\lambda)G\big(f(y)\big)\big).$$

Definition 2.7 ([4]) Let $K \subseteq R^n$ be an invex set (with respect to η). The function $f : K \to R$ is said to be strictly G-preinvex on K iff there exist a continuous real-valued increasing function $G : I_f(K) \to R$ and a vector-valued function $\eta : K \times K \to R^n$ such that, $\forall x, y \in K$, $x \neq y$, $\forall\lambda \in (0,1)$,

$$f\big(y + \lambda\eta(x,y)\big) < G^{-1}\big(\lambda G\big(f(x)\big) + (1-\lambda)G\big(f(y)\big)\big).$$

We also observe that the semistrictly G-preinvex function in Definition 2.6 is a generalization of the semistrictly preinvex function in Definition 2.3, and the strictly G-preinvex function in Definition 2.7 is a generalization of the strictly preinvex function in Definition 2.4 when taking $G(t) = t$.

Example 2.1 This example illustrates that a G-preinvex function is not necessarily a strictly G-preinvex function. Let

$$f(x) = \ln\big(|x| + 1\big),$$
$$\eta(x,y) = \begin{cases} x - y & \text{if } xy < 0, \\ -x - y & \text{if } xy \geq 0. \end{cases}$$

Then f is a G-preinvex function on R with respect to η, where $G(t) = e^t$. But if we let $x = 0$, $y = 1$, $\lambda = \frac{1}{2}$, we have

$$f(y) = f(1) = \ln 2, \qquad f(x) = f(0) = 0$$

and

$$f\big(y + \lambda\eta(x,y)\big) = f\left(\frac{1}{2}\right) = \ln\left(\frac{3}{2}\right) = G^{-1}\left(\frac{3}{2}\right) = G^{-1}\big(\lambda G\big(f(x)\big) + (1-\lambda)G\big(f(y)\big)\big).$$

So, f is not a strictly G-preinvex function with respect to η on R.

Example 2.2 This example illustrates that a semistrictly G-preinvex function is not necessarily a strictly G-preinvex function. Let

$$f(x) = \begin{cases} 1 & \text{if } x = 0, \\ 0 & \text{if } x \neq 0, x \in [-6,-2] \cup [-1,6], \end{cases}$$

$$\eta(x,y) = \begin{cases} x - y & \text{if } -1 \leq x \leq 6, -1 \leq y \leq 6, \\ x - y & \text{if } -6 \leq x \leq -2, -6 \leq y \leq -2, \\ -7 - y & \text{if } -1 \leq x \leq 6, -6 \leq y \leq -2, \\ -y & \text{if } -6 \leq x \leq -2, -1 \leq y \leq 6, y \neq 0, \\ \frac{x}{6} & \text{if } -6 \leq x \leq -2, y = 0. \end{cases}$$

Then it is not difficult to show that f is a semistrictly G-preinvex function on $x \in [-6,-2] \cup [-1,6]$ with respect to η, where $G(t) = t$. However, let $x = -1 \neq y = 1$, $\lambda = \frac{1}{2}$, and we have

$$f\bigl(y + \lambda\eta(x,y)\bigr) = f(0) = 1 > 0 = G^{-1}\bigl(\lambda G\bigl(f(x)\bigr) + (1-\lambda)G\bigl(f(y)\bigr)\bigr).$$

Thus, f is not a strictly G-preinvex function with respect to the same η on $x \in [-6,-2] \cup [-1,6]$.

Example 2.3 This example illustrates that a strictly G-preinvex function is not necessarily a strictly preinvex function. Let

$$f(x) = \frac{1}{|x|+1},$$

$$\eta(x,y) = \begin{cases} x - y & \text{if } xy < 0, \\ -x - y & \text{if } xy \geq 0. \end{cases}$$

Then it is not difficult to show that f is a strictly G-preinvex function on R with respect to η, where $G(t) = \ln t$. But if we let $x = -1$, $y = 1$, $\lambda = \frac{1}{2}$, we have

$$f(y) = f(1) = f(x) = f(-1) = \frac{1}{2}$$

and

$$f\bigl(y + \lambda\eta(x,y)\bigr) = f(0) = 1 > \frac{1}{2} = \lambda f(x) + (1-\lambda)f(y).$$

So, f is not a strictly preinvex function with respect to η on R.

Remark 2.1 From Example 2.1 and Example 2.2, we know that strictly G-preinvex functions are different from semistrictly G-preinvex functions and G-preinvex functions. By Example 2.3, we know that the definition of strictly G-preinvex functions is a true generalization of that of strictly preinvex functions.

3 Properties of strictly *G*-preinvex functions

In [7], the authors discussed the properties of G-preinvex functions and semistrictly G-preinvex functions. In this section, we derive some properties of strictly G-preinvex functions. We assume always that:

(i) $K \subseteq R^n$ is an invex set with respect to $\eta : K \times K \to R^n$;

(ii) η satisfies Condition C; f is a real-valued function on K.

Theorem 3.1 *Let f be a semistrictly G-preinvex function with respect to η on K. If $\bar{x} \in K$ is a local minimum to the problem of minimizing $f(x)$ subject to $x \in K$; then $\bar{x}$ is a global one.*

Proof Suppose that $\bar{x} \in K$ is a local minimum to the problem of minimizing $f(x)$ subject to $x \in K$. Then there is an ϵ-neighborhood $N_\epsilon(\bar{x})$ around $\bar{x}$ such that

$$f(\bar{x}) \leq f(x), \quad \forall x \in K \cap N_\epsilon(\bar{x}). \tag{3.1}$$

If $\bar{x}$ is not a global minimum of f over K, then there exists $x^* \in K$ such that

$$f(x^*) < f(\bar{x}).$$

By assumption, $f : K \to R$ is a semistrictly G-preinvex function with respect to η on K. Thus,

$$f(\bar{x} + \lambda\eta(x^*, \bar{x})) < G^{-1}(\lambda G(f(x^*)) + (1-\lambda)G(f(\bar{x}))) < f(\bar{x})$$

for all $\lambda \in (0,1)$. For a sufficiently small $\lambda > 0$, it follows that

$$\bar{x} + \lambda\eta(x^*, \bar{x}) \in K \cap N_\epsilon(\bar{x}),$$

which is a contradiction to (3.1). This completes the proof. □

Remark 3.1 It is easy to see that strict G-preinvexity implies semistrict G-preinvexity, so by Theorem 3.1, we have the following.

Corollary 3.1 *Let $f : K \to R$ be a strictly G-preinvex function with respect to η on K. If $\bar{x} \in K$ is a local minimum to the problem of minimizing $f(x)$ subject to $x \in K$, then $\bar{x}$ is a global one.*

By Corollary 3.1, we can conclude that strictly G-preinvex functions constitute an important class of generalized convex functions in mathematical programming.

Theorem 3.2 *Let f be a G-preinvex function on K. For each pair $x, y \in K$, $x \neq y$, if there exists $\alpha \in (0,1)$ such that*

$$f(y + \alpha\eta(x,y)) < G^{-1}(\alpha G(f(x)) + (1-\alpha)G(f(y))), \tag{3.2}$$

then f is a strictly G-preinvex function on K.

Proof By contradiction: suppose that there exist $x, y \in K$, $x \neq y$, $\lambda \in (0,1)$ such that

$$f(y + \lambda\eta(x,y)) \geq G^{-1}(\lambda G(f(x)) + (1-\lambda)G(f(y))). \tag{3.3}$$

Denote

$$z = y + \lambda\eta(x,y).$$

Since f is G-preinvex, we have

$$f(z) \leq G^{-1}(\lambda G(f(x)) + (1-\lambda)G(f(y))). \tag{3.4}$$

The above two inequalities imply that

$$f(z) = G^{-1}(\lambda G(f(x)) + (1-\lambda)G(f(y))). \tag{3.5}$$

We note that the pair x, z and the pair z, y are both distinct under (3.2), there exists $\beta_1, \beta_2 \in (0,1)$ such that

$$f\big(z + \beta_1\eta(x,z)\big) < G^{-1}\big(\beta_1 G\big(f(x)\big) + (1-\beta_1)G\big(f(z)\big)\big), \tag{3.6}$$

$$f\big(y + \beta_2\eta(z,y)\big) < G^{-1}\big(\beta_2 G\big(f(z)\big) + (1-\beta_2)G\big(f(y)\big)\big). \tag{3.7}$$

Denote

$$\bar{x} = z + \beta_1\eta(x,z), \qquad \bar{y} = y + \beta_2\eta(z,y).$$

From Condition C

$$\begin{aligned}
\bar{x} &= z + \beta_1\eta(x,z) = y + \lambda\eta(x,y) + \beta_1\eta\big(x, y + \lambda\eta(x,y)\big) \\
&= y + \lambda\eta(x,y) + (1-\lambda)\beta_1\eta(x,y) \\
&= y + \big(\lambda + (1-\lambda)\beta_1\big)\eta(x,y), \\
\bar{y} &= y + \beta_2\eta(z,y) = y + \beta_2\eta\big(y + \lambda\eta(x,y), y\big) \\
&= y + \beta_2\eta\big(y + \lambda\eta(x,y), y + \lambda\eta(x,y) - \lambda\eta(x,y)\big) \\
&= y + \beta_2\eta\big(y + \lambda\eta(x,y), y + \lambda\eta(x,y) + \eta\big(y, y + \lambda\eta(x,y)\big)\big) \\
&= y - \beta_2\eta\big(y, y + \lambda\eta(x,y)\big) \\
&= y + \lambda\beta_2\eta(x,y).
\end{aligned}$$

Let $\mu_1 = \lambda + (1-\lambda)\beta_1$, $\mu_2 = \lambda\beta_2$, $\mu = \frac{\lambda-\mu_2}{\mu_1-\mu_2}$. It is easy to verify that $\mu_1, \mu_2, \mu \in (0,1)$. Again from Condition C,

$$\begin{aligned}
\bar{y} + \mu\eta(\bar{x},\bar{y}) &= y + \mu_2\eta(x,y) + \mu\eta\big(y + \mu_1\eta(x,y), y + \mu_2\eta(x,y)\big) \\
&= y + \mu_2\eta(x,y) + \mu\eta\big(y + \mu_1\eta(x,y), y + \mu_1\eta(x,y) + (\mu_2 - \mu_1)\eta(x,y)\big) \\
&= y + \mu_2\eta(x,y) + \mu\eta\Bigg(y + \mu_1\eta(x,y), y + \mu_1\eta(x,y) \\
&\quad + \left(\frac{\mu_2-\mu_1}{1-\mu_1}\right)\eta\big(x, y + \mu_1\eta(x,y)\big)\Bigg) \\
&= y + \mu_2\eta(x,y) - \left(\frac{\mu(\mu_2-\mu_1)}{1-\mu_1}\right)\eta\big(x, y + \mu_1\eta(x,y)\big) \\
&= y + \mu_2\eta(x,y) - \mu(\mu_2-\mu_1)\eta(x,y) \\
&= y + \big(\mu_2 - \mu(\mu_2-\mu_1)\big)\eta(x,y) \\
&= y + \lambda\eta(x,y) \\
&= z.
\end{aligned}$$

Since f is G-preinvex, we have

$$f(z) \le G^{-1}\big(\mu G\big(f(\bar{x})\big) + (1-\mu)G\big(f(\bar{y})\big)\big). \tag{3.8}$$

By the assumption, G is an increasing function and hence, by Lemma 1 of [4], G^{-1} is also increasing. Thus, from (3.5)-(3.8), we have

$$\begin{aligned}
f(z) &\le G^{-1}\bigl(\mu G\bigl(f(\bar{x})\bigr) + (1-\mu)G\bigl(f(\bar{y})\bigr)\bigr) \\
&< G^{-1}\bigl(\mu\bigl(\beta_1 G\bigl(f(x)\bigr) + (1-\beta_1)G\bigl(f(z)\bigr)\bigr) \\
&\quad + (1-\mu)\bigl(\beta_2 G\bigl(f(z)\bigr) + (1-\beta_2)G\bigl(f(y)\bigr)\bigr)\bigr) \\
&= G^{-1}\bigl(\mu\beta_1 G\bigl(f(x)\bigr) + \bigl(\mu(1-\beta_1) + (1-\mu)\beta_2\bigr)G\bigl(f(z)\bigr) \\
&\quad + (1-\mu)(1-\beta_2)G\bigl(f(y)\bigr)\bigr) \\
&= G^{-1}\bigl(\bigl(\mu\beta_1 + \lambda\mu(1-\beta_1) + \lambda\beta_2(1-\mu)\bigr)G\bigl(f(x)\bigr) \\
&\quad + \bigl((1-\mu)(1-\beta_2) + (1-\lambda)\mu(1-\beta_1) + (1-\lambda)\beta_2(1-\mu)\bigr)G\bigl(f(y)\bigr)\bigr) \\
&= G^{-1}\bigl(\lambda G\bigl(f(x)\bigr) + (1-\lambda)G\bigl(f(y)\bigr)\bigr),
\end{aligned}$$

where

$$\begin{aligned}
&\mu\beta_1 + \lambda\mu(1-\beta_1) + \lambda\beta_2(1-\mu) \\
&\quad = \mu\bigl(\lambda + \beta_1(1-\lambda) - \lambda\beta_2\bigr) + \lambda\beta_2 \\
&\quad = \mu\bigl(\lambda + \beta_1(1-\lambda) - \mu_2\bigr) + \mu_2 \\
&\quad = \mu(\mu_1 - \mu_2) + \mu_2 \\
&\quad = \lambda, \\
&(1-\mu)(1-\beta_2) + (1-\lambda)\mu(1-\beta_1) + (1-\lambda)\beta_2(1-\mu) \\
&\quad = (1-\mu)(1-\mu_2) + \mu(1-\mu_1) \\
&\quad = 1 - \mu_2 + \mu(\mu_2 - \mu_1) \\
&\quad = 1 - \lambda,
\end{aligned}$$

which contradicts (3.3). This completes the proof. □

Corollary 3.2 *Let f be a G-preinvex function on K. If there exists $\alpha \in (0,1)$ such that, for each pair $x, y \in K$, $x \neq y$,*

$$f\bigl(y + \alpha\eta(x,y)\bigr) < G^{-1}\bigl(\alpha G\bigl(f(x)\bigr) + (1-\alpha)G\bigl(f(y)\bigr)\bigr),$$

then f is a strictly G-preinvex function on K.

Corollary 3.3 *Let f be a preinvex function on K. For each pair $x, y \in K$, $x \neq y$, if there exists $\alpha \in (0,1)$ such that*

$$f\bigl(y + \alpha\eta(x,y)\bigr) < \alpha f(x) + (1-\alpha)f(y),$$

then f is a strictly preinvex function on K.

Remark 3.2 Even Corollary 3.3 is a new result, it is an improvement of [12, Theorem 1].

Corollary 3.4 ([12]) *Let f be a preinvex function on K. If there exists $\alpha \in (0,1)$ such that, for each pair $x, y \in K$, $x \neq y$,*

$$f\bigl(y + \alpha\eta(x,y)\bigr) < \alpha f(x) + (1-\alpha)f(y),$$

then f is a strictly preinvex function on K.

Theorem 3.3 *f is a strictly G-preinvex function on K if and only if f is a semistrictly G-preinvex function on K and satisfies the following condition: there exists $\alpha \in (0,1)$ such that, for each pair $x, y \in K$, $x \neq y$,*

$$f\bigl(y + \alpha\eta(x,y)\bigr) < G^{-1}\bigl(\alpha G\bigl(f(x)\bigr) + (1-\alpha)G\bigl(f(y)\bigr)\bigr). \tag{3.9}$$

Proof The necessity is obvious from Definition 2.7. So we only prove the sufficiency. Since f is a semistrictly G-preinvex function on K, we only show that $f(x) = f(y)$, $x \neq y$ implies that

$$f\bigl(y + \lambda\eta(x,y)\bigr) < G^{-1}\bigl(\lambda G\bigl(f(x)\bigr) + (1-\lambda)G\bigl(f(y)\bigr)\bigr) = f(x), \quad \forall\lambda \in (0,1).$$

Let $\bar{x} = y + \alpha\eta(x,y)$. From (3.9) and for each $x, y \in K$, $f(x) = f(y)$, $x \neq y$, we have

$$f(\bar{x}) = f\bigl(y + \alpha\eta(x,y)\bigr) < G^{-1}\bigl(\alpha G\bigl(f(x)\bigr) + (1-\alpha)G\bigl(f(y)\bigr)\bigr) = f(x). \tag{3.10}$$

For each $\lambda \in (0,1)$, if $\lambda < \alpha$, taking $\mu = \frac{\alpha-\lambda}{\alpha}$, then $\mu \in (0,1)$, and from Condition C, we have

$$\begin{aligned}
\bar{x} + \mu\eta(y,\bar{x}) &= y + \alpha\eta(x,y) + \frac{\alpha-\lambda}{\alpha}\eta\bigl(y, y + \eta(x,y)\bigr) \\
&= y + \alpha\eta(x,y) + (\lambda-\alpha)\eta(x,y) \\
&= y + \lambda\eta(x,y).
\end{aligned}$$

From the semistrict G-preinvexity of f and (3.10), it follows that

$$\begin{aligned}
f\bigl(y + \lambda\eta(x,y)\bigr) &= f\bigl(\bar{x} + \mu\eta(y,\bar{x})\bigr) \\
&< G^{-1}\bigl(\mu G\bigl(f(y)\bigr) + (1-\mu)G\bigl(f(\bar{x})\bigr)\bigr) \\
&\leq G^{-1}\bigl(\mu G\bigl(f(y)\bigr) + (1-\mu)G\bigl(f(x)\bigr)\bigr) \\
&= f(x).
\end{aligned}$$

If $\lambda > \alpha$, taking $\nu = \frac{\lambda-\alpha}{1-\alpha}$, then $\nu \in (0,1)$, and from Condition C, we have

$$\begin{aligned}
\bar{x} + \nu\eta(x,\bar{x}) &= y + \alpha\eta(x,y) + \frac{\lambda-\alpha}{1-\alpha}\eta\bigl(x, y + \alpha\eta(x,y)\bigr) \\
&= y + \alpha\eta(x,y) + (\lambda-\alpha)\eta(x,y) \\
&= y + \lambda\eta(x,y).
\end{aligned}$$

From the semistrict G-preinvexity of f and (3.10), it follows that

$$\begin{aligned} f\big(y+\lambda\eta(x,y)\big) &= f\big(\bar{x}+\nu\eta(x,\bar{x})\big) \\ &< G^{-1}\big(\nu G\big(f(x)\big)+(1-\nu)G\big(f(\bar{x})\big)\big) \\ &\leq G^{-1}\big(\mu G\big(f(x)\big)+(1-\mu)G\big(f(x)\big)\big) \\ &= f(x). \end{aligned}$$

This completes the proof. □

Theorems 3.1-3.3 improve and generalize the corresponding ones given in [12, 13] from the strictly preinvex case to the strictly G-preinvex case.

4 Conclusions

In this paper, we firstly obtain one property of strictly G-preinvex functions, we consider the strictly G-preinvex functions introduced by Antczak [4]. The relationships between semistrictly G-preinvex functions and strictly G-preinvex functions, G-preinvex functions and strictly G-preinvex functions are investigated under weaker conditions. Our results improve and extend the existing ones in the literature.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The main theorems are proved by TYL. Both authors drafted the manuscript, read and approved the final manuscript.

Acknowledgements
The research was supported by the National Natural Science Foundation of China Grant 10461007, the Foundation of Department of Education of Zhejiang Province under Grant Y201121204 and the Foundation of Tianmu College Grant TMYY1210. The authors would like to express their thanks to the referees for helpful suggestions.

Received: 11 February 2014 Accepted: 13 May 2014 Published: 23 May 2014

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10.1186/1029-242X-2014-210
Cite this article as: Li and Hu: **On the strictly *G*-preinvex function.** *Journal of Inequalities and Applications* 2014, 2014:210