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Generalizations of Hölder inequality and some related results on time scales

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Abstract
In this paper, we establish some generalizations and refinements of the Hölder inequality on time scales via the diamond-α dynamic integral, which is defined as a linear combination of the delta and nabla integrals. Some related inequalities are also considered.
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Keywords: Hölder inequality; generalization; refinement; diamond-α integral; time scale

1 Introduction

Let $\sum_{j=1}^{m}\frac{1}{p_j}=1$. If $f_j(x)>0$ and f_j $(j=1,2,\ldots,m)$ is a continuous real-valued function on $[a,b]$, then

(1) for $p_j>0$, we have the following Hölder inequality (see [1]):

$$\int_a^b\prod_{j=1}^{m}f_j(x)\,dx\le\prod_{j=1}^{m}\left(\int_a^b f_j^{p_j}(x)\,dx\right)^{1/p_j};\tag{1.1}$$

(2) for $0<p_m<1$, $p_j<0$ $(j=1,2,\ldots,m-1)$, we have the following reverse Hölder inequality (see [2]):

$$\int_a^b\prod_{j=1}^{m}f_j(x)\,dx\ge\prod_{j=1}^{m}\left(\int_a^b f_j^{p_j}(x)\,dx\right)^{1/p_j}.\tag{1.2}$$

If $m=2$ and $p_1=p_2=2$, then inequality (1.1) reduces to the famous Cauchy-Schwarz inequality (see [3]). Both the Cauchy-Schwarz inequality and the Hölder inequality play a significant role in different branches of modern mathematics. A great number of generalizations, refinements, variations, and applications of these inequalities have been studied in the literature (see [3–16] and the references therein).

The aim of this paper is to derive some new generalizations and refinements of the diamond-α integral Hölder inequality on time scales. Some related inequalities are also considered. The paper is organized as follows. In Section 2, we recall the basic definitions of time scale calculus, which can also be found in [13, 17–32], and of delta, nabla, and diamond-α dynamic derivatives. In Section 3, we will give the main results.

2 Preliminaries

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of $\mathbb{R}$. The set of the real numbers, the integers, the natural numbers, and the Cantor set are examples of time scales. But the rational numbers, the irrational numbers, the complex numbers, and the open interval between 0 and 1 are not time scales. We first recall some basic concepts from the theory of time scales.

For $t \in \mathbb{T}$, we define the forward jump operator $\sigma : \mathbb{T} \to \mathbb{T}$ by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}$$

and the backward jump operator $\rho : \mathbb{T} \to \mathbb{T}$ by

$$\rho(t) = \sup\{s \in \mathbb{T} : s < t\},$$

where $\inf \emptyset := \sup \mathbb{T}$ and $\sup \emptyset := \inf \mathbb{T}$, $\emptyset$ denotes the empty set.

Definition 2.1 A point $t \in \mathbb{T}$, $t > \inf \mathbb{T}$, is said to be left-dense if $\rho(t) = t$, right-dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, left-scattered if $\rho(t) < t$, and right-scattered if $\sigma(t) > t$.

Definition 2.2 A function $f : \mathbb{T} \to \mathbb{R}$ is called rd-continuous if it is continuous at right-dense points and has finite left-sided limits at left-dense points. A function is called ld-continuous if it is continuous at left-dense points and has finite right-sided limits at right-dense points.

Definition 2.3 Assume that $f : \mathbb{T} \to \mathbb{R}$ is a function, then we define the functions $f^\sigma = f \circ \sigma$ and $f^\rho = f \circ \rho$.

Definition 2.4 Suppose that $f : \mathbb{T} \to \mathbb{R}$ is a function, then for $t \in \mathbb{T}$ we define $f^\Delta(t)$ to be the number, if one exists, such that for all $\varepsilon > 0$ there is a neighborhood U of t such that for all $s \in U$

$$\left|f\left(\sigma(t)\right) - f(s) - f^\Delta(t)\left(\sigma(t) - s\right)\right| \leq \varepsilon\left|\sigma(t) - s\right|.$$

We say that f is delta differentiable on $\mathbb{T}$ provided $f^\Delta(t)$ exists for all $t \in \mathbb{T}$. Similarly, for $t \in \mathbb{T}$ we define $f^\nabla(t)$ to be the number value, if one exists, such that for all $\varepsilon > 0$ there is a neighborhood V of t such that for all $s \in V$

$$\left|f\left(\rho(t)\right) - f(s) - f^\nabla(t)\left(\rho(t) - s\right)\right| \leq \varepsilon\left|\rho(t) - s\right|.$$

We say that f is nabla differentiable on $\mathbb{T}$ provided $f^\nabla(t)$ exists for all $t \in \mathbb{T}$.

We now introduce the basic notions of delta and nabla integrations.

Definition 2.5 An $F : \mathbb{T} \to \mathbb{R}$ with $F^\Delta = f$ is called a Δ-antiderivative of f, and then the Δ-integral of f is defined by $\int_a^t f(s)\Delta s = F(t) - F(a)$ for any $a, t \in \mathbb{T}$. Also, $G : \mathbb{T} \to \mathbb{R}$ with $G^\nabla = f$ is called a ∇-antiderivative of f, and then the ∇-integral of f is defined by $\int_a^t f(s)\nabla s = G(t) - G(a)$ for any $a, t \in \mathbb{T}$. It is known that rd-continuous functions have Δ-antiderivatives and ld-continuous functions have ∇-antiderivatives.

Recently, using the above derivatives and integrations, Sheng *et al.* [30] (see also [27–29]) have established the diamond-α derivative and the diamond-α integration on time scales.

Definition 2.6 If $\alpha \in [0,1]$ and $f : \mathbb{T} \to \mathbb{R}$ is a function, then the $\diamondsuit_\alpha$-differentiation of f at a point $t \in \mathbb{T}$ is defined by

$$f^{\diamondsuit_\alpha}(t) = \alpha f^{\Delta}(t) + (1-\alpha) f^{\nabla}(t).$$

The $\diamondsuit_\alpha$-integral of f is defined by

$$\int_a^t h(\tau)\diamondsuit_\alpha \tau = \alpha \int_a^t h(\tau)\Delta\tau + (1-\alpha)\int_a^t h(\tau)\nabla\tau.$$

Proposition 2.1 (see [30]) *Let $\mathbb{T}$ be a time scale $a,b \in \mathbb{T}$ with $a < b$. Assume that f and g are continuous functions on $[a,b]_{\mathbb{T}}$. Let $c \in [a,b]_{\mathbb{T}}$ and $k \in \mathbb{R}$. Then*

(1) $\int_a^b (f(t)+g(t))\diamondsuit_\alpha t = \int_a^b f(t)\diamondsuit_\alpha t + \int_a^b g(t)\diamondsuit_\alpha t$;
(2) $\int_a^b kf(t)\diamondsuit_\alpha t = k\int_a^b f(t)\diamondsuit_\alpha t$;
(3) $\int_a^b f(t)\diamondsuit_\alpha t = -\int_b^a f(t)\diamondsuit_\alpha t$;
(4) $\int_a^b f(t)\diamondsuit_\alpha t = \int_a^c f(t)\diamondsuit_\alpha t + \int_c^b f(t)\diamondsuit_\alpha t$;
(5) $\int_a^a f(t)\diamondsuit_\alpha t = 0$.

Proposition 2.2 (see [30]) *Let $\mathbb{T}$ be a time scale $a,b \in \mathbb{T}$ with $a < b$. Assume that f and g are continuous functions on $[a,b]_{\mathbb{T}}$,*

(1) *if $f(t) \geq 0$ for all $t \in [a,b]_{\mathbb{T}}$, then $\int_a^b f(t)\diamondsuit_\alpha t \geq 0$;*
(2) *if $f(t) \leq g(t)$ for all $t \in [a,b]_{\mathbb{T}}$, then $\int_a^b f(t)\diamondsuit_\alpha t \leq \int_a^b g(t)\diamondsuit_\alpha t$;*
(3) *if $f(t) \geq 0$ for all $t \in [a,b]_{\mathbb{T}}$, then $f(t) = 0$ if and only if $\int_a^b f(t)\diamondsuit_\alpha t = 0$.*

Results about $\diamondsuit_\alpha$-derivatives and $\diamondsuit_\alpha$-integrals may be found in the papers [28–30, 32].

Throughout this work, we suppose that $\mathbb{T}$ is a time scale, $a,b \in \mathbb{T}$ with $a < b$ and an interval $[a,b]$ means the intersection of a real interval with the given time scale.

3 Main results

In this section, we introduce the following lemma first before we give our results.

Lemma 3.1 (see [33]) *Let $\sum_{j=1}^m \frac{1}{p_j} = 1$, $a_j \geq 0$ $(j = 1,2,\ldots,m)$. Then*

(1) *for $p_j > 1$, we have*

$$\prod_{j=1}^m a_j \leq \sum_{j=1}^m \frac{a_j^{p_j}}{p_j}; \tag{3.1}$$

(2) *for $0 < p_m < 1$, $p_j < 0$ $(j = 1,2,\ldots,m-1)$, we have*

$$\prod_{j=1}^m a_j \geq \sum_{j=1}^m \frac{a_j^{p_j}}{p_j}. \tag{3.2}$$

Theorem 3.2 *Let $\mathbb{T}$ be a time scale, $a,b \in \mathbb{T}$ with $a < b$ and $\sum_{j=1}^m \frac{1}{p_j} = 1$. If $f_j(x) > 0$, and f_j $(j = 1,2,\ldots,m)$ is continuous real-valued function on $[a,b]_{\mathbb{T}}$, then*

(1) *for* $p_j > 1$, *we have*

$$\int_a^b \prod_{j=1}^m f_j(x)\diamond_\alpha x \le \prod_{j=1}^m \left(\int_a^b f_j^{p_j}(x)\diamond_\alpha x\right)^{1/p_j}; \tag{3.3}$$

(2) *for* $0 < p_m < 1$, $p_j < 0$ $(j = 1, 2, \ldots, m-1)$, *we have*

$$\int_a^b \prod_{j=1}^m f_j(x)\diamond_\alpha x \ge \prod_{j=1}^m \left(\int_a^b f_j^{p_j}(x)\diamond_\alpha x\right)^{1/p_j}. \tag{3.4}$$

Proof (1) Let $\varphi_j(x) = \frac{f_j(x)}{(\int_a^b f_j^{p_j}(x)\diamond_\alpha x)^{1/p_j}}$, by (3.1), we have

$$\begin{aligned}\int_a^b \prod_{j=1}^m \varphi_j(x)\diamond_\alpha x &\le \int_a^b \sum_{j=1}^m \frac{\varphi_j^{p_j}(x)}{p_j}\diamond_\alpha x \\ &= \sum_{j=1}^m \frac{1}{p_j}\int_a^b \frac{f_j^{p_j}(x)}{\int_a^b f_j^{p_j}(x)\diamond_\alpha x}\diamond_\alpha x = \sum_{j=1}^m \frac{1}{p_j} = 1.\end{aligned}$$

Therefore, we obtain the desired inequality.

(2) Set $\varphi_j(x) = \frac{f_j(x)}{(\int_a^b f_j^{p_j}(x)\diamond_\alpha x)^{1/p_j}}$, by (3.2), we obtain

$$\begin{aligned}\int_a^b \prod_{j=1}^m \varphi_j(x)\diamond_\alpha x &\ge \int_a^b \sum_{j=1}^m \frac{\varphi_j^{p_j}(x)}{p_j}\diamond_\alpha x \\ &= \sum_{j=1}^m \frac{1}{p_j}\int_a^b \frac{f_j^{p_j}(x)}{\int_a^b f_j^{p_j}(x)\diamond_\alpha x}\diamond_\alpha x = \sum_{j=1}^m \frac{1}{p_j} = 1.\end{aligned}$$

Hence, we have the desired result. □

Theorem 3.3 *Let* $\mathbb{T}$ *be a time scale* $a, b \in \mathbb{T}$ *with* $a < b$ *and* $\alpha_{kj} \in \mathbb{R}$ $(j = 1, 2, \ldots, m$, $k = 1, 2, \ldots, s)$, $\sum_k^s \frac{1}{p_k} = 1$, $\sum_{k=1}^s \alpha_{kj} = 0$. *If* $f_j(x) > 0$, *and* f_j $(j = 1, 2, \ldots, m)$ *is a continuous real-valued function on* $[a, b]_\mathbb{T}$, *then*

(1) *for* $p_k > 1$, *we have the following inequality*:

$$\int_a^b \prod_{j=1}^m f_j(x)\diamond_\alpha x \le \prod_{k=1}^s \left(\int_a^b \prod_{j=1}^m f_j^{1+p_k\alpha_{kj}}(x)\diamond_\alpha x\right)^{1/p_k}; \tag{3.5}$$

(2) $0 < p_s < 1$, $p_k < 0$ $(k = 1, 2, \ldots, s-1)$, *we have the following reverse inequality*:

$$\int_a^b \prod_{j=1}^m f_j(x)\diamond_\alpha x \ge \prod_{k=1}^s \left(\int_a^b \prod_{j=1}^m f_j^{1+p_k\alpha_{kj}}(x)\diamond_\alpha x\right)^{1/p_k}. \tag{3.6}$$

Proof (1) Set

$$g_k(x) = \left(\prod_{j=1}^m f_j^{1+p_k\alpha_{kj}}(x)\right)^{1/p_k}. \tag{3.7}$$

Applying the assumptions $\sum_{k}^{s}\frac{1}{p_k}=1$ and $\sum_{k=1}^{s}\alpha_{kj}=0$, by computing, we can observe that

$$\begin{aligned}\prod_{k=1}^{s}g_k(t)&=g_1g_2\cdots g_s\\&=\left(\prod_{j=1}^{m}f_j^{1+a_1\alpha_{1j}}(t)\right)^{1/a_1}\left(\prod_{j=1}^{m}f_j^{1+a_2\alpha_{2j}}(t)\right)^{1/a_2}\cdots\left(\prod_{j=1}^{m}f_j^{1+a_s\alpha_{sj}}(t)\right)^{1/a_s}\\&=\prod_{j=1}^{m}f_j^{1/a_1+\alpha_{1j}}(t)\prod_{j=1}^{m}f_j^{1/a_2+\alpha_{2j}}(t)\cdots\prod_{j=1}^{m}f_j^{1/a_s+\alpha_{sj}}(t)\\&=\prod_{j=1}^{m}f_j^{1/a_1+1/a_2+\cdots+1/a_s+\alpha_{1j}+\alpha_{2j}+\cdots+\alpha_{sj}}(t)=\prod_{j=1}^{m}f_j(t).\end{aligned}$$

That is,

$$\prod_{k=1}^{s}g_k(t)=\prod_{j=1}^{m}f_j(t).$$

Hence, we obtain

$$\int_a^b\prod_{j=1}^{m}f_j(x)\diamond_\alpha x=\int_a^b\prod_{k=1}^{s}g_k(x)\diamond_\alpha x. \tag{3.8}$$

By the Hölder inequality (3.3), we find

$$\int_a^b\prod_{k=1}^{s}g_k(x)\diamond_\alpha x\le\prod_{k=1}^{s}\left(\int_a^b g_k^{p_k}(x)\diamond_\alpha x\right)^{1/p_k}. \tag{3.9}$$

Substitution of $g_k(x)$ in (3.9) conduce to inequality (3.5) immediately. This proves inequality (3.5).

(2) This proof is similar to the proof of inequality (3.5), by (3.7), (3.8), and the reverse Hölder inequality (3.4), we have

$$\int_a^b\prod_{k=1}^{s}g_k(x)\diamond_\alpha x\ge\prod_{k=1}^{s}\left(\int_a^b g_k^{p_k}(x)\diamond_\alpha x\right)^{1/p_k}. \tag{3.10}$$

Substitution of $g_k(x)$ in (3.10) leads to inequality (3.6) immediately. □

Corollary 3.1 *Under the conditions of Theorem* 3.3, *let* $s=m$, $\alpha_{kj}=-t/p_k$ *for* $k\neq j$ *and* $\alpha_{jj}=t(1-1/p_j)$ *with* $t\in\mathrm{R}$, *then*

(1) *for* $p_k>1$, *we have the following inequality*:

$$\int_a^b\prod_{j=1}^{m}f_j(x)\diamond_\alpha x\le\prod_{k=1}^{m}\left(\int_a^b\left(\prod_{j=1}^{m}f_j(x)\right)^{1-t}\left(f_k^{p_k}\right)^t(x)\diamond_\alpha x\right)^{1/p_k}; \tag{3.11}$$

(2) $0 < p_m < 1$, $p_k < 0$ $(k = 1, 2, \ldots, m-1)$, *we have the following reverse inequality*:

$$\int_a^b \prod_{j=1}^m f_j(x) \diamondsuit_\alpha x \geq \prod_{k=1}^m \left(\int_a^b \left(\prod_{j=1}^m f_j(x) \right)^{1-t} \left(f_k^{p_k} \right)^t (x) \diamondsuit_\alpha x \right)^{1/p_k}. \tag{3.12}$$

Theorem 3.4 *Let $\mathbb{T}$ be a time scale, $a, b \in \mathbb{T}$ with $a < b$ and $r \in \mathrm{R}$, $\alpha_{kj} \in \mathrm{R}$ $(j = 1, 2, \ldots, m$, $k = 1, 2, \ldots, s)$, $\sum_k^s \frac{1}{p_k} = r, \sum_{k=1}^s \alpha_{kj} = 0$. If $f_j(x) > 0$, and f_j $(j = 1, 2, \ldots, m)$ is a continuous real-valued function on $[a, b]_\mathbb{T}$, then*

(1) *for $rp_k > 1$, we have the following inequality*:

$$\int_a^b \prod_{j=1}^m f_j(x) \diamondsuit_\alpha x \leq \prod_{k=1}^s \left(\int_a^b \prod_{j=1}^m f_j^{1+rp_k\alpha_{kj}}(x) \diamondsuit_\alpha x \right)^{1/rp_k}; \tag{3.13}$$

(2) *for $0 < rp_s < 1$, $rp_k < 0$ $(k = 1, 2, \ldots, s-1)$, we have the following reverse inequality*:

$$\int_a^b \prod_{j=1}^m f_j(x) \diamondsuit_\alpha x \geq \prod_{k=1}^s \left(\int_a^b \prod_{j=1}^m f_j^{1+rp_k\alpha_{kj}}(x) \diamondsuit_\alpha x \right)^{1/rp_k}. \tag{3.14}$$

Proof (1) Since $rp_k > 1$ and $\sum_k^s \frac{1}{p_k} = r$, we get $\sum_k^s \frac{1}{rp_k} = 1$. Then by (3.5), we immediately obtain the inequality (3.13).

(2) Since $0 < rp_s < 1$, $rp_k < 0$ $(k = 1, 2, \ldots, s-1)$ and $\sum_k^s \frac{1}{p_k} = r$, we have $\sum_k^s \frac{1}{rp_k} = 1$, by (3.6), we immediately have the inequality (3.14). This completes the proof. □

Recently, Yang [11] established an extension of the Callebaut inequality, that is,

$$\int_a^b \prod_{j=1}^m f_j(x) \diamondsuit_\alpha x \leq \left(\int_a^b \prod_{j=1}^m f_j^{1+\alpha_j}(x) \diamondsuit_\alpha x \right)^{1/2} \left(\int_a^b \prod_{j=1}^m f_j^{1-\alpha_j}(x) \diamondsuit_\alpha x \right)^{1/2}. \tag{3.15}$$

From Theorem 3.4, we obtain a Hölder type generalization of (3.15) as follows.

Corollary 3.2 *Under the conditions of Theorem* 3.4, *and taking $s = 2$, $p_1 = p$, $p_2 = q$, $\alpha_{1j} = -\alpha_{2j} = \alpha_j$, then*

(1) *for $rp, rq > 1$, we have the following inequality*:

$$\int_a^b \prod_{j=1}^m f_j(x) \diamondsuit_\alpha x \leq \left(\int_a^b \prod_{j=1}^m f_j^{1+rp\alpha_j}(x) \diamondsuit_\alpha x \right)^{1/rp} \left(\int_a^b \prod_{j=1}^m f_j^{1-rq\alpha_j}(x) \diamondsuit_\alpha x \right)^{1/rq}; \tag{3.16}$$

(2) *for $0 < rp < 1$, $rq < 0$, we have the following reverse inequality*:

$$\int_a^b \prod_{j=1}^m f_j(x) \diamondsuit_\alpha x \geq \left(\int_a^b \prod_{j=1}^m f_j^{1+rp\alpha_j}(x) \diamondsuit_\alpha x \right)^{1/rp} \left(\int_a^b \prod_{j=1}^m f_j^{1-rq\alpha_j}(x) \diamondsuit_\alpha x \right)^{1/rq}. \tag{3.17}$$

Now we present a refinement of inequality (3.13) and (3.14), respectively.

Theorem 3.5 *Under the conditions of Theorem* 3.4, *we have*

(1) *for* $rp_k > 1$, *we have the following inequality*:

$$\int_a^b \prod_{j=1}^m f_j(x)\diamond_\alpha x \le \varphi(c) \le \prod_{k=1}^s \left(\int_a^b \prod_{j=1}^m f_j^{1+rp_k\alpha_{kj}}(x)\diamond_\alpha x\right)^{1/rp_k}, \tag{3.18}$$

where

$$\varphi(c) \equiv \int_a^c \prod_{j=1}^m f_j(x)\diamond_\alpha x + \prod_{k=1}^s \left(\int_c^b \prod_{j=1}^m f_j^{1+rp_k\alpha_{kj}}(x)\diamond_\alpha x\right)^{1/rp_k}$$

is a nonincreasing function with $a \le c \le b$;

(2) *for* $0 < rp_s < 1$, $rp_k < 0$ $(k = 1, 2, \ldots, s-1)$, *we have the following reverse inequality*:

$$\int_a^b \prod_{j=1}^m f_j(x)\diamond_\alpha x \ge \phi(c) \ge \prod_{k=1}^s \left(\int_a^b \prod_{j=1}^m f_j^{1+rp_k\alpha_{kj}}(x)\diamond_\alpha x\right)^{1/rp_k}, \tag{3.19}$$

where

$$\phi(c) \equiv \int_a^c \prod_{j=1}^m f_j(x)\diamond_\alpha x + \prod_{k=1}^s \left(\int_c^b \prod_{j=1}^m f_j^{1+rp_k\alpha_{kj}}(x)\diamond_\alpha x\right)^{1/rp_k}$$

is a nondecreasing function with $a \le c \le b$.

Proof (1) Let

$$g_k(x) = \left(\prod_{j=1}^m f_j^{1+rp_k\alpha_{kj}}(x)\right)^{1/rp_k}.$$

By rearrangement, using the assumptions of Theorem 3.4, we have

$$\prod_{j=1}^m f_j(x) = \prod_{k=1}^s g_k(x).$$

Then by the Hölder inequality (3.3), we obtain

$$\begin{aligned}\int_a^b \prod_{j=1}^m f_j(x)\diamond_\alpha x &= \int_a^b \prod_{k=1}^s g_k(x)\diamond_\alpha x = \int_a^c \prod_{k=1}^s g_k(x)\diamond_\alpha x + \int_c^b \prod_{k=1}^s g_k(x)\diamond_\alpha x \\ &\le \int_a^c \prod_{k=1}^s g_k(x)\diamond_\alpha x + \prod_{k=1}^s \left(\int_c^b g_k^{rp_k}(x)\diamond_\alpha x\right)^{1/rp_k} \\ &\le \prod_{k=1}^s \left(\int_a^c g_k^{rp_k}(x)\diamond_\alpha x + \int_c^b g_k^{rp_k}(x)\diamond_\alpha x\right)^{1/rp_k}\end{aligned}$$

$$= \prod_{k=1}^{s} \left(\int_a^b g_k^{rp_k}(x) \diamond_\alpha x \right)^{1/rp_k}$$

$$= \prod_{k=1}^{s} \left(\int_a^b \prod_{j=1}^{m} f_j^{1+rp_k\alpha_{kj}}(x) \diamond_\alpha x \right)^{1/rp_k}.$$

Therefore, we obtain the desired result.

(2) This proof is similar to the proof of inequality (3.18), we have inequality (3.19). □

Remark 3.1 Taking $s = 2$, Theorem 3.5 presents refinement of (3.16) and (3.17). Moreover, letting $\mathbb{T} = \mathbb{R}$ and $\mathbb{T} = \mathbb{Z}$, then the results of this paper lead to the main results of [13].

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
This paper is the result of joint work of all authors who contributed equally to the final version of this paper. All authors read and approved the final manuscript.

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