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Multivalued fixed point theorems in cone b-metric spaces

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Abstract
In this paper we extend the Banach contraction for multivalued mappings in a cone b-metric space without the assumption of normality on cones and generalize some attractive results in literature.
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Keywords: cone b-metric space; non-normal cones; multivalued contraction; fixed points

1 Introduction

The analysis on existence of linear and nonlinear operators was developed after the Banach contraction theorem [1] presented in 1922. Many generalizations are available with applications in the literature [2–13]. Nadler [14] gave its set-valued form in his classical paper in 1969 on multivalued contractions. A real generalization of Nadler's theorem was presented by Mizoguchi and Takahashi [15] as follows.

Theorem 1.1 [15] *Let (X,d) be a complete metric space and let $T : X \to 2^X$ be a multivalued map such that Tx is a closed bounded subset of X for all $x \in X$. If there exists a function $\varphi : (0,\infty) \to [0,1)$ such that $\lim_{r\to t^+} \sup \varphi(r) < 1$ for all $t \in [0,\infty)$ and if*

$$H(Tx, Ty) \le \varphi\big(d(x,y)\big)\big(d(x,y)\big) \quad \textit{for all } x, y \in X,$$

then T has a fixed point in X.

Huang and Zhang [10] introduced a cone metric space with normal cone as a generalization of a metric space. Rezapour and Hamlbarani [16] presented the results of [10] for the case of a cone metric space without normality in cone. Many authors worked on it (see [17]). Cho and Bae [18] presented the result of [15] for multivalued mappings in cone metric spaces with normal cone.

Recently Hussain and Shah [19] introduced the notion of cone b-metric spaces as a generalization of b-metric and cone metric spaces. In [20] the authors presented some fixed point results in cone b-metric spaces without assumption of normality on cone.

In this article we present the generalized form of Cho and Bae [18] for the case of cone b-metric spaces without normality on cone. We also give an example to support our main theorem.

2 Preliminaries

Let $\mathbb{E}$ be a real Banach space and P be a subset of $\mathbb{E}$. By θ we denote the zero element of $\mathbb{E}$ and by $\operatorname{int} P$ the interior of P. The subset P is called a cone if and only if:

(i) P is closed, nonempty, and $P = \{\theta\}$;
(ii) $a, b \in \mathbb{R}$, $a, b \geq 0$, $x, y \in P \Rightarrow ax + by \in P$;
(iii) $P \cap (-P) = \{\theta\}$.

For a given cone $P \subseteq \mathbb{E}$, we define a partial ordering $\preceq$ with respect to P by $x \preceq y$ if and only if $y - x \in P$; $x \prec y$ will stand for $x \preceq y$ and $x \neq y$, while $x \ll y$ will stand for $y - x \in \operatorname{int} P$, where $\operatorname{int} P$ denotes the interior of P. The cone P is said to be solid if it has a nonempty interior.

Definition 2.1 [19] Let X be a nonempty set and $r \geq 1$ be a given real number. A function $d : X \times X \to \mathbb{E}$ is said to be a cone b-metric if the following conditions hold:

(C1) $\theta \preceq d(x,y)$ for all $x, y \in X$ and $d(x,y) = \theta$ if and only if $x = y$;
(C2) $d(x,y) = d(y,x)$ for all $x, y \in X$;
(C3) $d(x,z) \preceq r[d(x,y) + d(y,z)]$ for all $x, y, z \in X$.

The pair (X,d) is then called a cone b-metric space.

Example 2.1 [20] Let $X = l^p$ with $0 < p < 1$, where $l^p = \{\{x_n\} \subset \mathbb{R} : \sum_{n=1}^{+\infty} |x_n|^p < \infty\}$. Let $d : X \times X \to \mathbb{R}$ be defined as

$$d(x,y) = \left(\sum_{n=1}^{+\infty} |x_n - y_n|^p \right)^{\frac{1}{p}},$$

where $x = \{x_n\}, y = \{y_n\} \in l^p$. Then (X,d) is a b-metric space. Put $E = l^1$, $P = \{\{x_n\} \in E : x_n \geq 0 \text{ for all } n \geq 1\}$. Letting the map $d' : X \times X \to E$ be defined by $d'(x,y) = \{\frac{d(x,y)}{2^n}\}_{n \geq 1}$, we conclude that (X,d') is a cone b-metric space with the coefficient $r = 2^{\frac{1}{p}} > 1$, but is not a cone metric space.

Example 2.2 [20] Let $X = \{1,2,3,4\}$, $E = \mathbb{R}^2$, $P = \{(x,y) \in E : x \geq 0, y \geq 0\}$. Define $d : X \times X \to E$ by

$$d(x,y) = \begin{cases} (|x-y|^{-1}, |x-y|^{-1}) & \text{if } x \neq y, \\ \theta & \text{if } x = y. \end{cases}$$

Then (X,d) is a cone b-metric space with coefficient $r = \frac{6}{5}$. But it is not a cone metric space, because

$$d(1,2) > d(1,4) + d(4,2),$$
$$d(3,4) > d(3,1) + d(1,4).$$

Remark 2.1 [19] The class of cone b-metric spaces is larger than the class of cone metric spaces since any cone metric space must be a cone metric b-metric space. Therefore, it is obvious that cone b-metric spaces generalize b-metric spaces and cone metric spaces.

Definition 2.2 [19] Let (X,d) be a cone b-metric space, $x \in X$, let $\{x_n\}$ be a sequence in X. Then

(i) $\{x_n\}$ converges to x whenever for every $c \in \mathbb{E}$ with $\theta \ll c$ there is a natural number n_0 such that $d(x_n, x) \ll c$ for all $n \geq n_0$. We denote this by $\lim_{n\to\infty} x_n = x$;
(ii) $\{x_n\}$ is a Cauchy sequence whenever for every $c \in \mathbb{E}$ with $\theta \ll c$ there is a natural number n_0 such that $d(x_n, x_m) \ll c$ for all $n, m \geq n_0$;
(iii) (X, d) is complete cone b-metric if every Cauchy sequence in X is convergent.

Remark 2.2 [17] The results concerning fixed points and other results, in case of cone spaces with non-normal solid cones, cannot be provided by reducing to metric spaces, because in this case neither of the conditions of Lemmas 1-4 in [10] hold. Further, the vector cone metric is not continuous in the general case, *i.e.*, from $x_n \to x$, $y_n \to y$ it need not follow that $d(x_n, y_n) \to d(x, y)$.

Let $\mathbb{E}$ be an ordered Banach space with a positive cone P. The following properties hold [17, 19]:
(PT1) If $u \preceq v$ and $v \ll w$, then $u \ll w$.
(PT2) If $u \ll v$ and $v \preceq w$, then $u \ll w$.
(PT3) If $u \ll v$ and $v \ll w$, then $u \ll w$.
(PT4) If $\theta \preceq u \ll c$ for each $c \in \operatorname{int} P$, then $u = \theta$.
(PT5) If $a \preceq b + c$ for each $c \in \operatorname{int} P$, then $a \preceq b$.
(PT6) Let $\{a_n\}$ be a sequence in $\mathbb{E}$. If $c \in \operatorname{int} P$ and $a_n \to \theta$ (as $n \to \infty$), then there exists $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, we have $a_n \ll c$.

3 Main result

According to [18], we denote by Λ a family of nonempty closed and bounded subsets of X, and

$$s(p) = \{q \in \mathbb{E} : p \preceq q\} \quad \text{for } q \in \mathbb{E},$$
$$s(a, B) = \bigcup_{b\in B} s\big(d(a, b)\big) = \bigcup_{b\in B} \big\{x \in \mathbb{E} : d(a, b) \preceq x\big\} \quad \text{for } a \in X \text{ and } B \in \Lambda.$$

For $A, B \in \Lambda$, we define

$$s(A, B) = \left(\bigcap_{a\in A} s(a, B)\right) \cap \left(\bigcap_{b\in B} s(b, A)\right).$$

Remark 3.1 Let (X, d) be a cone b-metric space. If $\mathbb{E} = \mathbb{R}$ and $P = [0, +\infty)$, then (X, d) is a b-metric space. Moreover, for $A, B \in CB(X)$, $H(A, B) = \inf s(A, B)$ is the Hausdorff distance induced by d.

Now, we start with the main result of this paper.

Theorem 3.1 *Let (X, d) be a complete cone b-metric space with the coefficient $r \geq 1$ and cone P, and let $T : X \to \Lambda$ be a multivalued mapping. If there exists a function $\varphi : P \to [0, \frac{1}{r})$ such that*

$$\lim_{n\to\infty} \sup \varphi(a_n) < \frac{1}{r} \tag{a}$$

for any decreasing sequence $\{a_n\}$ *in* P. *If for all* $x, y \in X$,

$$\varphi\big(d(x,y)\big)d(x,y) \in s(Tx, Ty), \tag{b}$$

then T *has a fixed point in* X.

Proof Let x_0 be an arbitrary point in X, then $Tx_0 \in \Lambda$, so $Tx_0 \neq \phi$. Let $x_1 \in Tx_0$ and consider

$$\varphi\big(d(x_0,x_1)\big)d(x_0,x_1) \in s(Tx_0, Tx_1).$$

By definition we have

$$\varphi\big(d(x_0,x_1)\big)d(x_0,x_1) \in \Big(\bigcap_{x\in Tx_0} s(x, Tx_1)\Big) \cap \Big(\bigcap_{y\in Tx_1} s(y, Tx_0)\Big),$$

which implies

$$\varphi\big(d(x_0,x_1)\big)d(x_0,x_1) \in s(x, Tx_1) \quad \text{for all } x \in Tx_0.$$

Since $x_1 \in Tx_0$, so we have

$$\varphi\big(d(x_0,x_1)\big)d(x_0,x_1) \in s(x_1, Tx_1).$$

We have

$$\varphi\big(d(x_0,x_1)\big)d(x_0,x_1) \in \bigcup_{x\in Tx_1} s\big(d(x_1,x)\big).$$

So there exists some $x_2 \in Tx_1$ such that

$$\varphi\big(d(x_0,x_1)\big)d(x_0,x_1) \in s\big(d(x_1,x_2)\big).$$

It gives

$$d(x_1,x_2) \preceq \varphi\big(d(x_0,x_1)\big)d(x_0,x_1).$$

By induction we can construct a sequence $\{x_n\}$ in X such that

$$d(x_n,x_{n+1}) \preceq \varphi\big(d(x_{n-1},x_n)\big)d(x_{n-1},x_n), \quad x_{n+1} \in Tx_n \text{ for } n \in \mathbb{N}. \tag{c}$$

If $x_n = x_{n+1}$ for some $n \in \mathbb{N}$, then T has a fixed point. Assume that $x_n \neq x_{n+1}$, then from (c) the sequence $\{d(x_n,x_{n+1})\}$ is decreasing in P. Hence from (a) there exists $a \in (0, \frac{1}{r})$ such that

$$\lim_{n\to\infty} \sup \varphi\big(d(x_n,x_{n+1})\big) < a.$$

Thus, for any $k \in (a, \frac{1}{r})$, there exists some $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, implies $\varphi(d(x_n, x_{n+1})) < k$. Now consider, for all $n \geq n_0$,

$$\begin{aligned} d(x_n, x_{n+1}) &\preceq \varphi\bigl(d(x_{n-1}, x_n)\bigr)d(x_{n-1}, x_n) \prec kd(x_{n-1}, x_n) \prec k^{n-n_0}d(x_{n_0}, x_{n_0+1}) \\ &= k^n v_0, \end{aligned}$$

where $v_0 = k^{-n_0}d(x_{n_0}, x_{n_0+1})$.

Let $m > n \geq n_0$. Applying (C3) to triples $\{x_n, x_{n+1}, x_m\}, \{x_{n+1}, x_{n+2}, x_m\}, \ldots, \{x_{m-2}, x_{m-1}, x_m\}$, we obtain

$$\begin{aligned} d(x_n, x_m) &\preceq r\bigl[d(x_n, x_{n+1}) + d(x_{n+1}, x_m)\bigr] \\ &\preceq rd(x_n, x_{n+1}) + r^2\bigl[d(x_{n+1}, x_{n+2}) + d(x_{n+2}, x_m)\bigr] \\ &\preceq \cdots \\ &\preceq rd(x_n, x_{n+1}) + r^2 d(x_{n+1}, x_{n+2}) + \cdots + r^{m-n-1}\bigl[d(x_{m-2}, x_{m-1}) + d(x_{m-1}, x_m)\bigr] \\ &\preceq rd(x_n, x_{n+1}) + r^2 d(x_{n+1}, x_{n+2}) + \cdots + r^{m-n-1}d(x_{m-2}, x_{m-1}) + r^{m-n}d(x_{m-1}, x_m). \end{aligned}$$

Now $d(x_n, x_{n+1}) \preceq k^n v_0$ and $kr < 1$ imply that

$$\begin{aligned} d(x_n, x_m) &\preceq \bigl(rk^n + r^2k^{n+1} + \cdots + r^{m-n}k^{m-1}\bigr)v_0 \\ &= rk^n\bigl(1 + (rk) + \cdots + (rk)^{m-n-1}\bigr)v_0 \\ &\preceq \frac{rk^n}{1 - rk}v_0 \to \theta \quad \text{when } n \to \infty. \end{aligned}$$

Now, according to (PT6) and (PT1), we obtain that for a given $\theta \ll c$ there exists $m_0 \in \mathbb{N}$ such that

$$d(x_n, x_m) \ll c \quad \text{for all } m, n > m_0,$$

that is, $\{x_n\}$ is Cauchy sequence in (X, d). Since (X, d) is a complete cone b-metric space, so there exists some $u \in X$ such that $x_n \to u$. Take $k_0 \in \mathbb{N}$ such that $d(x_n, u) \ll \frac{c}{2r}$ for all $n \geq k_0$. Now we will prove $u \in Tu$. For this let us consider

$$\varphi\bigl(d(x_n, u)\bigr)d(x_n, u) \in s(Tx_n, Tu).$$

By definition we have

$$\varphi\bigl(d(x_n, u)\bigr)d(x_n, u) \in \biggl(\bigcap_{x \in Tx_n} s(x, Tu)\biggr) \cap \biggl(\bigcap_{y \in Tu} s(y, Tx_n)\biggr),$$

which implies

$$\varphi\bigl(d(x_n, u)\bigr)d(x_n, u) \in \biggl(\bigcap_{x \in Tx_n} s(x, Tu)\biggr),$$

$$\varphi\bigl(d(x_n, u)\bigr)d(x_n, u) \in s(x, Tu) \quad \text{for all } x \in Tx_n.$$

Since $x_{n+1} \in Tx_n$, so we have

$$\varphi\big(d(x_n, u)\big)d(x_n, u) \in s(x_{n+1}, Tu).$$

So there exists some $v_n \in Tu$ such that

$$\varphi\big(d(x_n, u)\big)d(x_n, u) \in s\big(d(x_{n+1}, v_n)\big).$$

It gives

$$d(x_{n+1}, v_n) \preceq \varphi\big(d(x_n, u)\big)d(x_n, u) \preceq d(x_n, u). \tag{d}$$

Now consider

$$\begin{aligned} d(u, v_n) &\preceq r\big[d(u, x_{n+1}) + d(x_{n+1}, v_n)\big] \\ &\preceq rd(u, x_{n+1}) + rd(x_n, u) \\ &\ll \frac{c}{2} + \frac{c}{2} = c \quad \text{for all } n \geq k_0, \end{aligned}$$

which means $v_n \to u$, since Tu is closed so $u \in Tu$. □

Corollary 3.1 [18] *Let (X, d) be a complete cone metric space with a normal cone P, and let $T : X \to CB(X)$ be a multivalued mapping. If there exists a function $\varphi : P \to [0, 1)$ such that*

$$\lim_{n\to\infty} \sup \varphi(a_n) < 1$$

for any decreasing sequence $\{a_n\}$ in P. If for all $x, y \in X$,

$$\varphi\big(d(x, y)\big)d(x, y) \in s(Tx, Ty),$$

then T has a fixed point in X.

Corollary 3.2 [15] *Let (X, d) be a complete metric space and let $T : X \to 2^X$ be a multivalued map such that Tx is a closed bounded subset of X for all $x \in X$. If there exists a function $\varphi : (0, \infty) \to [0, 1)$ such that* $\limsup_{r\to t^+} \varphi(r) < 1$ *for all $t \in [0, \infty)$ and if*

$$H(Tx, Ty) \leq \varphi\big(d(x, y)\big)\big(d(x, y)\big) \quad \textit{for all } x, y \in X,$$

then T has a fixed point in X.

The following is Nadler's theorem for multivalued mappings in a complete metric space.

Corollary 3.3 [14] *Let (X, d) be a complete metric space and let $T : X \to 2^X$ be a multivalued map such that Tx is a closed bounded subset of X for all $x \in X$. If there exists $k \in [0, 1)$ such that*

$$H(Tx, Ty) \leq kd(x, y) \quad \textit{for all } x, y \in X,$$

then T has a fixed point in X.

Example 3.1 Let $X = [0,1]$ and $\mathbb{E}$ be the set of all real-valued functions on X which also have continuous derivatives on X. Then $\mathbb{E}$ is a vector space over $\mathbb{R}$ under the following operations:

$$(x+y)(t) = x(t) + y(t), \qquad (\alpha x)(t) = \alpha x(t)$$

for all $x, y \in \mathbb{E}$, $\alpha \in \mathbb{R}$. That is, $E = C_R^1[0,1]$ with the norm $\|f\| = \|f\|_\infty + \|f'\|_\infty$ and

$$P = \{x \in \mathbb{E} : \theta \preceq x\}, \quad \text{where } \theta(t) = 0 \text{ for all } t \in X,$$

then P is a non-normal cone. Define $d : X \times X \to \mathbb{E}$ as follows:

$$\bigl(d(x,y)\bigr)(t) = |x-y|^p e^t \quad \text{for } p > 1.$$

Then (X,d) is a cone b-metric space but not a cone metric space. For $x, y, z \in X$, set $u = x - z$, $v = z - y$, so $x - y = u + v$. From the inequality

$$(a+b)^p \le \bigl(2\max\{a,b\}\bigr)^p \le 2^p\bigl(a^p + b^p\bigr) \quad \text{for all } a, b \ge 0,$$

we have

$$|x-y|^p = |u+v|^p \le \bigl(|u| + |v|\bigr)^p \le 2^p\bigl(|u|^p + |v|^p\bigr) = 2^p\bigl(|x-z|^p + |z-y|^p\bigr),$$
$$|x-y|^p e^t \le 2^p\bigl(|x-z|^p e^t + |z-y|^p e^t\bigr),$$

which implies that

$$d(x,y) \preccurlyeq r\bigl[d(x,z) + d(y,z)\bigr] \quad \text{with } r = 2^p > 1.$$

But

$$|x-y|^p e^t \le |x-z|^p e^t + |z-y|^p e^t$$

is impossible for all $x > z > y$. Indeed, taking advantage of the inequality

$$(a+b)^p > a^p + b^p,$$

we have

$$|x-y|^p > |x-z|^p + |z-y|^p,$$
$$|x-y|^p e^t > |x-z|^p e^t + |z-y|^p e^t$$

for all $x > z > y$. Thus the triangular inequality in a cone metric space is not satisfied, so (X,d) is not a cone metric space but is a cone b-metric space.

Let $T : X \to \Lambda$ be such that

$$Tx = \left[0, \frac{x}{30}\right],$$

then we have, for $x < y$,

$$s(Tx, Ty) = s\left(\left|\frac{x}{30} - \frac{y}{30}\right|^p e^t\right).$$

Since

$$\left|\frac{x}{30} - \frac{y}{30}\right|^p e^t \preceq \frac{1}{3^p}|x-y|^p e^t,$$

so

$$\frac{1}{3^p}\left(|x-y|^p e^t\right) \in s\left(\left|\frac{x}{30} - \frac{y}{30}\right|^p e^t\right).$$

Hence, for $\varphi(d(x,y)) = \frac{1}{3^p}$, we have

$$\varphi\big(d(x,y)\big)d(x,y) \in s(Tx, Ty).$$

All conditions of our main theorems are satisfied, so T has a fixed point.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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