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Asymptotic behavior of second-order nonlinear neutral dynamic equations

Yefei Jing, Chenghui Zhang* and Tongxing Li

*Correspondence:
zchui@sdu.edu.cn
School of Control Science and Engineering, Shandong University, Jinan, Shandong 250061, P.R. China

Abstract

This paper is concerned with oscillation and asymptotic behavior of a second-order neutral delay dynamic equation on an arbitrary time scale. We obtain two theorems which guarantee that every solution of the studied equation oscillates or converges to zero. These results improve and complement some known results given in the literature.

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Keywords: asymptotic behavior; oscillation; neutral delay dynamic equation; time scale

1 Introduction

In this paper, we study oscillation and asymptotic behavior of a second-order nonlinear neutral delay dynamic equation

$$\Bigl(r(t)\bigl(\bigl(x(t)+p(t)x\bigl(\eta(t)\bigr)\bigr)^{\Delta}\bigr)^{\gamma}\Bigr)^{\Delta}+f\bigl(t,x\bigl(g(t)\bigr)\bigr)=0 \tag{1.1}$$

on an arbitrary time scale $\mathbb{T}$, where γ is a quotient of odd positive integers, r and p are positive rd-continuous functions on $\mathbb{T}$, $0 \le p(t) \le p_1 < 1$. Also, we assume that $\eta, g : \mathbb{T} \to \mathbb{T}$ are rd-continuous, $\eta(t) \le t$, $g(t) \le t$, $\lim_{t\to\infty} \eta(t) = \lim_{t\to\infty} g(t) = \infty$, $uf(t,u) > 0$ for all $u \neq 0$, and there exists a positive rd-continuous function q defined on $\mathbb{T}$ such that $|f(t,u)| \ge q(t)|u|^{\gamma}$.

The theory of dynamic equations on time scales, which goes back to its founder Hilger [1], has recently attracted attention of researchers. Several authors have expounded on various aspects of this new theory; see the survey paper written by Agarwal *et al.* [2] and the references cited therein. The books on the subject of time scales, by Bohner and Peterson [3, 4], present much of time scale calculus.

Since we are interested in oscillatory and asymptotic properties, we assume throughout this paper that the given time scale $\mathbb{T}$ is unbounded above. We assume that $t_0 \in \mathbb{T}$, and it is convenient to assume that $t_0 > 0$, and define the time scale interval of the form $[t_0,\infty)_{\mathbb{T}}$ by $[t_0,\infty)_{\mathbb{T}} := [t_0,\infty) \cap \mathbb{T}$. Throughout, we use the notation $z := x + px \circ \eta$. By a solution of equation (1.1), we mean a non-trivial real-valued function $x \in C^1_{\mathrm{rd}}\,[T_x,\infty)_{\mathbb{T}}$, $T_x \in [t_0,\infty)_{\mathbb{T}}$ which has the property that z and $r(z^{\Delta})^{\gamma}$ are defined and Δ-differentiable for $t \in \mathbb{T}$ and satisfies equation (1.1) on $[T_x,\infty)_{\mathbb{T}}$. The solutions vanishing in some neighborhood of infinity will be excluded from our consideration. A solution x of equation (1.1) is said to be oscillatory if it is neither eventually positive nor eventually negative; otherwise, it is called nonoscillatory.

In recent years, there has been much research activity concerning oscillation and nonoscillation of solutions to neutral differential and dynamic equations on time scales, we refer the reader to [5, 6] and [7–22], and the references cited therein. Han *et al.* [6] studied a second-order nonlinear neutral equation

$$\left(r(t)\left|u'(t)\right|^{\alpha-1}u'(t)\right)' + q(t)f\left(x\left(\delta(t)\right)\right) = 0, \tag{1.2}$$

where $u := x + px \circ \tau$, and established two results which guarantee that every solution of equation (1.2) is oscillatory under the assumptions that

$$p'(t) \geq 0, \qquad \sigma(t) \leq \tau(t) = t - \tau_0 \leq t, \tag{1.3}$$

and

$$\int_{t_0}^{\infty} \frac{\mathrm{d}t}{r^{1/\gamma}(t)} < \infty.$$

Agarwal *et al.* [7], Erbe *et al.* [8], Şahiner [9], Saker [10], Saker *et al.* [11], Saker and O'Regan [12], Chen [13], Zhang and Wang [14], Wu *et al.* [15], Candan [17], and Li *et al.* [19] investigated equation (1.1) and obtained some oscillation criteria in the case

$$\int_{t_0}^{\infty} \frac{\Delta t}{r^{1/\gamma}(t)} = \infty. \tag{1.4}$$

As yet, there are few results regarding the study of asymptotic behavior of equation (1.1) under the assumption that

$$\int_{t_0}^{\infty} \frac{\Delta t}{r^{1/\gamma}(t)} < \infty. \tag{1.5}$$

In 2007, Saker *et al.* [11] posed an open problem as follows: How to establish oscillation criteria for equation (1.1) when condition (1.5) holds? Assuming (1.5), Zhang *et al.* [21, 22] obtained some sufficient conditions which insure that all solutions of equation (1.1) are oscillatory.

The purpose of this paper is to present some asymptotic tests for equation (1.1) in the case where (1.5) holds. This paper is organized as follows: In the next section, we shall establish the main results. In Section 3, two examples are provided to illustrate the results obtained.

In the sequel, when we write a functional inequality without specifying its domain of validity, we assume that it holds for all sufficiently large t.

2 Main results

In what follows, we use the notation

$$\delta_+^{\Delta}(t) := \max\left\{0, \delta^{\Delta}(t)\right\}, \qquad Q(t) := q(t)\left(1 - p\left(g(t)\right)\right)^{\gamma}, \qquad \theta(t,u) := \frac{\int_u^{g(t)} \Delta s/r^{1/\gamma}(s)}{\int_u^t \Delta s/r^{1/\gamma}(s)},$$

and, for sufficiently large T_*, $\beta(t, T_*) := \theta^{\gamma}(t, T_*)$.

In order to prove our main results, we will use the following result; see [8, Theorem 2.1].

Theorem 2.1 *Let* (1.4) *hold. Suppose that there exists a positive* Δ*-differentiable function* δ *such that for all sufficiently large* T_* *and for* $g(T) > T_*$,

$$\limsup_{t\to\infty}\int_T^t\left(\beta(s,T_*)\delta(s)Q(s)-\frac{r(s)(\delta_+^\Delta(s))^{\gamma+1}}{(\gamma+1)^{\gamma+1}\delta^\gamma(s)}\right)\Delta s=\infty. \tag{2.1}$$

Then every solution x *of equation* (1.1) *is oscillatory.*

Theorem 2.2 *Let* (1.5) *hold. Assume that there exists a positive* Δ*-differentiable function* δ *such that for all sufficiently large* T_* *and for* $g(T) > T_*$, *one has* (2.1). *If*

$$\int_{t_0}^\infty\frac{1}{r^{1/\gamma}(s)}\left(\int_{t_0}^s q(u)\left(\int_{g(u)}^\infty\frac{\Delta v}{r^{1/\gamma}(v)}\right)^\gamma\Delta u\right)^{1/\gamma}\Delta s=\infty, \tag{2.2}$$

then every solution x *of equation* (1.1) *is oscillatory or* $\lim_{t\to\infty}x(t)=0$.

Proof Let x be a nonoscillatory solution of equation (1.1). Without loss of generality, we assume that $x(t) > 0$, $x(\eta(t)) > 0$, and $x(g(t)) > 0$ for $t\in[t_0,\infty)_{\mathbb{T}}$. Then $z(t) > 0$ for $t\in[t_0,\infty)_{\mathbb{T}}$. In view of (1.1), we get

$$\bigl(r(t)\bigl(z^\Delta(t)\bigr)^\gamma\bigr)^\Delta\le -q(t)x^\gamma\bigl(g(t)\bigr)<0,\quad t\in[t_0,\infty)_{\mathbb{T}}. \tag{2.3}$$

Therefore, $r(z^\Delta)^\gamma$ is strictly decreasing, and there exists a $t_1\in[t_0,\infty)_{\mathbb{T}}$ such that $z^\Delta(t)>0$ or $z^\Delta(t)<0$ for $t\in[t_1,\infty)_{\mathbb{T}}$. We consider each of two cases separately.

Case 1. Assume that $z^\Delta(t)>0$ for $t\in[t_1,\infty)_{\mathbb{T}}$. As in the proof of [8, Theorem 2.1], we can obtain a contradiction to (2.1).

Case 2. Assume that $z^\Delta(t)<0$ for $t\in[t_1,\infty)_{\mathbb{T}}$. Then, there exists a finite limit

$$\lim_{t\to\infty}z(t)=l,$$

where $l\ge 0$. Now, we claim that $l=0$. If not, then for any $\epsilon>0$, we have $l<z(t)<l+\epsilon$, eventually. Take $0<\epsilon<l(1-p_1)/p_1$. We calculate

$$x(t)=z(t)-p(t)x\bigl(\eta(t)\bigr)>l-p_1z\bigl(\eta(t)\bigr)>l-p_1(l+\epsilon)=m(l+\epsilon)>mz(t), \tag{2.4}$$

where

$$m:=\frac{l}{l+\epsilon}-p_1=\frac{l(1-p_1)-\epsilon p_1}{l+\epsilon}>0.$$

Since $r(z^\Delta)^\gamma$ is strictly decreasing,

$$z^\Delta(s)\le\frac{r^{1/\gamma}(t)z^\Delta(t)}{r^{1/\gamma}(s)},\quad s\in[t,\infty)_{\mathbb{T}}.$$

Integrating the inequality above from t to l and letting $l\to\infty$, we have by (2.3) that

$$z(t)\ge -r^{1/\gamma}(t)z^\Delta(t)\int_t^\infty\frac{\Delta s}{r^{1/\gamma}(s)}\ge -r^{1/\gamma}(t_1)z^\Delta(t_1)\int_t^\infty\frac{\Delta s}{r^{1/\gamma}(s)}=k\int_t^\infty\frac{\Delta s}{r^{1/\gamma}(s)}, \tag{2.5}$$

where $k := -r^{1/\gamma}(t_1)z^{\Delta}(t_1) > 0$. Combining (2.4) and (2.5), we get

$$x(g(t)) \geq mz(g(t)) \geq mk \int_{g(t)}^{\infty} \frac{\Delta s}{r^{1/\gamma}(s)}.$$

Then by (2.3), we obtain

$$\big(r(t)\big(-z^{\Delta}(t)\big)^{\gamma}\big)^{\Delta} \geq (mk)^{\gamma} q(t)\bigg(\int_{g(t)}^{\infty} \frac{\Delta s}{r^{1/\gamma}(s)}\bigg)^{\gamma}.$$

Integrating the inequality above from t_2 ($t_2 \in [t_1, \infty)_{\mathbb{T}}$) to t, we have

$$\begin{aligned} r(t)\big(-z^{\Delta}(t)\big)^{\gamma} &\geq r(t_2)\big(-z^{\Delta}(t_2)\big)^{\gamma} + (mk)^{\gamma} \int_{t_2}^{t} q(s)\bigg(\int_{g(s)}^{\infty} \frac{\Delta u}{r^{1/\gamma}(u)}\bigg)^{\gamma} \Delta s \\ &\geq (mk)^{\gamma} \int_{t_2}^{t} q(s)\bigg(\int_{g(s)}^{\infty} \frac{\Delta u}{r^{1/\gamma}(u)}\bigg)^{\gamma} \Delta s, \end{aligned}$$

which implies that

$$z^{\Delta}(t) \leq -\frac{mk}{r^{1/\gamma}(t)}\bigg(\int_{t_2}^{t} q(s)\bigg(\int_{g(s)}^{\infty} \frac{\Delta u}{r^{1/\gamma}(u)}\bigg)^{\gamma} \Delta s\bigg)^{1/\gamma}.$$

Integrating the latter inequality from t_2 to t, we get

$$z(t) \leq z(t_2) - mk \int_{t_2}^{t} \frac{1}{r^{1/\gamma}(s)}\bigg(\int_{t_2}^{s} q(u)\bigg(\int_{g(u)}^{\infty} \frac{\Delta v}{r^{1/\gamma}(v)}\bigg)^{\gamma} \Delta u\bigg)^{1/\gamma} \Delta s,$$

which yields $\lim_{t\to\infty} z(t) = -\infty$, this is a contradiction. Hence, $\lim_{t\to\infty} z(t) = 0$. By virtue of $0 < x(t) \leq z(t)$, $\lim_{t\to\infty} x(t) = 0$. The proof is complete. □

Next, we establish another criterion which improves Theorem 2.2.

Theorem 2.3 *Let* (1.5) *hold. Suppose that there exists a positive* Δ*-differentiable function* δ *such that for all sufficiently large* T_* *and for* $g(T) > T_*$, *one has* (2.1). *If*

$$\int_{t_0}^{\infty} \bigg(\frac{1}{r(s)} \int_{t_0}^{s} q(u)\Delta u\bigg)^{1/\gamma} \Delta s = \infty, \tag{2.6}$$

then every solution x *of equation* (1.1) *is oscillatory or* $\lim_{t\to\infty} x(t) = 0$.

Proof Let x be a nonoscillatory solution of equation (1.1). Without loss of generality, we assume that $x(t) > 0$, $x(\eta(t)) > 0$, and $x(g(t)) > 0$ for $t \in [t_0, \infty)_{\mathbb{T}}$. Then $z(t) > 0$ for $t \in [t_0, \infty)_{\mathbb{T}}$. In view of (1.1), we get (2.3). Thus, $r(z^{\Delta})^{\gamma}$ is strictly decreasing, and there exists a $t_1 \in [t_0, \infty)_{\mathbb{T}}$ such that $z^{\Delta}(t) > 0$ or $z^{\Delta}(t) < 0$ for $t \in [t_1, \infty)_{\mathbb{T}}$. We consider each of two cases separately.

Case 1. Assume that $z^{\Delta}(t) > 0$ for $t \in [t_1, \infty)_{\mathbb{T}}$. Similarly to the proof of [8, Theorem 2.1], we can obtain a contradiction to (2.1).

Case 2. Assume that $z^{\Delta}(t) < 0$ for $t \in [t_1, \infty)_{\mathbb{T}}$. Then there exists a finite limit

$$\lim_{t\to\infty} z(t) = l,$$

where $l \geq 0$. Next, we claim that $l = 0$. If not, then for any $\epsilon > 0$, we have $l < z(t) < l + \epsilon$, eventually. Take $0 < \epsilon < l(1 - p_1)/p_1$. Then we have (2.4). It follows from (2.3), (2.4), and $z(g(t)) > l$ that

$$\bigl(r(t)\bigl(z^{\Delta}(t)\bigr)^{\gamma}\bigr)^{\Delta} \leq -q(t)x^{\gamma}\bigl(g(t)\bigr) \leq -m^{\gamma}q(t)z^{\gamma}\bigl(g(t)\bigr) \leq -(ml)^{\gamma}q(t).$$

Integrating the inequality above from t_2 ($t_2 \in [t_1, \infty)_{\mathbb{T}}$) to t, we get

$$r(t)\bigl(z^{\Delta}(t)\bigr)^{\gamma} - r(t_2)\bigl(z^{\Delta}(t_2)\bigr)^{\gamma} \leq -(ml)^{\gamma}\int_{t_2}^{t} q(s)\Delta s,$$

which yields

$$z^{\Delta}(t) \leq -ml\left(\frac{1}{r(t)}\int_{t_2}^{t} q(s)\Delta s\right)^{1/\gamma}.$$

Integrating the latter inequality from t_2 to t, we have

$$z(t) \leq z(t_2) - ml\int_{t_2}^{t}\left(\frac{1}{r(s)}\int_{t_2}^{s} q(u)\Delta u\right)^{1/\gamma}\Delta s,$$

which implies that $\lim_{t\to\infty} z(t) = -\infty$, this is a contradiction. Hence, $\lim_{t\to\infty} z(t) = 0$. By $0 < x(t) \leq z(t)$, $\lim_{t\to\infty} x(t) = 0$. This completes the proof. □

Remark 2.1 When $\mathbb{T} = \mathbb{R}$, Theorems 2.2 and 2.3 improve results of Han *et al.* [6, Theorems 2.1 and 2.2] since our results do not require condition (1.3).

Remark 2.2 The results obtained in this paper complement the recent results given in [7–19] in the sense that these results can be applied to case (1.5).

3 Applications

In this section, we give two examples to illustrate applications of results in the previous section.

Example 3.1 For $t \in [1, \infty)_{\mathbb{T}}$, consider a second-order neutral delay dynamic equation

$$\left(\frac{1}{t\sigma(t)}\left(x(t) + \frac{1}{2}x\bigl(\eta(t)\bigr)\right)^{\Delta}\right)^{\Delta} + q(t)x\bigl(g(t)\bigr) = 0, \tag{3.1}$$

where $q(t) \geq \beta > 0$ satisfying $\int_1^t (q(u)/g(u))\Delta u \geq \sigma(t)$, $\eta(t) \leq t$, $g(t) \leq t$, and $\lim_{t\to\infty}\eta(t) = \lim_{t\to\infty} g(t) = \infty$. Let $\gamma = 1$ and $r(t) = 1/(t\sigma(t))$. Then, we have

$$\int_{t_0}^{\infty}\frac{\Delta t}{r^{1/\gamma}(t)} = \int_1^{\infty}\frac{\Delta t}{t\sigma(t)} = 1,$$

that is, (1.5) holds. Note that $Q(t) = q(t)/2$, and for every constant $k \in (0,1)$ and for $t \in [t_k, \infty)_{\mathbb{T}}$,

$$\beta(t, T_*) = \left(\frac{\frac{1}{T_*} - \frac{1}{g(t)}}{\frac{1}{T_*} - \frac{1}{t}}\right)^{\gamma} = \frac{t}{g(t)}\frac{g(t) - T_*}{t - T_*} \geq k > 0.$$

Choose $\delta(t) = 1$. It is not difficult to verify that (2.1) holds. On the other hand,

$$\int_{t_0}^{\infty} \frac{1}{r^{1/\gamma}(s)} \left(\int_{t_0}^{s} q(u) \left(\int_{g(u)}^{\infty} \frac{\Delta v}{r^{1/\gamma}(v)} \right)^{\gamma} \Delta u \right)^{1/\gamma} \Delta s \geq \int_{1}^{\infty} \frac{\Delta s}{s} = \infty.$$

Thus, we have by Theorem 2.2 that every solution x of (3.1) is oscillatory or $\lim_{t\to\infty} x(t) = 0$.

Example 3.2 For $t \in [1,\infty)_{\mathbb{T}}$, consider a second-order neutral delay dynamic equation

$$\left(\frac{1}{(t\sigma(t))^{\gamma}} \left(\left(x(t) + \frac{1}{2} x(\eta(t)) \right)^{\Delta} \right)^{\gamma} \right)^{\Delta} + q(t) x^{\gamma}(g(t)) = 0, \tag{3.2}$$

where $q(t) \geq \beta > 0$ satisfying $\int_1^t q(u)\Delta u \geq \sigma^{\gamma}(t)$, $\eta(t) \leq t$, $g(t) \leq t$, and $\lim_{t\to\infty} \eta(t) = \lim_{t\to\infty} g(t) = \infty$. Let $r(t) = 1/(t\sigma(t))^{\gamma}$. Then we have

$$\int_{t_0}^{\infty} \frac{\Delta t}{r^{1/\gamma}(t)} = \int_{1}^{\infty} \frac{\Delta t}{t\sigma(t)} = 1,$$

that is, (1.5) holds. Note that $Q(t) = q(t)/2^{\gamma}$, and for every constant $k \in (0,1)$ and for $t \in [t_k,\infty)_{\mathbb{T}}$,

$$\beta(t, T_*) = \left(\frac{\frac{1}{T_*} - \frac{1}{g(t)}}{\frac{1}{T_*} - \frac{1}{t}} \right)^{\gamma} = \left(\frac{t}{g(t)} \frac{g(t) - T_*}{t - T_*} \right)^{\gamma} \geq k > 0.$$

Choose $\delta(t) = 1$. It is easy to verify that (2.1) holds. On the other hand,

$$\int_{t_0}^{\infty} \left(\frac{1}{r(s)} \int_{t_0}^{s} q(u)\Delta u \right)^{1/\gamma} \Delta s \geq \int_{1}^{\infty} \frac{\Delta s}{s} = \infty.$$

Hence, by Theorem 2.3, every solution x of (3.2) is oscillatory or $\lim_{t\to\infty} x(t) = 0$.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of the paper. All authors read and approved the final manuscript.

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