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On λ-statistical convergence and strongly λ-summable functions of order α

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Abstract

In this study, using the notion of (V,λ)-summability and λ-statistical convergence, we introduce the concepts of strong (V,λ,p)-summability and λ-statistical convergence of order α of real-valued functions which are measurable (in the Lebesgue sense) in the interval $(1,\infty)$. Also some relations between λ-statistical convergence of order α and strong (V,λ,p)-summability of order β are given.

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1 Introduction

The idea of statistical convergence was given by Zygmund [1] in the first edition of his monograph published in Warsaw in 1935. The concept of statistical convergence was introduced by Steinhaus [2] and Fast [3] and then reintroduced by Schoenberg [4] independently. Over the years and under different names, statistical convergence has been discussed in the theory of Fourier analysis, ergodic theory, number theory, measure theory, trigonometric series, turnpike theory and Banach spaces. Later on it was further investigated from the sequence space point of view and linked with summability theory by Alotaibi and Alroqi [5], Belen and Mohiuddine [6], Connor [7], Dutta *et al.* [8–11], Et *et al.* [12–14], Fridy [15], Güngör *et al.* [16, 17], Kolk [18], Mohiuddine *et al.* [19–28], Mursaleen *et al.* [29–32], Nuray [33], Rath and Tripathy [34], Salat [35], Savaş *et al.* [36, 37], Tripathy [38] and many others. In recent years, generalizations of statistical convergence have appeared in the study of strong integral summability and the structure of ideals of bounded continuous functions on locally compact spaces. Statistical convergence and its generalizations are also connected with subsets of the Stone-Čech compactification of the natural numbers. Moreover, statistical convergence is closely related to the concept of convergence in probability.

2 Definition and preliminaries

The definitions of statistical convergence and strong p-Cesàro convergence of a sequence of real numbers were introduced in the literature independently of one another and followed different lines of development since their first appearance. It turns out, however, that the two definitions can be simply related to one another in general and are equivalent for bounded sequences. The idea of statistical convergence depends on the density of subsets of the set $\mathbb{N}$ of natural numbers. The density of a subset E of $\mathbb{N}$ is defined by

$$\delta(E) = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \chi_E(k) \text{ provided the limit exists,}$$

where χ_E is the characteristic function of E. It is clear that any finite subset of $\mathbb{N}$ has zero natural density and $\delta(E^c) = 1 - \delta(E)$.

A sequence $x = (x_k)$ is said to be statistically convergent to L if for every $\varepsilon > 0$, $\delta(\{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}) = 0$. In this case we write $x_k \overset{\text{stat}}{\longrightarrow} L$ or $S\text{-}\lim x_k = L$. The set of all statistically convergent sequences will be denoted by S.

The order of statistical convergence of a sequence of numbers was given by Gadjiev and Orhan in [39] and after then statistical convergence of order α and strong p-Cesàro summability of order α were studied by Çolak [40, 41] and generalized by Çolak and Bektaş [42].

Let $\lambda = (\lambda_n)$ be a non-decreasing sequence of positive real numbers tending to ∞ such that $\lambda_{n+1} \leq \lambda_n + 1$, $\lambda_1 = 1$. The generalized de la Vallée-Poussin mean is defined by $t_n(x) = \frac{1}{\lambda_n} \sum_{k\in I_n} x_k$, where $I_n = [n - \lambda_n + 1, n]$ for $n = 1, 2, \ldots$. A sequence $x = (x_k)$ is said to be (V, λ)-summable to a number L if $t_n(x) \to L$ as $n \to \infty$ [43]. If $\lambda_n = n$, then (V, λ)-summability is reduced to Cesàro summability. By Λ we denote the class of all non-decreasing sequence of positive real numbers tending to ∞ such that $\lambda_{n+1} \leq \lambda_n + 1$, $\lambda_1 = 1$.

In [44] Borwein introduced and studied strongly summable functions. A real-valued function $x(t)$, measurable (in the Lebesgue sense) in the interval $(1, \infty)$, is said to be strongly summable to $L = L_x$ if

$$\lim_{n\to\infty} \frac{1}{n} \int_1^n \left|x(t) - L\right|^p dt = 0, \quad 1 \leq p < \infty.$$

$[W_p]$ will denote the space of real-valued function x, measurable (in the Lebesgue sense) in the interval $(1, \infty)$. The space $[W_p]$ is a normed space with the norm

$$\|x\| = \sup_{n\geq 1} \left(\frac{1}{n} \int_1^n \left|x(t)\right|^p dt \right)^{\frac{1}{p}}.$$

In this paper, using the notion of (V, λ)-summability and λ-statistical convergence, we introduce and study the concepts of strong (V, λ, p)-summability and λ-statistical convergence of order α of real-valued functions $x(t)$, measurable (in the Lebesgue sense) in the interval $(1, \infty)$.

Throughout the paper, unless stated otherwise, by 'for all $n \in \mathbb{N}_{n_o}$' we mean 'for all $n \in \mathbb{N}$ except finite numbers of positive integers' where $\mathbb{N}_{n_o} = \{n_o, n_o + 1, n_o + 2, \ldots\}$ for some $n_o \in \mathbb{N} = \{1, 2, 3, \ldots\}$.

3 Main results

In this section we give the main results of this paper. In Theorem 3.4 we give the inclusion relations between λ-statistically convergent functions of order α for different α's of real-valued functions which are measurable (in the Lebesgue sense) in the interval $(1, \infty)$. In Theorem 3.8 we give the relationship between the strong (V, λ, p)-summability of order α for different α's of real-valued functions which are measurable (in the Lebesgue sense) in the interval $(1,\infty)$. In Theorem 3.10 we give the relationship between the strong $[W^{\beta}_{\lambda p}]$-summability and $[S^{\alpha}_{\lambda}]$-statistical convergence of real-valued functions which are measurable (in the Lebesgue sense) in the interval $(1,\infty)$.

Definition 3.1 Let the sequence $\lambda = (\lambda_n)$ be as above, $\alpha \in (0,1]$ and $x(t)$ be a real-valued function which is measurable (in the Lebesgue sense) in the interval $(1,\infty)$. A real-valued function $x(t)$ is said to be strongly (V,λ,p)-summable of order α (or $[W^{\alpha}_{\lambda p}]$-summable) if there is a number $L = L_x$ such that

$$\lim_{n\to\infty} \frac{1}{\lambda_n^{\alpha}} \int_{n-\lambda_n+1}^{n} \left|x(t) - L\right|^p dt = 0, \quad 1 \le p < \infty,$$

where $I_n = [n - \lambda_n + 1, n]$ and λ_n^{α} denote the αth power $(\lambda_n)^{\alpha}$ of λ_n, that is, $\lambda^{\alpha} = (\lambda_n^{\alpha}) = (\lambda_1^{\alpha}, \lambda_2^{\alpha}, \dots, \lambda_n^{\alpha}, \dots)$. In this case we write $[W^{\alpha}_{\lambda p}]$-$\lim x(t) = L$. The set of all strongly (V,λ,p)-summable functions of order α will be denoted by $[W^{\alpha}_{\lambda p}]$. For $\lambda_n = n$ for all $n \in \mathbb{N}$, we shall write $[W^{\alpha}_{p}]$ instead of $[W^{\alpha}_{\lambda p}]$ and in the special case $\alpha = 1$ we shall write $[W_{\lambda p}]$ instead of $[W^{\alpha}_{\lambda p}]$, and also in the special case $\alpha = 1$ and $\lambda_n = n$ for all $n \in \mathbb{N}$ we shall write $[W_p]$ instead of $[W^{\alpha}_{\lambda p}]$.

Definition 3.2 Let the sequence $\lambda = (\lambda_n)$ be as above, $\alpha \in (0,1]$ and $x(t)$ be a real-valued function which is measurable (in the Lebesgue sense) in the interval $(1,\infty)$. A real-valued function $x(t)$ is said to be λ-statistically convergent of order α (or $[S^{\alpha}_{\lambda}]$-statistical convergence) to a number $L = L_x$ for every $\varepsilon > 0$,

$$\lim_{n} \frac{1}{\lambda_n^{\alpha}} \left|\left\{t \in I_n : \left|x(t) - L\right| \ge \varepsilon\right\}\right| = 0.$$

The set of all λ-statistically convergent functions of order α will be denoted by $[S^{\alpha}_{\lambda}]$. In this case we write $[S^{\alpha}_{\lambda}]$-$\lim x(t) = L$. For $\lambda_n = n$, for all $n \in \mathbb{N}$, we shall write $[S^{\alpha}]$ instead of $[S^{\alpha}_{\lambda}]$ and in the special case $\alpha = 1$, we shall write $[S_{\lambda}]$ instead of $[S^{\alpha}_{\lambda}]$.

The λ-statistical convergence of order α is well defined for $0 < \alpha \le 1$, but it is not well defined for $\alpha > 1$ in general. For this let $\lambda_n = n$ for all $n \in \mathbb{N}$ and $x(t)$ be defined as follows:

$$x(t) = \begin{cases} 1 & \text{if } t = 3n, \\ 0 & \text{if } t \neq 3n, \end{cases} \qquad n = 1, 2, 3, \dots,$$

then both

$$\lim_{n\to\infty} \frac{1}{\lambda_n^{\alpha}} \left|\left\{t \in I_n : \left|x(t) - 1\right| \ge \varepsilon\right\}\right| \le \lim_{n\to\infty} \frac{[\lambda_n] + 1}{3\lambda_n^{\alpha}} = 0$$

and

$$\lim_{n\to\infty} \frac{1}{\lambda_n^{\alpha}} \left|\left\{t \in I_n : \left|x(t) - 0\right| \ge \varepsilon\right\}\right| \le \lim_{n\to\infty} \frac{2([\lambda_n] + 1)}{3\lambda_n^{\alpha}} = 0$$

for $\alpha > 1$, and so λ-statistically converges of order α both to 1 and 0, *i.e.*, $[S^{\alpha}_{\lambda}]$-$\lim x(t) = 1$ and $[S^{\alpha}_{\lambda}]$-$\lim x(t) = 0$. But this is impossible.

The proof of the following two results is easy, so we state without proof.

Theorem 3.3 *Let* $0 < \alpha \leq 1$ *and* $x(t)$ *and* $y(t)$ *be real-valued functions which are measurable* (*in the Lebesgue sense*) *in the interval* $(1,\infty)$, *then*

(i) *If* $[S_\lambda^\alpha]\text{-}\lim x(t) = L$ *and* $c \in \mathbb{R}$, *then* $[S_\lambda^\alpha]\text{-}\lim cx(t) = cL$;

(ii) *If* $[S_\lambda^\alpha]\text{-}\lim x(t) = L_1$ *and* $[S_\lambda^\alpha]\text{-}\lim y(t) = L_2$, *then* $[S_\lambda^\alpha]\text{-}\lim(x(t) + y(t)) = L_1 + L_2$.

Theorem 3.4 *Let* $\lambda = (\lambda_n) \in \Lambda$ $(0 < \alpha \leq \beta \leq 1)$ *and* $x(t)$ *be a real-valued function which is measurable* (*in the Lebesgue sense*) *in the interval* $(1,\infty)$, *then* $[S_\lambda^\alpha] \subseteq [S_\lambda^\beta]$.

From Theorem 3.4 we have the following.

Corollary 3.5 *If* $x(t)$ *is* λ*-statistically convergent of order* α *to* L, *then it is statistically convergent to* L *for each* $\alpha \in (0,1]$.

Theorem 3.6 *Let* $\lambda = (\lambda_n) \in \Lambda$ $(0 < \alpha \leq 1)$ *and* $x(t)$ *be a real-valued function which is measurable* (*in the Lebesgue sense*) *in the interval* $(1,\infty)$, *then* $[S^\alpha] \subseteq [S_\lambda^\alpha]$ *if*

$$\liminf_{n\to\infty} \frac{\lambda_n^\alpha}{n^\alpha} > 0. \tag{1}$$

Proof For given $\varepsilon > 0$, we have

$$\{t \leq n : |x(t) - L| \geq \varepsilon\} \supset \{t \in I_n : |x(t) - L| \geq \varepsilon\},$$

and so

$$\frac{1}{n^\alpha}\bigl|\{t \leq n : |x(t) - L| \geq \varepsilon\}\bigr| \geq \frac{\lambda_n^\alpha}{n^\alpha}\frac{1}{\lambda_n^\alpha}\bigl|\{t \in I_n : |x(t) - L| \geq \varepsilon\}\bigr|.$$

Taking the limit as $n \to \infty$ and using (1), we get $[S^\alpha]\text{-}\lim x_k = L \Longrightarrow [S_\lambda^\alpha]\text{-}\lim x_k = L$. □

From Theorem 3.6 we have the following result.

Corollary 3.7 *Let* $\lambda = (\lambda_n) \in \Lambda$ $(0 < \alpha \leq 1)$ *and* $x(t)$ *be a real-valued function which is measurable* (*in the Lebesgue sense*) *in the interval* $(1,\infty)$, *then* $[S] \subseteq [S_\lambda^\alpha]$ *if*

$$\liminf_{n\to\infty} \frac{\lambda_n^\alpha}{n} > 0.$$

Theorem 3.8 *Let* $\lambda = (\lambda_n) \in \Lambda$, $0 < \alpha \leq \beta \leq 1$, p *be a positive real number and* $x(t)$ *be a real-valued function which is measurable* (*in the Lebesgue sense*) *in the interval* $(1,\infty)$, *then* $[W_{\lambda p}^\alpha] \subseteq [W_{\lambda p}^\beta]$.

Proof Omitted. □

From Theorem 3.8 we have the following result.

Corollary 3.9 *Let* $\lambda = (\lambda_n) \in \Lambda$ *and* p *be a positive real number and* $x(t)$ *be a real-valued function which is measurable* (*in the Lebesgue sense*) *in the interval* $(1,\infty)$, *then* $[W_{\lambda p}^\alpha] \subseteq [W_{\lambda p}]$.

Theorem 3.10 *Let $\lambda = (\lambda_n) \in \Lambda$, $0 < \alpha \leq \beta \leq 1$, p be a positive real number and $x(t)$ be a real-valued function which is measurable (in the Lebesgue sense) in the interval $(1,\infty)$. If a function $x(t)$ is $[W^{\alpha}_{\lambda p}]$-summable, then it is λ-statistically convergent of order β.*

Proof For any sequence $x(t) \in [W^{\alpha}_{\lambda p}]$ and $\varepsilon > 0$, we have

$$\begin{aligned}\int_{n-\lambda_n+1}^{n} \left|x(t)-L\right|^p dt &= \int_{\substack{n-\lambda_n+1\\ |x(t)-L|\geq\varepsilon}}^{n} \left|x(t)-L\right|^p dt + \int_{\substack{n-\lambda_n+1\\ |x(t)-L|<\varepsilon}}^{n} \left|x(t)-L\right|^p dt \\ &\geq \int_{\substack{n-\lambda_n+1\\ |x(t)-L|\geq\varepsilon}}^{n} \left|x(t)-L\right|^p dt \\ &\geq \left|\left\{t \in I_n : \left|x(t)-L\right| \geq \varepsilon\right\}\right| \cdot \varepsilon^p\end{aligned}$$

and so that

$$\frac{1}{\lambda_n^{\alpha}} \int_{n-\lambda_n+1}^{n} \left|x(t)-L\right|^p dt \geq \frac{1}{\lambda_n^{\beta}} \left|\left\{t \in I_n : \left|x(t)-L\right| \geq \varepsilon\right\}\right| \cdot \varepsilon^p.$$

From this it follows that if $x(t)$ is $[W^{\alpha}_{\lambda p}]$-summable, then it is λ-statistically convergent of order β. □

From Theorem 3.10 we have the following results.

Corollary 3.11 *Let α be a fixed real number such that $0 < \alpha \leq 1$ and $0 < p < \infty$. The following statements hold:*

(i) *If $x(t)$ is strongly $[W^{\alpha}_{\lambda p}]$-summable of order α, then it is λ-statistically convergent of order α.*
(ii) *If $x(t)$ is strongly $[W^{\alpha}_{\lambda p}]$-summable of order α, then it is λ-statistically convergent.*

Theorem 3.12 *Let $\lambda = (\lambda_n)$ and $\mu = (\mu_n)$ be two sequences in Λ such that $\lambda_n \leq \mu_n$ for all $n \in \mathbb{N}_{n_o}$ $(0 < \alpha \leq \beta \leq 1)$, and let $x(t)$ be a real-valued function which is measurable (in the Lebesgue sense) in the interval $(1,\infty)$. If*

$$\liminf_{n\to\infty} \frac{\lambda_n^{\alpha}}{\mu_n^{\beta}} > 0, \tag{2}$$

then $[S^{\beta}_{\mu}] \subseteq [S^{\alpha}_{\lambda}]$.

Proof Suppose that $\lambda_n \leq \mu_n$ for all $n \in \mathbb{N}_{n_o}$ and let (2) be satisfied. Then $I_n \subset J_n$ and so that $\varepsilon > 0$ we may write

$$\left\{t \in J_n : \left|x(t)-L\right| \geq \varepsilon\right\} \supset \left\{t \in I_n : \left|x(t)-L\right| \geq \varepsilon\right\}$$

and so

$$\frac{1}{\mu_n^{\beta}} \left|\left\{t \in J_n : \left|x(t)-L\right| \geq \varepsilon\right\}\right| \geq \frac{\lambda_n^{\alpha}}{\mu_n^{\beta}} \frac{1}{\lambda_n^{\alpha}} \left|\left\{t \in I_n : \left|x(t)-L\right| \geq \varepsilon\right\}\right|$$

for all $n \in \mathbb{N}_{n_o}$, where $J_n = [n - \mu_n + 1, n]$. Now, taking the limit as $n \to \infty$ in the last inequality and using (2), we get $[S_\mu^\beta] \subseteq [S_\lambda^\alpha]$. □

From Theorem 3.12 we have the following results.

Corollary 3.13 *Let $\lambda = (\lambda_n)$ and $\mu = (\mu_n)$ be two sequences in Λ such that $\lambda_n \leq \mu_n$ for all $n \in \mathbb{N}_{n_o}$, and let $x(t)$ be a real-valued function which is measurable (in the Lebesgue sense) in the interval $(1, \infty)$. If (2) holds, then*

(i) $[S_\mu^\alpha] \subseteq [S_\lambda^\alpha]$ *for each* $\alpha \in (0,1]$,
(ii) $[S_\mu] \subseteq [S_\lambda^\alpha]$ *for each* $\alpha \in (0,1]$,
(iii) $[S_\mu] \subseteq [S_\lambda]$.

Theorem 3.14 *Let $\lambda = (\lambda_n)$ and $\mu = (\mu_n)$ be two sequences in Λ such that $\lambda_n \leq \mu_n$ for all $n \in \mathbb{N}_{n_o}$ $(0 < \alpha \leq \beta \leq 1)$, and let $x(t)$ be a real-valued function which is measurable (in the Lebesgue sense) in the interval $(1, \infty)$. If (2) holds, then $[W_{\mu p}^\beta] \subseteq [W_{\lambda p}^\alpha]$.*

Proof Omitted. □

Corollary 3.15 *Let $\lambda = (\lambda_n)$ and $\mu = (\mu_n)$ be two sequences in Λ such that $\lambda_n \leq \mu_n$ for all $n \in \mathbb{N}_{n_o}$, and let $x(t)$ be a real-valued function which is measurable (in the Lebesgue sense) in the interval $(1, \infty)$. If (2) holds, then*

(i) $[W_{\mu p}^\alpha] \subseteq [W_{\lambda p}^\alpha]$ *for each* $\alpha \in (0,1]$,
(ii) $[W_{\mu p}] \subseteq [W_{\lambda p}^\alpha]$ *for each* $\alpha \in (0,1]$,
(iii) $[W_{\mu p}] \subseteq [W_{\lambda p}]$.

Theorem 3.16 *Let $\lambda = (\lambda_n)$ and $\mu = (\mu_n)$ be two sequences in Λ such that $\lambda_n \leq \mu_n$ for all $n \in \mathbb{N}_{n_o}$ $(0 < \alpha \leq \beta \leq 1)$, and let $x(t)$ be a real-valued function which is measurable (in the Lebesgue sense) in the interval $(1, \infty)$ and (2) holds. If a real-valued function $x(t)$ is strongly (V, μ, p)-summable of order β to L, then it is λ-statistically convergent of order α to L.*

Proof Let $x(t)$ be a real-valued function such that $x(t)$ is strongly (V, μ, p)-summable of order β to L and $\varepsilon > 0$. Then we have

$$\begin{aligned}
\int_{t \in J_n} \left|x(t) - L\right|^p dt &= \int_{\substack{t \in J_n \\ |x(t)-L| \geq \varepsilon}} \left|x(t) - L\right|^p dt + \int_{\substack{t \in J_n \\ |x(t)-L| < \varepsilon}} \left|x(t) - L\right|^p dt \\
&\geq \int_{\substack{t \in I_n \\ |x(t)-L| \geq \varepsilon}} \left|x(t) - L\right|^p dt \\
&\geq \left|\left\{t \in I_n : \left|x(t) - L\right| \geq \varepsilon\right\}\right| \cdot \varepsilon^p
\end{aligned}$$

and so that

$$\frac{1}{\mu_n^\beta} \int_{t \in J_n} \left|x(t) - L\right|^p dt \geq \frac{\lambda_n^\alpha}{\mu_n^\beta} \frac{1}{\lambda_n^\alpha} \left|\left\{t \in I_n : \left|x(t) - L\right| \geq \varepsilon\right\}\right| \cdot \varepsilon^p.$$

Since (2) holds, it follows that if $x(t)$ is strongly (V, μ, p)-summable of order β to L, then it is λ-statistically convergent of order α to L. □

Corollary 3.17 *Let $\lambda = (\lambda_n)$ and $\mu = (\mu_n)$ be two sequences in Λ such that $\lambda_n \leq \mu_n$ for all $n \in \mathbb{N}_{n_o}$, and let $x(t)$ be a real-valued function which is measurable* (*in the Lebesgue sense*) *in the interval* $(1,\infty)$. *If* (2) *holds, then*

(i) *A real-valued function $x(t)$ is strongly (V,μ,p)-summable of order α to L, then it is λ-statistically convergent of order α to L;*
(ii) *A real-valued function $x(t)$ is strongly (V,μ,p)-summable to L, then it is λ-statistically convergent of order α to L;*
(iii) *A real-valued function $x(t)$ is strongly (V,μ,p)-summable to L, then it is λ-statistically convergent to L.*

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors contributed equally and significantly in writing this paper. All the authors read and approved the final manuscript.

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