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Periodic solutions for a class of diffusive Nicholson's blowflies model with Dirichlet boundary conditions

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Abstract

In this paper, we study the problem of periodic solutions for a class of diffusive Nicholson's blowflies model with Dirichlet boundary conditions. By applying the Schauder fixed point theorem, the existence of nontrivial nonnegative periodic solutions of the considered model is established. Our results complement with some recent ones.

MSC: 35B05; 35B10

Keywords: diffusive Nicholson's model; periodic solution; Schauder fixed point theorem

1 Introduction

In the classic study of population dynamics, Gurney *et al.* [1] proposed the following delay equation:

$$\frac{du(t)}{dt} = -\delta u(t) + pu(t-\tau)e^{-au(t-\tau)} \tag{1.1}$$

to describe the population of the Australian sheep-blowfly *Lucilia cuprina*, where $u(t)$ is the population of the adult flies at the time t, p is the maximum per capita daily egg production rate, $1/a$ is the size at which the blowfly population reproduces at its maximum rate, δ is the per capita daily egg adult death rate, and τ is the maturation time. Since this equation explains Nicholson's data of blowfly more accurately, the model and its modifications have been now refereed to as Nicholson's blowflies model, we refer the reader to [2–6] and the references therein for some interesting results on this issue.

When the model is used to describe the population dynamics of a species in a nonlaboratory habitat, spatial heterogeneity exists, and spatial variables are needed. In this case, the immature individuals do not diffuse, but the matured ones do, and a diffusion term is needed to describe the random movement of individuals. Then, model (1.1) can be naturally extended to the following delayed reaction diffusion equation:

$$\frac{\partial u(t,x)}{\partial t} = d\Delta u(t,x) - \delta u(t,x) + pu(t-\tau,x)e^{-au(t-\tau,x)}, \quad x \in \Omega \subset \mathbb{R}^{\mathbf{n}}. \tag{1.2}$$

For the model above with either Dirichlet boundary conditions or Neumann boundary conditions, some results have been obtained. For example, So and Yang in [7] studied the asymptotic behavior of solutions of equation (1.2) under the Dirichlet boundary condition. In [8], Gourley considered the existence of travelling front solutions and their qualitative form for equation (1.2). The nonlinear stability of travelling wavefronts of equation (1.2) was investigated by Mei *et al.* in [9]. Yi and Zou [10] also established the global attractivity of the positive steady state of equation (1.2). Yi *et al.* in [11] established the threshold dynamics of equation (1.2) subject to the homogeneous Dirichlet boundary condition when the delayed reaction term is non-monotone.

One characteristic phenomenon of population dynamics is the often observed oscillatory behavior of the population densities. To better understand such a phenomenon, one mechanism is to introduce time delays in the models. As pointed out by Gopalsamy in [12], more realistic and interesting models of single or multiple species growth should take into account both the seasonality of the changing environment and the effects of time delays. In particular, the effects of a periodically varying environment are important for the evolutionary theory, as the selective forces on systems in a fluctuating environment differ from those in a stable environment. Hence, the effects of the periodic environment on evolutionary equations with delays have been the object of intensive analysis by numerous authors, some of the results can be found in [13–18] and the references therein. Recently, the authors in [19] obtained the existence of periodic solutions for the following nonlinear diffusive Nicholson's blowflies model with constant delays and constant coefficients:

$$\frac{\partial u}{\partial t} = \Delta u^m - \delta u + pu(t-\tau,x)e^{-au(t-\tau,x)} + g(t,x) + \beta\int_{t-\tau}^{t} e^{-\alpha(t-s)u(s,x)}\,ds, \tag{1.3}$$

however, the existence of periodic solutions for the diffusive Nicholson's blowflies model with time-varying coefficients and delays has not been sufficiently researched. On the other hand, as mentioned above, the coefficients and delays are usually periodically time-varying in the population and ecology models with a periodically varying environment. Hence, it is more natural to consider diffusive Nicholson's blowflies model with time-varying coefficients and delays. To do this, in the present paper, we shall consider the following diffusive Nicholson's blowflies model with time-varying coefficients and delays:

$$\begin{aligned}\frac{\partial u(t,x)}{\partial t} &= d(t)\Delta u(t,x) - \delta(t)u(t,x)\\ &\quad + p(t)u\bigl(t-\tau(t),x\bigr)e^{-a(t)u(t-\tau(t),x)} + g(t,x), \quad (x,t)\in Q_T,\end{aligned} \tag{1.4}$$

under the Dirichlet boundary conditions:

$$u(t,x) = 0, \quad (t,x)\in[0,T]\times\partial\Omega \tag{1.5}$$

with

$$u(0,x) = u(T,x), \quad x\in\Omega, \tag{1.6}$$

where $d(t)$, $\delta(t)$, $p(t)$, $\tau(t)$, $a(t)$ are positive, continuous and T-periodic in t, $g(t,x)$ is the known function, which satisfies some structure conditions (see [19]), $g(t,x)$ is T-periodic in the first argument, $\Omega\subset\mathbb{R}^{\mathbf{n}}$ is a bounded domain with smooth boundary $\partial\Omega$, $Q_T =$

$\Omega\times[0,T]$, $\partial Q_T=\partial\Omega\times[0,T]$, $x=(x_1,\dots,x_n)$ denotes the spatial variable vector in $\mathbb{R}^n$, and Δ is the Laplacian operator in $\mathbb{R}^n$.

The rest of this paper is organized as follows. In Section 2, we introduce some further notations and recall some useful results, which will be used in the later section. In Section 3, we give our main result and its proof.

2 Preliminaries

(a) Let us first introduce certain notations and definitions, constantly used throughout the present paper.

Let Ω be an n-dimensional bounded domain in $\mathbb{R}^n$ with a boundary $\partial\Omega$ and a closure $\bar{\Omega}$. Assume throughout that $\partial\Omega$ can be covered by a finite number of balls S_k such that the portion $\partial\Omega\cap S_k$ may be represented in the form $x_i=h(x_1,\dots,x_{i-1},x_{i+1},\dots,x_n)$ for some i, where h has the Hölder continuous (exponent α, $0<\alpha<1$) second derivatives.

Following the usual notations, we introduce the following norms on a function $u(t,x)$ defined in $\bar{Q}_T$:

$$|u|_0=\sup_{P\in\bar{Q}_T}\big|u(P)\big|,$$

$$|u|_\alpha=|u|_0+\sup_{P,P'\in\bar{Q}_T}\frac{|u(P)-u(P')|}{[d(P,P')]^\alpha},\quad 0<\alpha<1,$$

$$|u|_{1+\alpha}=|u|_\alpha+\sum_{i=1}^n|u_{x_i}|_\alpha,$$

$$|u|_{2+\alpha}=|u|_\alpha+\sum_{i=1}^n|u_{x_i}|_\alpha+\sum_{i,j=1}^n|u_{x_ix_j}|_\alpha+|u_t|_\alpha,$$

where $P=(t,x)$, $P'=(t',x')$ and $d(P,P')=[|t-t'|+|x-x'|^2]^{1/2}$.

We shall say that $u\in C^q(\bar{Q}_T)$ when the norm $|u|_q$ is finite ($q=0,\alpha,1+\alpha,2+\alpha$). The set of all functions in $C^q(\bar{Q}_T)$ that are periodic in t (period T) will be denoted by $C^q_T(\bar{Q}_T)$.

(b) Let us now consider the linear boundary problem

$$\sum_{i,j=1}^n a_{ij}(t,x)u_{x_ix_j}+\sum_{i=1}^n b_i(t,x)u_{x_i}+c(t,x)u-u_t=f(t,x)\quad\text{in } Q_T,\tag{2.1}$$

$$u(t,x)=0\quad\text{on } \partial Q_T.\tag{2.2}$$

Assume that the functions $a_{ij}(t,x)$, $b_i(t,x)$, $c(t,x)$, $f(t,x)$ are all T-periodic in t, and that there exists a positive number a_0 such that, for each $(t,x)\in\bar{Q}_T$ and for any real vector ξ,

$$\sum_{i,j=1}^n a_{ij}(t,x)\xi_i\xi_j\geq a_0\sum_{i=1}^n\xi_i^2.$$

The following fundamental results concerning (2.1), (2.2), which are due to Fife [20] and Shmulev [21], are stated in a form suitable to our purpose.

Lemma 2.1 (See [20]) *Assume that*

(i) $c(t,x)\leq 0$ *in* $\bar{Q}_T$;

(ii) $\sum_{i,j=1}^{n}|a_{ij}|_\alpha + \sum_{i=1}^{n}|b_i|_\alpha + |c|_\alpha \le M$, $|f|_\alpha < \infty$ *for some constant* M;

(iii) $\varphi(t,x)$ *is the trace on* ∂Q *of a function* $\Phi(t,x)$ *of class* $C_T^{2+\alpha}(\bar{Q}_T)$.

Then there exists a unique periodic solution $u(t,x) \in C_T^{2+\alpha}(\bar{Q}_T)$ *of problem* (2.1), (2.2). *Furthermore, the following estimate holds*:

$$|u|_{2+\alpha} \le K\big(|f|_\alpha + |\Phi|_{2+\alpha}\big), \tag{2.3}$$

where K *depends only on* a_0, M *and* Q_T.

Lemma 2.2 (See [21]) *Assume that*

(i) $a_{ij}(t,x)$, $b_i(t,x)$, $c(t,x)$ *and* $f(t,x)$ *are continuous in* $\bar{Q}_T$;

(ii) $\sum_{i,j=1}^{n}|a_{ij}|_\alpha + \sum_{i=1}^{n}|b_i|_0 + |c|_0 \le M_1$ *for some constant* M_1;

(iii) $\sum_{i,j=1}^{n}(\sup_{\partial Q_T}|a_{ij}| + \sup_{(t,x),(t',x')\in\partial Q_T}\frac{|a_{ij}(t,x)-a_{ij}(t',x')|}{|t-t'|+|x-x'|}) \le M_2$ *for some constant* M_2.

If $u(t,x)$ *is a smooth* T*-periodic solution of* (2.1), (2.2) *with* $\varphi(t,x) = 0$, *then there exists, for any* $0 < \nu < 1$, *a constant* K *depending only on* ν, a_0, M_1, M_2 *and* Q_T *such that*

$$|u|_{1+\nu} \le K|f|_0. \tag{2.4}$$

Lemma 2.3 (See [22]) *For any* $0 < \alpha < \beta < 1$, $0 \le p \le q$, *the bounded subsets of the space* $C_{q+\beta}(Q_T)$ *are precompact subsets of* $C_{p+\alpha}(Q_T)$.

Lemma 2.4 (The Schauder fixed point theorem, see [22]) *Let* X *be a closed convex subset of the Banach space* Y, *and let* $\mathbf{T}$ *be a continuous operator on* X *such that* $\mathbf{T}X$ *is contained in* X, *and* $\mathbf{T}X$ *is precompact. Then* $\mathbf{T}$ *has a fixed point, i.e., there exists a point* $x \in X$ *such that* $\mathbf{T}x = x$.

3 Main result and its proof

The purpose of the present section is to prove the existence of a nonnegative periodic solution $u(t,x)$ of (1.4)-(1.6).

For convenience, we rescale (1.4)-(1.6) to the following:

$$\begin{aligned} d(t)\Delta u(t,x) - \delta(t)u(t,x) - u_t &= -g(t,x) - p(t)u\big(t-\tau(t),x\big)e^{-a(t)u(t-\tau(t),x)} \\ &\triangleq -g(t,x) - \psi(t,x,u) \quad \text{in } Q_T, \end{aligned} \tag{3.1}$$

satisfying the boundary condition

$$u(t,x) = 0, \quad (t,x) \in [0,T] \times \partial\Omega, \tag{3.2}$$

$$u(0,x) = u(T,x), \quad x \in \Omega. \tag{3.3}$$

In order to give the basic constructive existence theorem of this paper, we shall impose the following hypotheses on problem (3.1)-(3.3):

(H_1) $d(t)$, $\delta(t)$ are Hölder continuous (exponent α, $0 < \alpha < 1$) in $[0,T]$, and there exist positive constants a_0, a_1 such that

$$d(t) \ge a_0 > 0, \qquad a(t) \ge a_1 > 0.$$

(H_2) $0 \le g(t,x) \in C^{\alpha}(\bar{Q}_T)$, $g(t,x) \not\equiv 0$, $\psi(t,x,u) \in C^{\alpha}(\bar{Q}_T)$, $|u| < \infty$, *i.e.*, $g(t,x)$, $\psi(t,x,u)$ satisfy the local Hölder condition in (t,x) with the exponent α, $\psi(t,x,u)$ satisfies a local Hölder condition in u, uniformly with respect to t.

(H_3) there exists a function $\Phi(t,x) \in C_T^{2+\alpha}(\bar{Q}_T)$ such that $\Phi(t,x)|_{\partial\Omega}=0$.

We shall now prove an existence theorem by employing the Schauder fixed point theorem.

Theorem 3.1 *Suppose that the assumptions* (H_1)-(H_3) *are satisfied, then there exists a nonnegative T-periodic solution $u(t,x)$ of problem* (3.1)-(3.3). *Furthermore, $u(t,x)$ belongs to $C_T^{1+\nu}(\bar{Q}_T)$ for any $0 < \nu < 1$ and to $C_T^{2+\gamma}(\bar{Q}_T)$ for some $0 < \gamma < 1$.*

Proof Let us first consider the following auxiliary problem:

$$d(t)\Delta u(t,x) - \delta(t)u(t,x) - u_t = -g(t,x) - p(t)v_+\bigl(t-\tau(t),x\bigr)e^{-a(t)v_+(t-\tau(t),x)} \quad \text{in } Q_T, \tag{3.4}$$

satisfying the boundary condition

$$u(t,x) = 0, \quad (t,x) \in [0,T] \times \partial\Omega, \tag{3.5}$$

$$u(0,x) = u(T,x), \quad x \in \Omega, \tag{3.6}$$

where $v_+ = \max\{0, v(t-\tau(t),x)\}$, $(x,t) \in Q_T$, obviously, $v_+ = \max\{0, v(t-\tau(t),x)\} \ge 0$.

On the one hand, we prove the existence of the periodic solution of problem (3.4)-(3.6) under the conditions of Theorem 3.1, we split the proof into two cases: (i) $v_+ = 0$; (ii) $v_+ > 0$.

Case (i). If $v_+ = 0$, the coefficients are periodic and Hölder continuous on $\bar{Q}_T$, a unique periodic solution u exists according to Lemma 2.1.

Case (ii). If $v_+ > 0$, then $v_+ = v$, we define a mapping $u = \mathbf{T}v$ as follows:

$$\begin{aligned} d(t)\Delta u(t,x) - \delta(t)u(t,x) - u_t &= -g(t,x) - p(t)v\bigl(t-\tau(t),x\bigr)e^{-a(t)v(t-\tau(t),x)} \\ &\triangleq G(t,x) \quad \text{in } Q_T, \end{aligned} \tag{3.7}$$

satisfying the boundary condition: $u(t,x) = 0$ on ∂Q_T. Since the coefficients and right members are periodic and Hölder continuous on $\bar{Q}_T$, a unique periodic solution $u \in C_T^{2+\alpha}(\bar{Q}_T)$ exists according to Lemma 2.1. Moreover, by (H_2)-(H_3), and in view of the fact that $\sup_{u\ge 0} ue^{-a(t)u} = \frac{1}{a(t)e}$, we have the following estimate:

$$\begin{aligned} |u|_{1+\nu} &\le K\bigl(\bigl|g(t,x)\bigr|_0 + \bigl|p(t)v\bigl(t-\tau(t),x\bigr)e^{-a(t)v(t-\tau(t),x)}\bigr|_0\bigr) \\ &\le K\bigl(\bigl|g(t,x)\bigr|_0 + \bigl|p(t)v\bigl(t-\tau(t),x\bigr)e^{-a(t)v(t-\tau(t),x)}\bigr|_0\bigr) \\ &\le K\left(\bigl|g(t,x)\bigr|_0 + \sup_{t\in[0,T]} \frac{p(t)}{a(t)e}\right) \\ &= N. \end{aligned} \tag{3.8}$$

Set

$$\mathbf{V} = \bigl\{v \mid |v|_{1+\nu} \le N, v \in C_T^{1+\nu}(\bar{Q}_T)\bigr\}.$$

Obviously, $\mathbf{V}$ is a closed convex set in the Banach space $C_T^{1+\nu}(\bar{Q}_T)$, from (3.8), we can see that the mapping $\mathbf{T}$ maps $\mathbf{V}$ into itself.

Applying again Lemma 2.2 to (3.7), we obtain, for any $0 < \nu < \nu' < 1$ and for any $v \in \mathbf{V}$,

$$|u|_{1+\nu'} \le N_1 \quad \bigl(N_1 \text{ depends on } \nu'\bigr), \tag{3.9}$$

in view of Lemma 2.3, we know that the bounded subsets of the space $C_T^{1+\nu'}(\bar{Q}_T)$ are precompact subsets of $C_T^{1+\nu}(\bar{Q}_T)$, so we conclude that $\mathbf{T}$ maps $\mathbf{V}$ into a compact subset of $\mathbf{V}$.

To show that $\mathbf{T}$ is a continuous mapping, we observed that if $u_i = \mathbf{T}v_i$, $v_i \in \mathbf{V}$ $(i = 1, 2)$, then the periodic function $u = u_1 - u_2$ satisfies the linear parabolic equations

$$d(t)\Delta u - \delta(t)u - u_t = p(t)v_2\bigl(t-\tau(t)\bigr)e^{-a(t)v_2(t-\tau(t))} - p(t)v_1\bigl(t-\tau(t)\bigr)e^{-a(t)v_1(t-\tau(t))} \tag{3.10}$$

together with the boundary condition $u = 0$ on ∂Q. By Lemma 2.2 applied to (3.10), we obtain

$$|u|_{1+\nu} \le K' \sup_{\bar{Q}} \bigl| p(t)v_2\bigl(t-\tau(t)\bigr)e^{-a(t)v_2(t-\tau(t))} - p(t)v_1\bigl(t-\tau(t)\bigr)e^{-a(t)v_1(t-\tau(t))}\bigr|, \tag{3.11}$$

from $\sup_{u\ge 0} |\frac{1-u}{e^u}| = 1$ and the inequality

$$\begin{aligned} \bigl|xe^{-x} - ye^{-y}\bigr| &= \left| \frac{1-(x+\theta(y-x))}{e^{x+\theta(y-x)}} \right| |x-y| \\ &\le |x-y|, \end{aligned}$$

where $x, y \ge 0$, $0 < \theta < 1$.

From (3.11), we have

$$\begin{aligned} |u|_{1+\nu} &\le K' \sup_{t\in[0,T]} \frac{p(t)}{a(t)} |v_1 - v_2|_0 \\ &\le M_3 |v_1 - v_2|_0, \end{aligned}$$

which means that

$$|v_1 - v_2|_{1+\nu} \to 0,$$

and so

$$|\mathbf{T}v_1 - \mathbf{T}v_2|_{1+\nu} \to 0,$$

then the continuity of $\mathbf{T}$ in the $(1+\nu)$-norms is proved.

By Lemma 2.4, it follows that $\mathbf{T}$ has a fixed point $u^* \in C_T^{1+\nu}(Q_T)$, u^* is then a periodic solution of (3.4)-(3.6), and it belongs to $C_T^{1+\nu}(\bar{Q}_T)$, ν is arbitrary. Moreover, u^* also belongs to $C_T^{2+\gamma}(\bar{Q}_T)$ for some $0 < \gamma < 1$, following directly from Lemma 2.1.

On the other hand, we claim that the T-periodic solution u of problem (3.4)-(3.6) is nonnegative and nontrival.

Multiplying equation (3.4) by u_- and making use of integral by part over Q_T, we have

$$\begin{aligned}\iint_{Q_T} \frac{\partial u}{\partial t} u_- \,dx\,dt = &-\iint_{Q_T} d(t)\nabla u \cdot \nabla u_- \,dx\,dt \\ &- \iint_{Q_T} \delta(t)(u_-)^2 \,dx\,dt + \iint_{Q_T} g u_- \,dx\,dt \\ &+ \iint_{Q_T} p(t)u_+\bigl(t-\tau(t),x\bigr)e^{-a(t)u_+(t-\tau(t),x)} u_- \,dx\,dt,\end{aligned}$$

where $u_-(t,x) = \min\{0, u(t,x), (t,x) \in Q_T\}$, due to the periodicity of u with respect to t, we obtain

$$\iint_{Q_T} \frac{\partial u}{\partial t} u_- \,dx\,dt = 0,$$

then we get

$$\begin{aligned}\iint_{Q_T} \delta(t)u_-^2 \,dx\,dt = &-\iint_{Q_T} d(t)|\nabla u_-|^2 \,dx\,dt + \iint_{Q_T} g u_- \,dx\,dt \\ &+ \iint_{Q_T} p(t)u_+\bigl(t-\tau(t),x\bigr)e^{-a(t)u_+(t-\tau(t),x)} u_- \,dx\,dt \\ \le{}& 0,\end{aligned}$$

which combined with the continuity of u_- implies

$$u_- = 0 \quad \text{in } Q_T.$$

By the definition of u^-, we see that

$$u \ge 0 \quad \text{in } Q_T.$$

Consequently, we can rewrite (3.4) as

$$\frac{\partial u(t,x)}{\partial t} = d(t)\Delta u(t,x) - \delta(t)u(t,x) + p(t)u\bigl(t-\tau(t),x\bigr)e^{-a(t)u(t-\tau(t),x)} + g(t,x),$$

since $g(t,x) \not\equiv 0$, we see that T-periodic solution is nontrivial. The proof of Theorem 3.1 is completed. □

Remark 3.1 In [2–6, 13, 15], the authors studied the dynamical behavior of delayed Nicholson's blowflies equation, however, the effect of diffusive term was seldom considered in these references. In this sense, the main result obtained in the present paper extended the mentioned results.

Remark 3.2 It is worth pointing out that the coefficients of equation (1.4) are time-varying, which is more difficult to research than the constant case. To the best of our knowledge, there are very few works concerning the studied problem, which implies that the results of this paper are more general, and they effectually complement the previously known results.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
BL recalled Lemmas 2.1-2.4 and drafted the manuscript. JM and WJ proved Theorem 3.1 and gave two remarks to illustrate the effectiveness of the obtained results. All authors read and approved the final manuscript.

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