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Some topological and geometrical properties of new Banach sequence spaces

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Abstract

In the present paper, we introduce a new band matrix $\hat{F}$ and define the sequence space

$$\ell_p(\hat{F}) = \left\{ x = (x_k) \in \omega : \sum_k \left| \frac{f_k}{f_{k+1}} x_k - \frac{f_{k+1}}{f_k} x_{k-1} \right|^p < \infty; 1 \le p \le \infty \right\},$$

where f_k is the kth Fibonacci number for every $k \in \mathbb{N}$. We also establish some inclusion relations concerning this space and determine its α-, β-, γ-duals. Further, we characterize some matrix classes on the space $\ell_p(\hat{F})$ and examine some geometric properties of this space.

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1 Introduction

Let ω be the space of all real-valued sequences. Any vector subspace of ω is called a *sequence space*. By ℓ_∞, c, c_0 and ℓ_p $(1 \le p < \infty)$, we denote the sets of all bounded, convergent, null sequences and p-absolutely convergent series, respectively. Also, we use the conventions that $e = (1, 1, \ldots)$ and $e^{(n)}$ is the sequence whose only non-zero term is 1 in the nth place for each $n \in \mathbb{N}$, where $\mathbb{N} = \{0, 1, 2, \ldots\}$.

Let X and Y be two sequence spaces and $A = (a_{nk})$ be an infinite matrix of real numbers a_{nk}, where $n, k \in \mathbb{N}$. We write $A = (a_{nk})$ instead of $A = (a_{nk})_{n,k=0}^\infty$. Then we say that A defines a matrix mapping from X into Y and we denote it by writing $A : X \to Y$ if for every sequence $x = (x_k)_{k=0}^\infty \in X$, the sequence $Ax = \{A_n(x)\}_{n=0}^\infty$, the A-transform of x, is in Y, where

$$A_n(x) = \sum_{k=0}^\infty a_{nk} x_k \quad (n \in \mathbb{N}). \tag{1.1}$$

For simplicity in notation, here and in what follows, the summation without limits runs from 0 to ∞. Also, if $x \in \omega$, then we write $x = (x_k)$ instead of $x = (x_k)_{k=0}^\infty$.

By (X, Y), we denote the class of all matrices A such that $A : X \to Y$. Thus, $A \in (X, Y)$ if and only if the series on the right-hand side of (1.1) converges for each $n \in \mathbb{N}$ and every $x \in X$ and we have $Ax \in Y$ for all $x \in X$.

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The matrix domain X_A of an infinite matrix A in a sequence space X is defined by

$$X_A = \{x = (x_k) \in \omega : Ax \in X\} \tag{1.2}$$

which is a sequence space.

The approach constructing a new sequence space by means of the matrix domain of a particular limitation method has recently been employed by several authors; see, for instance, [1–12].

Let Δ denote the matrix $\Delta = (\Delta_{nk})$ defined by

$$\Delta_{nk} = \begin{cases} (-1)^{n-k} & (n-1 \le k \le n), \\ 0 & (0 \le k < n-1 \text{ or } k > n) \end{cases}$$

or

$$\Delta_{nk} = \begin{cases} (-1)^{n-k} & (n \le k \le n+1), \\ 0 & (0 \le k < n \text{ or } k > n+1). \end{cases}$$

In the literature, the matrix domain λ_Δ is called the *difference sequence space* whenever λ is a normed or paranormed sequence space. The idea of difference sequence spaces was introduced by Kızmaz [13]. In 1981, Kızmaz [13] defined the sequence spaces

$$X(\Delta) = \{x = (x_k) \in \omega : (x_k - x_{k+1}) \in X\}$$

for $X = \ell_\infty$, c and c_0. The difference space bv_p, consisting of all sequences (x_k) such that $(x_k - x_{k-1})$ is in the sequence space ℓ_p, was studied in the case $0 < p < 1$ by Altay and Başar [14] and in the case $1 \le p \le \infty$ by Başar and Altay [4] and Çolak *et al.* [15]. The paranormed difference sequence space

$$\Delta\lambda(p) = \{x = (x_k) \in \omega : (x_k - x_{k+1}) \in \lambda(p)\}$$

was examined by Ahmad and Mursaleen [16] and Malkowsky [17], where $\lambda(p)$ is any of the paranormed spaces $\ell_\infty(p)$, $c(p)$ and $c_0(p)$ defined by Simons [18] and Maddox [19].

Recently, Başar *et al.* [20] have defined the sequence spaces $bv(u,p)$ and $bv_\infty(u,p)$ by

$$bv(u,p) = \left\{x = (x_k) \in \omega : \sum_k \left|u_k(x_k - x_{k-1})\right|^{p_k} < \infty\right\}$$

and

$$bv_\infty(u,p) = \left\{x = (x_k) \in \omega : \sup_{k\in\mathbb{N}} \left|u_k(x_k - x_{k-1})\right|^{p_k} < \infty\right\},$$

where $u = (u_k)$ is an arbitrary fixed sequence and $0 < p_k \le H < \infty$ for all $k \in \mathbb{N}$. These spaces are generalization of the space bv_p for $1 \le p \le \infty$. Quite recently, Kirişçi and Başar [21] have introduced and studied the generalized difference sequence spaces

$$\hat{X} = \{x = (x_k) \in \omega : B(r,s)x \in X\}$$

for $X = \ell_\infty$, ℓ_p, c and c_0, where $1 \le p < \infty$ and $B(r,s)x = (sx_{k-1} + rx_k)$ $(r,s \neq 0)$. Following Kirişçi and Başar [21], Sönmez [22] has examined the sequence space $X(B)$ as the set of all sequences whose $B(r,s,t)$-transforms are in the space $X \in \{\ell_\infty, \ell_p, c, c_0\}$, where $B(r,s,t)$ denotes the triple band matrix $B(r,s,t) = \{b_{nk}(r,s,t)\}$ defined by

$$b_{nk}(r,s,t) = \begin{cases} r & (n = k), \\ s & (n = k+1), \\ t & (n = k+2), \\ 0 & \text{otherwise} \end{cases}$$

for all $n,k \in \mathbb{N}$ and $r,s,t \in \mathbb{R} - \{0\}$. Also in [23–34], the authors studied some difference sequence spaces.

In this paper, we define the Fibonacci difference matrix $\hat{F}$ by using the Fibonacci sequence $\{f_n\}_{n=0}^{\infty}$ and introduce new sequence spaces $\ell_p(\hat{F})$ and $\ell_\infty(\hat{F})$ related to the matrix domain of $\hat{F}$ in the sequence spaces ℓ_p and ℓ_∞, respectively, where $1 \le p < \infty$. This study is organized as follows.

In Section 2, we give some notations and basic concepts including the Fibonacci sequence and a BK-space. In Section 3, we define a new band matrix with Fibonacci numbers and introduce the sequence spaces $\ell_p(\hat{F})$ and $\ell_\infty(\hat{F})$. Also, we establish some inclusion relations concerning these spaces and construct the basis of the space $\ell_p(\hat{F})$ for $1 \le p < \infty$. In Section 4, we determine the α-, β-, γ-duals of the spaces $\ell_p(\hat{F})$ and $\ell_\infty(\hat{F})$. In Section 5, we characterize the classes $(\ell_p(\hat{F}), X)$ and $(\ell_\infty(\hat{F}), X)$, where $1 \le p < \infty$ and X is any of the spaces ℓ_∞, ℓ_1, c and c_0. In the final section of the paper, we investigate some geometric properties of the space $\ell_p(\hat{F})$ for $1 < p < \infty$.

2 The Fibonacci difference sequence space $\ell_p(\hat{F})$

Define the sequence $\{f_n\}_{n=0}^{\infty}$ of Fibonacci numbers given by the linear recurrence relations

$$f_0 = f_1 = 1 \quad \text{and} \quad f_n = f_{n-1} + f_{n-2}, \quad n \ge 2.$$

Fibonacci numbers have many interesting properties and applications in arts, sciences and architecture. For example, the ratio sequences of Fibonacci numbers converges to the golden ratio which is important in sciences and arts. Also, some basic properties of Fibonacci numbers [35] are given as follows:

$$\lim_{n\to\infty} \frac{f_{n+1}}{f_n} = \frac{1+\sqrt{5}}{2} = \alpha \quad \text{(golden ratio)},$$
$$\sum_{k=0}^{n} f_k = f_{n+2} - 1 \quad (n \in \mathbb{N}),$$
$$\sum_k \frac{1}{f_k} \quad \text{converges},$$
$$f_{n-1}f_{n+1} - f_n^2 = (-1)^{n+1} \quad (n \ge 1) \text{ (Cassini formula)}.$$

Substituting for f_{n+1} in Cassini's formula yields $f_{n-1}^2 + f_n f_{n-1} - f_n^2 = (-1)^{n+1}$.

A sequence space X is called a *FK-space* if it is a complete linear metric space with continuous coordinates $p_n : X \to \mathbb{R}$ ($n \in \mathbb{N}$), where $\mathbb{R}$ denotes the real field and $p_n(x) = x_n$ for all $x = (x_k) \in X$ and every $n \in \mathbb{N}$. A *BK space* is a normed *FK space*, that is, a *BK-space* is a Banach space with continuous coordinates. The space ℓ_p ($1 \leq p < \infty$) is a BK-space with $\|x\|_p = (\sum_{k=0}^{\infty} |x_k|^p)^{1/p}$ and c_0, c and ℓ_∞ are BK-spaces with $\|x\|_\infty = \sup_k |x_k|$.

A sequence (b_n) in a normed space X is called a *Schauder basis* for X if for every $x \in X$, there is a unique sequence (α_n) of scalars such that $x = \sum_n \alpha_n b_n$, *i.e.*, $\lim_{m\to\infty} \|x - \sum_{n=0}^{m} \alpha_n b_n\| = 0$.

The α-, β- and γ-duals of the sequence space X are respectively defined by

$$X^{\alpha} = \{a = (a_k) \in \omega : ax = (a_k x_k) \in \ell_1 \text{ for all } x = (x_k) \in X\},$$
$$X^{\beta} = \{a = (a_k) \in \omega : ax = (a_k x_k) \in cs \text{ for all } x = (x_k) \in X\}$$

and

$$X^{\gamma} = \{a = (a_k) \in \omega : ax = (a_k x_k) \in bs \text{ for all } x = (x_k) \in X\},$$

where cs and bs are the sequence spaces of all convergent and bounded series, respectively [36].

We assume throughout that $p, q \geq 1$ with $p^{-1} + q^{-1} = 1$ and denote the collection of all finite subsets of $\mathbb{N}$ by $\mathcal{F}$.

3 The Fibonacci difference sequence spaces $\ell_p(\hat{F})$ and $\ell_\infty(\hat{F})$

In this section, we define the Fibonacci band matrix $\hat{F} = (\hat{f}_{nk})$ and introduce the sequence spaces $\ell_p(\hat{F})$ and $\ell_\infty(\hat{F})$, where $1 \leq p < \infty$. Also, we present some inclusion theorems and construct the Schauder basis of the space $\ell_p(\hat{F})$ for$1 \leq p < \infty$.

Let f_n be the nth Fibonacci number for every $n \in \mathbb{N}$. Then we define the infinite matrix $\hat{F} = (\hat{f}_{nk})$ by

$$\hat{f}_{nk} = \begin{cases} -\frac{f_{n+1}}{f_n} & (k = n-1), \\ \frac{f_n}{f_{n+1}} & (k = n), \\ 0 & (0 \leq k < n-1 \text{ or } k > n) \end{cases} \qquad (n, k \in \mathbb{N}).$$

Now, we introduce the Fibonacci difference sequence spaces $\ell_p(\hat{F})$ and $\ell_\infty(\hat{F})$ as the set of all sequences such that their $\hat{F}$-transforms are in the space ℓ_p and ℓ_∞, respectively, *i.e.*,

$$\ell_p(\hat{F}) = \left\{x = (x_n) \in \omega : \sum_n \left|\frac{f_n}{f_{n+1}} x_n - \frac{f_{n+1}}{f_n} x_{n-1}\right|^p < \infty\right\},$$
$$1 \leq p < \infty,$$

and

$$\ell_\infty(\hat{F}) = \left\{x = (x_n) \in \omega : \sup_{n\in\mathbb{N}} \left|\frac{f_n}{f_{n+1}} x_n - \frac{f_{n+1}}{f_n} x_{n-1}\right| < \infty\right\}.$$

With the notation of (1.2), the sequence spaces $\ell_p(\hat{F})$ and $\ell_\infty(\hat{F})$ may be redefined by

$$\ell_p(\hat{F}) = (\ell_p)_{\hat{F}} \quad (1 \leq p < \infty) \quad \text{and} \quad \ell_\infty(\hat{F}) = (\ell_\infty)_{\hat{F}}. \tag{3.1}$$

Define the sequence $y = (y_n)$, which will be frequently used, by the $\hat{F}$-transform of a sequence $x = (x_n)$, *i.e.*,

$$y_n = \hat{F}_n(x) = \begin{cases} \frac{f_0}{f_1}x_0 = x_0 & (n = 0), \\ \frac{f_n}{f_{n+1}}x_n - \frac{f_{n+1}}{f_n}x_{n-1} & (n \geq 1) \end{cases} \quad (n \in \mathbb{N}). \tag{3.2}$$

Now, we may begin with the following theorem which is essential in the text.

Theorem 3.1 *Let $1 \leq p \leq \infty$. Then $\ell_p(\hat{F})$ is a BK-space with the norm $\|x\|_{\ell_p(\hat{F})} = \|\hat{F}x\|_p$, that is,*

$$\|x\|_{\ell_p(\hat{F})} = \left(\sum_n \left|\hat{F}_n(x)\right|^p\right)^{1/p} \quad (1 \leq p < \infty)$$

and

$$\|x\|_{\ell_\infty(\hat{F})} = \sup_{n\in\mathbb{N}}\left|\hat{F}_n(x)\right|.$$

Proof Since (3.1) holds, ℓ_p and ℓ_∞ are BK-spaces with respect to their natural norms and the matrix $\hat{F}$ is a triangle; Theorem 4.3.12 of Wilansky [37, p.63] gives the fact that the spaces $\ell_p(\hat{F})$ and $\ell_\infty(\hat{F})$ are BK-spaces with the given norms, where $1 \leq p < \infty$. This completes the proof. □

Remark 3.2 One can easily check that the absolute property does not hold on the spaces $\ell_p(\hat{F})$ and $\ell_\infty(\hat{F})$, that is, $\|x\|_{\ell_p(\hat{F})} \neq \||x|\|_{\ell_p(\hat{F})}$ and $\|x\|_{\ell_\infty(\hat{F})} \neq \||x|\|_{\ell_\infty(\hat{F})}$ for at least one sequence in the spaces $\ell_p(\hat{F})$ and $\ell_\infty(\hat{F})$, and this shows that $\ell_p(\hat{F})$ and $\ell_\infty(\hat{F})$ are the sequence spaces of non-absolute type, where $|x| = (|x_k|)$ and $1 \leq p < \infty$.

Theorem 3.3 *The Fibonacci difference sequence space $\ell_p(\hat{F})$ of non-absolute type is linearly isomorphic to the space ℓ_p, that is, $\ell_p(\hat{F}) \cong \ell_p$ for $1 \leq p \leq \infty$.*

Proof To prove this, we should show the existence of a linear bijection between the spaces $\ell_p(\hat{F})$ and ℓ_p for $1 \leq p \leq \infty$. Consider the transformation T defined, with the notation of (3.2), from $\ell_p(\hat{F})$ to ℓ_p by $x \to y = Tx$. Then $Tx = y = \hat{F}x \in \ell_p$ for every $x \in \ell_p(\hat{F})$. Also, the linearity of T is clear. Further, it is trivial that $x = 0$ whenever $Tx = 0$ and hence T is injective.

Furthermore, let $y = (y_k) \in \ell_p$ for $1 \leq p \leq \infty$ and define the sequence $x = (x_k)$ by

$$x_k = \sum_{j=0}^{k} \frac{f_{k+1}^2}{f_j f_{j+1}} y_j \quad (k \in \mathbb{N}). \tag{3.3}$$

Then, in the cases $1 \leq p < \infty$ and $p = \infty$, we get

$$\|x\|_{\ell_p(\hat{F})} = \left(\sum_k \left|\frac{f_k}{f_{k+1}}x_k - \frac{f_{k+1}}{f_k}x_{k-1}\right|^p\right)^{1/p} = \left(\sum_k \left|\frac{f_k}{f_{k+1}}\sum_{j=0}^{k}\frac{f_{k+1}^2}{f_j f_{j+1}}y_j - \frac{f_{k+1}}{f_k}\sum_{j=0}^{k-1}\frac{f_k^2}{f_j f_{j+1}}y_j\right|^p\right)^{1/p}$$
$$= \left(\sum_k |y_k|^p\right)^{1/p} = \|y\|_p < \infty$$

and

$$\|x\|_{\ell_\infty(\hat{F})} = \sup_{k\in\mathbb{N}}\left|\hat{F}_k(x)\right| = \|y\|_\infty < \infty,$$

respectively. Thus, we have $x \in \ell_p(\hat{F})$ $(1 \leq p \leq \infty)$. Hence, T is surjective and norm preserving. Consequently, T is a linear bijection which shows that the spaces $\ell_p(\hat{F})$ and ℓ_p are linearly isomorphic for $1 \leq p \leq \infty$. This concludes the proof. □

Now, we give some inclusion relations concerning the space $\ell_p(\hat{F})$.

Theorem 3.4 *The inclusion $\ell_p \subset \ell_p(\hat{F})$ strictly holds for $1 \leq p \leq \infty$.*

Proof To prove the validity of the inclusion $\ell_p \subset \ell_p(\hat{F})$ for $1 \leq p \leq \infty$, it suffices to show the existence of a number $M > 0$ such that $\|x\|_{\ell_p(\hat{F})} \leq M\|x\|_p$ for every $x \in \ell_p$.

Let $x \in \ell_p$ and $1 < p \leq \infty$. Since the inequalities $\frac{f_k}{f_{k+1}} \leq 1$ and $\frac{f_{k+1}}{f_k} \leq 2$ hold for every $k \in \mathbb{N}$, we obtain with the notation of (3.2),

$$\sum_k \left|\hat{F}_k(x)\right|^p \leq \sum_k 2^{p-1}\left(|x_k|^p + |2x_{k-1}|^p\right) \leq 2^{2p-1}\left(\sum_k |x_k|^p + \sum_k |x_{k-1}|^p\right)$$

and

$$\sup_{k\in\mathbb{N}}\left|\hat{F}_k(x)\right| \leq 3\sup_{k\in\mathbb{N}}|x_k|,$$

which together yield, as expected,

$$\|x\|_{\ell_p(\hat{F})} \leq 4\|x\|_p \tag{3.4}$$

for $1 < p \leq \infty$. Further, since the sequence $x = (x_k) = (f_{k+1}^2) = (1, 2^2, 3^2, 5^2, \ldots)$ is in $\ell_p(\hat{F}) - \ell_p$, the inclusion $\ell_p \subset \ell_p(\hat{F})$ is strict for $1 < p \leq \infty$. Similarly, one can easily prove that inequality (3.4) also holds in the case $p = 1$, and so we omit the details. This completes the proof. □

Theorem 3.5 *Neither of the spaces bv_p and $\ell_p(\hat{F})$ includes the other one, where $1 \leq p < \infty$.*

Proof Let $e = (1, 1, 1, \ldots)$ and $x = (x_k) = (f_{k+1}^2)$. Then, since $\hat{F}x = (1, 0, 0, \ldots) \in \ell_p$ and $\Delta x = (1, f_0f_3, f_1f_4, \ldots, f_{k-1}f_{k+2}, \ldots) \notin \ell_p$, we conclude that x is in $\ell_p(\hat{F})$ but not in bv_p. Now, consider the equation

$$\left|\frac{f_k}{f_{k+1}} - \frac{f_{k+1}}{f_k}\right| = \frac{|f_k^2 - f_{k+1}^2|}{f_k f_{k+1}} = \frac{|(-1)^k - f_k f_{k+1}|}{f_k f_{k+1}} \quad (k \in \mathbb{N}).$$

Then $|(-1)^k - f_k f_{k+1}| > f_k f_{k+1}$ whenever k is odd, which implies that the series $\sum_k |\frac{f_k}{f_{k+1}} - \frac{f_{k+1}}{f_k}|^p$ is not convergent, where $1 \leq p < \infty$. Thus, $\hat{F}e = (\frac{f_k}{f_{k+1}} - \frac{f_{k+1}}{f_k})$ is not in ℓ_p for $1 \leq p < \infty$. Additionally, since $\Delta e = (1, 0, 0, \dots)$, the sequence e is in ℓ_p. Hence, the sequence spaces $\ell_p(\hat{F})$ and bv_p overlap but neither contains the other, as asserted. □

Theorem 3.6 *If* $1 \leq p < s$, *then* $\ell_p(\hat{F}) \subset \ell_s(\hat{F})$.

Proof Let $1 \leq p < s$ and $x \in \ell_p(\hat{F})$. Then we obtain from Theorem 3.1 that $y \in \ell_p$, where y is the sequence given by (3.2). Thus, the well-known inclusion $\ell_p \subset \ell_s$ yields $y \in \ell_s$. This means that $x \in \ell_s(\hat{F})$ and hence, the inclusion $\ell_p(\hat{F}) \subset \ell_s(\hat{F})$ holds. This completes the proof. □

Now, we give a sequence of the points of the space $\ell_p(\hat{F})$ which forms a basis for the space $\ell_p(\hat{F})$ $(1 \leq p < \infty)$.

Theorem 3.7 *Let* $1 \leq p < \infty$ *and define the sequence* $c^{(k)} \in \ell_p(\hat{F})$ *for every fixed* $k \in \mathbb{N}$ *by*

$$\left(c^{(k)}\right)_n = \begin{cases} 0 & (n < k), \\ \frac{f_{n+1}^2}{f_k f_{k+1}} & (n \geq k) \end{cases} \quad (n \in \mathbb{N}). \tag{3.5}$$

Then the sequence $(c^{(k)})_{k=0}^{\infty}$ *is a basis for the space* $\ell_p(\hat{F})$, *and every* $x \in \ell_p(\hat{F})$ *has a unique representation of the form*

$$x = \sum_k \hat{F}_k(x) c^{(k)}. \tag{3.6}$$

Proof Let $1 \leq p < \infty$. Then it is obvious by (3.5) that $\hat{F}(c^{(k)}) = e^{(k)} \in \ell_p$ $(k \in \mathbb{N})$ and hence $c^{(k)} \in \ell_p(\hat{F})$ for all $k \in \mathbb{N}$.

Further, let $x \in \ell_p(\hat{F})$ be given. For every non-negative integer m, we put

$$x^{(m)} = \sum_{k=0}^{m} \hat{F}_k(x) c^{(k)}.$$

Then we have that

$$\hat{F}\left(x^{(m)}\right) = \sum_{k=0}^{m} \hat{F}_k(x) \hat{F}\left(c^{(k)}\right) = \sum_{k=0}^{m} \hat{F}_k(x) e^{(k)}$$

and hence

$$\hat{F}_n\left(x - x^{(m)}\right) = \begin{cases} 0 & (0 \leq n \leq m), \\ \hat{F}_n(x) & (n > m) \end{cases} \quad (n, m \in \mathbb{N}).$$

Now, for any given $\varepsilon > 0$, there is a non-negative integer m_0 such that

$$\sum_{n=m_0+1}^{\infty} \left|\hat{F}_n(x)\right|^p \leq \left(\frac{\varepsilon}{2}\right)^p.$$

Therefore, we have for every $m \geq m_0$ that

$$\left\|x - x^{(m)}\right\|_{\ell_p(\hat{F})} = \left(\sum_{n=m+1}^{\infty} \left|\hat{F}_n(x)\right|^p\right)^{1/p} \leq \left(\sum_{n=m_0+1}^{\infty} \left|\hat{F}_n(x)\right|^p\right)^{1/p} \leq \frac{\varepsilon}{2} < \varepsilon,$$

which shows that $\lim_{m\to\infty} \|x - x^{(m)}\|_{\ell_p(\hat{F})} = 0$ and hence x is represented as in (3.6).

Finally, let us show the uniqueness of the representation (3.6) of $x \in \ell_p(\hat{F})$. For this, suppose that $x = \sum_k \mu_k(x)c^{(k)}$. Since the linear transformation T defined from $\ell_p(\hat{F})$ to ℓ_p in the proof of Theorem 3.3 is continuous, we have

$$\hat{F}_n(x) = \sum_k \mu_k(x)\hat{F}_n\left(c^{(k)}\right) = \sum_k \mu_k(x)\delta_{nk} = \mu_n(x) \quad (n \in \mathbb{N}).$$

Hence, the representation (3.6) of $x \in \ell_p(\hat{F})$ is unique. This concludes the proof. □

4 The α-, β- and γ-duals of the space $\ell_p(\hat{F})$

In this section, we determine the α-, β- and γ-duals of the sequence space $\ell_p(\hat{F})$ of non-absolute type. Since the case $p = 1$ can be proved by analogy, we omit the proof of that case and consider only the case $1 < p \leq \infty$ in the proof of Theorems 4.5 and 4.6, respectively.

The following known results [38] are fundamental for our investigation.

Lemma 4.1 *$A = (a_{nk}) \in (\ell_p, \ell_1)$ if and only if*

$$\sup_{K\in\mathcal{F}} \sum_k \left|\sum_{n\in K} a_{nk}\right| < \infty, \quad 1 < p \leq \infty.$$

Lemma 4.2 *$A = (a_{nk}) \in (\ell_p, c)$ if and only if*

$$\lim_{n\to\infty} a_{nk} \quad \textit{exists for all } k \in \mathbb{N}, \tag{4.1}$$

$$\sup_{n\in\mathbb{N}} \sum_k |a_{nk}|^q < \infty, \quad 1 < p < \infty. \tag{4.2}$$

Lemma 4.3 *$A = (a_{nk}) \in (\ell_\infty, c)$ if and only if (4.1) holds and*

$$\lim_{n\to\infty} \sum_k |a_{nk}| = \sum_k \left|\lim_{n\to\infty} a_{nk}\right|. \tag{4.3}$$

Lemma 4.4 *$A = (a_{nk}) \in (\ell_p, \ell_\infty)$ if and only if (4.2) holds with $1 < p \leq \infty$.*

Theorem 4.5 *The α-dual of the space $\ell_p(\hat{F})$ is the set*

$$\hat{d}_1 = \left\{a = (a_k) \in \omega : \sup_{K\in\mathcal{F}} \sum_k \left|\sum_{n\in K} \frac{f_{n+1}^2}{f_k f_{k+1}} a_n\right|^q < \infty\right\},$$

where $1 < p \leq \infty$.

Proof Let $1 < p \leq \infty$. For any fixed sequence $a = (a_n) \in \omega$, we define the matrix $B = (b_{nk})$ by

$$b_{nk} = \begin{cases} \frac{f_{n+1}^2}{f_k f_{k+1}} a_n & (0 \leq k \leq n), \\ 0 & (k > n) \end{cases}$$

for all $n, k \in \mathbb{N}$. Also, for every $x = (x_n) \in \omega$, we put $y = \hat{F}x$. Then it follows by (3.2) that

$$a_n x_n = \sum_{k=0}^{n} \frac{f_{n+1}^2}{f_k f_{k+1}} a_n y_k = B_n(y) \quad (n \in \mathbb{N}). \tag{4.4}$$

Thus, we observe by (4.4) that $ax = (a_n x_n) \in \ell_1$ whenever $x \in \ell_p(\hat{F})$ if and only if $By \in \ell_1$ whenever $y \in \ell_p$. Therefore, we derive by using Lemma 4.1 that

$$\sup_{K \in \mathcal{F}} \sum_k \left| \sum_{n \in K} \frac{f_{n+1}^2}{f_k f_{k+1}} a_n \right|^q < \infty,$$

which implies that $(\ell_p(\hat{F}))^\alpha = \hat{d}_1$. □

Theorem 4.6 *Define the sets* $\hat{d}_2$, $\hat{d}_3$ *and* $\hat{d}_4$ *by*

$$\hat{d}_2 = \left\{ a = (a_k) \in \omega : \sum_{j=k}^{\infty} \frac{f_{j+1}^2}{f_k f_{k+1}} a_j \text{ exists for all } k \in \mathbb{N} \right\},$$

$$\hat{d}_3 = \left\{ a = (a_k) \in \omega : \sup_{n \in \mathbb{N}} \sum_{k=0}^{n} \left| \sum_{j=k}^{n} \frac{f_{j+1}^2}{f_k f_{k+1}} a_j \right|^q < \infty \right\}$$

and

$$\hat{d}_4 = \left\{ a = (a_k) \in \omega : \lim_{n \to \infty} \sum_{k=0}^{n} \left| \sum_{j=k}^{n} \frac{f_{j+1}^2}{f_k f_{k+1}} a_j \right| = \sum_k \left| \sum_{j=k}^{\infty} \frac{f_{j+1}^2}{f_k f_{k+1}} a_j \right| < \infty \right\}.$$

Then $(\ell_p(\hat{F}))^\beta = \hat{d}_2 \cap \hat{d}_3$ *and* $(\ell_\infty(\hat{F}))^\beta = \hat{d}_2 \cap \hat{d}_4$, *where* $1 < p < \infty$.

Proof Let $a = (a_k) \in \omega$ and consider the equality

$$\sum_{k=0}^{n} a_k x_k = \sum_{k=0}^{n} a_k \left(\sum_{j=0}^{n} \frac{f_{k+1}^2}{f_j f_{j+1}} y_j \right) = \sum_{k=0}^{n} \left(\sum_{j=k}^{n} \frac{f_{j+1}^2}{f_k f_{k+1}} a_j \right) y_k = D_n(y), \tag{4.5}$$

where $D = (d_{nk})$ is defined by

$$d_{nk} = \begin{cases} \sum_{j=k}^{n} \frac{f_{j+1}^2}{f_k f_{k+1}} a_j & (0 \leq k \leq n), \\ 0 & (k > n), \end{cases} \qquad n, k \in \mathbb{N}.$$

Then we deduce from Lemma 4.2 with (3.2) that $ax = (a_k x_k) \in cs$ whenever $x = (x_k) \in \ell_p(\hat{F})$ if and only if $Dy \in c$ whenever $y = (y_k) \in \ell_p$. Thus, $(a_k) \in (\ell_p(\hat{F}))^\beta$ if and only if $(a_k) \in$

$\hat{d}_2$ and $(a_k) \in \hat{d}_3$ by (4.1) and (4.2), respectively. Hence, $(\ell_p(\hat{F}))^\beta = \hat{d}_2 \cap \hat{d}_3$. It is clear that one can also prove the case $p = \infty$ by the technique used in the proof of the case $1 < p < \infty$ with Lemma 4.3 instead of Lemma 4.2. So, we leave the detailed proof to the reader. □

Theorem 4.7 $(\ell_p(\hat{F}))^\gamma = \hat{d}_3$, *where* $1 < p \leq \infty$.

Proof This result can be obtained from Lemma 4.4 by using (4.5). □

5 Some matrix transformations related to the sequence space $\ell_p(\hat{F})$

In this section, we characterize the classes $(\ell_p(\hat{F}), X)$, where $1 \leq p \leq \infty$ and X is any of the spaces ℓ_∞, ℓ_1, c and c_0.

For simplicity in notation, we write

$$\widetilde{a}_{nk} = \sum_{j=k}^{\infty} \frac{f_{j+1}^2}{f_k f_{k+1}} a_{nj}$$

for all $k, n \in \mathbb{N}$.

The following lemma is essential for our results.

Lemma 5.1 (see [21, Theorem 4.1]) *Let λ be an FK-space, U be a triangle, V be its inverse and μ be an arbitrary subset of ω. Then we have $A = (a_{nk}) \in (\lambda_U, \mu)$ if and only if*

$$C^{(n)} = \bigl(c_{mk}^{(n)}\bigr) \in (\lambda, c) \quad \textit{for all } n \in \mathbb{N}$$

and

$$C = (c_{nk}) \in (\lambda, \mu),$$

where

$$c_{mk}^{(n)} = \begin{cases} \sum_{j=k}^{m} a_{nj} v_{jk} & (0 \leq k \leq m), \\ 0 & (k > m) \end{cases}$$

and $c_{nk} = \sum_{j=k}^{\infty} a_{nj} v_{jk}$ for all $k, m, n \in \mathbb{N}$.

Now, we list the following conditions:

$$\sup_{m \in \mathbb{N}} \sum_{k=0}^{m} \left| \sum_{j=k}^{m} \frac{f_{j+1}^2}{f_k f_{k+1}} a_{nj} \right|^q < \infty, \tag{5.1}$$

$$\lim_{m \to \infty} \sum_{j=k}^{m} \frac{f_{j+1}^2}{f_k f_{k+1}} a_{nj} = \widetilde{a}_{nk}, \quad \forall n, k \in \mathbb{N}, \tag{5.2}$$

$$\lim_{m \to \infty} \sum_{k=0}^{m} \left| \sum_{j=k}^{m} \frac{f_{j+1}^2}{f_k f_{jk+1}} a_{nj} \right| = \sum_{k} |\widetilde{a}_{nk}| \quad \text{for each } n \in \mathbb{N}, \tag{5.3}$$

$$\sup_{n \in \mathbb{N}} \sum_{k} |\widetilde{a}_{nk}|^q < \infty, \tag{5.4}$$

$$\sup_{N\in\mathcal{F}}\sum_k\left|\sum_{n\in\mathbb{N}}\widetilde{a}_{nk}\right|^q<\infty, \tag{5.5}$$

$$\lim_{n\to\infty}\widetilde{a}_{nk}=\widetilde{\alpha}_k;\quad k\in\mathbb{N}, \tag{5.6}$$

$$\lim_{n\to\infty}\sum_k|\widetilde{a}_{nk}|=\sum_k|\widetilde{\alpha}_k|, \tag{5.7}$$

$$\lim_{n\to\infty}\sum_k\widetilde{a}_{nk}=0, \tag{5.8}$$

$$\sup_{n,k\in\mathbb{N}}|\widetilde{a}_{nk}|<\infty, \tag{5.9}$$

$$\sup_{k,m\in\mathbb{N}}\left|\sum_{j=k}^m\frac{f_{j+1}^2}{f_kf_{k+1}}a_{nj}\right|<\infty, \tag{5.10}$$

$$\sup_{k\in\mathbb{N}}\sum_n|\widetilde{a}_{nk}|<\infty, \tag{5.11}$$

$$\sup_{N,K\in\mathcal{F}}\left|\sum_{n\in N}\sum_{k\in K}\widetilde{a}_{nk}\right|<\infty. \tag{5.12}$$

Then, by combining Lemma 5.1 with the results in [38], we immediately derive the following results.

Theorem 5.2
(a) $A=(a_{nk})\in(\ell_1(\hat{F}),\ell_\infty)$ *if and only* (5.2), (5.9) *and* (5.10) *hold.*
(b) $A=(a_{nk})\in(\ell_1(\hat{F}),c)$ *if and only if* (5.2), (5.6), (5.9) *and* (5.10) *hold.*
(c) $A=(a_{nk})\in(\ell_1(\hat{F}),c_0)$ *if and only if* (5.2), (5.6) *with* $\widetilde{\alpha}_k=0$, (5.9) *and* (5.10) *hold.*
(d) $A=(a_{nk})\in(\ell_1(\hat{F}),\ell_1)$ *if and only* (5.2), (5.10) *and* (5.11) *hold.*

Theorem 5.3 *Let* $1<p<\infty$. *Then we have*
(a) $A=(a_{nk})\in(\ell_p(\hat{F}),\ell_\infty)$ *if and only if* (5.1), (5.2) *and* (5.4) *hold.*
(b) $A=(a_{nk})\in(\ell_p(\hat{F}),c)$ *if and only if* (5.1), (5.2), (5.4) *and* (5.6) *hold.*
(c) $A=(a_{nk})\in(\ell_p(\hat{F}),c_0)$ *if and only if* (5.1), (5.2), (5.4) *and* (5.6) *with* $\widetilde{\alpha}_k=0$ *hold.*
(d) $A=(a_{nk})\in(\ell_p(\hat{F}),\ell_1)$ *if and only if* (5.1), (5.2) *and* (5.5) *hold.*

Theorem 5.4
(a) $A=(a_{nk})\in(\ell_\infty(\hat{F}),\ell_\infty)$ *if and only* (5.2), (5.3) *and* (5.4) *with* $q=1$ *hold.*
(b) $A=(a_{nk})\in(\ell_\infty(\hat{F}),c)$ *if and only* (5.2), (5.3), (5.6) *and* (5.7) *hold.*
(c) $A=(a_{nk})\in(\ell_\infty(\hat{F}),c_0)$ *if and only* (5.2), (5.3) *and* (5.8) *hold.*
(d) $A=(a_{nk})\in(\ell_\infty(\hat{F}),\ell_1)$ *if and only* (5.2), (5.3) *and* (5.12) *hold.*

6 Some geometric properties of the space $\ell_p(\hat{F})$ ($1<p<\infty$)

In this section, we study some geometric properties of the space $\ell_p(\hat{F})$ for $1<p<\infty$.

For these properties, we refer to [3, 39–47].

A Banach space X is said to have the *Banach-Saks property* if every bounded sequence (x_n) in X admits a subsequence (z_n) such that the sequence $\{t_k(z)\}$ is convergent in the norm in X [40], where

$$t_k(z)=\frac{1}{k+1}(z_0+z_1+\cdots+z_k)\quad(k\in\mathbb{N}). \tag{6.1}$$

A Banach space X is said to have the *weak Banach-Saks property* whenever, given any weakly null sequence $(x_n) \subset X$, there exists a subsequence (z_n) of (x_n) such that the sequence $\{t_k(z)\}$ is strongly convergent to zero.

In [43], García-Falset introduces the following coefficient:

$$R(X) = \sup\Big\{\liminf_{n\to\infty}\|x_n - x\| : (x_n) \subset B(X), x_n \xrightarrow{w} 0, x \in B(X)\Big\}, \tag{6.2}$$

where $B(X)$ denotes the unit ball of X.

Remark 6.1 A Banach space X with $R(X) < 2$ has the weak fixed point property [44].

Let $1 < p < \infty$. A Banach space is said to have the *Banach-Saks type p* or the property $(BS)_p$ if every weakly null sequence (x_k) has a subsequence (x_{k_l}) such that for some $C > 0$,

$$\left\|\sum_{l=0}^{n} x_{k_l}\right\| < C(n+1)^{1/p} \tag{6.3}$$

for all $n \in \mathbb{N}$ (see [45]).

Now, we may give the following results related to some geometric properties, mentioned above, of the space $\ell_p(\hat{F})$, where $1 < p < \infty$.

Theorem 6.2 *Let $1 < p < \infty$. Then the space $\ell_p(\hat{F})$ has the Banach-Saks type p.*

Proof Let (ε_n) be a sequence of positive numbers for which $\sum \varepsilon_n \leq 1/2$, and also let (x_n) be a weakly null sequence in $B(\ell_p(\hat{F}))$. Set $z_0 = x_0 = 0$ and $z_1 = x_{n_1} = x_1$. Then there exists $m_1 \in \mathbb{N}$ such that

$$\left\|\sum_{i=m_1+1}^{\infty} z_1(i)e^{(i)}\right\|_{\ell_p(\hat{F})} < \varepsilon_1. \tag{6.4}$$

Since (x_n) being a weakly null sequence implies $x_n \to 0$ coordinatewise, there is an $n_2 \in \mathbb{N}$ such that

$$\left\|\sum_{i=0}^{m_1} x_n(i)e^{(i)}\right\|_{\ell_p(\hat{F})} < \varepsilon_1,$$

when $n \geq n_2$. Set $z_2 = x_{n_2}$. Then there exists an $m_2 > m_1$ such that

$$\left\|\sum_{i=m_2+1}^{\infty} z_2(i)e^{(i)}\right\|_{\ell_p(\hat{F})} < \varepsilon_2.$$

Again using the fact that $x_n \to 0$ coordinatewise, there exists an $n_3 \geq n_2$ such that

$$\left\|\sum_{i=0}^{m_2} x_n(i)e^{(i)}\right\|_{\ell_p(\hat{F})} < \varepsilon_2,$$

when $n \geq n_3$.

If we continue this process, we can find two increasing subsequences (m_i) and (n_i) such that

$$\left\| \sum_{i=0}^{m_j} x_n(i)e^{(i)} \right\|_{\ell_p(\hat{F})} < \varepsilon_j$$

for each $n \geq n_{j+1}$ and

$$\left\| \sum_{i=m_j+1}^{\infty} z_j(i)e^{(i)} \right\|_{\ell_p(\hat{F})} < \varepsilon_j,$$

where $b_j = x_{n_j}$. Hence,

$$\begin{aligned} \left\| \sum_{j=0}^{n} z_j \right\|_{\ell_p(\hat{F})} &= \left\| \sum_{j=0}^{n} \left(\sum_{i=0}^{m_{j-1}} z_j(i)e^{(i)} + \sum_{i=m_{j-1}+1}^{m_j} z_j(i)e^{(i)} + \sum_{i=m_j+1}^{\infty} z_j(i)e^{(i)} \right) \right\|_{\ell_p(\hat{F})} \\ &\leq \left\| \sum_{j=0}^{n} \left(\sum_{i=m_{j-1}+1}^{m_j} z_j(i)e^{(i)} \right) \right\|_{\ell_p(\hat{F})} + 2\sum_{j=0}^{n} \varepsilon_j. \end{aligned}$$

On the other hand, it can be seen that $\|x\|_{\ell_p(\hat{F})} < 1$. Therefore, we have that

$$\begin{aligned} \left\| \sum_{j=0}^{n} \left(\sum_{i=m_{j-1}+1}^{m_j} z_j(i)e^{(i)} \right) \right\|_{\ell_p(\hat{F})}^{p} &= \sum_{j=0}^{n} \sum_{i=m_{j-1}+1}^{m_j} \left| \frac{f_i}{f_{i+1}} z_j(i) - \frac{f_{i+1}}{f_i} z_j(i-1) \right|^p \\ &\leq \sum_{j=0}^{n} \sum_{i=0}^{\infty} \left| \frac{f_i}{f_{i+1}} z_j(i) - \frac{f_{i+1}}{f_i} z_j(i-1) \right|^p \\ &\leq (n+1). \end{aligned}$$

Hence, we obtain

$$\left\| \sum_{j=0}^{n} \left(\sum_{i=m_{j-1}+1}^{m_j} z_j(i)e^{(i)} \right) \right\| \leq (n+1)^{1/p}.$$

By using the fact that $1 \leq (n+1)^{1/p}$ for all $n \in \mathbb{N}$ and $1 < p < \infty$, we have

$$\left\| \sum_{j=0}^{n} z_j \right\|_{\ell_p(\hat{F})} \leq (n+1)^{1/p} + 1 \leq 2(n+1)^{1/p}.$$

Hence, $\ell_p(\hat{F})$ has the Banach-Saks type p. This concludes the proof. □

Remark 6.3 Note that $R(\ell_p(\hat{F})) = R(\ell_p) = 2^{1/p}$ since $\ell_p(\hat{F})$ is linearly isomorphic to ℓ_p.

Hence, by Remarks 6.1 and 6.3, we have the following theorem.

Theorem 6.4 *The space $\ell_p(\hat{F})$ has the weak fixed point property, where $1 < p < \infty$.*

Competing interests
The author declares that they have no competing interests.

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