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A damped algorithm for the split feasibility and fixed point problems

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Abstract
The purpose of this paper is to study the split feasibility problem and the fixed point problem. We suggest a damped algorithm. Convergence theorem is proven.
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Keywords: split feasibility problem; fixed point problem; nonexpansive mapping; damped algorithm

1 Introduction

Let C and Q be two closed convex subsets of two Hilbert spaces H_1 and H_2, respectively, and let $A : H_1 \to H_2$ be a bounded linear operator. Finding a point x^* satisfies

$$x^* \in C \quad \text{and} \quad Ax^* \in Q. \tag{1.1}$$

This problem, referred to as the split problem, has been studied by some authors. See, *e.g.*, [1–8] and [9]. Some algorithms for solving (1.1) have been presented. One is Byrne's CQ algorithm [1]

$$x_{n+1} = P_C\big(x_n - \tau A^*(I - P_Q)Ax_n\big), \quad n \in \mathbb{N},$$

where $\tau \in (0, \frac{2}{L})$ with L being the largest eigenvalue of the matrix A^*A, I is the unit matrix or operator, and P_C and P_Q denote the orthogonal projections onto C and Q, respectively. Motivated by Byrne's CQ algorithm, Xu [6] suggested a single step regularized method

$$x_{n+1} = P_C\big((1 - \alpha_n\gamma_n)x_n - \gamma_n A^*(I - P_Q)Ax_n\big), \quad n \in \mathbb{N}. \tag{1.2}$$

Very recently, Dang and Gao [5] introduced the following damped projection algorithm

$$x_{n+1} = (1 - \beta_n)x_n + \beta_n P_C\big((1 - \alpha_n)\big(x_n - \tau A^*(I - P_Q)Ax_n\big)\big), \quad n \in \mathbb{N}.$$

If every closed convex subset of a Hilbert space is the fixed point set of its associating projection, then the split feasibility problem becomes a special case of the split common fixed point problem of finding a point x^* with the property

$$x^* \in \mathrm{Fix}(U) \quad \text{and} \quad Ax^* \in \mathrm{Fix}(T).$$

This problem was first introduced by Censor and Segal [10], who invented an algorithm, which generates a sequence $\{x_n\}$ according to the iterative procedure

$$x_{n+1} = U\big(x_n - \gamma A^*(I - T)Ax_n\big), \quad n \in \mathbb{N}.$$

Recently, Cui, Su and Wang [11] extended the damped projection algorithm to the split common fixed point problems. For some related work, please refer to [12] and [13, 14].

Motivated by these results, the purpose of this paper is to study the following split feasibility problem and fixed point problem

$$\text{Find } x^* \in C \cap \operatorname{Fix}(T) \text{ such that } Ax^* \in Q \cap \operatorname{Fix}(S), \tag{1.3}$$

where $S : Q \to Q$ and $T : C \to C$ are two nonexpansive mappings. We suggest a damped algorithm for solving (1.3). Convergence theorem is proven.

2 Preliminaries

Let H be a real Hilbert space with the inner product $\langle \cdot, \cdot \rangle$ and the norm $\| \cdot \|$, respectively. Let C be a nonempty closed convex subset of H.

Definition 2.1 A mapping $T : C \to C$ is called *nonexpansive* if

$$\|Tx - Ty\| \le \|x - y\|$$

for all $x, y \in C$.

We will use $\operatorname{Fix}(T)$ to denote the set of fixed points of T, that is, $\operatorname{Fix}(T) = \{x \in C : x = Tx\}$.

Definition 2.2 We call $P_C : H \to C$ the metric projection if for each $x \in H$

$$\big\|x - P_C(x)\big\| = \inf\big\{\|x - y\| : y \in C\big\}.$$

It is well known that the metric projection $P_C : H \to C$ is characterized by

$$\big\langle x - P_C(x), y - P_C(x)\big\rangle \le 0$$

for all $x \in H$, $y \in C$. From this, we can deduce that P_C is firmly-nonexpansive, that is,

$$\big\|P_C(x) - P_C(y)\big\|^2 \le \big\langle x - y, P_C(x) - P_C(y)\big\rangle \tag{2.1}$$

for all $x, y \in H$. Hence P_C is also nonexpansive.

It is well known that in a real Hilbert space H, the following two equalities hold

$$\big\|tx + (1 - t)y\big\|^2 = t\|x\|^2 + (1 - t)\|y\|^2 - t(1 - t)\|x - y\|^2 \tag{2.2}$$

for all $x, y \in H$ and $t \in [0, 1]$, and

$$\|x + y\|^2 = \|x\|^2 + 2\langle x, y\rangle + \|y\|^2 \tag{2.3}$$

for all $x, y \in H$. It follows that

$$\|x+y\|^2 \leq \|x\|^2 + 2\langle y, x+y\rangle \tag{2.4}$$

for all $x, y \in H$.

Lemma 2.3 [15] *Let C be a closed convex subset of a real Hilbert space H, and let $S : C \to C$ be a nonexpansive mapping. Then, the mapping $I - S$ is demiclosed. That is, if $\{x_n\}$ is a sequence in C such that $x_n \to x^*$ weakly and $(I-S)x_n \to y$ strongly, then $(I-S)x^* = y$.*

Lemma 2.4 [16] *Assume that $\{a_n\}$ is a sequence of nonnegative real numbers such that*

$$a_{n+1} \leq (1-\gamma_n)a_n + \delta_n, \quad n \in \mathbb{N},$$

where $\{\gamma_n\}$ is a sequence in $(0,1)$, and $\{\delta_n\}$ is a sequence such that
(1) $\sum_{n=1}^{\infty} \gamma_n = \infty$;
(2) $\limsup_{n\to\infty} \frac{\delta_n}{\gamma_n} \leq 0$ *or* $\sum_{n=1}^{\infty} |\delta_n| < \infty$.
Then $\lim_{n\to\infty} a_n = 0$.

3 Main results

Let C and Q be two nonempty closed convex subsets of real Hilbert spaces H_1 and H_2, respectively. Let $A : H_1 \to H_2$ be a bounded linear operator with its adjoint A^*. Let $S : Q \to Q$ and $T : C \to C$ be two nonexpansive mappings. We use Γ to denote the set of solutions of (1.3), that is, $\Gamma = \{x^* | x^* \in C \cap \mathrm{Fix}(T), Ax^* \in Q \cap \mathrm{Fix}(S)\}$. Now, we present our algorithm.

Algorithm 3.1 For $x_0 \in H_1$ arbitrarily, let $\{x_n\}$ be a sequence defined by

$$x_{n+1} = TP_C\big((1-\alpha_n)\big(x_n - \delta A^*(I - SP_Q)Ax_n\big)\big) \quad \text{for all } n \in \mathbb{N}, \tag{3.1}$$

where $\{\alpha_n\}_{n\in\mathbb{N}}$ and $\{\beta_n\}_{n\in\mathbb{N}}$ are two real number sequences in $(0,1)$ and $\delta \in (0, \frac{1}{\|A\|^2})$.

Theorem 3.2 *Suppose $\Gamma \neq \emptyset$. Assume the sequence $\{\alpha_n\}_{n\in\mathbb{N}}$ satisfies three conditions*
(C1) $\lim_{n\to\infty} \alpha_n = 0$;
(C2) $\sum_{n=1}^{\infty} \alpha_n = \infty$;
(C3) $\lim_{n\to\infty} \frac{\alpha_{n+1}}{\alpha_n} = 1$.
Then the sequence $\{x_n\}$, generated by algorithm (3.1), *converges strongly to $x^* = P_\Gamma(0)$.*

Proof For the convenience, we write $z_n = P_Q Ax_n$, $y_n = (1-\alpha_n)(x_n - \delta A^*(I - SP_Q)Ax_n)$ and $u_n = P_C((1-\alpha_n)(x_n - \delta A^*(I - SP_Q)Ax_n))$ for all $n \in \mathbb{N}$. Thus $u_n = P_C y_n$ for all $n \in \mathbb{N}$.

Let $x^* = P_\Gamma(0)$. Hence, $x^* \in C \cap \mathrm{Fix}(T)$ and $Ax^* \in Q \cap \mathrm{Fix}(S)$. By the firmly-nonexpansivity of P_C and P_Q, we can deduce the following conclusions

$$\|z_n - Ax^*\| = \|P_Q Ax_n - P_Q Ax^*\| \leq \|Ax_n - Ax^*\|, \tag{3.2}$$

$$\|u_n - x^*\| = \|P_C y_n - P_C x^*\| \leq \|y_n - x^*\|, \tag{3.3}$$

$$\|Sz_n - Ax^*\|^2 \leq \|z_n - Ax^*\|^2 \leq \|Ax_n - Ax^*\|^2 - \|z_n - Ax_n\|^2, \tag{3.4}$$

$$\|u_{n+1} - u_n\| = \|P_C y_{n+1} - P_C y_n\| \leq \|y_{n+1} - y_n\| \tag{3.5}$$

and

$$\|z_{n+1}-z_n\| = \|P_QAx_{n+1}-P_QAx_n\| \le \|Ax_{n+1}-Ax_n\|. \tag{3.6}$$

From (3.1) and (3.3), we have

$$\|x_{n+1}-x^*\| = \|Tu_n-x^*\| \le \|u_n-x^*\| \le \|y_n-x^*\|. \tag{3.7}$$

Using (2.3), we get

$$\begin{aligned}\|y_n-x^*\|^2 &= \|(1-\alpha_n)\big(x_n-x^*+\delta A^*(Sz_n-Ax_n)\big)-\alpha_n x^*\|^2\\ &\le (1-\alpha_n)\|(x_n-x^*+\delta A^*(Sz_n-Ax_n)\|^2+\alpha_n\|x^*\|^2\\ &= (1-\alpha_n)\big[\|x_n-x^*\|+\delta^2\|A^*(Sz_n-Ax_n)\|^2\\ &\quad +2\delta\langle x_n-x^*,A^*(Sz_n-Ax_n)\rangle\big]+\alpha_n\|x^*\|^2.\end{aligned} \tag{3.8}$$

Since A is a linear operator with its adjoint A^*, we have

$$\begin{aligned}&\langle x_n-x^*,A^*(Sz_n-Ax_n)\rangle\\ &\quad=\langle A(x_n-x^*),Sz_n-Ax_n\rangle\\ &\quad=\langle Ax_n-Ax^*+Sz_n-Ax_n-(Sz_n-Ax_n),Sz_n-Ax_n\rangle\\ &\quad=\langle Sz_n-Ax^*,Sz_n-Ax_n\rangle-\|Sz_n-Ax_n\|^2.\end{aligned} \tag{3.9}$$

Again using (2.3), we obtain

$$\langle Sz_n-Ax^*,Sz_n-Ax_n\rangle=\frac{1}{2}\big(\|Sz_n-Ax^*\|^2+\|Sz_n-Ax_n\|^2-\|Ax_n-Ax^*\|^2\big). \tag{3.10}$$

By (3.4), (3.9) and (3.10), we get

$$\begin{aligned}\langle x_n-x^*,A^*(Sz_n-Ax_n)\rangle &= \frac{1}{2}\big(\|Sz_n-Ax^*\|^2+\|Sz_n-Ax_n\|^2-\|Ax_n-Ax^*\|^2\big)\\ &\quad-\|Sz_n-Ax_n\|^2\\ &\le \frac{1}{2}\big(\|Ax_n-Ax^*\|^2-\|z_n-Ax_n\|^2+\|Sz_n-Ax_n\|^2\\ &\quad-\|Ax_n-Ax^*\|^2\big)-\|Sz_n-Ax_n\|^2\\ &= -\frac{1}{2}\|z_n-Ax_n\|^2-\frac{1}{2}\|Sz_n-Ax_n\|^2.\end{aligned} \tag{3.11}$$

Substituting (3.11) into (3.8), we deduce

$$\begin{aligned}\|y_n-x^*\|^2 &\le (1-\alpha_n)\bigg[\|x_n-x^*\|^2+\delta^2\|A\|^2\|Sz_n-Ax_n\|^2\\ &\quad+2\delta\bigg(-\frac{1}{2}\|z_n-Ax_n\|^2-\frac{1}{2}\|Sz_n-Ax_n\|^2\bigg)\bigg]+\alpha_n\|x^*\|^2\\ &= (1-\alpha_n)\big[\|x_n-x^*\|^2+\big(\delta^2\|A\|^2-\delta\big)\|Sz_n-Ax_n\|^2\end{aligned}$$

$$\begin{aligned} &- \delta\|z_n - Ax_n\|^2\big] + \alpha_n\|x^*\|^2 \\ &\leq (1-\alpha_n)\|x_n - x^*\|^2 + \alpha_n\|x^*\|^2. \end{aligned} \tag{3.12}$$

It follows from (3.7) that

$$\begin{aligned} \|x_{n+1} - x^*\|^2 &\leq \|y_n - x^*\|^2 \\ &\leq (1-\alpha_n)\|x_n - x^*\|^2 + \alpha_n\|x^*\|^2 \\ &\leq \max\{\|x_n - x^*\|^2, \|x^*\|^2\}. \end{aligned}$$

The boundedness of the sequence $\{x_n\}$ yields.

Next, we estimate $\|x_{n+1} - x_n\|$. Set $v_n = x_n - \delta A^*(I - SP_Q)Ax_n$. According to (2.3) and (3.5), we have

$$\begin{aligned} \|v_{n+1} - v_n\|^2 &= \big\|x_{n+1} - x_n + \delta\big[A^*(SP_Q - I)Ax_{n+1} - A^*(SP_Q - I)Ax_n\big]\big\|^2 \\ &= \|x_{n+1} - x_n\|^2 + \delta^2\big\|A^*\big[(SP_Q - I)Ax_{n+1} - (SP_Q - I)Ax_n\big]\big\|^2 \\ &\quad + 2\delta\big\langle x_{n+1} - x_n, A^*\big[(SP_Q - I)Ax_{n+1} - (SP_Q - I)Ax_n\big]\big\rangle \\ &\leq \|x_{n+1} - x_n\|^2 + \delta^2\|A\|^2\big\|Sz_{n+1} - Sz_n - (Ax_{n+1} - Ax_n)\big\|^2 \\ &\quad + 2\delta\big\langle Ax_{n+1} - Ax_n, Sz_{n+1} - Sz_n - (Ax_{n+1} - Ax_n)\big\rangle \\ &= \|x_{n+1} - x_n\|^2 + \delta^2\|A\|^2\big\|Sz_{n+1} - Sz_n - (Ax_{n+1} - Ax_n)\big\|^2 \\ &\quad + 2\delta\big\langle Sz_{n+1} - Sz_n, Sz_{n+1} - Sz_n - (Ax_{n+1} - Ax_n)\big\rangle \\ &\quad - 2\delta\big\|Sz_{n+1} - Sz_n - (Ax_{n+1} - Ax_n)\big\|^2 \\ &= \|x_{n+1} - x_n\|^2 + \delta^2\|A\|^2\big\|Sz_{n+1} - Sz_n - (Ax_{n+1} - Ax_n)\big\|^2 \\ &\quad + \delta\big(\|Sz_{n+1} - Sz_n\|^2 + \big\|Sz_{n+1} - Sz_n - (Ax_{n+1} - Ax_n)\big\|^2 \\ &\quad - \|Ax_{n+1} - Ax_n\|^2\big) - 2\delta\big\|Sz_{n+1} - Sz_n - (Ax_{n+1} - Ax_n)\big\|^2 \\ &= \|x_{n+1} - x_n\|^2 + \big(\delta^2\|A\|^2 - \delta\big)\big\|Sz_{n+1} - Sz_n - (Ax_{n+1} - Ax_n)\big\|^2 \\ &\quad + \delta\big(\|Sz_{n+1} - Sz_n\|^2 - \|Ax_{n+1} - Ax_n\|^2\big) \\ &\leq \|x_{n+1} - x_n\|^2 + \big(\delta^2\|A\|^2 - \delta\big)\big\|Sz_{n+1} - Sz_n - (Ax_{n+1} - Ax_n)\big\|^2 \\ &\quad + \delta\big(\|z_{n+1} - z_n\|^2 - \|Ax_{n+1} - Ax_n\|^2\big). \end{aligned} \tag{3.13}$$

Since $\delta \in (0, \frac{1}{\|A\|^2})$, we derive by virtue of (3.6) and (3.13) that

$$\|v_{n+1} - v_n\| \leq \|x_{n+1} - x_n\|. \tag{3.14}$$

From (3.5) and (3.14), we have

$$\begin{aligned} \|x_{n+1} - x_n\| &\leq \|y_{n+1} - y_n\| \\ &= \big\|(1-\alpha_{n+1})v_{n+1} - (1-\alpha_n)v_n\big\| \\ &= \big\|(1-\alpha_{n+1})(v_{n+1} - v_n) + (\alpha_n - \alpha_{n+1})v_n\big\| \end{aligned}$$

$$\begin{aligned} &\le (1-\alpha_{n+1})\|v_{n+1}-v_n\| + |\alpha_{n+1}-\alpha_n|\|v_n\| \\ &\le (1-\alpha_{n+1})\|x_{n+1}-x_n\| + |\alpha_{n+1}-\alpha_n|\|v_n\|. \end{aligned}$$

It follows that

$$\|x_{n+1}-x_n\| \le \frac{|\alpha_{n+1}-\alpha_n|}{\alpha_{n+1}}\|v_n\|.$$

This, together with condition (C3), implies that

$$\lim_{n\to\infty}\|x_{n+1}-x_n\| = 0. \tag{3.15}$$

That is,

$$\lim_{n\to\infty}\|x_n - Tu_n\| = 0. \tag{3.16}$$

Using the firmly-nonexpansiveness of P_C, we have

$$\begin{aligned} \|u_n - x^*\|^2 &= \|P_C y_n - x^*\|^2 \\ &\le \|y_n - x^*\|^2 - \|P_C y_n - y_n\|^2 \\ &= \|y_n - x^*\|^2 - \|u_n - y_n\|^2. \end{aligned} \tag{3.17}$$

Thus,

$$\begin{aligned} \|x_{n+1} - x^*\|^2 &\le \|u_n - x^*\|^2 \\ &\le \|y_n - x^*\|^2 - \|u_n - y_n\|^2 \\ &\le (1-\alpha_n)\|x_n - x^*\|^2 + \alpha_n\|x^*\|^2 - \|u_n - y_n\|^2. \end{aligned} \tag{3.18}$$

It follows that

$$\begin{aligned} \|u_n - y_n\|^2 &\le \|x_n - x^*\|^2 - \|x_{n+1} - x^*\|^2 + \alpha_n\|x^*\|^2 \\ &\le \left(\|x_n - x^*\| + \|x_{n+1} - x^*\|\right)\|x_{n+1} - x_n\| + \alpha_n\|x^*\|^2. \end{aligned}$$

This, together with (3.15) and (C1), implies that

$$\lim_{n\to\infty}\|u_n - y_n\| = 0. \tag{3.19}$$

Returning to (3.18) and using (3.12), we have

$$\begin{aligned} \|x_{n+1} - x^*\|^2 &\le \|y_n - x^*\|^2 \\ &\le (1-\alpha_n)\|x_n - x^*\|^2 + (1-\alpha_n)\left(\delta^2\|A\|^2 - \delta\right)\|Sz_n - Ax_n\|^2 \\ &\quad - (1-\alpha_n)\delta\|z_n - Ax_n\|^2 + \alpha_n\|x^*\|^2. \end{aligned}$$

Hence,

$$\begin{aligned}(1-\alpha_n)\bigl(\delta-\delta^2\|A\|^2\bigr)\|Sz_n-Ax_n\|^2+(1-\alpha_n)\delta\|z_n-Ax_n\|^2\\ \le \|x_n-x^*\|^2-\|x_{n+1}-x^*\|^2+\alpha_n\|x^*\|^2\\ \le \bigl(\|x_n-x^*\|+\|x_{n+1}-x^*\|\bigr)\|x_{n+1}-x_n\|+\alpha_n\|x^*\|^2,\end{aligned}$$

which implies that

$$\lim_{n\to\infty}\|Sz_n-Ax_n\|=\lim_{n\to\infty}\|z_n-Ax_n\|=0. \tag{3.20}$$

So,

$$\lim_{n\to\infty}\|Sz_n-z_n\|=0. \tag{3.21}$$

Note that

$$\begin{aligned}\|y_n-x_n\| &= \bigl\|\delta A^*(SP_Q-I)Ax_n+\alpha_n v_n\bigr\|\\ &\le \delta\|A\|\|Sz_n-Ax_n\|+\alpha_n\|v_n\|.\end{aligned}$$

It follows from (3.20) that

$$\lim_{n\to\infty}\|x_n-y_n\|=0. \tag{3.22}$$

From (3.16), (3.19) and (3.22), we get

$$\lim_{n\to\infty}\|x_n-Tx_n\|=0. \tag{3.23}$$

Now, we show that

$$\limsup_{n\to\infty}\langle x^*,y_n-x^*\rangle\ge 0.$$

Choose a subsequence $\{y_{n_i}\}$ of $\{y_n\}$ such that

$$\limsup_{n\to\infty}\langle x^*,y_n-x^*\rangle=\lim_{i\to\infty}\langle x^*,y_{n_i}-x^*\rangle. \tag{3.24}$$

Since the sequence $\{y_{n_i}\}$ is bounded, we can choose a subsequence $\{y_{n_{i_j}}\}$ of $\{y_{n_i}\}$ such that $y_{n_{i_j}}\rightharpoonup z$. For the sake of convenience, we assume (without loss of generality) that $y_{n_i}\rightharpoonup z$. Consequently, we derive from the above conclusions that

$$x_{n_i}\rightharpoonup z,\qquad u_{n_i}\rightharpoonup z,\qquad Ax_{n_i}\rightharpoonup Az\quad\text{and}\quad z_{n_i}\rightharpoonup Az. \tag{3.25}$$

By the demiclosed principle of the nonexpansive mappings S and T (see Lemma 2.3), we deduce that $z\in \operatorname{Fix}(T)$ and $Az\in\operatorname{Fix}(S)$ (according to (3.23) and (3.21), respectively). Note that $u_{n_i}=P_Cy_{n_i}\in C$ and $z_{n_i}=P_QAx_{n_i}\in Q$. From (3.25), we deduce $z\in C$ and $Az\in Q$.

To this end, we deduce that $z \in C \cap \mathrm{Fix}(T)$ and $Az \in Q \cap \mathrm{Fix}(S)$. That is to say, $z \in \Gamma$. Therefore,

$$\begin{aligned}\limsup_{n\to\infty}\langle x^*, y_n - x^*\rangle &= \lim_{i\to\infty}\langle x^*, y_{n_i} - x^*\rangle \\ &= \lim_{i\to\infty}\langle x^*, z - x^*\rangle \\ &\geq 0. \end{aligned}\tag{3.26}$$

Finally, we prove that $x_n \to x^*$. From (3.1), we have

$$\begin{aligned}\|x_{n+1} - x^*\|^2 &\leq \|y_n - x^*\|^2 \\ &= \|(1-\alpha_n)(v_n - x^*) - \alpha_n x^*\|^2 \\ &\leq (1-\alpha_n)\|v_n - x^*\|^2 - 2\alpha_n\langle x^*, y_n - x^*\rangle \\ &\leq (1-\alpha_n)\|x_n - x^*\|^2 - 2\alpha_n\langle x^*, y_n - x^*\rangle. \end{aligned}\tag{3.27}$$

Applying Lemma 2.4 and (3.26) to (3.27), we deduce that $x_n \to x^*$. The proof is completed. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors read and approved the final manuscript.

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