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Notes on analytic functions with a bounded positive real part

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Dedicated to Professor Hari M Srivastava

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Abstract

For real α and β such that $0 \le \alpha < 1 < \beta$, we denote by $\mathcal{S}(\alpha, \beta)$ the class of normalized analytic functions f such that $\alpha < \operatorname{Re}\{zf'(z)/f(z)\} < \beta$ in $\mathbb{U}$. We find some properties, including inclusion properties, Fekete-Szegö problem and coefficient problems of inverse functions.

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1 Introduction

Let $\mathcal{H}$ denote the class of analytic functions in the unit disc $\mathbb{U} = \{z : |z| < 1\}$ on the complex plane $\mathbb{C}$. Let $\mathcal{A}$ denote the subclass of $\mathcal{H}$ consisting of functions normalized by $f(0) = 0$ and $f'(0) = 1$. Let $\mathcal{S}$ denote the subclass of $\mathcal{A}$ consisting of univalent functions. Denote by $\mathcal{S}^*$ and $\mathcal{K}$, the class of starlike functions and convex functions, respectively. It is well-known that $\mathcal{K} \subset \mathcal{S}^* \subset \mathcal{S}$.

We say that f is subordinate to F in $\mathbb{U}$, written as $f \prec F$ if and only if $f(z) = F(w(z))$ for some Schwarz function with $w(0) = 0$ and $|w(z)| < 1$, $z \in \mathbb{U}$. If $F(z)$ is univalent in $\mathbb{U}$, then the subordination $f \prec F$ is equivalent to $f(0) = F(0)$ and $f(\mathbb{U}) \subset F(\mathbb{U})$.

We denote by $\mathcal{S}^*(A, B)$, the class of Janowski starlike functions, namely, the functions satisfying the subordination equation: $zf'(z)/f(z) \prec (1 + Az)/(1 + Bz)$. Note that $\mathcal{S}^*(1, -1) = \mathcal{S}^*$.

Now, we shall introduce the class of analytic functions used in the sequel.

Definition 1 Let α and β be real numbers such that $0 \le \alpha < 1 < \beta$. The function $f \in \mathcal{A}$ belongs to the class $\mathcal{S}(\alpha, \beta)$ if f satisfies the following inequality:

$$\alpha < \operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\} < \beta \quad (z \in \mathbb{U}).$$

We remark that for given α, β $(0 \le \alpha < 1 < \beta)$, $f \in \mathcal{S}(\alpha, \beta)$ if and only if f satisfies the following two subordination equations:

$$\frac{zf'(z)}{f(z)} \prec \frac{1 + (1 - 2\alpha)z}{1 - z} \quad \text{and} \quad \frac{zf'(z)}{f(z)} \prec \frac{1 + (1 - 2\beta)z}{1 - z}, \tag{1}$$

since the functions $(1+(1-2\alpha)z)/(1-z)$ and $(1+(1-2\beta)z)/(1-z)$ map $\mathbb{U}$ onto the right half plane, having real part greater than α, and the left half plane, having real part smaller than β, respectively. The above class $\mathcal{S}(\alpha,\beta)$ is introduced by Kuroki and Owa [1]. They investigated coefficient estimates for $f \in \mathcal{S}(\alpha,\beta)$ and found the necessary and sufficient condition for $f \in \mathcal{S}(\alpha,\beta)$ using the following subordination.

Lemma 1 (Kuroki and Owa [1]) *Let $f \in \mathcal{A}$ and $0 \le \alpha < 1 < \beta$. Then $f \in \mathcal{S}(\alpha,\beta)$ if and only if*

$$\frac{zf'(z)}{f(z)} \prec 1 + \frac{\beta-\alpha}{\pi} i \log\left(\frac{1-e^{2\pi i \frac{1-\alpha}{\beta-\alpha}} z}{1-z}\right) \quad (z \in \mathbb{U}).$$

Lemma 1 means that the function p defined by

$$p(z) = 1 + \frac{\beta-\alpha}{\pi} i \log\left(\frac{1-e^{2\pi i \frac{1-\alpha}{\beta-\alpha}} z}{1-z}\right) \tag{2}$$

maps the unit disk $\mathbb{U}$ onto the strip domain w with $\alpha < \operatorname{Re}(w) < \beta$. We note that the function $f \in \mathcal{A}$, given by

$$f(z) = z \exp\left\{\frac{\beta-\alpha}{\pi} i \int_0^z \frac{1}{t} \log\left(\frac{1-e^{2\pi i \frac{1-\alpha}{\beta-\alpha}} t}{1-t}\right) dt\right\},$$

is in the class $\mathcal{S}(\alpha,\beta)$.

2 Inclusion properties

Theorem 1 *For given $0 \le \alpha < 1 < \beta$, let A and B be real numbers such that*

$$\frac{2-\alpha-\beta}{\beta-\alpha} \le B < A \le \frac{\beta-2\alpha\beta+\alpha}{\beta-\alpha}. \tag{3}$$

Then $\mathcal{S}^*(A,B) \subset \mathcal{S}(\alpha,\beta)$.

Proof At first, we note that

$$-1 < \frac{2-\alpha-\beta}{\beta-\alpha} \quad \text{and} \quad \frac{\beta-2\alpha\beta+\alpha}{\beta-\alpha} < 1.$$

For $f \in \mathcal{S}^*(A,B)$, we know that the following inequality holds:

$$\frac{1-A}{1-B} < \operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\} < \frac{1+A}{1+B} \quad (z \in \mathbb{U}).$$

Therefore, it suffices to show that α and β satisfy the following inequalities:

$$\alpha \le \frac{1-A}{1-B} \quad \text{and} \quad \frac{1+A}{1+B} \le \beta. \tag{4}$$

Using inequality (4), we can derive that

$$1+B \ge \frac{2(1-\alpha)}{\beta-\alpha} \quad \text{and} \quad 1+A \le \frac{2\beta(\beta-\alpha)}{\beta-\alpha}. \tag{5}$$

Also,

$$1 - B \le \frac{2(\beta - 1)}{\beta - \alpha} \quad \text{and} \quad 1 - A \ge \frac{2\alpha(\beta - a)}{\beta - \alpha}. \tag{6}$$

By the above inequalities (5) and (6), we can easily obtain the inequalities (3), so the proof of Theorem 1 is completed. □

Lemma 2 (Miller and Mocanu [2]) *Let Ξ be a set in the complex plane $\mathbb{C}$ and let b be a complex number such that* $\mathrm{Re}(b) > 0$. *Suppose that a function $\psi : \mathbb{C}^2 \times \mathbb{U} \to \mathbb{C}$ satisfies the condition*

$$\psi(i\rho, \sigma; z) \notin \Xi$$

for all real ρ, $\sigma \le -|b - i\rho|^2/(2\,\mathrm{Re}(b))$ and all $z \in \mathbb{U}$. If the function $p(z)$ defined by $p(z) = b + b_1 z + b_2 z^2 + \cdots$ is analytic in $\mathbb{U}$ and if

$$\psi\bigl(p(z), zp'(z); z\bigr) \in \Xi,$$

then $\mathrm{Re}(p(z)) > 0$ *in* $\mathbb{U}$.

Theorem 2 *Let $f \in \mathcal{A}$, $1/2 \le \alpha < 1$ and* $\mathrm{Re}\{zf'(z)/f(z)\} > \alpha$ *in $\mathbb{U}$. Then*

$$\mathrm{Re}\left\{\frac{f(z)}{z}\right\} > \gamma(\alpha) := \frac{1}{3 - 2\alpha} \quad (z \in \mathbb{U}). \tag{7}$$

Proof Write $\gamma(\alpha) := \gamma$ and note that $\frac{1}{2} \le \gamma < 1$ for $\frac{1}{2} \le \alpha < 1$. Let p be defined by

$$p(z) = \frac{1}{1 - \gamma}\left(\frac{zf'(z)}{f(z)} - \gamma\right).$$

Then p is analytic in $\mathbb{U}$, $p(0) = 1$ and

$$\frac{zf'(z)}{f(z)} = 1 + \frac{(1 - \gamma)zp'(z)}{(1 - \gamma)p(z) + \gamma} = \psi\bigl(p(z), zp'(z)\bigr),$$

where

$$\psi(r, s) = 1 + \frac{(1 - \gamma)s}{(1 - \gamma)r + \gamma}. \tag{8}$$

Also,

$$\bigl\{\psi\bigl(p(z), zp'(z)\bigr) : z \in \mathbb{U}\bigr\} \subset \bigl\{w \in \mathbb{C} : \mathrm{Re}(w) > \alpha\bigr\} := \Omega_\alpha.$$

Now for all real ρ, $\sigma \le -\frac{1}{2}(1 + \rho^2)$,

$$\begin{aligned}\mathrm{Re}\bigl(\psi(i\rho, \sigma)\bigr) &= \mathrm{Re}\left(1 + \frac{(1 - \gamma)\sigma}{(1 - \gamma)i\rho + \gamma}\right) = 1 + \frac{\gamma(1 - \gamma)\sigma}{\gamma^2 + (1 - \gamma)^2\rho^2} \\ &\le 1 - \frac{1}{2}\gamma(1 - \gamma)\frac{1 + \rho^2}{\gamma^2 + (1 - \gamma)^2\rho^2}.\end{aligned}$$

Now, we let

$$g(\rho) = \frac{1+\rho^2}{\gamma^2 + (1-\gamma)^2\rho^2}. \tag{9}$$

Then

$$g'(\rho) = \frac{2(2\gamma-1)\rho}{(\gamma^2 + (1-\gamma)^2\rho^2)^2},$$

hence $g'(\rho) = 0$ occurs at only $\rho = 0$ and g satisfies

$$g(0) = \frac{1}{\gamma^2}$$

and

$$\lim_{\rho\to\infty} g(\rho) = \frac{1}{(1-\gamma)^2}.$$

Since $1/2 \le \gamma < 1$, we have

$$\frac{1}{\gamma^2} \le g(\rho) < \frac{1}{(1-\gamma)^2},$$

hence we get

$$\operatorname{Re}\bigl(\psi(i\rho,\sigma)\bigr) \le 1 - \frac{1}{2}\gamma(1-\gamma)\frac{1}{\gamma^2} = \frac{3\gamma-1}{2\gamma} = \alpha.$$

This shows that $\operatorname{Re}\{\psi(i\rho,\sigma)\} \notin \Omega_\alpha$. By Lemma 2, we get $\operatorname{Re}(p(z)) > 0$ in $\mathbb{U}$, and this shows that inequality (7) holds and the proof of Theorem 2 is completed. □

Theorem 3 *Let* $f \in \mathcal{A}$, $1 < \beta < 3/2$ *and* $\operatorname{Re}\{zf'(z)/f(z)\} < \beta$ *in* $\mathbb{U}$. *Then*

$$\operatorname{Re}\left\{\frac{f(z)}{z}\right\} < \delta(\beta) := \frac{1}{3-2\beta} \quad (z \in \mathbb{U}). \tag{10}$$

Proof Note that $\delta := \delta(\beta) = \frac{1}{3-2\beta} > 1$ for $\beta > 1$. Let p be defined by

$$p(z) = \frac{1}{1-\delta}\left(\frac{zf'(z)}{f(z)} - \delta\right).$$

Then p is analytic in $\mathbb{U}$, $p(0) = 1$ and

$$\frac{zf'(z)}{f(z)} = \psi\bigl(p(z), zp'(z)\bigr),$$

where ψ is given in (8). Also

$$\bigl\{\psi\bigl(p(z), zp'(z)\bigr) : z \in \mathbb{U}\bigr\} \subset \bigl\{w \in \mathbb{C} : \operatorname{Re}(w) < \beta\bigr\} := \Omega_\beta.$$

Now, for all real ρ, $\sigma \le -\frac{1}{2}(1+\rho^2)$,

$$\operatorname{Re}\bigl(\psi(i\rho,\sigma)\bigr) \ge 1 - \frac{1}{2}\delta(1-\delta)g(\rho),$$

where $g(\rho)$ is given (9). Since

$$\frac{1}{(1-\delta)^2} < g(\rho) \le \frac{1}{\delta^2}$$

for all $\delta > 1$, we have

$$\operatorname{Re}\bigl(\psi(i\rho,\sigma)\bigr) \ge \frac{3\delta-1}{2\delta} = \beta.$$

This shows that $\operatorname{Re}\{\psi(i\rho,\sigma)\} \notin \Omega_\beta$. By Lemma 2, we get $\operatorname{Re}(p(z)) > 0$ in $\mathbb{U}$, and this is equivalent to

$$\operatorname{Re}\left\{\frac{f(z)}{z}\right\} < \delta \quad (z \in \mathbb{U}),$$

and the proof of Theorem 3 is completed. □

Combining the above Theorems 2 and 3, we can obtain the following result:

Theorem 4 *Let* $f \in \mathcal{A}$, $1/2 \le \alpha < 1 < \beta < 3/2$ *and* $\alpha < \operatorname{Re}\{zf'(z)/f(z)\} < \beta$ *in* $\mathbb{U}$. *Then*

$$\gamma(\alpha) < \operatorname{Re}\left\{\frac{f(z)}{z}\right\} < \delta(\beta) \quad (z \in \mathbb{U}),$$

where $\gamma(\alpha)$ *and* $\delta(\beta)$ *is given* (7) *and* (10).

3 Some coefficient problems

In this section, we investigate coefficient problems for functions in the class $\mathcal{S}(\alpha,\beta)$. In [1], Kuroki and Owa investigated the coefficient of the function p given by (2); the function p can be written as

$$p(z) = 1 + \sum_{n=1}^{\infty} B_n z^n,$$

where

$$B_n = \frac{2(\beta-\alpha)}{n\pi}\sin\left(\frac{1-\alpha}{\beta-\alpha}\pi\right).$$

We denote by $\mathcal{S}_\sigma(\alpha,\beta)$ the class of bi-univalent functions f such that $f \in \mathcal{S}(\alpha,\beta)$ and the inverse function $f^{-1} \in \mathcal{S}(\alpha,\beta)$. Srivastava *et al.* investigated the estimates on the initial coefficient for certain subclasses of analytic and bi-univalent functions in [3, 4]. Ali *et al.* have studied similar problems in [5].

In theorem, we shall solve the Fekete-Szegö problem for $f \in S(\alpha,\beta)$. We need the following lemma:

Lemma 3 (Keogh and Merkers [6]) *Let $p(z) = 1 + c_1z + c_2z^2 + \cdots$ be a function with positive real part in $\mathbb{U}$. Then, for any complex number ν,*

$$\left|c_2 - \nu c_1^2\right| \le 2\max\left\{1, |1-2\nu|\right\}.$$

The following result holds for the coefficient of $f \in S(\alpha,\beta)$.

Theorem 5 *Let $0 \le \alpha < 1 < \beta$ and let the function f given by $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ be in the class $\mathcal{S}(\alpha,\beta)$. Then, for a complex number μ,*

$$\begin{aligned}\left|a_3 - \mu a_2^2\right| \le{}& \frac{\beta-\alpha}{2\pi}\sqrt{2 - 2\cos\left(\frac{1-\alpha}{\beta-\alpha}\cdot 2\pi\right)} \\ &\cdot \max\left\{1; \left|\frac{1}{2} + (1-2\mu)\frac{\beta-\alpha}{\pi}i + \left(\frac{1}{2} - (1-2\mu)\frac{\beta-\alpha}{\pi}i\right)e^{2\pi i\frac{1-\alpha}{\beta-\alpha}}\right|\right\}. \end{aligned} \tag{11}$$

Proof Let us consider a function q given by $q(z) = zf'(z)/f(z)$. Then, since $f \in \mathcal{S}(\alpha,\beta)$, we have $q(z) \prec p(z)$, where

$$p(z) = 1 + \frac{\beta-\alpha}{\pi} i \log\left(\frac{1 - e^{2\pi i\frac{1-\alpha}{\beta-\alpha}} z}{1-z}\right).$$

Let

$$h(z) = \frac{1 + p^{-1}(q(z))}{1 - p^{-1}(q(z))} = 1 + h_1 z + h_2 z^2 + \cdots \quad (z \in \mathbb{U}).$$

Then h is analytic and has positive real part in the open unit disk $\mathbb{U}$. We also have

$$q(z) = p\left(\frac{h(z)-1}{h(z)+1}\right) \quad (z \in \mathbb{U}). \tag{12}$$

We find from equation (12) that

$$a_2 = \frac{1}{2}B_1 h_1$$

and

$$a_3 = \frac{1}{4}B_1 h_2 - \frac{1}{8}B_1 h_1^2 + \frac{1}{8}B_2 h_1^2 + \frac{1}{8}B_1^2 h_1^2,$$

which imply that

$$a_3 - \mu a_2^2 = \frac{1}{4}B_1\left(h_2 - \nu h_1^2\right),$$

where

$$\nu = \frac{1}{2}\left(1 - \frac{B_2}{B_1} - B_1 + 2\mu B_1\right).$$

Applying Lemma 3, we can obtain

$$\begin{aligned}\left|a_3 - \mu a_2^2\right| &= \frac{1}{4}|B_1|\left|h_2 - \nu h_1^2\right| \\ &\le \frac{1}{2}\cdot \max\left\{1; |1-2\nu|\right\}. \end{aligned} \tag{13}$$

And substituting

$$B_1 = \frac{\beta-\alpha}{\pi} i\left(1 - e^{2\pi i \frac{1-\alpha}{\beta-\alpha}}\right) \tag{14}$$

and

$$B_2 = \frac{\beta-\alpha}{2\pi} i\left(1 - e^{4\pi i \frac{1-\alpha}{\beta-\alpha}}\right) \tag{15}$$

in (13), we can obtain the result as asserted. □

Using the above Theorem 5, we can get the following result.

Corollary 1 *Let the function f, given by $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$, be in the class $\mathcal{S}(\alpha,\beta)$. Also let the function f^{-1}, defined by*

$$f^{-1}\big(f(z)\big) = z = f\big(f^{-1}(z)\big), \tag{16}$$

be the inverse of f. If

$$f^{-1}(w) = w + \sum_{n=2}^{\infty} b_n w^n \quad \left(|w| < r_0; r_0 > \frac{1}{4}\right), \tag{17}$$

then

$$|b_2| \le \frac{2(\beta-\alpha)}{\pi}\sin\left(\frac{1-\alpha}{\beta-\alpha}\pi\right)$$

and

$$\begin{aligned}|b_3| \le{}& \frac{\beta-\alpha}{2\pi}\sqrt{2 - 2\cos\left(\frac{1-\alpha}{\beta-\alpha}\cdot 2\pi\right)} \\ &\cdot \max\left\{1; \left|\frac{1}{2} - 3\frac{\beta-\alpha}{\pi} i + \left(\frac{1}{2} + 3\frac{\beta-\alpha}{\pi} i\right) e^{2\pi i \frac{1-\alpha}{\beta-\alpha}}\right|\right\}.\end{aligned}$$

Proof Relations (16) and (17) give

$$b_2 = -a_2 \quad \text{and} \quad b_3 = 2a_2^2 - a_3.$$

Thus, we can get the estimate for $|b_2|$ by

$$|b_2| = |a_2| \le \frac{2(\beta-\alpha)}{\pi}\sin\left(\frac{1-\alpha}{\beta-\alpha}\pi\right),$$

immediately. An application of Theorem 5 (with $\mu = 2$) gives the estimates for $|b_3|$, hence the proof of Corollary 1 is completed. □

Next, we shall estimate on some initial coefficient for the bi-univalent functions $f \in \mathcal{S}_\sigma(\alpha,\beta)$.

Theorem 6 *Let f be given by $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ be in the class $\mathcal{S}_\sigma(\alpha,\beta)$. Then*

$$|a_2| \le \frac{|B_1|\sqrt{|B_1|}}{|B_1^2 + B_1 - B_2|} \tag{18}$$

and

$$|a_3| \le |B_1| + |B_2 - B_1|, \tag{19}$$

where B_1 and B_2 are given by (14) *and* (15).

Proof If $f \in \mathcal{S}_\sigma(\alpha,\beta)$, then $f \in \mathcal{S}(\alpha,\beta)$ and $g \in \mathcal{S}(\alpha,\beta)$, where $g = f^{-1}$. Hence

$$Q(z) := \frac{zf'(z)}{f(z)} \prec p(z) \quad \text{and} \quad L(z) := \frac{zg'(z)}{g(z)} \prec p(z),$$

where $p(z)$ is given by (2). Let

$$h(z) = \frac{1 + p^{-1}(Q(z))}{1 - p^{-1}(Q(z))} = 1 + h_1 z + h_2 z^2 + \cdots$$

and

$$k(z) = \frac{1 + p^{-1}(L(z))}{1 - p^{-1}(L(z))} = 1 + k_1 z + k_2 z^2 + \cdots.$$

Then h and k are analytic and have positive real part in $\mathbb{U}$. Also, we have

$$Q(z) = p\left(\frac{h(z)-1}{h(z)+1}\right) \quad \text{and} \quad L(z) = p\left(\frac{k(z)-1}{k(z)+1}\right).$$

By suitably comparing coefficients, we get

$$a_2 = \frac{1}{2}B_1 h_1, \tag{20}$$

$$2a_3 - a_2^2 = \frac{1}{2}B_1 h_2 - \frac{1}{4}B_1 h_1^2 + \frac{1}{4}B_2 h_1^2, \tag{21}$$

$$-a_2 = \frac{1}{2}B_1 k_1 \tag{22}$$

and

$$3a_2^2 - 2a_3 = \frac{1}{2}B_1 k_2 - \frac{1}{4}B_1 k_1^2 + \frac{1}{4}B_2 k_1^2, \tag{23}$$

where B_1 and B_2 are given by (14) and (15), respectively. Now, considering (20) and (22), we get

$$h_1 = -k_1. \tag{24}$$

Also, from (21), (22), (23) and (24), we find that

$$a_2^2 = \frac{B_1^3(h_2 + k_2)}{4(B_1^2 + B_1 - B_2)}. \tag{25}$$

Therefore, we have

$$\left|a_2^2\right| \le \frac{|B_1|^3}{4|B_1^2 + B_1 - B_2|}\bigl(|h_2| + |k_2|\bigr) \le \frac{|B_1|^3}{|B_1^2 + B_1 - B_2|}.$$

This gives the bound on $|a_2|$ as asserted in (18). Now, further computations from (21), (23), (24) and (25) lead to

$$a_3 = \frac{1}{8}\bigl(B_1(h_2 + 3k_2) + 2h_1^2(B_2 - B_1)\bigr).$$

This equation, together with the well-known estimates:

$$|h_1| \le 2, \qquad |h_2| \le 2 \quad \text{and} \quad |k_2| \le 2$$

lead us to inequality (19). Therefore, the proof of Theorem 6 is completed. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The corresponding author, OSK carried out the subclasses of analytic functions studies and conceived of the study. YJS participated in the sequence alignment and drafted the manuscript. All authors read and approved the final manuscript.

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