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A note on complete convergence of weighted sums for array of rowwise AANA random variables

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Abstract

In this paper, we consider complete convergence and complete moment convergence of weighted sums for an array of rowwise AANA random variables. The main result of the paper generalizes the Baum-Katz theorem on AANA random variables. Our results extend and improve the corresponding ones of Wang *et al.* (Abstr. Appl. Anal. 2012:315138, 2012).

MSC: 60B10; 60F15

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1 Introduction

Assume that random variables $X_n, n \in \mathbb{N} = \{1, 2, \dots\}$ are defined on a fixed probability space $(\Omega, \mathcal{A}, P)$.

First, we recall two definitions as follows.

Definition 1.1 Random variables $X_1, X_2, \dots, X_n$, $n \geq 2$, are said to be negatively associated (NA, in short) if

$$\operatorname{Cov}\bigl(f(X_{i_1}, \dots, X_{i_k}), g(X_{j_1}, \dots, X_{j_m})\bigr) \leq 0$$

for any pair of nonempty disjoint subsets $A = \{i_1, \dots, i_k\}$ and $B = \{j_1, \dots, j_m\}$, $k + m \leq n$, of the set $\{1, 2, \dots, n\}$ and for any bounded coordinatewise increasing real functions $f(x_{i_1}, \dots, x_{i_k})$ and $g(x_{j_1}, \dots, x_{j_m})$, $x_1, \dots, x_n \in \mathbb{R} = (-\infty, \infty)$. Random variables X_n, $n \in \mathbb{N}$, are NA if every $n \in \mathbb{N}$ random variables $X_1, X_2, \dots, X_n$ are NA.

Random variables X_{ni}, $i, n \in \mathbb{N}$, are called an array of rowwise NA random variables if for every $n \in \mathbb{N}$ random variables X_{ni}, $i \in \mathbb{N}$ are, NA.

The concept of NA random variables was introduced by Block *et al.* [1] and carefully studied by Joav-Dev and Proschan [2]. Primarily motivated by this, Chandra and Ghosal [3, 4] introduced the following dependence.

Definition 1.2 Random variables X_n, $n \in \mathbb{N}$, are said to be asymptotically almost negatively associated (AANA, in short) if there exists a nonnegative sequence $q(n) \to 0$ as

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$n \to \infty$ such that

$$\operatorname{Cov}\big(f(X_n), g(X_{n+1}, \ldots, X_{n+k})\big) \le q(n)\big\{\operatorname{Var}\big(f(X_n)\big)\operatorname{Var}\big(g(X_{n+1}, \ldots, X_{n+k})\big)\big\}^{1/2}$$

for all $n, k \in \mathbb{N}$ and for all coordinatewise nondecreasing continuous functions f and g for which $\operatorname{Var} f(X_n)$ and $\operatorname{Var} g(X_{n+1}, \ldots, X_{n+k})$ exist.

Random variables X_{ni}, $i, n \in \mathbb{N}$, are called an array of rowwise AANA random variables if for every $n \in \mathbb{N}$, random variables X_{ni}, $i \in \mathbb{N}$, are AANA.

The family of AANA random variables contains NA (in particular, independent) random variables (with $q(n) = 0$, $n \ge 1$) and some more kinds of random variables which are not much deviated from being negatively associated. An example of AANA random variables which are not NA was constructed by Chandra and Ghosal [3]. For various results and applications of AANA random variables, one can refer to Chandra and Ghosal [4], Wang *et al.* [5], Ko *et al.* [6], Yuan and An [7], Wang *et al.* [8, 9] and Wang *et al.* [10], Yang *et al.* [11], Shen and Wu [12] among others.

The concept of complete convergence was introduced by Hsu and Robbins [13] as follows. Random variables U_n, $n \in \mathbb{N}$, are said to converge completely to a constant C if $\sum_{n=1}^{\infty} P(|U_n - C| > \varepsilon) < \infty$ for all $\varepsilon > 0$. In view of the Borel-Cantelli lemma, this implies that $U_n \to C$ almost surely (a.s.). The converse is true if random variables U_n, $n \in \mathbb{N}$, are independent. Hsu and Robbins [13] proved that arithmetic means of independent and identically distributed (i.i.d.) random variables converges completely to the expected value if the variance of the summands is finite. Erdös [14] proved the converse. The result of Hsu, Robbins and Erdös is a fundamental theorem in probability theory and has been generalized and extended in several directions by many authors. One of the most important generalizations was provided by Baum and Katz [15] for the strong law of large numbers as follows.

Theorem A *Let* $1/2 < \alpha \le 1$ *and* $\alpha p > 1$. *Let* X_n, $n \in \mathbb{N}$ *be i.i.d. random variables with zero means. Then the following statements are equivalent*:

(i) $E|X_1|^p < \infty$,

(ii) $\sum_{n=1}^{\infty} n^{\alpha p-2} P(\max_{1 \le j \le n} |\sum_{i=1}^{j} X_i| > \varepsilon n^{\alpha}) < \infty$ *for all* $\varepsilon > 0$.

Motivated by Baum and Katz [15] for i.i.d. random variables, many authors studied the Baum-Katz-type theorem for dependent random variables. One can refer to Peligrad [16], Shao [17], Peligrad and Gut [18], Kruglov *et al.* [19], Wang and Hu [20], Shen *et al.* [21], Wang *et al.* [22], *etc.*

Next, we will give the definition of stochastic domination which is used frequently in the paper.

Definition 1.3 Random variables X_n, $n \in \mathbb{N}$, are said to be stochastically dominated by a random variable X if for every $n \in \mathbb{N}$ there exists a positive constant C such that

$$P\big(|X_n| > x\big) \le CP\big(|X| > x\big)$$

for all $x \ge 0$.

An array of rowwise random variables X_{ni}, $i, n \in \mathbb{N}$, is said to be stochastically dominated by a random variable X if for every $n \in \mathbb{N}$ there exists a positive constant C such that

$$\sup_{i\geq 1} P\big(|X_{ni}| > x\big) \leq CP\big(|X| > x\big)$$

for all $x \geq 0$.

Wang *et al.* [10] discussed the complete convergence for an array of rowwise AANA random variables which are stochastically dominated by a random variable X and obtained the following result.

Theorem B *Let X_{ni}, $i, n \in \mathbb{N}$, be an array of rowwise AANA random variables which are stochastically dominated by a random variable X and $EX_{ni} = 0$ for every $i, n \in \mathbb{N}$ with $q(n)$ from Definition* 1.2.

(i) *Let $1/2 < \alpha \leq 1$, $p > 1$ and $\alpha p > 1$. If $E|X|^p < \infty$ and $\sum_{n=1}^{\infty} q^{s/r}(n) < \infty$ for some $r \in (3 \cdot 2^{k-1}, 4 \cdot 2^{k-1}]$ and*

$$r > \max\left(2, \frac{\alpha p - 1}{\alpha - 1/2}, p\right),$$

where integer number $k \geq 1$ and $s \doteq r/(r-1)$ for $r > 1$, then for all $\varepsilon > 0$,

$$\sum_{n=1}^{\infty} n^{\alpha p-2} P\left(\max_{1\leq j\leq n}\left|\sum_{i=1}^{j} X_{ni}\right| > \varepsilon n^{\alpha}\right) < \infty.$$

(ii) *If $E|X| \log|X| < \infty$ and $\sum_{n=1}^{\infty} q^2(n) < \infty$, then for all $\varepsilon > 0$,*

$$\sum_{n=1}^{\infty} n^{-1} P\left(\max_{1\leq j\leq n}\left|\sum_{i=1}^{j} X_{ni}\right| > \varepsilon n\right) < \infty.$$

The complete convergence for an array of rowwise random variables was studied by many authors. See, for example, the complete convergence for an array of rowwise independent random variables was studied by Hu *et al.* [23], Sung *et al.* [24], Kruglov *et al.* [19] and others. Recently, many authors extended the complete convergence for an array of rowwise independent random variables to the cases of dependent random variables. One can refer to Kuczmaszewska [25, 26], Chen *et al.* [27], Kruglov [28], Zhou and Lin [29], Guo [30], Wu [31], and so on.

The main purpose of the paper is to further study the complete convergence and complete moment convergence of weighted sums for an array of rowwise AANA random variables. The result of the paper generalizes the Baum-Katz theorem on AANA random variables in different methods. As an application, we get the Marcinkiewicz-Zygmund type strong law of large numbers for weighted sums on AANA random variables. Our results extend and improve the corresponding ones of [10].

Throughout this paper, for $r > 1$, let $s \doteq r/(r-1)$ be the dual number of r. The symbols $C, C_1, C_2, \ldots$ denote positive constants which may be different in various places. Assume that $I(A)$ is the indicator function of the set A. Let $x^+ = \max(0, x)$ and $\log x = \ln \max(x, e)$, where $\ln x$ denotes the natural logarithm. $a_n = O(b_n)$ stands for $|a_n| \leq C|b_n|$.

2 Preliminaries

To prove the main results of the paper, we need the following lemmas.

Lemma 2.1 (*cf.* [32, Lemma 4.1.6]) *Let X_n, $n \in \mathbb{N}$, be random variables, which are stochastically dominated by a random variable X. Then, for any $a > 0$ and $b > 0$, the following two statements hold*:

$$E|X_n|^a I(|X_n| \le b) \le C_1 \{E|X|^a I(|X| \le b) + b^a P(|X| > b)\}$$

and

$$E|X_n|^a I(|X_n| > b) \le C_2 E|X|^a I(|X| > b),$$

where C_1 and C_2 are positive constants.

Lemma 2.2 (*cf.* [7, Lemma 2.1]) *Let X_n, $n \in \mathbb{N}$, be AANA random variables with $q(n)$ from Definition* 1.2. *Assume that f_n, $n \in \mathbb{N}$ are all nondecreasing* (*or all nonincreasing*) *and continuous functions, then $f_n(X_n)$, $n \in \mathbb{N}$, are still AANA random variables with $q(n)$.*

Lemma 2.3 (*cf.* [7, Theorem 2.1]) *Let $r > 1$ and X_n, $n \in \mathbb{N}$, be AANA random variables with $q(n)$ from Definition* 1.2.

If $\sum_{n=1}^{\infty} q^2(n) < \infty$, then there exists a positive constant C_r depending only on r such that for all $n \ge 1$ and $1 < r \le 2$,

$$E\left(\max_{1\le j\le n}\left|\sum_{i=1}^{j} X_i\right|^r\right) \le C_r \sum_{i=1}^{n} E|X_i|^r.$$

If $\sum_{n=1}^{\infty} q^{s/r}(n) < \infty$ for some $r \in (3 \cdot 2^{k-1}, 4 \cdot 2^{k-1}]$, where integer number $k \ge 1$, then there exists a positive constant D_r depending only on r such that for all $n \ge 1$,

$$E\left(\max_{1\le j\le n}\left|\sum_{i=1}^{j} X_i\right|^r\right) \le D_r \left\{\sum_{i=1}^{n} E|X_i|^r + \left(\sum_{i=1}^{n} EX_i^2\right)^{r/2}\right\}.$$

Lemma 2.4 (*cf.* [33, Lemma 2.4]) *Let Y_n, Z_n, $n \in \mathbb{N}$ be random variables. Then, for any $q > 1$, $\varepsilon > 0$ and $a > 0$,*

$$E\left(\max_{1\le j\le n}\left|\sum_{i=1}^{j} (Y_i + Z_i)\right| - \varepsilon a\right)^+ \le \left(\frac{1}{\varepsilon^q} + \frac{1}{q-1}\right)\frac{1}{a^{q-1}} E \max_{1\le j\le n}\left|\sum_{i=1}^{j} Y_i\right|^q + E \max_{1\le j\le n}\left|\sum_{i=1}^{j} Z_i\right|.$$

3 Main results and their proofs

In this section, let X_{ni}, $i, n \in \mathbb{N}$, be an array of rowwise AANA random variables, *i.e.*, for every $n \in \mathbb{N}$, X_{ni}, $i \in \mathbb{N}$, are AANA random variables with the identical mixing coefficient $q(i)$ and let a_{ni}, $i, n \in \mathbb{N}$, be an array of real numbers. Let X_n, $n \in \mathbb{N}$, be AANA random variables with $q(n)$ from Definition 1.2.

In the following, let $\psi(x) = 1$ or $\psi(x) = \log x$. Note that the function $\psi(x)$ has the following properties (see [34]):

(a) for all $m \geq k \geq 1$,

$$\sum_{n=k}^{m} n^{r-1}\psi(n) \leq Cm^r\psi(m) \quad \text{if } r > 0 \tag{3.1}$$

and

$$\sum_{n=m}^{\infty} n^{r-1}\psi(n) \leq Cm^r\psi(m) \quad \text{if } r < 0; \tag{3.2}$$

(b) for all $p > 0$,

$$\psi\left(|x|^p\right) \leq C(p)\psi\left(|x|\right) \leq C(p)\psi\left(1 + |x|\right). \tag{3.3}$$

We will consider the following conditions.

(H_1) $\sum_{n=1}^{\infty} q^{s/r}(n) < \infty$ for some $r \in (3 \cdot 2^{k-1}, 4 \cdot 2^{k-1}]$ and $r > \frac{\alpha p-1}{\alpha-1/2}$, where integer number $k \geq 1$ if $\alpha > 1/2$, $\alpha p > 1$ and $p \geq 2$.

(H_2) $\sum_{n=1}^{\infty} q^2(n) < \infty$ if $\alpha > 1/2$, $\alpha p > 1$ and $1 \leq p < 2$ or $\alpha > 1/2$ and $\alpha p = 1$.

Theorem 3.1 *Let $\alpha > \frac{1}{2}$ and $\alpha p \geq 1$. Assume that X_{ni}, $i, n \in \mathbb{N}$, are an array of rowwise AANA random variables which are stochastically dominated by a random variable X, a_{ni}, $i, n \in \mathbb{N}$, are an array of real numbers with $\sum_{i=1}^{n} |a_{ni}|^q = O(n)$ for some $q > \max\{\frac{\alpha p-1}{\alpha-1/2}, 2\}$. Let $EX_{ni} = 0$ for all $i, n \in \mathbb{N}$, if $p \geq 1$ and the conditions (H_1) and (H_2) are satisfied. If*

$$E|X|^p\psi\left(|X|\right) < \infty, \tag{3.4}$$

then

$$\sum_{n=1}^{\infty} n^{\alpha p-2}\psi(n)P\left(\max_{1\leq j\leq n}\left|\sum_{i=1}^{j} a_{ni}X_{ni}\right| > \varepsilon n^{\alpha}\right) < \infty \quad \textit{for all } \varepsilon > 0. \tag{3.5}$$

Proof Without loss of generality, we can assume that $a_{ni} > 0$ for all $i, n \in \mathbb{N}$. For fixed $n \in \mathbb{N}$, let $X'_{ni} = -n^{\alpha}I(X_{ni} < -n^{\alpha}) + X_{ni}I(|X_{ni}| \leq n^{\alpha}) + n^{\alpha}I(X_{ni} > n^{\alpha})$ and $X''_{ni} = X_{ni} - X'_{ni}$, $i \geq 1$. We will consider the following three cases.

(i) Let $p > 1$. It is easy to check that

$$\begin{aligned}
&\sum_{n=1}^{\infty} n^{\alpha p-2}\psi(n)P\left(\max_{1\leq j\leq n}\left|\sum_{i=1}^{j} a_{ni}X_{ni}\right| > \varepsilon n^{\alpha}\right) \\
&\quad\leq \sum_{n=1}^{\infty} n^{\alpha p-2}\psi(n)P\left(\max_{1\leq j\leq n}\left|\sum_{i=1}^{j} a_{ni}\left(X'_{ni} - EX'_{ni}\right)\right| > \varepsilon n^{\alpha}/2\right) \\
&\qquad+ \sum_{n=1}^{\infty} n^{\alpha p-2}\psi(n)P\left(\max_{1\leq j\leq n}\left|\sum_{i=1}^{j} a_{ni}\left(X''_{ni} - EX''_{ni}\right)\right| > \varepsilon n^{\alpha}/2\right) \\
&\quad:= I^* + J^*.
\end{aligned}$$

By C_r inequality and $\sum_{i=1}^n a_{ni}^q = O(n)$, it is easy to check that for all $0 < \gamma \le q$,

$$\frac{1}{n}\sum_{i=1}^n a_{ni}^\gamma \le \left(\frac{1}{n}\sum_{i=1}^n a_{ni}^q\right)^{\gamma/q} = O(1). \tag{3.6}$$

For J^*, noting that $|X''_{ni}| \le |X_{ni}| I(|X_{ni}| > n^\alpha)$, we have by Markov's inequality, Lemma 2.1 and (3.3), that

$$\begin{aligned}
J^* &\le C\sum_{n=1}^\infty n^{\alpha p-2-\alpha}\psi(n)\sum_{i=1}^n a_{ni} E\left|X''_{ni}\right| \\
&\le C\sum_{n=1}^\infty n^{\alpha p-2-\alpha}\psi(n)\sum_{i=1}^n a_{ni} E|X_{ni}| I\left(|X_{ni}| > n^\alpha\right) \\
&\le C\sum_{n=1}^\infty n^{\alpha p-1-\alpha}\psi(n) E|X| I\left(|X| > n^\alpha\right) \\
&= C\sum_{n=1}^\infty n^{\alpha p-1-\alpha}\psi(n)\sum_{j=n}^\infty E|X| I\left(j < |X|^{1/\alpha} \le j+1\right) \\
&= C\sum_{j=1}^\infty E|X| I\left(j < |X|^{1/\alpha} \le j+1\right)\sum_{n=1}^j n^{\alpha p-1-\alpha}\psi(n) \\
&\le C\sum_{j=1}^\infty j^{\alpha p-\alpha}\psi(j) E|X| I\left(j < |X|^{1/\alpha} \le j+1\right) \\
&\le CE|X|^p \psi\left(|X|^{1/\alpha}\right) \le CE|X|^p\psi(|X|) < \infty.
\end{aligned} \tag{3.7}$$

For I^*, note that for every $n \in \mathbb{N}$, $a_{ni}X'_{ni} - Ea_{ni}X'_{ni}$, $i \in \mathbb{N}$, are AANA random variables from Lemma 2.2. By Markov's inequality, Lemma 2.3 and Jensen's inequality, we have that for any $r \ge 2$,

$$\begin{aligned}
I^* &\le C_r\sum_{n=1}^\infty n^{\alpha p-2-\alpha r}\psi(n) E\left(\max_{1\le j\le n}\left|\sum_{i=1}^j \left(a_{ni}X'_{ni} - Ea_{ni}X'_{ni}\right)\right|^r\right) \\
&\le C_r\sum_{n=1}^\infty n^{\alpha p-2-\alpha r}\psi(n)\sum_{i=1}^n a_{ni}^r E\left|X'_{ni}\right|^r + C_r\sum_{n=1}^\infty n^{\alpha p-2-\alpha r}\psi(n)\left(\sum_{i=1}^n a_{ni}^2 E\left(X'_{ni}\right)^2\right)^{r/2} \\
&:= I_1^* + I_2^*.
\end{aligned} \tag{3.8}$$

We consider the following three cases.

Case 1. $\alpha > 1/2$, $\alpha p > 1$ and $p \ge 2$.

Take $r = q$. By $q > \max\{\frac{\alpha p-1}{\alpha-1/2}, 2\}$, it follows that $q > p$ and $\alpha p - 2 - \alpha q + q/2 < -1$.

For I_1^*, we have by C_r inequality that

$$\begin{aligned}
I_1^* &\le C\sum_{n=1}^\infty n^{\alpha p-2-\alpha q}\psi(n)\sum_{i=1}^n a_{ni}^q\left(E|X_{ni}|^q I\left(|X_{ni}| \le n^\alpha\right) + n^{\alpha q} P\left(|X_{ni}| > n^\alpha\right)\right) \\
&\le C\sum_{n=1}^\infty n^{\alpha p-2-\alpha q}\psi(n)\sum_{i=1}^n a_{ni}^q\left(E|X|^q I\left(|X| \le n^\alpha\right) + n^{\alpha q} P\left(|X| > n^\alpha\right)\right)
\end{aligned}$$

$$\begin{aligned}
&\le C\sum_{n=1}^{\infty} n^{\alpha p-1-\alpha q}\psi(n)E|X|^q I\big(|X|\le n^{\alpha}\big) + C\sum_{n=1}^{\infty} n^{\alpha p-1-\alpha}\psi(n)E|X|I\big(|X|>n^{\alpha}\big)\\
&\le C\sum_{n=1}^{\infty} n^{\alpha(p-q)-1}\psi(n)\sum_{j=1}^{n} j^{\alpha q}P\big(j-1<|X|^{1/\alpha}\le j\big) + CE|X|^p\psi\big(|X|\big)\\
&\le C\sum_{j=1}^{\infty} j^{\alpha q}P\big(j-1<|X|^{1/\alpha}\le j\big)\sum_{n=j}^{\infty} n^{\alpha(p-q)-1}\psi(n) + CE|X|^p\psi\big(|X|\big)\\
&\le C\sum_{j=1}^{\infty} j^{\alpha p}\psi(j)P\big(j-1<|X|^{1/\alpha}\le j\big) + CE|X|^p\psi\big(|X|\big)\\
&\le CE|X|^p\psi\big(|X|^{1/\alpha}\big)\le CE|X|^p\psi\big(|X|\big)<\infty. \qquad (3.9)
\end{aligned}$$

For I_2^*, note that $EX^2<\infty$ if $E|X|^p\psi(|X|)<\infty$ for $p\ge 2$. We have by (3.6) that

$$\begin{aligned}
I_2^* &\le C\sum_{n=1}^{\infty} n^{\alpha p-2-\alpha q}\psi(n)\left(\sum_{i=1}^{n} a_{ni}^2 EX_{ni}^2\right)^{q/2}\\
&\le C\sum_{n=1}^{\infty} n^{\alpha p-2-\alpha q}\psi(n)\left(\sum_{i=1}^{n} a_{ni}^2 EX^2\right)^{q/2}\\
&\le C\sum_{n=1}^{\infty} n^{\alpha p-2-\alpha q+q/2}\psi(n)<\infty.
\end{aligned}$$

Case 2. $\alpha>1/2$, $\alpha p>1$ and $1<p<2$.

Take $r=2$. Similar to the proofs of (3.8), (3.9) and (3.7), we have that

$$\begin{aligned}
I^* &\le C\sum_{n=1}^{\infty} n^{\alpha p-2-2\alpha}\psi(n)\sum_{i=1}^{n} a_{ni}^2\big(EX_{ni}^2 I\big(|X_{ni}|\le n^{\alpha}\big) + n^{2\alpha}P\big(|X_{ni}|>n^{\alpha}\big)\big)\\
&\le C\sum_{n=1}^{\infty} n^{\alpha p-1-2\alpha}\psi(n)EX^2 I\big(|X|\le n^{\alpha}\big) + C\sum_{n=1}^{\infty} n^{\alpha p-1-\alpha}\psi(n)E|X|I\big(|X|>n^{\alpha}\big)\\
&<\infty. \qquad (3.10)
\end{aligned}$$

Case 3. $\alpha>1/2$, $\alpha p=1$ and $p>1$.

Take $r=2$. Note that $1/2<\alpha<1$ if $\alpha p=1$. Similar to the proof of (3.10), it follows that $I^*<\infty$.

(ii) Let $p=1$. Note that $\alpha\ge 1$ from $\alpha p\ge 1$. By $EX_{ni}=0$ for $i,n\in\mathbb{N}$, Lemma 2.1, (3.6) and (3.4), we have that

$$\begin{aligned}
n^{-\alpha}\max_{1\le j\le n}\left|\sum_{i=1}^{j} a_{ni}EX'_{ni}\right| &\le n^{-\alpha}\sum_{i=1}^{n} a_{ni}E|X_{ni}|I\big(|X_{ni}|>n^{\alpha}\big)\\
&\le n^{1-\alpha}E|X|I\big(|X|>n^{\alpha}\big)\to 0 \quad \text{as } n\to\infty.
\end{aligned}$$

Hence for n large enough, we have

$$n^{-\alpha}\max_{1\le j\le n}\left|\sum_{i=1}^{j} a_{ni}X'_{ni}\right|<\frac{\varepsilon}{2}. \qquad (3.11)$$

It follows that

$$\begin{aligned}
&\sum_{n=1}^{\infty} n^{\alpha-2}\psi(n)P\left(\max_{1\le j\le n}\left|\sum_{i=1}^{j} a_{ni}X_{ni}\right| > \varepsilon n^{\alpha}\right)\\
&\quad\le \sum_{n=1}^{\infty} n^{\alpha-2}\psi(n)\sum_{i=1}^{n} P\bigl(|X_{ni}| > n^{\alpha}\bigr)\\
&\qquad + \sum_{n=1}^{\infty} n^{\alpha-2}\psi(n)P\left(\max_{1\le j\le n}\left|\sum_{i=1}^{j} a_{ni}X'_{ni}\right| > \varepsilon n^{\alpha}\right)\\
&\quad\le C\sum_{n=1}^{\infty} n^{\alpha-1}\psi(n)P\bigl(|X| > n^{\alpha}\bigr)\\
&\qquad + C\sum_{n=1}^{\infty} n^{\alpha-2}\psi(n)P\left(\max_{1\le j\le n}\left|\sum_{i=1}^{j} a_{ni}\bigl(X'_{ni} - EX'_{ni}\bigr)\right| > \frac{\varepsilon n^{\alpha}}{2}\right)\\
&\quad:= CI_1 + CI_2.
\end{aligned} \tag{3.12}$$

For I_1, we have by (3.1) and (3.4) that

$$\begin{aligned}
I_1 &= \sum_{n=1}^{\infty} n^{\alpha-1}\psi(n)\sum_{i=n}^{\infty} P\bigl(i^{\alpha} < |X| \le (i+1)^{\alpha}\bigr)\\
&= \sum_{i=1}^{\infty} P\bigl(i^{\alpha} < |X| \le (i+1)^{\alpha}\bigr)\sum_{n=1}^{i} n^{\alpha-1}\psi(n)\\
&\le C\sum_{i=1}^{\infty} P\bigl(i^{\alpha} < |X| \le (i+1)^{\alpha}\bigr) i^{\alpha}\psi(i)\\
&\le CE|X|\psi\bigl(|X|^{1/\alpha}\bigr) \le CE|X|\psi\bigl(|X|\bigr) < \infty.
\end{aligned} \tag{3.13}$$

For I_2, we have by Markov's inequality, Lemma 2.3, Lemma 2.1, (3.2) and (3.3) that

$$\begin{aligned}
I_2 &\le C\sum_{n=1}^{\infty} n^{-\alpha-2}\psi(n)E\max_{1\le j\le n}\left(\sum_{i=1}^{j} a_{ni}\bigl(X'_{ni} - EX'_{ni}\bigr)\right)^2\\
&\le C\sum_{n=1}^{\infty} n^{-\alpha-2}\psi(n)\sum_{i=1}^{n} a_{ni}^2 E\bigl(X'_{ni}\bigr)^2\\
&= C\sum_{n=1}^{\infty} n^{-\alpha-2}\psi(n)\left\{\sum_{i=1}^{n} a_{ni}^2 EX_{ni}^2 I\bigl(|X_{ni}| \le n^{\alpha}\bigr) + n^{2\alpha}\sum_{i=1}^{n} a_{ni}^2 P\bigl(|X_{ni}| > n^{\alpha}\bigr)\right\}\\
&\le C\sum_{n=1}^{\infty} n^{-\alpha-1}\psi(n)EX^2 I\bigl(|X| \le n^{\alpha}\bigr) + C\sum_{n=1}^{\infty} n^{\alpha-1}\psi(n)P\bigl(|X| > n^{\alpha}\bigr)\\
&= C\sum_{n=1}^{\infty} n^{-\alpha-1}\psi(n)\sum_{k=1}^{n} EX^2 I\bigl((k-1)^{\alpha} < |X| \le k^{\alpha}\bigr) + C\\
&= C\sum_{k=1}^{\infty} EX^2 I\bigl((k-1)^{\alpha} < |X| \le k^{\alpha}\bigr)\sum_{n=k}^{\infty} n^{-\alpha-1}\psi(n) + C
\end{aligned}$$

$$\begin{aligned}&\le C\sum_{k=1}^{\infty}k^{-\alpha}\psi(k)EX^2I\big((k-1)^{\alpha}<|X|\le k^{\alpha}\big)+C\\&\le CE|X|\psi\big(|X|\big)+C<\infty.\end{aligned}\tag{3.14}$$

By (3.12)-(3.14), (3.5) holds for the case $p = 1$.

(iii) Let $0 < p < 1$. Denote

$$\sum_{i=1}^{j}a_{ni}X_{ni}=\sum_{i=1}^{j}a_{ni}X_{ni}I\big(|X_{ni}|\le n^{\alpha}\big)+\sum_{i=1}^{j}a_{ni}X_{ni}I\big(|X_{ni}|>n^{\alpha}\big)=:S'_{nj}+S''_{nj}.\tag{3.15}$$

Noting that $E|X|^p\psi(|X|)<\infty$, we have by Markov's inequality, Lemma 2.1 and (3.2)-(3.6), that

$$\begin{aligned}&\sum_{n=1}^{\infty}n^{\alpha p-2}\psi(n)P\Big(\max_{1\le j\le n}\big|S'_{nj}\big|>\varepsilon n^{\alpha}\Big)\\
&\quad\le\varepsilon^{-1}\sum_{n=1}^{\infty}n^{\alpha p-2-\alpha}\psi(n)E\left(\max_{1\le j\le n}\left|\sum_{i=1}^{j}a_{ni}X_{ni}I\big(|X_{ni}|\le n^{\alpha}\big)\right|\right)\\
&\quad\le\varepsilon^{-1}\sum_{n=1}^{\infty}n^{\alpha p-2-\alpha}\psi(n)\sum_{i=1}^{n}a_{ni}E|X_{ni}|I\big(|X_{ni}|\le n^{\alpha}\big)\\
&\quad\le C\varepsilon^{-1}\sum_{n=1}^{\infty}n^{\alpha p-1-\alpha}\psi(n)E|X|I\big(|X|\le n^{\alpha}\big)+C\varepsilon^{-1}\sum_{n=1}^{\infty}n^{\alpha p-1}\psi(n)P\big(|X|>n^{\alpha}\big)\\
&\quad=C\varepsilon^{-1}\sum_{n=1}^{\infty}n^{\alpha p-1-\alpha}\psi(n)\sum_{j=1}^{n}E|X|I\big(j-1<|X|^{1/\alpha}\le j\big)\\
&\qquad+C\varepsilon^{-1}\sum_{n=1}^{\infty}n^{\alpha p-1}\psi(n)\sum_{j=n}^{\infty}P\big(j<|X|^{1/\alpha}\le j+1\big)\\
&\quad\le C\varepsilon^{-1}\sum_{j=1}^{\infty}j^{\alpha}P\big(j-1<|X|^{1/\alpha}\le j\big)\sum_{n=j}^{\infty}n^{\alpha p-1-\alpha}\psi(n)\\
&\qquad+C\varepsilon^{-1}\sum_{j=1}^{\infty}P\big(j<|X|^{1/\alpha}\le j+1\big)\sum_{n=1}^{j}n^{\alpha p-1}\psi(n)\\
&\quad\le C\varepsilon^{-1}\sum_{j=1}^{\infty}j^{\alpha p}\psi(j)P\big(j-1<|X|^{1/\alpha}\le j\big)+C\varepsilon^{-1}\sum_{j=1}^{\infty}j^{\alpha p}\psi(j)P\big(j<|X|^{1/\alpha}\le j+1\big)\\
&\quad\le CE|X|^p\psi\big(|X|^{1/\alpha}\big)\le CE|X|^p\psi\big(|X|\big)<\infty\end{aligned}\tag{3.16}$$

and

$$\begin{aligned}&\sum_{n=1}^{\infty}n^{\alpha p-2}\psi(n)P\Big(\max_{1\le j\le n}\big|S''_{nj}\big|>\varepsilon n^{\alpha}\Big)\\
&\quad\le\varepsilon^{-p/2}\sum_{n=1}^{\infty}n^{\alpha p/2-2}\psi(n)E\left(\max_{1\le j\le n}\left|\sum_{i=1}^{j}a_{ni}X_{ni}I\big(|X_{ni}|>n^{\alpha}\big)\right|\right)^{p/2}\end{aligned}$$

$$
\begin{aligned}
&\le \varepsilon^{-p/2} \sum_{n=1}^{\infty} n^{\alpha p/2-2} \psi(n) \sum_{i=1}^{n} a_{ni}^{p/2} E|X_{ni}|^{p/2} I\big(|X_{ni}| > n^{\alpha}\big) \\
&\le C\varepsilon^{-p/2} \sum_{n=1}^{\infty} n^{\alpha p/2-1} \psi(n) E|X|^{p/2} I\big(|X| > n^{\alpha}\big) \\
&= C\varepsilon^{-p/2} \sum_{n=1}^{\infty} n^{\alpha p/2-1} \psi(n) \sum_{j=n}^{\infty} E|X|^{p/2} I\big(j < |X|^{1/\alpha} \le j+1\big) \\
&\le C\varepsilon^{-p/2} \sum_{j=1}^{\infty} j^{\alpha p/2} P\big(j < |X|^{1/\alpha} \le j+1\big) \sum_{n=1}^{j} n^{\alpha p/2-1} \psi(n) \\
&\le C\varepsilon^{-p/2} \sum_{j=1}^{\infty} j^{\alpha p} \psi(j) P\big(j-1 < |X|^{1/\alpha} \le j\big) \\
&\le CE|X|^{p} \psi\big(|X|^{1/\alpha}\big) \le CE|X|^{p} \psi\big(|X|\big) < \infty. \qquad (3.17)
\end{aligned}
$$

Hence (3.15)-(3.17) imply (3.5). From all the statements above, we have proved (3.5). □

Remark 3.1 Taking $\psi(x) \equiv 1$ and $a_{ni} \equiv 1$ in Theorem 3.1, we can get (i) of Theorem B; meanwhile, relax the mixing coefficient condition $\sum_{n=1}^{\infty} q^{s/r}(n) < \infty$ to $\sum_{n=1}^{\infty} q^{2}(n) < \infty$ for the case $\alpha > 1/2$, $\alpha p > 1$ and $1 < p < 2$. In addition, we extend the case $1/2 < \alpha \le 1$, $p > 1$ and $\alpha p > 1$ to the case $\alpha > 1/2$, $\alpha p \ge 1$. Taking $\psi(x) \equiv 1$, $a_{ni} \equiv 1$ and $\alpha = 1$, $p = 1$ in Theorem 3.1, we can get (ii) of Theorem B and weaken the condition $E|X| \log|X| < \infty$ to the condition $E|X| < \infty$. Hence we extend and improve the corresponding results of [10].

Remark 3.2 Under the conditions of Theorem 3.1, we have that for $p > 1$,

$$
\sum_{n=1}^{\infty} n^{\alpha p-2-\alpha} \psi(n) E\left(\max_{1 \le j \le n} \left| \sum_{i=1}^{j} a_{ni} X_{ni} \right| - \varepsilon n^{\alpha} \right)^{+} < \infty. \qquad (3.18)
$$

In fact, by Lemma 2.4 with $r \ge 2$, we get

$$
\begin{aligned}
&\sum_{n=1}^{\infty} n^{\alpha p-2-\alpha} \psi(n) E\left(\max_{1 \le j \le n} \left| \sum_{i=1}^{j} a_{ni} X_{ni} \right| - \varepsilon n^{\alpha} \right)^{+} \\
&\quad \le C \sum_{n=1}^{\infty} n^{\alpha p-2-\alpha r} \psi(n) E\left(\max_{1 \le j \le n} \left| \sum_{i=1}^{j} \big(a_{ni} X'_{ni} - E a_{ni} X'_{ni}\big) \right| \right)^{r} \\
&\qquad + \sum_{n=1}^{\infty} n^{\alpha p-2-\alpha} \psi(n) E\left(\max_{1 \le j \le n} \left| \sum_{i=1}^{j} \big(a_{ni} X''_{ni} - E a_{ni} X''_{ni}\big) \right| \right).
\end{aligned}
$$

By the process of the proof of Theorem 3.1 in the case $p > 1$, it follows that (3.18) holds.

Similar to the proof of Theorem 3.1, we can get easily the following result.

Theorem 3.2 *Let $\alpha > \frac{1}{2}$ and $\alpha p \ge 1$. Let X_n, $n \in \mathbb{N}$, be AANA random variables which are stochastically dominated by a random variable X. Assume that a_n, $n \in \mathbb{N}$, are real numbers* *with* $\sum_{i=1}^{n}|a_i|^q = O(n)$ *for some* $q > \max\{\frac{\alpha p-1}{\alpha-1/2}, 2\}$, *the conditions* (H_1) *and* (H_2) *are satisfied. If* (3.4) *holds, then*

$$\sum_{n=1}^{\infty} n^{\alpha p-2}\psi(n)P\left(\max_{1\le j\le n}\left|\sum_{i=1}^{j} a_iX_i\right| > \varepsilon n^{\alpha}\right) < \infty \quad \text{for all } \varepsilon > 0.$$

Remark 3.3 Similar to Remark 3.1, taking $\psi(x) \equiv 1$ and $a_i \equiv 1$ in Theorem 3.2, we can get (i) of Theorem 3.4 in [10]; meanwhile, relax the mixing coefficient condition $\sum_{n=1}^{\infty} q^{s/r}(n) < \infty$ to $\sum_{n=1}^{\infty} q^2(n) < \infty$ for the case $\alpha > 1/2$, $\alpha p > 1$ and $1 < p < 2$. In addition, we extend the case $1/2 < \alpha \le 1$, $p > 1$ and $\alpha p > 1$ to the case $\alpha > 1/2$, $\alpha p \ge 1$. Taking $\psi(x) \equiv 1$, $a_{ni} \equiv 1$ and $\alpha = 1$, $p = 1$ in Theorem 3.2, we can get (ii) of Theorem 3.4 in [10] and weaken the condition $E|X|\log|X| < \infty$ to the condition $E|X| < \infty$. Hence, we extend and improve the corresponding results of [10].

In the following, we give the Marcinkiewicz-Zygmund type strong law of large numbers of weights sums on AANA random variables.

Corollary 3.1 *Let* $\alpha > \frac{1}{2}$ *and* $\alpha p \ge 1$. *Let* X_n, $n \in \mathbb{N}$, *be AANA random variables which are stochastically dominated by a random variable* X. *Assume that* a_n, $n \in \mathbb{N}$ *are real numbers with* $\sum_{i=1}^{n}|a_i|^q = O(n)$ *for some* $q > \max\{\frac{\alpha p-1}{\alpha-1/2}, 2\}$, *the conditions* (H_1) *and* (H_2) *are satisfied. If* $E|X|^p < \infty$, *then*

$$\sum_{n=1}^{\infty} n^{\alpha p-2}P\left(\max_{1\le j\le n}\left|\sum_{i=1}^{j} a_iX_i\right| > \varepsilon n^{\alpha}\right) < \infty \tag{3.19}$$

and

$$n^{-\alpha}\sum_{i=1}^{n} a_iX_i \to 0 \quad \text{a.s. } n\to\infty. \tag{3.20}$$

Further, for $p > 1$,

$$\sum_{n=1}^{\infty} n^{\alpha p-2-\alpha}E\left(\max_{1\le j\le n}\left|\sum_{i=1}^{j} a_iX_i\right| - \varepsilon n^{\alpha}\right)^{+} < \infty. \tag{3.21}$$

Proof Taking $\psi(x) = 1$ in Theorem 3.2, we get (3.19) easily. Similar to the proof of (3.18), (3.21) is obtained immediately. We only need to prove (3.20).

By (3.20), it follows that for all $\varepsilon > 0$,

$$\begin{aligned}
\infty &> \sum_{n=1}^{\infty} n^{\alpha p-2}P\left(\max_{1\le j\le n}\left|\sum_{i=1}^{j} a_iX_i\right| > \varepsilon n^{\alpha}\right) = \sum_{k=0}^{\infty}\sum_{n=2^k}^{2^{k+1}-1} n^{\alpha p-2}P\left(\max_{1\le j\le n}\left|\sum_{i=1}^{j} a_iX_i\right| > \varepsilon n^{\alpha}\right) \\
&\ge \begin{cases} \sum_{k=0}^{\infty}(2^k)^{\alpha p-2}2^kP(\max_{1\le j\le 2^k}|\sum_{i=1}^{j} a_iX_i| > \varepsilon 2^{(k+1)\alpha}) & \text{if } \alpha p \ge 2, \\ \sum_{k=0}^{\infty}(2^{k+1})^{\alpha p-2}2^kP(\max_{1\le j\le 2^k}|\sum_{i=1}^{j} a_iX_i| > \varepsilon 2^{(k+1)\alpha}) & \text{if } 1 \le \alpha p < 2 \end{cases} \\
&\ge \begin{cases} \sum_{k=0}^{\infty}P(\max_{1\le j\le 2^k}|\sum_{i=1}^{j} a_iX_i| > \varepsilon 2^{(k+1)\alpha}) & \text{if } \alpha p \ge 2, \\ \frac{1}{2}\sum_{k=0}^{\infty}P(\max_{1\le j\le 2^k}|\sum_{i=1}^{j} a_iX_i| > \varepsilon 2^{(k+1)\alpha}) & \text{if } 1 \le \alpha p < 2. \end{cases}
\end{aligned}$$

By the Borel-Cantelli lemma, we obtain that

$$\frac{\max_{1\le j\le 2^k}|\sum_{i=1}^{j}a_iX_i|}{2^{(k+1)\alpha}}\to 0\quad \text{a.s. } k\to\infty. \tag{3.22}$$

For all positive integers n, there exists a positive integer k such that $2^{k-1}\le n\le 2^k$. We have by (3.22) that

$$n^{-\alpha}\left|\sum_{i=1}^{n}a_iX_i\right|\le \max_{2^{k-1}\le n\le 2^k} n^{-\alpha}\left|\sum_{i=1}^{n}a_iX_i\right|\le \frac{2^\alpha\max_{1\le j\le 2^k}|\sum_{i=1}^{j}a_iX_i|}{2^{(k+1)\alpha}}\to 0\quad \text{a.s. } k\to\infty,$$

which implies that

$$n^{-\alpha}\sum_{i=1}^{n}a_iX_i\to 0\quad \text{a.s. } n\to\infty.$$

This completes the proof of the corollary. □

Remark 3.4 Taking $a_n\equiv 1$ in Corollary 3.1, we can get the Baum-Katz result on AANA random variables. Comparing with Theorem 3.4 and Corollary 3.5 of [10], Corollary 3.1 relaxes the mixing coefficient condition $\sum_{n=1}^{\infty}q^{s/r}(n)<\infty$ to $\sum_{n=1}^{\infty}q^2(n)<\infty$ for the case $\alpha>1/2$, $\alpha p>1$ and $1<p<2$. In addition, we also consider the case $\alpha p=1$ and the case $\alpha p\ge 1$ and $0<p\le 1$. Taking $\alpha=1$ and $p=2$ in Corollary 3.1, we can get the Hsu-Robbins-type theorem (see [13]) on AANA random variables. Taking $\alpha=1$ and $p=1$ in Corollary 3.1, we improve (ii) of Theorem 3.4 and (ii) of Corollary 3.5 in [10].

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors read and approved the final manuscript.

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