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Fixed point results for Meir-Keeler-type ϕ-α-contractions on partial metric spaces

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Abstract
The purpose of this paper is to study fixed point theorems for a mapping satisfying the generalized Meir-Keeler-type ϕ-α-contractions in complete partial metric spaces. Our results generalize or improve many recent fixed point theorems in the literature.
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1 Introduction and preliminaries

Throughout this paper, by $\mathbb{R}^+$ we denote the set of all nonnegative real numbers, while $\mathbb{N}$ is the set of all natural numbers. In 1994, Mattews [1] introduced the following notion of partial metric spaces.

Definition 1 [1] A partial metric on a nonempty set X is a function $p : X \times X \to \mathbb{R}^+$ such that for all $x, y, z \in X$,

(p_1) $x = y$ if and only if $p(x,x) = p(x,y) = p(y,y)$;
(p_2) $p(x,x) \le p(x,y)$;
(p_3) $p(x,y) = p(y,x)$;
(p_4) $p(x,y) \le p(x,z) + p(z,y) - p(z,z)$.

A partial metric space is a pair (X,p) such that X is a nonempty set and p is a partial metric on X.

Remark 1 It is clear that if $p(x,y) = 0$, then from (p_1) and (p_2), $x = y$. But if $x = y$, $p(x,y)$ may not be 0.

Each partial metric p on X generates a $\mathcal{T}_0$ topology τ_p on X which has as a base the family of open p-balls $\{B_p(x,\gamma) : x \in X, \gamma > 0\}$, where $B_p(x,\gamma) = \{y \in X : p(x,y) < p(x,x) + \gamma\}$ for all $x \in X$ and $\gamma > 0$. If p is a partial metric on X, then the function $d_p : X \times X \to \mathbb{R}^+$ given by

$$d_p(x,y) = 2p(x,y) - p(x,x) - p(y,y)$$

is a metric on X.

We recall some definitions of a partial metric space as follows.

Definition 2 [1] Let (X, p) be a partial metric space. Then

(1) a sequence $\{x_n\}$ in a partial metric space (X, p) converges to $x \in X$ if and only if $p(x, x) = \lim_{n\to\infty} p(x, x_n)$;

(2) a sequence $\{x_n\}$ in a partial metric space (X, p) is called a Cauchy sequence if and only if $\lim_{m,n\to\infty} p(x_m, x_n)$ exists (and is finite);

(3) a partial metric space (X, p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p, to a point $x \in X$ such that $p(x, x) = \lim_{m,n\to\infty} p(x_m, x_n)$;

(4) a subset A of a partial metric space (X, p) is closed if whenever $\{x_n\}$ is a sequence in A such that $\{x_n\}$ converges to some $x \in X$, then $x \in A$.

Remark 2 The limit in a partial metric space is not unique.

Lemma 1 [1, 2]

(1) *$\{x_n\}$ is a Cauchy sequence in a partial metric space (X, p) if and only if it is a Cauchy sequence in the metric space (X, d_p);*

(2) *a partial metric space (X, p) is complete if and only if the metric space (X, d_p) is complete. Furthermore, $\lim_{n\to\infty} d_p(x_n, x) = 0$ if and only if $p(x, x) = \lim_{n\to\infty} p(x_n, x) = \lim_{n\to\infty} p(x_n, x_m)$.*

In recent years, fixed point theory has developed rapidly on partial metric spaces, see [2–10].

In this study, we also recall the Meir-Keeler-type contraction [11] and α-admissible one [12]. In 1969, Meir and Keeler [11] introduced the following notion of Meir-Keeler-type contraction in a metric space (X, d).

Definition 3 Let (X, d) be a metric space, $f : X \to X$. Then f is called a Meir-Keeler-type contraction whenever, for each $\eta > 0$, there exists $\gamma > 0$ such that

$$\eta \le d(x, y) < \eta + \gamma \quad \Longrightarrow \quad d(fx, fy) < \eta.$$

The following definition was introduced in [12].

Definition 4 Let $f : X \to X$ be a self-mapping of a set X and $\alpha : X \times X \to \mathbb{R}^+$. Then f is called α-admissible if

$$x, y \in X, \quad \alpha(x, y) \ge 1 \quad \Longrightarrow \quad \alpha(fx, fy) \ge 1.$$

The purpose of this paper is to study fixed point theorems for a mapping satisfying the generalized Meir-Keeler-type ϕ-α-contractions in complete partial metric spaces. Our results generalize or improve many recent fixed point theorems in the literature.

2 Main results

In the article, we denote by Φ the class of functions $\phi : \mathbb{R}^{+4} \to \mathbb{R}^+$ satisfying the following conditions:

(ϕ_1) ϕ is an increasing and continuous function in each coordinate;

(ϕ_2) for $t \in \mathbb{R}^+ \setminus \{0\}$, $\phi(t, t, t, t) \le t$, $\phi(t, 0, 0, t) \le t$, $\phi(0, 0, t, \frac{t}{2}) \le t$; and $\phi(t_1, t_2, t_3, t_4) = 0$ iff $t_1 = t_2 = t_3 = t_4 = 0$.

We now state the new notions of generalized Meir-Keeler-type ϕ-contractions and generalized Meir-Keeler-type ϕ-α-contractions in partial metric spaces as follows.

Definition 5 Let (X,p) be a partial metric space, $f : X \to X$ and $\phi \in \Phi$. Then f is called a generalized Meir-Keeler-type ϕ-contraction whenever, for each $\eta > 0$, there exists $\delta > 0$ such that

$$\begin{aligned} &\eta \le \phi\left(p(x,y), p(x,fx), p(y,fy), \frac{1}{2}\bigl[p(x,fy) + p(y,fx)\bigr]\right) < \eta + \delta \\ &\quad \Longrightarrow \quad p(fx,fy) < \eta. \end{aligned}$$

Definition 6 Let (X,p) be a partial metric space, $f : X \to X$, $\phi \in \Phi$ and $\alpha : X \times X \to \mathbb{R}^+$. Then f is called a generalized Meir-Keeler-type ϕ-α-contraction if the following conditions hold:

(1) f is α-admissible;

(2) for each $\eta > 0$, there exists $\delta > 0$ such that

$$\begin{aligned} &\eta \le \phi\left(p(x,y), p(x,fx), p(y,fy), \frac{1}{2}\bigl[p(x,fy) + p(y,fx)\bigr]\right) < \eta + \delta \\ &\quad \Longrightarrow \quad \alpha(x,x)\alpha(y,y)p(fx,fy) < \eta. \end{aligned} \tag{2.1}$$

Remark 3 Note that if f is a generalized Meir-Keeler-type ϕ-α-contraction, then we have that for all $x, y \in X$,

$$\begin{aligned} &\alpha(x,x)\alpha(y,y)p(fx,fy) \\ &\quad \le \phi\left(p(x,y), p(x,fx), p(y,fy), \frac{1}{2}\bigl[p(x,fy) + p(y,fx)\bigr]\right). \end{aligned}$$

Further, if $\phi(p(x,y), p(x,fx), p(y,fy), \frac{1}{2}[p(x,fy) + p(y,fx)]) = 0$, then $p(fx,fy) = 0$. On the other hand, if $\phi(p(x,y), p(x,fx), p(y,fy), \frac{1}{2}[p(x,fy) + p(y,fx)]) > 0$, then $\alpha(x,x)\alpha(y,y)p(fx,fy) < \phi(p(x,y), p(x,fx), p(y,fy), \frac{1}{2}[p(x,fy) + p(y,fx)])$.

We now state our main result for the generalized Meir-Keeler-type ϕ-α-contraction as follows.

Theorem 1 *Let (X,p) be a complete partial metric space, and $\phi \in \Phi$. If $\alpha : X \times X \to \mathbb{R}^+$ satisfies the following conditions*:

(α_1) *there exists $x_0 \in X$ such that $\alpha(x_0,x_0) \ge 1$*;

(α_2) *if $\alpha(x_n,x_n) \ge 1$ for all $n \in \mathbb{N}$, then $\lim_{n\to\infty} \alpha(x_n,x_n) \ge 1$*;

(α_3) *$\alpha : X \times X \to \mathbb{R}^+$ is a continuous function in each coordinate.*

Suppose that $f : X \to X$ is a generalized Meir-Keeler-type ϕ-α-contraction. Then f has a fixed point in X.

Proof Let x_0 and let $x_{n+1} = fx_n = f^n x_0$ for $n = 0, 1, 2, \ldots$. Since f is α-admissible and $\alpha(x_0,x_0) \ge 1$, we have

$$\alpha(fx_0,fx_0) = \alpha(x_1,x_1) \ge 1.$$

By continuing this process, we get

$$\alpha(x_n, x_n) \geq 1 \quad \text{for all } n \in \mathbb{N} \cup \{0\}. \tag{2.2}$$

If there exists $n_0 \in \mathbb{N}$ such that $x_{n_0+1} = x_{n_0}$, then we finished the proof. Suppose that $x_{n+1} \neq x_n$ for any $n = 0, 1, 2, \ldots$. By the definition of the function ϕ, we have $\phi(p(x_n, x_{n+1}), p(x_n, fx_n), p(x_{n+1}, fx_{n+1}), \frac{1}{2}[p(x_n, fx_{n+1}) + p(x_{n+1}, fx_n)]) > 0$ for all $n \in \mathbb{N} \cup \{0\}$.

Step 1. We shall prove that

$$\lim_{n\to\infty} p(x_n, x_{n+1}) = 0, \quad \text{that is} \quad \lim_{n\to\infty} d_p(x_n, x_{n+1}) = 0.$$

By Remark 3 and (p_4), using (2.2), we have

$$\begin{aligned}
&p(x_{n+1}, x_{n+2}) \\
&\quad = p(fx_n, fx_{n+1}) \\
&\quad \leq \alpha(x_n, x_n)\alpha(x_{n+1}, x_{n+1})p(fx_n, fx_{n+1}) \\
&\quad < \phi\left(p(x_n, x_{n+1}), p(x_n, fx_n), p(x_{n+1}, fx_{n+1}), \frac{1}{2}\big[p(x_n, fx_{n+1}) + p(x_{n+1}, fx_n)\big]\right) \\
&\quad = \phi\left(p(x_n, x_{n+1}), p(x_n, x_{n+1}), p(x_{n+1}, x_{n+2}), \frac{1}{2}\big[p(x_n, x_{n+2}) + p(x_{n+1}, x_{n+1})\big]\right) \\
&\quad \leq \phi\left(p(x_n, x_{n+1}), p(x_n, x_{n+1}), p(x_{n+1}, x_{n+2}), \frac{1}{2}\big[p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2})\big]\right).
\end{aligned} \tag{2.3}$$

If $p(x_n, x_{n+1}) \leq p(x_{n+1}, x_{n+2})$, then

$$\begin{aligned}
p(x_{n+1}, x_{n+2}) &= p(fx_n, fx_{n+1}) \\
&< \phi\big(p(x_{n+1}, x_{n+2}), p(x_{n+1}, x_{n+2}), p(x_{n+1}, x_{n+2}), p(x_{n+1}, x_{n+2})\big) \\
&\leq p(x_{n+1}, x_{n+2}),
\end{aligned}$$

which implies a contradiction, and hence $p(x_n, x_{n+1}) < p(x_{n-1}, x_n)$. From the argument above, we also have that for each $n \in \mathbb{N}$,

$$\begin{aligned}
p(x_{n+1}, x_{n+2}) &= p(fx_n, fx_{n+1}) \\
&< \phi\big(p(x_n, x_{n+1}), p(x_n, x_{n+1}), p(x_n, x_{n+1}), p(x_n, x_{n+1})\big) \\
&\leq p(x_n, x_{n+1}).
\end{aligned} \tag{2.4}$$

Since the sequence $\{p(x_n, x_{n+1})\}$ is decreasing, it must converge to some $\eta \geq 0$, that is,

$$\lim_{n\to\infty} p(x_n, x_{n+1}) = \eta. \tag{2.5}$$

It follows from (2.4) and (2.5) that

$$\lim_{n\to\infty} \phi\big(p(x_n, x_{n+1}), p(x_n, x_{n+1}), p(x_n, x_{n+1}), p(x_n, x_{n+1})\big) = \eta. \tag{2.6}$$

Notice that $\eta = \inf\{p(x_n, x_{n+1}) : n \in \mathbb{N}\}$. We claim that $\eta = 0$. Suppose, to the contrary, that $\eta > 0$. Since f is a generalized Meir-Keeler-type ϕ-contraction, corresponding to η use, and taking into account the above inequality (2.6), there exist $\delta > 0$ and a natural number k such that

$$\begin{aligned} &\eta \le \phi\big(p(x_k, x_{k+1}), p(x_k, x_{k+1}), p(x_k, x_{k+1}), p(x_k, x_{k+1})\big) < \eta + \delta \\ &\quad \Longrightarrow \quad \alpha(x_k, x_k)\alpha(x_{k+1}, x_{k+1})p(fx_k, fx_{k+1}) < \eta, \end{aligned}$$

which implies

$$p(x_{k+1}, x_{k+2}) = p(fx_k, fx_{k+1}) \le \alpha(x_k, x_k)\alpha(x_{k+1}, x_{k+1})p(fx_k, fx_{k+1}) < \eta.$$

So, we get a contradiction since $\eta = \inf\{p(x_n, x_{n+1}) : n \in \mathbb{N}\}$. Thus we have that

$$\lim_{n\to\infty} p(x_n, x_{n+1}) = 0. \tag{2.7}$$

By (p_2), we also have

$$\lim_{n\to\infty} p(x_n, x_n) = 0. \tag{2.8}$$

Since $d_p(x, y) = 2p(x, y) - p(x, x) - p(y, y)$ for all $x, y \in X$, using (2.7) and (2.8), we obtain that

$$\lim_{n\to\infty} d_p(x_n, x_{n+1}) = 0. \tag{2.9}$$

Step 2. We show that $\{x_n\}$ is a Cauchy sequence in the partial metric space (X, p), that is, it is sufficient to show that $\{x_n\}$ is a Cauchy sequence in the metric space (X, d_p).

Suppose that the above statement is false. Then there exists $\epsilon > 0$ such that for any $k \in \mathbb{N}$, there are $n_k, m_k \in \mathbb{N}$ with $n_k > m_k \ge k$ satisfying

$$d_p(x_{m_k}, x_{n_k}) \ge \epsilon. \tag{2.10}$$

Further, corresponding to $m_k \ge k$, we can choose n_k in such a way that it is the smallest integer with $n_k > m_k \ge k$ and $d(x_{2m_k}, x_{2n_k}) \ge \epsilon$. Therefore

$$d_p(x_{m_k}, x_{n_k-2}) < \epsilon. \tag{2.11}$$

Now we have that for all $k \in \mathbb{N}$,

$$\begin{aligned} \epsilon &\le d_p(x_{m_k}, x_{n_k}) \\ &\le d_p(x_{m_k}, x_{n_k-2}) + d_p(x_{n_k-2}, x_{n_k-1}) + d_p(x_{n_k-1}, x_{n_k}) \\ &< \epsilon + d_p(x_{n_k-2}, x_{n_k-1}) + d_p(x_{n_k-1}, x_{n_k}). \end{aligned} \tag{2.12}$$

Letting $k \to \infty$ in the above inequality and using (2.12), we get

$$\lim_{n\to\infty} d_p(x_{m_k}, x_{n_k}) = \epsilon. \tag{2.13}$$

On the other hand, we have

$$\begin{aligned}\epsilon &\le d_p(x_{m_k}, x_{n_k}) \\ &\le d_p(x_{m_k}, x_{m_{k+1}}) + d_p(x_{m_{k+1}}, x_{n_{k+1}}) + d_p(x_{n_{k+1}}, x_{n_k}) \\ &\le d_p(x_{m_k}, x_{m_{k+1}}) + d_p(x_{m_{k+1}}, x_{m_k}) + d_p(x_{m_k}, x_{n_k}) + d_p(x_{n_k}, x_{n_{k+1}}) + d_p(x_{n_{k+1}}, x_{n_k}).\end{aligned}$$

Letting $n \to \infty$, we obtain that

$$\lim_{n\to\infty} d_p(x_{m_{k+1}}, x_{n_{k+1}}) = \epsilon. \tag{2.14}$$

Since $d_p(x,y) = 2p(x,y) - p(x,x) - p(y,y)$ and using (2.13) and (2.14), we have that

$$\lim_{n\to\infty} p(x_{m_k}, x_{n_k}) = \frac{\epsilon}{2} \tag{2.15}$$

and

$$\lim_{n\to\infty} p(x_{m_{k+1}}, x_{n_{k+1}}) = \frac{\epsilon}{2} \tag{2.16}$$

By Remark 3 and (p_4), we have

$$\begin{aligned}&p(x_{m_{k+1}}, x_{n_{k+1}}) \\ &\quad= p(fx_{m_k}, fx_{n_k}) \\ &\quad\le \alpha(x_{m_k}, x_{m_k})\alpha(x_{n_k}, x_{n_k})p(fx_{m_k}, fx_{n_k}) \\ &\quad< \phi\left(p(x_{m_k}, x_{n_k}), p(x_{m_k}, fx_{m_k}), p(x_{n_k}, fx_{n_k}), \frac{1}{2}\left[p(x_{m_k}, fx_{n_k}) + p(x_{n_k}, fx_{m_k})\right]\right) \\ &\quad= \phi\Bigg(p(x_{m_k}, x_{n_k}), p(x_{m_k}, x_{m_k+1}), p(x_{n_k}, x_{n_{k+1}}), \\ &\qquad \frac{1}{2}\left[p(x_{m_k}, x_{n_k+1}) + p(x_{n_k}, x_{m_k+1})\right]\Bigg).\end{aligned} \tag{2.17}$$

Since

$$p(x_{m_k}, x_{n_k+1}) \le p(x_{m_k}, x_{m_k+1}) + p(x_{m_k+1}, x_{n_k+1}) - p(x_{m_k+1}, x_{m_k+1}) \tag{2.18}$$

and

$$p(x_{n_k}, x_{m_k+1}) \le p(x_{n_k}, x_{n_k+1}) + p(x_{n_k+1}, x_{m_k+1}) - p(x_{n_k+1}, x_{n_k+1}). \tag{2.19}$$

Taking into account the above inequalities (2.8), (2.17), (2.18) and (2.19), letting $k \to \infty$, we have

$$\frac{\epsilon}{2} < \phi\left(\frac{\epsilon}{2}, 0, 0, \frac{\epsilon}{2}\right) \le \frac{\epsilon}{2},$$

which implies a contradiction. Thus, $\{x_n\}$ is a Cauchy sequence in the metric space (X, d_p).

Step 3. We show that f has a fixed point ν in $\bigcap_{i=1}^{m} A_i$.

Since (X,p) is complete, then from Lemma 1, we have that (X,d_p) is complete. Thus, there exists $\nu \in X$ such that

$$\lim_{n\to\infty} d_p(x_n,\nu) = 0.$$

Moreover, it follows from Lemma 1 that

$$p(\nu,\nu) = \lim_{n\to\infty} p(x_n,\nu) = \lim_{n,m\to\infty} p(x_n,x_m). \tag{2.20}$$

On the other hand, since the sequence $\{x_n\}$ is a Cauchy sequence in the metric space (X,d_p), we also have

$$\lim_{n\to\infty} d_p(x_n,x_m) = 0.$$

Since $d_p(x,y) = 2p(x,y) - p(x,x) - p(y,y)$, we can deduce that

$$\lim_{n\to\infty} p(x_n,x_m) = 0. \tag{2.21}$$

Using (2.20) and (2.21), we have

$$p(\nu,\nu) = \lim_{n\to\infty} p(x_n,\nu) = \lim_{n\to\infty} p(x_{n_k},\nu) = 0.$$

Again, by Remark 3, (p_4), and the conditions of the mapping α, we have

$$\begin{aligned}
p(x_{n+1},f\nu) &= p(fx_n,f\nu)\\
&\le \alpha(x_n,x_n)\alpha(\nu,\nu)p(fx_n,f\nu)\\
&< \phi\left(p(x_n,\nu),p(x_n,fx_n),p(\nu,f\nu),\frac{1}{2}\bigl[p(x_n,f\nu)+p(\nu,fx_n)\bigr]\right)\\
&= \phi\left(p(x_n,\nu),p(x_n,x_{n+1}),p(\nu,f\nu),\frac{1}{2}\bigl[p(x_n,f\nu)+p(\nu,x_{n+1})\bigr]\right).
\end{aligned} \tag{2.22}$$

Letting $n\to\infty$ in (2.22), we get

$$p(\nu,f\nu) < \phi\left(0,0,p(\nu,f\nu),\frac{1}{2}p(\nu,f\nu)\right) \le p(\nu,f\nu),$$

a contradiction. So, we have $p(\nu,f\nu) = 0$, that is, $f\nu = \nu$. □

We give the following example to illustrate Theorem 2.

Example 1 Let $X = [0,1]$. We define the partial metric p on X by

$$p(x,y) = \max\{x,y\}.$$

Let $\alpha : [0,1]\times[0,1] \to \mathbb{R}^+$ be defined as

$$\alpha(x,y) = 1 + x + y,$$

let $f : X \to X$ be defined as

$$f(x) = \frac{1}{16}x^2,$$

and, let $\phi : \mathbb{R}^{+4} \to \mathbb{R}^+$ denote

$$\psi(t_1, t_2, t_3, t_4) = \frac{1}{2} \cdot \max\left\{t_1, t_2, t_3, \frac{1}{2}t_4\right\}.$$

Then f is α-admissible.

Without loss of generality, we assume that $x > y$ and verify the inequality (2.1). For all $x, y \in [0,1]$ with $x > y$, we have

$$\alpha(x,x)\alpha(y,y)p(fx,fy) \geq \frac{1}{16}x^2,$$

$$p(x,y) = x, \qquad p(x,fx) = x, \qquad p(y,fy) = y \quad \text{and}$$

$$\begin{aligned} \frac{1}{2}\big[p(x,fy) + p(y,fx)\big] &= \frac{1}{2}\big[\max\{x,y^2\} + \max\{y,x^2\}\big] \\ &\leq \frac{1}{2}\big[\max\{x,y\} + \max\{y,x\}\big] \\ &< x, \end{aligned}$$

and hence $\phi(p(x,y), p(x,fx), p(y,fy), \frac{1}{2}[p(x,fy) + p(y,fx)]) = \frac{1}{2}x$. Therefore, all the conditions of Theorem 1 are satisfied, and we obtained that 0 is a fixed point of f.

If we let

$$\alpha(x,y) = 1 \quad \text{for } x, y \in X,$$

then it is easy to get the following theorem.

Theorem 2 *Let (X,p) be a complete partial metric space and $\phi \in \Phi$. Suppose that $f : X \to X$ is a generalized Meir-Keeler-type ϕ-contraction. Then f has a fixed point in X.*

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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