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Reduction formulae for the Lauricella functions in several variables

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Abstract

The main objective of this paper is to show how one can obtain several interesting reduction formulae for Lauricella functions from a multiple hypergeometric series identity established earlier by Jaimini *et al*. The results are derived with the help of generalized Kummer's second summation formulas obtained earlier by Lavoi *et al*. Some special cases of our main result are explicitly demonstrated.

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1 Introduction and results required

In the usual notation, let $\mathbb{C}$ denote the set of *complex* numbers. For

$$\alpha_j \in \mathbb{C} \quad (j = 1, \ldots, p) \quad \text{and} \quad \beta_j \in \mathbb{C} \setminus \mathbb{Z}_0^- \quad \bigl(\mathbb{Z}_0^- := \mathbb{Z} \cup \{0\} = \{0, -1, -2, \ldots\}\bigr),$$

the *generalized hypergeometric function* ${}_pF_q$ with p numerator parameters $\alpha_1, \ldots, \alpha_p$ and q denominator parameters $\beta_1, \ldots, \beta_q$ is defined by (see, for example, [1, Chapter 4]; see also [2, pp.71-72])

$$\begin{aligned} &{}_pF_q\left[\begin{matrix}\alpha_1, \ldots, \alpha_p; \\ \beta_1, \ldots, \beta_q;\end{matrix} \quad z\right] = \sum_{n=0}^{\infty} \frac{\prod_{j=1}^{p}(\alpha_j)_n}{\prod_{j=1}^{q}(\beta_j)_n} \frac{z^n}{n!} = {}_pF_q(\alpha_1, \ldots, \alpha_p; \quad \beta_1, \ldots, \beta_q; \quad z) \\ &\bigl(p, q \in \mathbb{N}_0 := \mathbb{N} \cup \{0\} = \{0, 1, 2, \ldots\}; p \leqq q + 1; p \leqq q \text{ and } |z| < \infty; \\ &p = q + 1 \text{ and } |z| < 1; p = q + 1, |z| = 1 \text{ and } \Re(\omega) > 0\bigr), \end{aligned} \tag{1.1}$$

where

$$\omega := \sum_{j=1}^{q} \beta_j - \sum_{j=1}^{p} \alpha_j \quad \bigl(\alpha_j \in \mathbb{C}(j = 1, \ldots, p); \beta_j \in \mathbb{C} \setminus \mathbb{Z}_0^-(j = 1, \ldots, q)\bigr) \tag{1.2}$$

and $(\lambda)_n$ is the Pochhammer symbol defined (for $\lambda \in \mathbb{C}$), in terms of the familiar gamma function Γ, by

$$(\lambda)_n := \frac{\Gamma(\lambda+n)}{\Gamma(\lambda)} = \begin{cases} 1 & (n=0), \\ \lambda(\lambda+1)\cdots(\lambda+n-1) & (n \in \mathbb{N}). \end{cases} \tag{1.3}$$

The generalized Lauricella series in several variables is defined and represented in the following manner (see, for example, [3, p.37]; see also [4]):

$$\begin{aligned} F^{A:B';\dots;B^{(n)}}_{C:D';\dots;D^{(n)}} \begin{pmatrix} z_1 \\ \vdots \\ z_n \end{pmatrix} &\equiv F^{A:B';\dots;B^{(n)}}_{C:D';\dots;D^{(n)}} \left(\begin{matrix} [(a):\theta',\dots,\theta^{(n)}]: \\ [(c):\psi',\dots,\psi^{(n)}]: \end{matrix} \right. \\ &\qquad \left. \begin{matrix} [(b':\phi')]; & \dots; & [(b^{(n)}):\phi^{(n)}]; \\ [(d':\delta')]; & \dots; & [(d^{(n)}):\delta^{(n)}]; \end{matrix} \; z_1,\dots,z_n \right) \\ &:= \sum_{m_1,\dots,m_n=0}^{\infty} \Lambda(m_1,\dots,m_n) \frac{z_1^{m_1}}{m_1!} \cdots \frac{z_n^{m_n}}{m_n!}, \end{aligned} \tag{1.4}$$

where, for convenience,

$$\begin{aligned} &\Lambda(m_1,\dots,m_n) \\ &\quad := \frac{\prod_{j=1}^{A}(a_j)_{m_1\theta_j'+\cdots+m_n\theta_j^{(n)}} \prod_{j=1}^{B'}(b_j')_{m_1\phi_j'} \cdots \prod_{j=1}^{B^{(n)}}(b_j^{(n)})_{m_n\phi_j^{(n)}}}{\prod_{j=1}^{C}(c_j)_{m_1\psi_j'+\cdots+m_n\psi_j^{(n)}} \prod_{j=1}^{D'}(d_j')_{m_1\delta_j'} \cdots \prod_{j=1}^{D^{(n)}}(d_j^{(n)})_{m_n\delta_j^{(n)}}}, \end{aligned} \tag{1.5}$$

the coefficients

$$\begin{aligned} &\theta_j^{(k)} \quad (j=1,\dots,A), \qquad \phi_j^{(k)} \quad \bigl(j=1,\dots,B^{(k)}\bigr), \qquad \psi_j^{(k)} \quad (j=1,\dots,C), \\ &\delta_j^{(k)} \quad \bigl(j=1,\dots,D^{(k)}\bigr) \quad \bigl(\forall k \in \{1,\dots,n\}\bigr); \end{aligned}$$

are real and nonnegative, and (a) abbreviates the array of A parameters $a_1,\dots,a_A$; $(b^{(k)})$ abbreviates the array of $B^{(k)}$ parameters

$$b_j^{(k)} \quad \bigl(j=1,\dots,B^{(k)}; \forall k \in \{1,\dots,n\}\bigr),$$

with similar interpretations for $(c^{(k)})$, *etc.*

In the course of study of hypergeometric functions of two or more variables, Srivastava [5, 6], Buschman and Srivastava [7], Grosjean and Sharma [8] and Grosjean and Srivastava [9] established a large number of double and multiple series identities involving essentially arbitrary coefficients (see, for example, [10]). Later Jaimini *et al.* [11] presented three substantially more general multiple series identities involving similar coefficients, one of which is recalled here as in the following theorem (see [11, Theorem 3]).

Theorem 1 *Let $\Omega(m)$ represent a single-valued, bounded and real or complex function of the nonnegative integer-valued parameter m. Then we have*

$$\begin{aligned}&\sum_{m_1,\ldots,m_r=0}^{\infty} \Omega(m_1+\cdots+m_r)\frac{(\alpha)_{m_1+m_2}}{(\alpha)_{m_1}(\alpha)_{m_2}}\prod_{j=1}^{r}\left\{\frac{(\mu_j)_{m_j}}{m_j!}x^{m_j}\right\}\\&\quad=\sum_{m,n=0}^{\infty}\Omega(m+2n)(\mu_1+\cdots+\mu_r+2n)_m\frac{(\mu_1)_n(\mu_2)_n}{(\alpha)_n}\frac{x^{m+2n}}{m!n!},\end{aligned} \tag{1.6}$$

provided that each of the series involved is absolutely convergent.

From Theorem 1, with

$$\Omega(n)=\frac{\prod_{j=1}^{p}(a_j)_n}{\prod_{j=1}^{q}(b_j)_n}\quad(n\in\mathbb{N}_0),$$

we arrive at the following multiple hypergeometric identity involving the generalized Lauricella function defined by (1.4) (see [11, Equation (3.1)]):

$$\begin{aligned}&F_{q:1;1;0;\ldots;0}^{p+1:1;1;1;\ldots;1}\left(\begin{matrix}[(a_p):1,\ldots,1],[\alpha:1,1,0,\ldots,0]:\\ [(b_q):1,\ldots,1]:\end{matrix}\right.\\&\qquad\left.\begin{matrix}(\mu_1:1); & (\mu_2:1); & (\mu_3:1); & \ldots; & (\mu_r:1);\\ (\alpha:1); & (\alpha:1); & \underline{\qquad}; & \ldots; & \underline{\qquad};\end{matrix}\ x,\ldots,x\right)\\&\quad=\sum_{n=0}^{\infty}\frac{(a_1)_{2n}\cdots(a_p)_{2n}}{(b_1)_{2n}\cdots(b_q)_{2n}}\frac{(\mu_1)_n(\mu_2)_n}{(\alpha)_n}\frac{x^{2n}}{n!}\\&\qquad\cdot {}_pF_q\left[\begin{matrix}\mu_1+\cdots+\mu_r+2n, & a_1+2n,\ldots,a_p+2n;\\ & b_1+2n,\ldots,b_q+2n;\end{matrix}\ x\right].\end{aligned} \tag{1.7}$$

For $p=q=1$, (1.7) reduces at once to (see [11, Equation (3.2)])

$$\begin{aligned}&F_{1:1;1;0;\ldots;0}^{2:1;1;1;\ldots;1}\left(\begin{matrix}(a:1,\ldots,1),(\alpha:1,1,0,\ldots,0):\\ (b:1,\ldots,1):\end{matrix}\right.\\&\qquad\left.\begin{matrix}(\mu_1:1); & (\mu_2:1); & (\mu_3:1); & \ldots; & (\mu_r:1);\\ (\alpha:1); & (\alpha:1); & \underline{\qquad}; & \ldots; & \underline{\qquad};\end{matrix}\ x,\ldots,x\right)\\&\quad=\sum_{n=0}^{\infty}\frac{(a)_{2n}(\mu_1)_n(\mu_2)_n}{(b)_{2n}(\alpha)_n}\frac{x^{2n}}{n!}{}_2F_1\left[\begin{matrix}\mu_1+\cdots+\mu_r+2n, & a+2n;\\ & b+2n;\end{matrix}\ x\right].\end{aligned} \tag{1.8}$$

Finally, if we use Kummer's second summation theorem (see, for example, [12, p.11, Equation 2.4(2)]; see also [13, Equation (1.4)])

$${}_2F_1\left[\begin{matrix}\alpha,\beta; & \frac{1}{2}\\ \frac{1}{2}(\alpha+\beta+1); & 2\end{matrix}\right]=\frac{\Gamma(\frac{1}{2})\Gamma[\frac{1}{2}(\alpha+\beta+1)]}{\Gamma[\frac{1}{2}(\alpha+1)]\Gamma[\frac{1}{2}(\beta+1)]} \tag{1.9}$$

in (1.8) when

$$x = \frac{1}{2} \quad \text{and} \quad b = \frac{1}{2}(a + \mu_1 + \cdots + \mu_r + 1),$$

Jaimini *et al.* [11, Equation (3.6)] established the following interesting reduction formula for the generalized Lauricella function:

$$\begin{aligned}
&F^{2:1;1;1;\ldots;1}_{1:1;1;0;\ldots;0}\left(\begin{matrix} (a:1,\ldots,1),(\alpha:1,1,0,\ldots,0): \\ (\frac{1}{2}(a+\mu_1+\cdots+\mu_r+1):1,\ldots,1): \end{matrix}\right. \\
&\qquad \left.\begin{matrix} (\mu_1:1); & (\mu_2:1); & (\mu_3:1); & \ldots; & (\mu_r:1); \\ (\alpha:1); & (\alpha:1); & \text{———}; & \ldots; & \text{———}; \end{matrix}\ \frac{1}{2},\ldots,\frac{1}{2}\right) \\
&= \frac{\Gamma(\frac{1}{2})\Gamma[\frac{1}{2}(a+\mu_1+\cdots+\mu_r+1)]}{\Gamma[\frac{1}{2}(\mu_1+\cdots+\mu_r+1)]\Gamma[\frac{1}{2}(a+1)]} \\
&\qquad \cdot {}_3F_2\left[\begin{matrix} \frac{a}{2},\mu_1,\mu_2; \\ \alpha,\frac{1}{2}(\mu_1+\cdots+\mu_r+1); \end{matrix}\ 1\right].
\end{aligned} \tag{1.10}$$

Here, in this paper, we aim mainly at showing how one can obtain several interesting reduction formulae for Lauricella functions from a multiple hypergeometric series identity (1.8). For this, we recall the following generalization of Kummer's second summation theorem (1.9) obtained earlier by Lavoie *et al.* [14]:

$$\begin{aligned}
&{}_2F_1\left[\begin{matrix} a,b; \\ \frac{1}{2}(a+b+\ell+1); \end{matrix}\ \frac{1}{2}\right] \\
&= \frac{\Gamma(\frac{1}{2})\Gamma(\frac{a}{2}+\frac{b}{2}+\frac{\ell}{2}+\frac{1}{2})\Gamma(\frac{a}{2}-\frac{b}{2}-\frac{\ell}{2}+\frac{1}{2})}{\Gamma(\frac{a}{2}-\frac{b}{2}+\frac{|\ell|}{2}+\frac{1}{2})} \\
&\qquad \cdot \left(\frac{A_\ell}{\Gamma(\frac{a}{2}+\frac{1}{2})\Gamma(\frac{b}{2}+\frac{\ell}{2}+\frac{1}{2}-[\frac{\ell+1}{2}])} + \frac{B_\ell}{\Gamma(\frac{a}{2})\Gamma(\frac{b}{2}+\frac{\ell}{2}-[\frac{\ell}{2}])}\right) \\
&\qquad (\ell = 0,\pm1,\pm2,\pm3,\pm4,\pm5),
\end{aligned} \tag{1.11}$$

where, and in what follows, $[x]$ denotes (as usual) the greatest integer less than or equal to x. The coefficients A_ℓ and B_ℓ are tabulated below.

It is remarked in passing that Equation (1.9) was *incorrectly* attributed to Gauss by Bailey [12, p.11, Equation 2.4(2)] (see, for details, [13, p.853]).

2 Main reduction formulae

The eleven reduction formulae in the form of a single result to be established are given in the following theorem.

Theorem 2 *The following reduction formula holds true*:

$$\begin{aligned}
&F^{2:1;1;1;\ldots;1}_{1:1;1;0;\ldots;0}\left[\begin{matrix} (a:1,\ldots,1),(\alpha:1,1,0,\ldots,0): \\ (\frac{1}{2}(a+\mu_1+\cdots+\mu_r+\ell+1):1,\ldots,1)): \end{matrix}\right. \\
&\qquad \left.\begin{matrix} (\mu_1:1); & (\mu_2:1); & (\mu_3:1); & \ldots; & (\mu_r:1); \\ (\alpha:1); & (\alpha:1); & \text{———}; & \ldots; & \text{———}; \end{matrix}\ \frac{1}{2},\ldots,\frac{1}{2}\right]
\end{aligned}$$

Table 1 The coefficients A_ℓ and B_ℓ

ℓ	A_ℓ	B_ℓ
5	$-(b+a+6)^2+\frac{1}{4}(b-a+6)^2+\frac{1}{2}(b-a+6)(b+a+6)+11(b+a+6)-\frac{13}{2}(b-a+6)-20$	$(b+a+6)^2-\frac{1}{4}(b-a+6)^2+\frac{1}{2}(b-a+6)(b+a+6)-17(b+a+6)-\frac{1}{2}(b-a+6)+62$
4	$\frac{1}{2}(b+a+1)(b+a-3)-\frac{1}{4}(b-a+3)(b-a-3)$	$-2(b+a-1)$
3	$-\frac{1}{2}(3a+b-2)$	$\frac{1}{2}(3b+a-2)$
2	$\frac{1}{2}(b+a-1)$	-2
1	-1	1
0	1	0
-1	1	1
-2	$\frac{1}{2}(b+a-1)$	2
-3	$\frac{1}{2}(3a+b-2)$	$\frac{1}{2}(3b+a-2)$
-4	$\frac{1}{2}(b+a+1)(b+a-3)-\frac{1}{4}(b-a+3)(b-a-3)$	$2(b+a-1)$
-5	$(b+a-4)^2-\frac{1}{4}(b-a-4)^2-\frac{1}{2}(b+a-4)(b-a-4)+4(b+a-4)-\frac{7}{2}(b-a-4)$	$(b+a-4)^2-\frac{1}{4}(b-a-4)^2+\frac{1}{2}(b+a-4)(b-a-4)+8(b+a-4)-\frac{1}{2}(b-a-4)+12$

$$
\begin{aligned}
&= \frac{\Gamma(\frac{1}{2})\Gamma(\frac{1}{2}(a+\mu_1+\cdots+\mu_r+\ell+1))\Gamma(\frac{1}{2}(\mu_1+\cdots+\mu_r)-\frac{a}{2}-\frac{\ell}{2}+\frac{1}{2})}{\Gamma(\frac{1}{2}(\mu_1+\cdots+\mu_r)-\frac{a}{2}+\frac{|\ell|}{2}+\frac{1}{2})} \\
&\quad\cdot \sum_{n=0}^{\infty} \frac{(\frac{a}{2})_n(\frac{a}{2}+\frac{1}{2})_n(\mu_1)_n(\mu_2)_n}{(\alpha)_n(\frac{1}{2}(a+\mu_1+\cdots+\mu_r+\ell+1))_{2n}n!} \\
&\quad\cdot \Bigg[\frac{A_\ell}{\Gamma(\frac{1}{2}(\mu_1+\cdots+\mu_r+1))\Gamma(\frac{a}{2}+\frac{\ell}{2}+\frac{1}{2}-[\frac{\ell+1}{2}])(\frac{1}{2}(\mu_1+\cdots+\mu_r+1))_n} \\
&\quad\cdot \frac{1}{(\frac{a}{2}+\frac{\ell}{2}+\frac{1}{2}-[\frac{\ell+1}{2}])_n} + \frac{B_\ell}{\Gamma(\frac{1}{2}(\mu_1+\cdots+\mu_r))\Gamma(\frac{a}{2}+\frac{\ell}{2}-[\frac{\ell}{2}])} \\
&\quad\cdot \frac{1}{(\frac{1}{2}(\mu_1+\cdots+\mu_r))_n(\frac{a}{2}+\frac{\ell}{2}-[\frac{\ell}{2}])_n}\Bigg],
\end{aligned}
\tag{2.1}
$$

where $\ell = 0, \pm1, \pm2, \pm3, \pm4, \pm5$ and, here, the coefficients A_ℓ and B_ℓ can be obtained in replacing a and b in Table 1 by $\mu_1+\cdots+\mu_r+2n$ and $a+2n$, respectively.

Proof The proof is quite straightforward. In fact, if we set $x = \frac{1}{2}$ and $b = \frac{1}{2}(a+\mu_1+\cdots+\mu_r+\ell+1)$ in Equation (1.8), we have the following form:

$$
\begin{aligned}
&F^{2:1;1;1;\ldots;1}_{1:1;1;0;\ldots;0}\left(\begin{matrix} (a:1,\ldots,1),(\alpha:1,1,0,\ldots,0): \\ (\frac{1}{2}(a+\mu_1+\cdots+\mu_r+\ell+1):1,\ldots,1): \end{matrix}\right. \\
&\qquad \left.\begin{matrix} (\mu_1:1); & (\mu_2:1); & (\mu_3:1); & \ldots; & (\mu_r:1); \\ (\alpha:1); & (\alpha:1); & \underline{\qquad}; & \ldots; & \underline{\qquad}; \end{matrix}\ \frac{1}{2},\ldots,\frac{1}{2}\right) \\
&= \sum_{n=0}^{\infty} \frac{(a)_{2n}(\mu_1)_n(\mu_2)_n}{(\alpha)_n(\frac{1}{2}(a+\mu_1+\cdots+\mu_r+\ell+1))_{2n}2^{2n}n!} \\
&\quad\cdot {}_2F_1\left[\begin{matrix} \mu_1+\cdots+\mu_r+2n, a+2n; \\ \frac{1}{2}(a+\mu_1+\cdots+\mu_r+\ell+1+4n); \end{matrix}\ \frac{1}{2}\right].
\end{aligned}
\tag{2.2}
$$

Now, we observe that the ${}_2F_1$ appearing on the right-hand side of (2.2) can be evaluated with the help of generalized Kummer's second summation theorem (1.11) in replacing a and b by $\mu_1+\cdots+\mu_r+2n$ and $a+2n$, respectively. And, after a little simplification, we easily arrive at the right-hand side of our main formula (2.1). The completes the proof of Theorem 2. □

3 Special cases

It is easy to see that the special case of (2.1) when $\ell = 0$ leads to Equation (1.10) due to Jaimini *et al.* [11]. Here we consider two interesting special cases of our main formula (2.1). Setting $\ell = -1$ and $\ell = 1$ in (2.1), we find Equations (3.1) and (3.2), respectively:

$$\begin{aligned}
&F^{2:1;1;1;\ldots;1}_{1:1;1;0;\ldots;0}\left[\begin{matrix}(a:1,\ldots,1),(\alpha:1,1,0,\ldots,0):\\ (\frac{1}{2}(a+\mu_1+\cdots+\mu_r):1,\ldots,1)):\end{matrix}\right.\\
&\qquad\left.\begin{matrix}(\mu_1:1); & (\mu_2:1); & (\mu_3:1); & \ldots; & (\mu_r:1);\\ (\alpha:1); & (\alpha:1); & \underline{\qquad}; & \ldots; & \underline{\qquad};\end{matrix}\ \frac{1}{2},\ldots,\frac{1}{2}\right]\\
&\quad=\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}(a+\mu_1+\cdots+\mu_r)\right)\\
&\qquad\cdot\left(\frac{1}{\Gamma(\frac{a}{2})\Gamma(\frac{1}{2}(\mu_1+\cdots+\mu_r+1))}\,{}_3F_2\left[\begin{matrix}\frac{a}{2}+\frac{1}{2},\mu_1,\mu_2;\\ \alpha,\frac{1}{2}(\mu_1+\cdots+\mu_r+1);\end{matrix}\ 1\right]\right.\\
&\qquad\left.+\frac{1}{\Gamma(\frac{a}{2}+\frac{1}{2})\Gamma(\frac{1}{2}(\mu_1+\cdots+\mu_r))}\,{}_3F_2\left[\begin{matrix}\frac{a}{2},\mu_1,\mu_2;\\ \alpha,\frac{1}{2}(\mu_1+\cdots+\mu_r);\end{matrix}\ 1\right]\right)
\end{aligned}\tag{3.1}$$

and

$$\begin{aligned}
&F^{2:1;1;1;\ldots;1}_{1:1;1;0;\ldots;0}\left[\begin{matrix}(a:1,\ldots,1),(\alpha:1,1,0,\ldots,0):\\ (\frac{1}{2}(a+\mu_1+\cdots+\mu_r+2):1,\ldots,1)):\end{matrix}\right.\\
&\qquad\left.\begin{matrix}(\mu_1:1); & (\mu_2:1); & (\mu_3:1); & \ldots; & (\mu_r:1);\\ (\alpha:1); & (\alpha:1); & \underline{\qquad}; & \ldots; & \underline{\qquad};\end{matrix}\ \frac{1}{2},\ldots,\frac{1}{2}\right]\\
&\quad=\frac{2\Gamma(\frac{1}{2})\Gamma(\frac{1}{2}(a+\mu_1+\cdots+\mu_r+2))}{\mu_1+\cdots+\mu_r-a}\\
&\qquad\cdot\left(\frac{1}{\Gamma(\frac{a}{2}+\frac{1}{2})\Gamma(\frac{1}{2}(\mu_1+\cdots+\mu_r))}\,{}_3F_2\left[\begin{matrix}\frac{a}{2}+\frac{1}{2},\mu_1,\mu_2;\\ \alpha,\frac{1}{2}(\mu_1+\cdots+\mu_r);\end{matrix}\ 1\right]\right.\\
&\qquad\left.-\frac{1}{\Gamma(\frac{a}{2})\Gamma(\frac{1}{2}(\mu_1+\cdots+\mu_r)+1)}\,{}_3F_2\left[\begin{matrix}\frac{a}{2},\mu_1,\mu_2;\\ \alpha,\frac{1}{2}(\mu_1+\cdots+\mu_r)+1;\end{matrix}\ 1\right]\right).
\end{aligned}\tag{3.2}$$

Clearly Equations (3.1) and (3.2) are closely related to Equation (1.10). The other special cases of (2.1) can also be obtained.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors have equal contributions to each part of this paper. All authors have read and approved the final manuscript.

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