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A new variant of statistical convergence

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Abstract

In this paper we study the notion of statistical (A, λ) -summability, which is a generalization of statistical A -summability. We study here many other related concepts and its relations with statistical convergence and λ -statistical convergence and provide some interesting examples.

Keywords: density; statistical convergence; de la Vallée-Poussin; regular matrix

1 Introduction and preliminaries

The concept of statistical convergence was first introduced by Fast [1]. In 1953 the concept arose as an example of convergence in density as introduced by Buck [2]. Schoenberg [3] studied statistical convergence as a summability method and Zygmund [4] established a relation between it and strong summability. This idea has grown a little faster after the papers of Šalát [5], Fridy [6], Connor [7, 8], Kolk [9], Mursaleen [10], Mursaleen and Edely [11, 12], Mursaleen and Mohiuddine [13–17] and many others. Its various generalizations, extensions and variants have been studied by various authors so far. For example, lacunary statistical convergence [18], λ -statistical convergence [10, 19–21], A -statistical convergence [9], statistical summability $(C, 1)$ [22–24]; statistical λ -summability [25], statistical lacunary summability [26], statistical A -summability [27] *etc.* For more details, related concepts and applications, we refer to [28–41] and references therein. Here we define the notion of statistical (A, λ) -summability as a λ -statistical convergence of A -transform of x and prove some results on some related sets of sequences. The results of this paper extend several ones obtained up to now and establish several inclusion relations, implications and other properties.

Let $K \subseteq \mathbb{N}$, the set of natural numbers. Then the *natural density* of K is defined by

$$\delta(K) = \lim_n \frac{1}{n} |\{k \leq n : k \in K\}|$$

if the limit exists, where the vertical bars denote the cardinality of the enclosed set.

The idea of λ -statistical convergence was introduced in [10] as follows:

Let $\lambda = (\lambda_n)$ be a non-decreasing sequence of positive λ_n numbers tending to ∞ such that

$$\lambda_{n+1} \leq \lambda_n + 1, \quad \lambda_1 = 0.$$

The generalized de la Vallée-Poussin mean is defined by

$$t_n(x) =: \frac{1}{\lambda_n} \sum_{j \in I_n} x_j,$$

where $I_n = [n - \lambda_n + 1, n]$.

Let $K \subseteq \mathbb{N}$. Then

$$\delta_\lambda(K) = \lim_n \frac{1}{\lambda_n} |\{n - \lambda_n + 1 \leq j \leq n : j \in K\}|$$

is said to be λ -density of K .

In case $\lambda_n = n$, λ -density reduces to the natural density. Also, since $(\lambda_n/n) \leq 1$, $\delta(K) \leq \delta_\lambda(K)$ for every $K \subseteq \mathbb{N}$.

A sequence $x = (x_k)$ is said to be λ -statistically convergent to L if for every $\epsilon > 0$ the set $K_\epsilon := \{k \in \mathbb{N} : |x_k - L| \geq \epsilon\}$ has λ -density zero, i.e., $\delta_\lambda(K_\epsilon) = 0$. That is,

$$\lim_n \frac{1}{\lambda_n} |\{n - \lambda_n + 1 \leq j \leq n : |x_k - L| \geq \epsilon\}| = 0.$$

In this case we write $st_\lambda\text{-}\lim x = L$.

Let $A = (a_{nk})$ be an infinite matrix of real or complex numbers and $x = (x_k)$ be a sequence of real or complex numbers. Then we write $A_n(x) = \sum_{k=1}^\infty a_{nk}x_k$, which is called the A -transform of the sequence $x = (x_k)$ whenever the series on the right converges for each $n = 1, 2, \dots$

We assume throughout this paper that the symbols ω and c denote the spaces of all sequences (real or complex numbers) and the space of all convergent sequences, respectively. Let X and Y be two nonempty subsets of the space ω . If $x \in X$ implies $Ax = (A_n(x)) \in Y$, then we say that A defines a *matrix transformation* from X into Y , and we denote by (X, Y) the class of matrices A which transform X into Y . By $(X, Y)_{\text{reg}}$ we denote the subset of (X, Y) for which limit or sum is preserved.

A matrix $A = (a_{nk})$ is said to be *conservative* if $Ax \in c$ for $x = (x_k) \in c$, and we denote this by $A \in (c, c)$.

A matrix $A = (a_{nk})$ is said to be *regular* if it is conservative and $\lim Ax = \lim x$, and we denote this by $A \in (c, c)_{\text{reg}}$.

The following are well-known Silverman-Toeplitz [42] conditions for the regularity of A .

A matrix $A = (a_{nk})$ is regular, i.e., $A \in (c, c)_{\text{reg}}$ if and only if

- (i) $\sup_n \sum_k |a_{nk}| < \infty$;
- (ii) $\lim_{n \rightarrow \infty} a_{nk} = 0$, for each k ;
- (iii) $\lim_{n \rightarrow \infty} \sum_k a_{nk} = 1$.

Let $A = (a_{ij})$ be a non-negative regular matrix. A sequence x is said to be *statistically A -summable* to L if, for every $\epsilon > 0$, $\delta(\{i \leq n : |y_i - L| \geq \epsilon\}) = 0$, i.e.,

$$\lim_n \frac{1}{n} |\{i \leq n : |y_i - L| \geq \epsilon\}| = 0,$$

where $y_i = A_i(x)$. Thus x is statistically A -summable to L if and only if Ax is statistically convergent to L . In this case we write $L = (A)_{st}\text{-}\lim x = st\text{-}\lim Ax$.

2 Statistical (A, λ) -summability

In [43], Malafosse and Rakočević presented the following definition of statistically (A, λ) -summable.

Definition 2.1 A sequence x is said to be *statistically (A, λ) -summable* to L if for every $\epsilon > 0$, $\delta_\lambda(\{n - \lambda_n + 1 \leq i \leq n : |y_i - L| \geq \epsilon\}) = 0$, i.e.,

$$\lim_n \frac{1}{\lambda_n} |\{n - \lambda_n + 1 \leq i \leq n : |y_i - L| \geq \epsilon\}| = 0.$$

Thus x is statistically (A, λ) -summable to L if and only if Ax is λ -statistically convergent to L . In this case we write $L = (A, \lambda)_{st}\text{-lim } x = st_\lambda\text{-lim } Ax$. By $(A, \lambda)_{st}$ we denote the set of all statistically (A, λ) -summable sequences.

We define the following.

Definition 2.2 A sequence $x = (x_k)$ is said to be *strongly (A, λ_q) -convergent* ($0 < q < \infty$) to the limit L if $\lim_n \frac{1}{\lambda_n} \sum_{i \in I_n} |y_i - L|^q = 0$, and we write it as $x_k \rightarrow L[A, \lambda]_q$. In this case L is called the $[A, \lambda]_q$ -limit of x .

Remarks 2.3

- (i) If $A = I$ (the unit matrix), then the statistical (A, λ) -summability is reduced to the λ -statistical convergence.
- (ii) If $\lambda_n = n$, then the statistical (A, λ) -summability is reduced to the statistical A -summability.
- (iii) If $\lambda_n = n$ and

$$a_{ik} = \begin{cases} \frac{1}{i+1}, & 0 \leq k \leq i, \\ 0, & \text{otherwise,} \end{cases}$$

then the statistical (A, λ) -summability is reduced to the statistical $(C, 1)$ -summability due to Moricz [22].

- (iv) If $\lambda_n = n$ and

$$a_{ik} = \begin{cases} \frac{p_k}{P_i}, & 0 \leq k \leq i, \\ 0, & \text{otherwise,} \end{cases}$$

then the statistical (A, λ) -summability is reduced to the statistical (\bar{N}, p) -summability due to Moricz and Orhan [44], where $p = (p_k)$ is a sequence of nonnegative numbers such that $p_0 > 0$ and

$$P_i = \sum_{k=0}^i p_k \rightarrow \infty \quad (i \rightarrow \infty).$$

- (v) If $\lambda_n = n$ and

$$a_{ik} = \begin{cases} \frac{1}{k!i}, & 0 \leq k \leq i, \\ 0, & \text{otherwise,} \end{cases}$$

where $l_i = \sum_{k=0}^i \frac{1}{(k+1)}$, then the statistical (A, λ) -summability is reduced to the statistical $(H, 1)$ -summability due to Moricz [45].

3 Main results

In this section, we establish the relation between statistical (A, λ) -summability and A -statistical convergence.

Theorem 3.1 *If a bounded sequence is A -statistically convergent to ℓ and $\liminf_{n \rightarrow \infty} \frac{\lambda_n}{n} > 0$, then it is A summable to ℓ , statistically A -summable to ℓ , and hence statistically (A, λ) -summable to ℓ but not conversely.*

Proof Let x be bounded and A -statistically convergent to L , and $K_\epsilon = \{k \leq n : |x_k - L| \geq \epsilon\}$. Then

$$\begin{aligned} |A_n(x) - L| &= \left| \sum_{k=1}^{\infty} a_{nk}(x_k - L) + L \left(\sum_{k=1}^{\infty} a_{nk} - 1 \right) \right| \\ &\leq \sum_{k=1}^{\infty} a_{nk}|x_k - L| + |L| \left| \sum_{k=1}^{\infty} a_{nk} - 1 \right| \\ &= \sum_{k \in K_\epsilon} a_{nk}|x_k - L| + \sum_{k \notin K_\epsilon} a_{nk}|x_k - L| + |L| \left| \sum_{k=1}^{\infty} a_{nk} - 1 \right| \\ &\leq \sup_k |x_k - L| \sum_{k \in K_\epsilon} a_{nk} + \epsilon \sum_{k \notin K_\epsilon} a_{nk} + |L| \left| \sum_{k=1}^{\infty} a_{nk} - 1 \right|. \end{aligned}$$

By using the definition of A -statistical convergence and the conditions of regularity of A , we get

$$\lim |A_n(x) - L| = 0 \quad \text{since } \epsilon \text{ was arbitrary,}$$

and hence $st\text{-}\lim |A_n(x) - L| = 0$, i.e., x is statistically A -summable to L . Now, using Theorem 3.1 of [10], we get $st_\lambda\text{-}\lim |A_n(x) - L| = 0$, i.e., x is statistically (A, λ) -summable to L .

To see that the converse does not hold, we construct the following example.

Let $\lambda_n = n$ and A be a Cesàro matrix, i.e.,

$$a_{nk} = \begin{cases} \frac{1}{n+1}, & 0 \leq n \leq k, \\ 0, & \text{otherwise.} \end{cases}$$

Let

$$x_k = \begin{cases} 1, & \text{if } k \text{ is odd,} \\ 0, & \text{if } k \text{ is even.} \end{cases}$$

Then x is A -summable to $1/2$ (and hence statistically (A, λ) -summable to $1/2$) but not A -statistically convergent.

This completes the proof of the theorem. □

Theorem 3.2 *If $\limsup_n(n - \lambda_n) < \infty$ and x is statistically (A, λ) -summable to L , then x is statistically A -summable to L .*

Proof Let $\limsup_n(n - \lambda_n) < \infty$. Then there exists $M > 0$ such that $n - \lambda_n \leq M$ for all n . Since $\frac{1}{n} \leq \frac{1}{\lambda_n}$ and

$$\{1 \leq i \leq n : |y_i - L| \geq \varepsilon\} \subseteq \{i \in I_n : |y_i - L| \geq \varepsilon\} \cup \{1 \leq i \leq n - \lambda_n : |y_i - L| \geq \varepsilon\},$$

we have

$$\begin{aligned} & \frac{1}{n} |\{1 \leq i \leq n : |y_i - L| \geq \varepsilon\}| \\ & \leq \frac{1}{\lambda_n} |\{1 \leq i \leq n : |y_i - L| \geq \varepsilon\}| \\ & \leq \frac{1}{\lambda_n} |\{i \in I_n : |y_i - L| \geq \varepsilon\}| + \frac{1}{\lambda_n} |\{i \leq n - \lambda_n : |y_i - L| \geq \varepsilon\}| \\ & \leq \frac{1}{\lambda_n} |\{i \in I_n : |y_i - L| \geq \varepsilon\}| + \frac{M}{\lambda_n}. \end{aligned}$$

Now, taking the limit as $n \rightarrow \infty$, we get the desired result. □

Theorem 3.3 *Statistical (A, λ) -summability implies statistical A -summability if and only if*

$$\liminf_{n \rightarrow \infty} \frac{\lambda_n}{n} > 0. \tag{3.1}$$

Proof For $\varepsilon > 0$, we have

$$\{i \in I_n : |y_i - L| \geq \varepsilon\} \subset \{i \leq n : |y_i - L| \geq \varepsilon\}.$$

Therefore

$$\begin{aligned} \frac{1}{n} |\{i \leq n : |y_i - L| \geq \varepsilon\}| & \geq \frac{1}{n} |\{i \in I_n : |y_i - L| \geq \varepsilon\}| \\ & \geq \frac{\lambda_n}{n} \cdot \frac{1}{\lambda_n} |\{i \in I_n : |y_i - L| \geq \varepsilon\}|. \end{aligned}$$

Taking the limit as $n \rightarrow \infty$ and using (3.1), we get that statistical (A, λ) -summability implies statistical A -summability.

Conversely, suppose that

$$\liminf_{n \rightarrow \infty} \frac{\lambda_n}{n} = 0.$$

Choose a subsequence $(n(j))_{j \geq 1}$ such that $\frac{\lambda_{n(j)}}{n(j)} < \frac{1}{j}$. Define a sequence $x = (x_k)_{k \geq 1}$ such that

$$y_i = \begin{cases} 1, & \text{for } i \in I_{n(j)}, j = 1, 2, 3, \dots, \\ 0, & \text{otherwise.} \end{cases}$$

Then, as in Theorem 3.1 of [10], we get that $y = (y_i)$ is not λ -statistically convergent, i.e., x is not statistically (A, λ) -summable. Hence (3.1) is necessary.

This completes the proof of the theorem. □

Theorem 3.4 (a) *If $0 < q < \infty$ and a sequence $x = (x_k)$ is strongly (A, λ_q) -convergent to the limit L , then x is statistically (A, λ) -convergent to L .*

(b) *If $x = (x_k)$ is bounded and statistically (A, λ) -convergent to L , then $x_k \rightarrow L[A, \lambda]_q$.*

Proof (a) It follows easily from the following:

$$\frac{1}{\lambda_n} \sum_{i \in I_n} |y_i - L|^q \geq \frac{\varepsilon^q}{\lambda_n} \left| \{i \in I_n : |y_i - L| \geq \varepsilon\} \right|.$$

The following example shows that the inclusion is proper. Let $x = (x_n)_{n \geq 1}$ be such that its A -transform is given by

$$y_i = \begin{cases} i, & \text{for } n - [\sqrt{\lambda_n}] + 1 \leq i \leq n, \\ 0, & \text{otherwise.} \end{cases}$$

Then $Ax \notin \ell_\infty$ and for $0 < \varepsilon \leq 1$,

$$\frac{1}{\lambda_n} \left| \{i \in I_n : |y_i - 0| \geq \varepsilon\} \right| = \frac{[\sqrt{\lambda_n}]}{\lambda_n} \rightarrow 0 \quad (n \rightarrow \infty),$$

i.e., x is statistically (A, λ) -convergent to 0. But

$$\frac{1}{\lambda_n} \sum_{i \in I_n} |y_i - 0|^q \not\rightarrow 0,$$

i.e., x is not strongly (A, λ_q) -convergent to the limit 0.

(b) Suppose $x = (x_k)$ is bounded and statistically (A, λ) -convergent to L . Then $|x_k - L| \leq M$ for all k , where $M > 0$. For $\varepsilon > 0$, we have

$$\begin{aligned} \frac{1}{\lambda_n} \sum_{k \in I_n} |y_i - L|^q &= \frac{1}{\lambda_n} \sum_{\substack{i \in I_n \\ |y_i - L|^q \geq \varepsilon}} |y_i - L|^q + \frac{1}{\lambda_n} \sum_{\substack{i \in I_n \\ |y_i - L|^q < \varepsilon}} |y_i - L|^q \\ &\leq \frac{M}{\lambda_n} \left| \{i \in I_n : |y_i - L| \geq \varepsilon\} \right| + \varepsilon^q. \end{aligned}$$

Hence $x_k \rightarrow L[A, \lambda]_q$ if x is statistically (A, λ) -convergent to L .

This completes the proof of the theorem. □

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

The authors contributed equally and significantly in writing this paper. All the authors read and approved the final manuscript.

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