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A generalization on $\mathcal{I}$-asymptotically lacunary statistical equivalent sequences

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Abstract

In this study we extend the concept of $\mathcal{I}$-asymptotically lacunary statistical equivalent sequences by using the sequence $p = (p_k)$ which is the sequence of positive real numbers where θ is a lacunary sequence and $\mathcal{I}$ is an ideal of the subset of positive integers.

MSC: 40G15; 40A35

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1 Introduction

The concept of $\mathcal{I}$-convergence was introduced by Kostyrko *et al.* in a metric space [1]. Later it was further studied by Dems [2], Das and Savaş [3], Savaş [4–7] and many others. $\mathcal{I}$-convergence is a generalization form of statistical convergence, which was introduced by Fast (see [8]) and that is based on the notion of an ideal of the subset of positive integers $\mathbb{N}$.

Definition 1.1 A family $\mathcal{I} \subset 2^{\mathbb{N}}$ is said to be an ideal of $\mathbb{N}$ if the following conditions hold:

(a) $A, B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$,

(b) $A \in \mathcal{I}$, $B \subset A$ implies $B \in \mathcal{I}$.

An ideal is called non-trivial if $\mathbb{N} \notin \mathcal{I}$, and a non-trivial ideal is called admissible if $\{n\} \in \mathcal{I}$ for each $n \in \mathbb{N}$.

Definition 1.2 A family of sets $F \subset 2^{\mathbb{N}}$ is a filter in $\mathbb{N}$ if and only if:

(i) $\emptyset \notin F$.

(ii) For each $A, B \in F$, we have $A \cap B \in F$.

(iii) For each $A \in F$ and each $B \supseteq A$, we have $B \in F$.

Proposition 1.1 *$\mathcal{I}$ is a non-trivial ideal in $\mathbb{N}$ if and only if*

$$F = F(\mathcal{I}) = \{M = \mathbb{N} \backslash A : A \in \mathcal{I}\}$$

is a filter in $\mathbb{N}$ (see [1]*).*

Definition 1.3 A real sequence $x = (x_k)$ is said to be $\mathcal{I}$-convergent to $L \in \mathbb{R}$ if and only if for each $\varepsilon > 0$ the set

$$A_\varepsilon = \{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}$$

belongs to $\mathcal{I}$. The number L is called the $\mathcal{I}$-limit of the sequence x (see [1]).

Remark 1 If we take $\mathcal{I} = \mathcal{I}_f = \{A \subseteq \mathbf{N} : A$ is a finite subset$\}$. Then $\mathcal{I}_f$ is a non-trivial admissible ideal of $\mathbf{N}$ and the corresponding convergence coincides with the usual convergence.

A lacunary sequence is an increasing integer sequence $\theta = (k_r)$ such that $k_0 = 0$ and $h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty$. The intervals determined by θ are denoted $I_r = (k_{r-1}, k_r]$ and the ratio $\frac{k_r}{k_{r-1}}$ is denoted by q_r.

In 1993, Marouf presented definitions for asymptotically equivalent sequences and asymptotic regular matrices.

Definition 1.4 [9] Two nonnegative sequences $x = (x_k)$ and $y = (y_k)$ are said to be asymptotically equivalent if

$$\lim_k \frac{x_k}{y_k} = 1$$

and it is denoted by $x \sim y$.

Definition 1.5 (Fridy [10]) The sequence $x = (x_k)$ has statistic limit L, denoted by $st\text{-}\lim x = L$, provided that for every $\epsilon > 0$,

$$\lim_n \frac{1}{n}\{\text{the number of } k \leq n : |x_k - L| \geq \epsilon\} = 0.$$

In 2003, Patterson defined asymptotically statistical equivalent sequences by using the definition of statistical convergence as follows.

Definition 1.6 (Patterson [11]) Two nonnegative sequences $x = (x_k)$ and $y = (y_k)$ are said to be asymptotically statistical equivalent of multiple L provided that for every $\epsilon > 0$,

$$\lim_n \frac{1}{n}\left\{\text{the number of } k < n : \left|\frac{x_k}{y_k} - L\right| \geq \epsilon\right\} = 0$$

(denoted by $x \overset{S_L}{\sim} y$), and simply asymptotically statistical equivalent if $L = 1$.

In 2006, Patterson and Savaş presented definitions for asymptotically lacunary statistical equivalent sequences (see [12]).

Definition 1.7 Let $\theta = (k_r)$ be a lacunary sequence, two nonnegative sequences $[x]$ and $[y]$ are said to be asymptotically lacunary statistical equivalent of multiple L provided that for every $\varepsilon > 0$

$$\lim_r \frac{1}{h_r}\left|\left\{k \in I_r : \left|\frac{x_k}{y_k} - L\right| \geq \varepsilon\right\}\right| = 0,$$

where the vertical bars indicate the number elements in the enclosed set.

Definition 1.8 Let $\theta = (k_r)$ be a lacunary sequence, two number sequences $x = (x_k)$ and $y = (y_k)$ are said to be strong asymptotically lacunary equivalent of multiple L provided that

$$\lim_r \frac{1}{h_r} \sum_{k \in I_r} \left| \frac{x_k}{y_k} - L \right| = 0.$$

In 2008, Savaş and Patterson gave an extension on asymptotically lacunary statistical equivalent sequences, and they investigated some relations between strongly asymptotically lacunary equivalent sequences and strongly Cesáro asymptotically equivalent sequences. More applications of the asymptotically statistical equivalent sequences can be seen in [13–16].

Definition 1.9 [17] Let $\theta = (k_r)$ be a lacunary sequence and let $p = (p_k)$ be a sequence of positive real numbers. Two number sequences $x = (x_k)$ and $y = (y_k)$ are said to be strongly asymptotically lacunary equivalent of multiple L provided that

$$\lim_r \frac{1}{h_r} \sum_{k \in I_r} \left| \frac{x_k}{y_k} - L \right|^{p_k} = 0$$

(denoted by $x \overset{N_\theta^{L(p)}}{\sim} y$) and simply strongly asymptotically lacunary equivalent if $L = 1$.

Definition 1.10 Let $p = (p_k)$ be a sequence of positive real numbers. Two number sequences $x = (x_k)$ and $y = (y_k)$ are said to be strongly Cesáro asymptotically equivalent to L provided that

$$\lim_n \frac{1}{n} \sum_{k=1}^{n} \left| \frac{x_k}{y_k} - L \right|^{p_k} = 0$$

(denoted by $x \overset{\sigma(p)}{\sim} y$) and simply strongly Cesáro asymptotically equivalent if $L = 1$.

The following definitions are given in [3].

Definition 1.11 A sequence $x = (x_k)$ is said to be $\mathcal{I}$-statistically convergent to L or $S(\mathcal{I})$-convergent to L if, for any $\varepsilon > 0$ and $\delta > 0$,

$$\left\{ n \in \mathbb{N} : \frac{1}{n} \left| \left\{ k \leq n : |x_k - L| \geq \varepsilon \right\} \right| \geq \delta \right\} \in \mathcal{I}.$$

In this case, we write $x_k \to L(S(\mathcal{I}))$. The class of all $\mathcal{I}$-statistically convergent sequences will be denoted by $S(\mathcal{I})$.

Definition 1.12 Let θ be a lacunary sequence. A sequence $x = (x_k)$ is said to be $\mathcal{I}$-lacunary statistically convergent to L or $S_\theta(\mathcal{I})$-convergent to L if, for any $\varepsilon > 0$ and $\delta > 0$,

$$\left\{ r \in \mathbb{N} : \frac{1}{h_r} \left| \left\{ k \in I_r : |x_k - L| \geq \varepsilon \right\} \right| \geq \delta \right\} \in \mathcal{I}.$$

In this case, we write $x_k \to L(S_\theta(\mathcal{I}))$. The class of all $\mathcal{I}$-lacunary statistically convergent sequences will be denoted by $S_\theta(\mathcal{I})$.

Definition 1.13 Let θ be a lacunary sequence. A sequence $x = (x_k)$ is said to be strong $\mathcal{I}$-lacunary convergent to L or $N_\theta(\mathcal{I})$-convergent to L if, for any $\varepsilon > 0$,

$$\left\{r \in N : \frac{1}{h_r}\sum_{k\in I_r}|x_k - L| \geq \varepsilon\right\} \in \mathcal{I}.$$

In this case, we write $x_k \to L(N_\theta(\mathcal{I}))$. The class of all strong $\mathcal{I}$-lacunary statistically convergent sequences will be denoted by $N_\theta(\mathcal{I})$.

Throughout $\mathcal{I}$ will stand for a proper admissible ideal of $\mathbb{N}$, and by sequence we always mean sequences of real numbers.

Recently, Savaş defined $\mathcal{I}$-asymptotically lacunary statistical equivalent sequences by using the definitions $\mathcal{I}$-convergence and asymptotically lacunary statistical equivalent sequences together.

Definition 1.14 [18] Let $\theta = (k_r)$ be a lacunary sequence; the two nonnegative sequences $x = (x_k)$ and $y = (y_k)$ are said to be $\mathcal{I}$-asymptotically lacunary statistical equivalent of multiple L provided that for every $\varepsilon > 0$ and $\delta > 0$,

$$\left\{r \in \mathbb{N} : \frac{1}{h_r}\left|\left\{k \in I_r : \left|\frac{x_k}{y_k} - L\right| \geq \varepsilon\right\}\right| \geq \delta\right\} \in \mathcal{I}.$$

In this case we write $x \overset{S_\theta^L(\mathcal{I})}{\sim} y$.

2 Main results

In this section we shall give some new definitions and also examine some inclusion relations.

Definition 2.1 Let $\theta = (k_r)$ be a lacunary sequence and let $p = (p_k)$ be a sequence of positive real numbers. Two number sequences $x = (x_k)$ and $y = (y_k)$ are said to be strongly $\mathcal{I}$-asymptotically lacunary equivalent of multiple L for the sequence p provided that

$$\left\{r \in \mathbb{N} : \frac{1}{h_r}\sum_{k\in I_r}\left|\frac{x_k}{y_k} - L\right|^{p_k} \geq \varepsilon\right\} \in \mathcal{I}.$$

In this situation we write $x \overset{N_\theta^{L(p)}(\mathcal{I})}{\sim} y$.

If we take $p_k = p$ for all $k \in \mathbb{N}$, we write $x \overset{N_\theta^{Lp}(\mathcal{I})}{\sim} y$ instead of $x \overset{N_\theta^{L(p)}(\mathcal{I})}{\sim} y$.

Definition 2.2 Let $\theta = (k_r)$ be a lacunary sequence and let $p = (p_k)$ be a sequence of positive real numbers. Two number sequences $x = (x_k)$ and $y = (y_k)$ are said to be strongly Cesáro $\mathcal{I}$-asymptotically equivalent of multiple L provided that

$$\left\{n \in \mathbb{N} : \frac{1}{n}\sum_{k=1}^{n}\left|\frac{x_k}{y_k} - L\right|^{p_k} \geq \varepsilon\right\} \in \mathcal{I}$$

(denoted by $x \overset{\sigma^{(p)}(\mathcal{I})}{\sim} y$) and simply strongly Cesáro $\mathcal{I}$-asymptotically equivalent if $L = 1$.

Theorem 2.1 *Let* $\theta = (k_r)$ *be a lacunary sequence. Then*:

(a) *If* $x \overset{N_\theta^{Lp}(\mathcal{I})}{\sim} y$, *then* $x \overset{S_\theta^L(\mathcal{I})}{\sim} y$;

(b) *If* $x, y \in l_\infty$ *and* $x \overset{S_\theta^L(\mathcal{I})}{\sim} y$, *then* $x \overset{N_\theta^{Lp}(\mathcal{I})}{\sim} y$;

(c) $(x \overset{S_\theta^L(\mathcal{I})}{\sim} y) \cap l_\infty = (x \overset{N_\theta^{Lp}(\mathcal{I})}{\sim} y) \cap l_\infty$.

Proof (a) Let $x \overset{N_\theta^{Lp}(\mathcal{I})}{\sim} y$ and $\varepsilon > 0$ be given. Then

$$\begin{aligned}\sum_{k\in I_r}\left|\frac{x_k}{y_k}-L\right|^p &\geq \sum_{k\in I_r \,\&\, |\frac{x_k}{y_k}-L|\geq\varepsilon}\left|\frac{x_k}{y_k}-L\right|^p \\ &\geq \varepsilon^p\left|\left\{k\in I_r : \left|\frac{x_k}{y_k}-L\right|\geq\varepsilon\right\}\right|\end{aligned}$$

and so

$$\frac{1}{\varepsilon^p h_r}\sum_{k\in I_r}\left|\frac{x_k}{y_k}-L\right|^p \geq \frac{1}{h_r}\left|\left\{k\in I_r : \left|\frac{x_k}{y_k}-L\right|\geq\varepsilon\right\}\right|.$$

Then, for any $\delta > 0$,

$$\begin{aligned}&\left\{r\in\mathbb{N} : \frac{1}{h_r}\left|\left\{k\in I_r : \left|\frac{x_k}{y_k}-L\right|\geq\varepsilon\right\}\right|\geq\delta\right\} \\ &\quad\subseteq\left\{r\in\mathbb{N} : \frac{1}{h_r}\sum_{k\in I_r}\left|\frac{x_k}{y_k}-L\right|^p\geq\varepsilon^p\delta\right\}\in\mathcal{I}.\end{aligned}$$

Therefore $x \overset{S_\theta^L(\mathcal{I})}{\sim} y$.

(b) Let x and y be bounded sequences and $x \overset{S_\theta^L(\mathcal{I})}{\sim} y$. Then there is an M such that $|\frac{x_k}{y_k} - L| \leq M$ for all k. For each $\varepsilon > 0$,

$$\begin{aligned}\frac{1}{h_r}\sum_{k\in I_r}\left|\frac{x_k}{y_k}-L\right|^p &= \frac{1}{h_r}\sum_{k\in I_r \,\&\, |\frac{x_k}{y_k}-L|\geq\varepsilon}\left|\frac{x_k}{y_k}-L\right|^p + \frac{1}{h_r}\sum_{k\in I_r \,\&\, |\frac{x_k}{y_k}-L|<\varepsilon}\left|\frac{x_k}{y_k}-L\right|^p \\ &\leq \frac{1}{h_r}M^p\left|\left\{k\in I_r : \left|\frac{x_k}{y_k}-L\right|\geq\varepsilon\right\}\right| + \frac{1}{h_r}\varepsilon^p\left|\left\{k\in I_r : \left|\frac{x_k}{y_k}-L\right|<\varepsilon\right\}\right| \\ &\leq \frac{M^p}{h_r}\left|\left\{k\in I_r : \left|\frac{x_k}{y_k}-L\right|\geq\varepsilon\right\}\right| + \varepsilon^p.\end{aligned}$$

Then, for any $\delta > 0$,

$$\begin{aligned}&\left\{r\in\mathbb{N} : \frac{1}{h_r}\sum_{k\in I_r}\left|\frac{x_k}{y_k}-L\right|^p\geq\varepsilon\right\} \\ &\quad\subseteq\left\{r\in\mathbb{N} : \frac{1}{h_r}\left|\left\{k\in I_r : \left|\frac{x_k}{y_k}-L\right|\geq\varepsilon\right\}\right|\geq\frac{\varepsilon^p}{M^p}\right\}\in\mathcal{I}.\end{aligned}$$

Therefore $x \overset{N_\theta^{Lp}(\mathcal{I})}{\sim} y$.

(c) This follows from (a) and (b). □

Theorem 2.2 *Let* $\theta = (k_r)$ *be a lacunary sequence*, $\inf_k p_k = h$ *and* $\sup_k p = H$. *Then*

$$x \overset{N_\theta^{L(p)}(\mathcal{I})}{\sim} y \quad \textit{implies} \quad x \overset{S_\theta^L(\mathcal{I})}{\sim} y.$$

Proof Assume that $x \overset{N_\theta^{L(p)}(\mathcal{I})}{\sim} y$ and $\varepsilon > 0$. Then

$$\begin{aligned}
\frac{1}{h_r}\sum_{k\in I_r}\left|\frac{x_k}{y_k}-L\right|^{p_k} &= \frac{1}{h_r}\sum_{k\in I_r \,\&\, |\frac{x_k}{y_k}-L|\geq\varepsilon}\left|\frac{x_k}{y_k}-L\right|^{p_k} + \frac{1}{h_r}\sum_{k\in I_r \,\&\, |\frac{x_k}{y_k}-L|<\varepsilon}\left|\frac{x_k}{y_k}-L\right|^{p_k} \\
&\geq \frac{1}{h_r}\sum_{k\in I_r \,\&\, |\frac{x_k}{y_k}-L|\geq\varepsilon}\left|\frac{x_k}{y_k}-L\right|^{p_k} \\
&\geq \frac{1}{h_r}\sum_{k\in I_r \,\&\, |\frac{x_k}{y_k}-L|\geq\varepsilon}(\varepsilon)^{p_k} \\
&\geq \frac{1}{h_r}\sum_{k\in I_r \,\&\, |\frac{x_k}{y_k}-L|\geq\varepsilon}\min\bigl\{(\varepsilon)^h,(\varepsilon)^H\bigr\} \\
&\geq \frac{1}{h_r}\min\bigl\{(\varepsilon)^h,(\varepsilon)^H\bigr\}\left|\left\{k\in I_r : \left|\frac{x_k}{y_k}-L\right|\geq\varepsilon\right\}\right|
\end{aligned}$$

and

$$\begin{aligned}
&\left\{r\in\mathbb{N} : \frac{1}{h_r}\left|\left\{k\in I_r : \left|\frac{x_k}{y_k}-L\right|\geq\varepsilon\right\}\right|\geq\delta\right\} \\
&\quad\subseteq\left\{r\in\mathbb{N} : \frac{1}{h_r}\sum_{k\in I_r}\left|\frac{x_k}{y_k}-L\right|^{p_k}\geq\delta\min\bigl\{(\varepsilon)^h,(\varepsilon)^H\bigr\}\right\}\in\mathcal{I}.
\end{aligned}$$

Thus we have $x \overset{S_\theta^L(\mathcal{I})}{\sim} y$. □

Theorem 2.3 *Let* x *and* y *be bounded sequences*, $\inf_k p_k = h$ *and* $\sup_k p = H$. *Then*

$$x \overset{S_\theta^L(\mathcal{I})}{\sim} y \quad \textit{implies} \quad x \overset{N_\theta^{L(p)}(\mathcal{I})}{\sim} y.$$

Proof Suppose that x and y are bounded and $\varepsilon > 0$. Then there is an integer K such that $|\frac{x_k}{y_k}-L|\leq K$ for all k,

$$\begin{aligned}
\frac{1}{h_r}\sum_{k\in I_r}\left|\frac{x_k}{y_k}-L\right|^{p_k} &= \frac{1}{h_r}\sum_{k\in I_r \,\&\, |\frac{x_k}{y_k}-L|\geq\varepsilon}\left|\frac{x_k}{y_k}-L\right|^{p_k} + \frac{1}{h_r}\sum_{k\in I_r \,\&\, |\frac{x_k}{y_k}-L|<\varepsilon}\left|\frac{x_k}{y_k}-L\right|^{p_k} \\
&\leq \frac{1}{h_r}\left|\left\{k\in I_r : \left|\frac{x_k}{y_k}-L\right|\geq\frac{\varepsilon}{2}\right\}\right|\max\bigl\{K^h,K^H\bigr\} \\
&\quad + \frac{1}{h_r}\left|\left\{k\in I_r : \left|\frac{x_k}{y_k}-L\right|<\frac{\varepsilon}{2}\right\}\right|\frac{\max(\varepsilon)^{p_k}}{2} \\
&\leq \max\bigl\{K^h,K^H\bigr\}\frac{1}{h_r}\left|\left\{k\in I_r : \left|\frac{x_k}{y_k}-L\right|\geq\frac{\varepsilon}{2}\right\}\right| + \frac{\max\{\varepsilon^h,\varepsilon^H\}}{2}
\end{aligned}$$

and

$$\left\{r \in \mathbb{N} : \frac{1}{h_r} \sum_{k \in I_r} \left| \frac{x_k}{y_k} - L \right|^{p_k} \geq \varepsilon \right\}$$
$$\subseteq \left\{ r \in \mathbb{N} : \frac{1}{h_r} \left| \left\{ k \in I_r : \left| \frac{x_k}{y_k} - L \right| \geq \frac{\varepsilon}{2} \right\} \right| \geq \frac{2\varepsilon - \max\{\varepsilon^h, \varepsilon^H\}}{2\max\{K^h, K^H\}} \right\} \in \mathcal{I}.$$

Thus we have $x \overset{N_\theta^{L(p)}(\mathcal{I})}{\sim} y$. □

Theorem 2.4 *Let* $\theta = (k_r)$ *be a lacunary sequence with* $\liminf_r q_r > 1$, *then*

$$x \overset{\sigma^{(p)}(\mathcal{I})}{\sim} y \quad \textit{implies} \quad x \overset{N_\theta^{L(p)}(\mathcal{I})}{\sim} y.$$

Proof If $\liminf_r q_r > 1$, then there exists $\delta > 0$ such that $q_r \geq 1 + \delta$ for all $r \geq 1$. Since $h_r = k_r - k_{r-1}$, we have $\frac{k_r}{h_r} \leq \frac{1+\delta}{\delta}$ and $\frac{k_{r-1}}{h_r} \leq \frac{1}{\delta}$. Let $\varepsilon > 0$ and define the set

$$S = \left\{ k_r \in \mathbb{N} : \frac{1}{k_r} \sum_{k=1}^{k_r} \left| \frac{x_k}{y_k} - L \right|^{p_k} < \varepsilon \right\}.$$

We can easily say that $S \in F(\mathcal{I})$, which is the filter of the ideal $\mathcal{I}$,

$$\begin{aligned} \frac{1}{h_r} \sum_{k \in I_r} \left| \frac{x_k}{y_k} - L \right|^{p_k} &= \frac{1}{h_r} \sum_{k=1}^{k_r} \left| \frac{x_k}{y_k} - L \right|^{p_k} - \frac{1}{h_r} \sum_{k=1}^{k_{r-1}} \left| \frac{x_k}{y_k} - L \right|^{p_k} \\ &= \frac{k_r}{h_r} \frac{1}{k_r} \sum_{k=1}^{k_r} \left| \frac{x_k}{y_k} - L \right|^{p_k} - \frac{k_{r-1}}{h_r} \frac{1}{k_{r-1}} \sum_{k=1}^{k_{r-1}} \left| \frac{x_k}{y_k} - L \right|^{p_k} \\ &\leq \left(\frac{1+\delta}{\delta} \right) \varepsilon - \frac{1}{\delta} \varepsilon' \end{aligned}$$

for each $k_r \in S$. Choose $\eta = (\frac{1+\delta}{\delta})\varepsilon - \frac{1}{\delta}\varepsilon'$. Therefore,

$$\left\{ r \in \mathbb{N} : \frac{1}{h_r} \sum_{k \in I_r} \left| \frac{x_k}{y_k} - L \right|^{p_k} < \eta \right\} \in F(\mathcal{I})$$

and it completes the proof. □

For the next result, we assume that the lacunary sequence θ satisfies the condition that for any set $C \in F(\mathcal{I})$, $\bigcup\{n : k_{r-1} < n < k_r, r \in C\} \in F(\mathcal{I})$.

Theorem 2.5 *Let* $\theta = (k_r)$ *be a lacunary sequence with* $\limsup q_r < \infty$, *then*

$$x \overset{N_\theta^{L(p)}(\mathcal{I})}{\sim} y \quad \textit{implies} \quad x \overset{\sigma^{(p)}(\mathcal{I})}{\sim} y.$$

Proof If $\limsup q_r < \infty$, then there exists $B > 0$ such that $q_r < B$ for all $r \geq 1$. Let $x \overset{N_\theta^{L(p)}(\mathcal{I})}{\sim} y$ and define the sets T and R such that

$$T = \left\{ r \in \mathbb{N} : \frac{1}{h_r} \sum_{k \in I_r} \left| \frac{x_k}{y_k} - L \right|^{p_k} < \varepsilon_1 \right\}$$

and

$$R = \left\{ n \in \mathbb{N} : \frac{1}{n} \sum_{k=1}^{n} \left| \frac{x_k}{y_k} - L \right|^{p_k} < \varepsilon_2 \right\}.$$

Let

$$A_j = \frac{1}{h_j} \sum_{k \in I_j} \left| \frac{x_k}{y_k} - L \right|^{p_k} < \varepsilon_1$$

for all $j \in T$. It is obvious that $T \in F(\mathcal{I})$. Choose n is any integer with $k_{r-1} < n < k_r$, where $r \in T$,

$$\begin{aligned}
\frac{1}{n} \sum_{k=1}^{n} \left| \frac{x_k}{y_k} - L \right|^{p_k} &\leq \frac{1}{k_{r-1}} \sum_{k=1}^{k_r} \left| \frac{x_k}{y_k} - L \right|^{p_k} \\
&= \frac{1}{k_{r-1}} \left(\sum_{k \in I_1} \left| \frac{x_k}{y_k} - L \right|^{p_k} + \sum_{k \in I_2} \left| \frac{x_k}{y_k} - L \right|^{p_k} + \cdots + \sum_{k \in I_r} \left| \frac{x_k}{y_k} - L \right|^{p_k} \right) \\
&= \frac{k_1}{k_{r-1}} \left(\frac{1}{h_1} \sum_{k \in I_1} \left| \frac{x_k}{y_k} - L \right|^{p_k} \right) + \frac{k_2 - k_1}{k_{r-1}} \left(\frac{1}{h_2} \sum_{k \in I_2} \left| \frac{x_k}{y_k} - L \right|^{p_k} \right) + \cdots \\
&\quad + \frac{k_r - k_{r-1}}{k_{r-1}} \left(\frac{1}{h_r} \sum_{k \in I_r} \left| \frac{x_k}{y_k} - L \right|^{p_k} \right) \\
&= \frac{k_1}{k_{r-1}} A_1 + \frac{k_2 - k_1}{k_{r-1}} A_2 + \cdots + \frac{k_r - k_{r-1}}{k_{r-1}} A_r \\
&\leq \left(\sup_{j \in T} A_j \right) \frac{k_r}{k_{r-1}} \\
&< \varepsilon_1 B.
\end{aligned}$$

Choose $\varepsilon_2 = \frac{\varepsilon_1}{B}$ and in view of the fact that $\bigcup \{ n : k_{r-1} < n < k_r, r \in T \} \subset R$, where $T \in F(\mathcal{I})$, it follows from our assumption on θ that the set R also belongs to $F(\mathcal{I})$ and this completes the proof of the theorem. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
Both authors completed the paper together. Both authors read and approved the final manuscript.

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