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Approximation of linear mappings in Banach modules over C^*-algebras

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Abstract

Let X, Y be Banach modules over a C^*-algebra and let $r_1, \ldots, r_n \in \mathbb{R}$ be given. Using fixed-point methods, we prove the stability of the following functional equation in Banach modules over a unital C^*-algebra:

$$\sum_{j=1}^{n} f\left(\frac{1}{2} \sum_{1\le i\le n, i\ne j} r_i x_i - \frac{1}{2} r_j x_j\right) + \sum_{i=1}^{n} r_i f(x_i) = nf\left(\frac{1}{2}\sum_{i=1}^{n} r_i x_i\right).$$

As an application, we investigate homomorphisms in unital C^*-algebras.

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1 Introduction and preliminaries

We say a functional equation (ζ) is stable if any function g satisfying the equation (ζ) approximately is near to the true solution of (ζ). We say that a functional equation is superstable if every approximate solution is an exact solution of it (see [1]). The stability problem of functional equations was originated from a question of Ulam [2] concerning the stability of group homomorphisms. Hyers [3] gave a first affirmative partial answer to the question of Ulam in Banach spaces. Hyers' theorem was generalized by Aoki [4] for additive mappings and by T.M. Rassias [5] for linear mappings by considering an unbounded Cauchy difference. A generalization of the T.M. Rassias theorem was obtained by Găvruta [6] by replacing the unbounded Cauchy difference by a general control function in the spirit of T.M. Rassias' approach.

The functional equation

$$f(x+y) + f(x-y) = 2f(x) + 2f(y)$$

is called a *quadratic functional equation*. In particular, every solution of the quadratic functional equation is said to be a *quadratic mapping*. A Hyers-Ulam stability problem for the quadratic functional equation was proved by Skof [7] for mappings $f : X \to Y$, where X is a normed space and Y is a Banach space. Cholewa [8] noticed that the theorem of Skof is still true if the relevant domain X is replaced by an Abelian group. Czerwik [9] proved the Hyers-Ulam stability of the quadratic functional equation. J.M. Rassias [10, 11] introduced and investigated the stability problem of Ulam for the Euler-Lagrange quadratic functional equation

$$f(a_1x_1 + a_2x_2) + f(a_2x_1 - a_1x_2) = \left(a_1^2 + a_2^2\right)\left[f(x_1) + f(x_2)\right]. \tag{1.1}$$

Grabiec [12] has generalized these results mentioned above.

The stability problems of several functional equations have been extensively investigated by a number of authors and there are many interesting results concerning this problem (see [13–43]).

Let X be a set. A function $d : X \times X \to [0,\infty]$ is called a *generalized metric* on X if d satisfies the following conditions:

(1) $d(x,y) = 0$ if and only if $x = y$;
(2) $d(x,y) = d(y,x)$ for all $x,y \in X$;
(3) $d(x,z) \le d(x,y) + d(y,z)$ for all $x,y,z \in X$.

We recall a fundamental result in fixed-point theory.

Theorem 1.1 [44, 45] *Let (X,d) be a complete generalized metric space and let $J : X \to X$ be a strictly contractive mapping with Lipschitz constant $L < 1$. Then, for each given element $x \in X$, either*

$$d\left(J^n x, J^{n+1}x\right) = \infty$$

for all nonnegative integers n or there exists a positive integer n_0 such that

(1) $d(J^n x, J^{n+1}x) < \infty$ *for all* $n \ge n_0$;
(2) *the sequence* $\{J^n x\}$ *converges to a fixed point* y^* *of* J;
(3) y^* *is the unique fixed point of J in the set* $Y = \{y \in X \mid d(J^{n_0}x, y) < \infty\}$;
(4) $d(y,y^*) \le \frac{1}{1-L}d(y,Jy)$ *for all* $y \in Y$.

In 1996, Isac and T.M. Rassias [46] were the first to provide applications of stability theory of functional equations for the proof of new fixed-point theorems with applications. By using fixed-point methods, the stability problems of several functional equations have been extensively investigated by a number of authors (see [47–58]).

Recently, Park and Park [59] introduced and investigated the following additive functional equation of Euler-Lagrange type:

$$\begin{aligned}&\sum_{i=1}^{n} r_i L\left(\sum_{j=1}^{n} r_j(x_i - x_j)\right) + \left(\sum_{i=1}^{n} r_i\right) L\left(\sum_{i=1}^{n} r_i x_i\right)\\&\quad = \left(\sum_{i=1}^{n} r_i\right)\sum_{i=1}^{n} r_i L(x_i), \quad r_1, \ldots, r_n \in (0,\infty)\end{aligned} \tag{1.2}$$

whose solution is said to be a *generalized additive mapping of Euler-Lagrange type*.

In this paper, we introduce the following additive functional equation of Euler-Lagrange type which is somewhat different from (1.2):

$$\sum_{j=1}^{n} f\left(\frac{1}{2}\sum_{1\le i\le n, i\ne j} r_i x_i - \frac{1}{2} r_j x_j\right) + \sum_{i=1}^{n} r_i f(x_i) = nf\left(\frac{1}{2}\sum_{i=1}^{n} r_i x_i\right), \tag{1.3}$$

where $r_1, \ldots, r_n \in \mathbb{R}$. Every solution of the functional equation (1.3) is said to be a *generalized Euler-Lagrange type additive mapping.*

Using fixed-point methods, we investigate the Hyers-Ulam stability of the functional equation (1.3) in Banach modules over a C^*-algebra. These results are applied to investigate C^*-algebra homomorphisms in unital C^*-algebras. Also, ones can get the superstability results after all theorems by putting the product of powers of norms as the control functions (see for more details [60, 61]).

Throughout this paper, assume that A is a unital C^*-algebra with the norm $\|\cdot\|_A$ and the unit e, B is a unital C^*-algebra with the norm $\|\cdot\|_B$, and X, Y are left Banach modules over a unital C^*-algebra A with the norms $\|\cdot\|_X$ and $\|\cdot\|_Y$, respectively. Let $U(A)$ be the group of unitary elements in A and let $r_1, \ldots, r_n \in \mathbb{R}$.

2 Hyers-Ulam stability of the functional equation (1.3) in Banach modules over a C^*-algebra

For any given mapping $f : X \to Y$, $u \in U(A)$ and $\mu \in \mathbb{C}$, we define $D_{u,r_1,\ldots,r_n}f$ and $D_{\mu,r_1,\ldots,r_n}f : X^n \to Y$ by

$$\begin{aligned} &D_{u,r_1,\ldots,r_n}f(x_1, \ldots, x_n) \\ &\quad := \sum_{j=1}^{n} f\left(\frac{1}{2}\sum_{1\leq i\leq n, i\neq j} r_i u x_i - \frac{1}{2} r_j u x_j\right) + \sum_{i=1}^{n} r_i u f(x_i) - nf\left(\frac{1}{2}\sum_{i=1}^{n} r_i u x_i\right) \end{aligned}$$

and

$$\begin{aligned} &D_{\mu,r_1,\ldots,r_n}f(x_1, \ldots, x_n) \\ &\quad := \sum_{j=1}^{n} f\left(\frac{1}{2}\sum_{1\leq i\leq n, i\neq j} \mu r_i x_i - \frac{1}{2} \mu r_j x_j\right) + \sum_{i=1}^{n} \mu r_i f(x_i) - nf\left(\frac{1}{2}\sum_{i=1}^{n} \mu r_i x_i\right) \end{aligned}$$

for all $x_1, \ldots, x_n \in X$.

Lemma 2.1 *Let X and Y be linear spaces and let $r_1, \ldots, r_n$ be real numbers with $\sum_{k=1}^{n} r_k \neq 0$ and $r_i \neq 0$, $r_j \neq 0$ for some $1 \leq i < j \leq n$. Assume that a mapping $L : X \to Y$ satisfies the functional equation* (1.3) *for all $x_1, \ldots, x_n \in X$. Then the mapping L is additive. Moreover, $L(r_k x) = r_k L(x)$ for all $x \in X$ and $1 \leq k \leq n$.*

Proof One can find a complete proof at [62]. □

Lemma 2.2 *Let X and Y be linear spaces and let $r_1, \ldots, r_n$ be real numbers with $r_i \neq 0$, $r_j \neq 0$ for some $1 \leq i < j \leq n$. Assume that a mapping $L : X \to Y$ with $L(0) = 0$ satisfies the functional equation* (1.3) *for all $x_1, \ldots, x_n \in X$. Then the mapping L is additive. Moreover, $L(r_k x) = r_k L(x)$ for all $x \in X$ and $1 \leq k \leq n$.*

Proof One can find a complete proof at [62]. □

We investigate the Hyers-Ulam stability of a generalized Euler-Lagrange type additive mapping in Banach modules over a unital C^*-algebra. Throughout this paper, let $r_1, \ldots, r_n$ be real numbers such that $r_i \neq 0$, $r_j \neq 0$ for fixed $1 \leq i < j \leq n$.

Theorem 2.3 *Let $f: X \to Y$ be a mapping satisfying $f(0) = 0$ for which there is a function $\varphi : X^n \to [0, \infty)$ such that*

$$\left\| D_{e,r_1,\ldots,r_n} f(x_1, \ldots, x_n) \right\|_Y \le \varphi(x_1, \ldots, x_n) \tag{2.1}$$

for all $x_1, \ldots, x_n \in X$. Let

$$\varphi_{ij}(x, y) := \varphi(0, \ldots, 0, \underbrace{x}_{i\text{th}}, 0, \ldots, 0, \underbrace{y}_{j\text{th}}, 0, \ldots, 0)$$

for all $x, y \in X$ and $1 \le i < j \le n$. If there exists $0 < C < 1$ such that

$$\varphi(2x_1, \ldots, 2x_n) \le 2C\varphi(x_1, \ldots, x_n)$$

for all $x_1, \ldots, x_n \in X$, then there exists a unique generalized Euler-Lagrange type additive mapping $L : X \to Y$ such that

$$\begin{aligned} \left\| f(x) - L(x) \right\|_Y \le \frac{1}{4-4C} \left\{ \varphi_{ij}\left(\frac{2x}{r_i}, \frac{2x}{r_j}\right) + 2\varphi_{ij}\left(\frac{x}{r_i}, -\frac{x}{r_j}\right) \right. \\ \left. + \varphi_{ij}\left(\frac{2x}{r_i}, 0\right) + 2\varphi_{ij}\left(\frac{x}{r_i}, 0\right) + \varphi_{ij}\left(0, \frac{2x}{r_j}\right) + 2\varphi_{ij}\left(0, -\frac{x}{r_j}\right) \right\} \end{aligned} \tag{2.2}$$

for all $x \in X$. Moreover, $L(r_k x) = r_k L(x)$ for all $x \in X$ and $1 \le k \le n$.

Proof For each $1 \le k \le n$ with $k \ne i, j$, let $x_k = 0$ in (2.1). Then we get the following inequality:

$$\begin{aligned} &\left\| f\left(\frac{-r_i x_i + r_j x_j}{2}\right) + f\left(\frac{r_i x_i - r_j x_j}{2}\right) - 2f\left(\frac{r_i x_i + r_j x_j}{2}\right) + r_i f(x_i) + r_j f(x_j) \right\|_Y \\ &\quad \le \varphi(0, \ldots, 0, \underbrace{x_i}_{i\text{th}}, 0, \ldots, 0, \underbrace{x_j}_{j\text{th}}, 0, \ldots, 0) \end{aligned} \tag{2.3}$$

for all $x_i, x_j \in X$. Letting $x_i = 0$ in (2.3), we get

$$\left\| f\left(-\frac{r_j x_j}{2}\right) - f\left(\frac{r_j x_j}{2}\right) + r_j f(x_j) \right\|_Y \le \varphi_{ij}(0, x_j) \tag{2.4}$$

for all $x_j \in X$. Similarly, letting $x_j = 0$ in (2.3), we get

$$\left\| f\left(-\frac{r_i x_i}{2}\right) - f\left(\frac{r_i x_i}{2}\right) + r_i f(x_i) \right\|_Y \le \varphi_{ij}(x_i, 0) \tag{2.5}$$

for all $x_i \in X$. It follows from (2.3), (2.4) and (2.5) that

$$\begin{aligned} &\left\| f\left(\frac{-r_i x_i + r_j x_j}{2}\right) + f\left(\frac{r_i x_i - r_j x_j}{2}\right) - 2f\left(\frac{r_i x_i + r_j x_j}{2}\right) \right. \\ &\qquad \left. + f\left(\frac{r_i x_i}{2}\right) + f\left(\frac{r_j x_j}{2}\right) - f\left(-\frac{r_i x_i}{2}\right) - f\left(-\frac{r_j x_j}{2}\right) \right\|_Y \\ &\quad \le \varphi_{ij}(x_i, x_j) + \varphi_{ij}(x_i, 0) + \varphi_{ij}(0, x_j) \end{aligned} \tag{2.6}$$

for all $x_i, x_j \in X$. Replacing x_i and x_j by $\frac{2x}{r_i}$ and $\frac{2y}{r_j}$ in (2.6), we get

$$\begin{aligned} &\left\|f(-x+y)+f(x-y)-2f(x+y)+f(x)+f(y)-f(-x)-f(-y)\right\|_Y \\ &\quad \le \varphi_{ij}\left(\frac{2x}{r_i},\frac{2y}{r_j}\right)+\varphi_{ij}\left(\frac{2x}{r_i},0\right)+\varphi_{ij}\left(0,\frac{2y}{r_j}\right) \end{aligned} \tag{2.7}$$

for all $x, y \in X$. Putting $y = x$ in (2.7), we get

$$\left\|2f(x)-2f(-x)-2f(2x)\right\|_Y \le \varphi_{ij}\left(\frac{2x}{r_i},\frac{2x}{r_j}\right)+\varphi_{ij}\left(\frac{2x}{r_i},0\right)+\varphi_{ij}\left(0,\frac{2x}{r_j}\right) \tag{2.8}$$

for all $x \in X$. Replacing x and y by $\frac{x}{2}$ and $-\frac{x}{2}$ in (2.7), respectively, we get

$$\left\|f(x)+f(-x)\right\|_Y \le \varphi_{ij}\left(\frac{x}{r_i},-\frac{x}{r_j}\right)+\varphi_{ij}\left(\frac{x}{r_i},0\right)+\varphi_{ij}\left(0,-\frac{x}{r_j}\right) \tag{2.9}$$

for all $x \in X$. It follows from (2.8) and (2.9) that

$$\left\|\frac{1}{2}f(2x)-f(x)\right\|_Y \le \frac{1}{4}\psi(x) \tag{2.10}$$

for all $x \in X$, where

$$\begin{aligned} \psi(x) := {}&\varphi_{ij}\left(\frac{2x}{r_i},\frac{2x}{r_j}\right)+2\varphi_{ij}\left(\frac{x}{r_i},-\frac{x}{r_j}\right) \\ &+\varphi_{ij}\left(\frac{2x}{r_i},0\right)+2\varphi_{ij}\left(\frac{x}{r_i},0\right)+\varphi_{ij}\left(0,\frac{2x}{r_j}\right)+2\varphi_{ij}\left(0,-\frac{x}{r_j}\right). \end{aligned}$$

Consider the set $\mathcal{W} := \{g : X \to Y\}$ and introduce the generalized metric on $\mathcal{W}$:

$$d(g,h) = \inf\left\{C \in \mathbb{R}_+ : \left\|g(x)-h(x)\right\|_Y \le C\psi(x), \forall x \in X\right\}.$$

It is easy to show that $(\mathcal{W}, d)$ is complete.

Now, we consider the linear mapping $J : \mathcal{W} \to \mathcal{W}$ such that

$$Jg(x) := \frac{1}{2}g(2x) \tag{2.11}$$

for all $x \in X$. By Theorem 3.1 of [44], $d(Jg, Jh) \le Cd(g,h)$ for all $g, h \in \mathcal{W}$. Hence, $d(f, Jf) \le \frac{1}{4}$.

By Theorem 1.1, there exists a mapping $L : X \to Y$ such that

(1) L is a fixed point of J, *i.e.*,

$$L(2x) = 2L(x) \tag{2.12}$$

for all $x \in X$. The mapping L is a unique fixed point of J in the set

$$Z = \left\{g \in \mathcal{W} : d(f,g) < \infty\right\}.$$

This implies that L is a unique mapping satisfying (2.12) such that there exists $C \in (0, \infty)$ satisfying

$$\|L(x) - f(x)\|_Y \le C\psi(x)$$

for all $x \in X$.

(2) $d(J^n f, L) \to 0$ as $n \to \infty$. This implies the equality

$$\lim_{n\to\infty} \frac{f(2^n x)}{2^n} = L(x)$$

for all $x \in X$.

(3) $d(f, L) \le \frac{1}{1-C} d(f, Jf)$, which implies the inequality $d(f, L) \le \frac{1}{4-4C}$. This implies that the inequality (2.2) holds.

Since $\varphi(2x_1, \dots, 2x_n) \le 2C\varphi(x_1, \dots, x_n)$, it follows that

$$\begin{aligned}\|D_{e,r_1,\dots,r_n} L(x_1, \dots, x_n)\|_Y &= \lim_{k\to\infty} \frac{1}{2^k} \|D_{e,r_1,\dots,r_n} f(2^k x_1, \dots, 2^k x_n)\|_Y \\ &\le \lim_{k\to\infty} \frac{1}{2^k} \varphi(2^k x_1, \dots, 2^k x_n) \\ &\le \lim_{k\to\infty} C^k \varphi(x_1, \dots, x_n) = 0\end{aligned}$$

for all $x_1, \dots, x_n \in X$. Therefore, the mapping $L : X \to Y$ satisfies the equation (1.3) and $L(0) = 0$. Hence, by Lemma 2.2, L is a generalized Euler-Lagrange type additive mapping and $L(r_k x) = r_k L(x)$ for all $x \in X$ and $1 \le k \le n$. This completes the proof. □

Theorem 2.4 *Let $f : X \to Y$ be a mapping satisfying $f(0) = 0$ for which there is a function $\varphi : X^n \to [0, \infty)$ satisfying*

$$\|D_{u,r_1,\dots,r_n} f(x_1, \dots, x_n)\| \le \varphi(x_1, \dots, x_n) \tag{2.13}$$

for all $x_1, \dots, x_n \in X$ and $u \in U(A)$. If there exists $0 < C < 1$ such that

$$\varphi(2x_1, \dots, 2x_n) \le 2C\varphi(x_1, \dots, x_n)$$

for all $x_1, \dots, x_n \in X$, then there exists a unique A-linear generalized Euler-Lagrange type additive mapping $L : X \to Y$ satisfying (2.2) *for all $x \in X$. Moreover, $L(r_k x) = r_k L(x)$ for all $x \in X$ and $1 \le k \le n$.*

Proof By Theorem 2.3, there exists a unique generalized Euler-Lagrange type additive mapping $L : X \to Y$ satisfying (2.2), and moreover $L(r_k x) = r_k L(x)$ for all $x \in X$ and $1 \le k \le n$. By the assumption, for each $u \in U(A)$, we get

$$\begin{aligned}&\|D_{u,r_1,\dots,r_n} L(0, \dots, 0, \underbrace{x}_{i\text{th}}, 0, \dots, 0)\|_Y \\ &\quad = \lim_{k\to\infty} \frac{1}{2^k} \|D_{u,r_1,\dots,r_n} f(0, \dots, 0, \underbrace{2^k x}_{i\text{th}}, 0, \dots, 0)\|_Y\end{aligned}$$

$$\leq \lim_{k\to\infty} \frac{1}{2^k}\varphi\bigl(0,\ldots,0,\underbrace{2^k x}_{i\text{th}},0,\ldots,0\bigr)$$

$$\leq \lim_{k\to\infty} C^k\varphi(0,\ldots,0,\underbrace{x}_{i\text{th}},0,\ldots,0) = 0$$

for all $x \in X$. So, we have

$$r_i uL(x) = L(r_i ux)$$

for all $u \in U(A)$ and $x \in X$. Since $L(r_i x) = r_i L(x)$ for all $x \in X$ and $r_i \neq 0$,

$$L(ux) = uL(x)$$

for all $u \in U(A)$ and $x \in X$. By the same reasoning as in the proofs of [63] and [64],

$$L(ax + by) = L(ax) + L(by) = aL(x) + bL(y)$$

for all $a, b \in A$ $(a, b \neq 0)$ and $x, y \in X$. Since $L(0x) = 0 = 0L(x)$ for all $x \in X$, the unique generalized Euler-Lagrange type additive mapping $L : X \to Y$ is an A-linear mapping. This completes the proof. □

Theorem 2.5 *Let $f : X \to Y$ be a mapping satisfying $f(0) = 0$ for which there is a function $\varphi : X^n \to [0,\infty)$ such that*

$$\bigl\|D_{e,r_1,\ldots,r_n} f(x_1,\ldots,x_n)\bigr\|_Y \leq \varphi(x_1,\ldots,x_n) \tag{2.14}$$

for all $x_1,\ldots,x_n \in X$. If there exists $0 < C < 1$ such that

$$\varphi(x_1,\ldots,2_n) \leq \frac{C}{2}\varphi(2x_1,\ldots,2x_n)$$

for all $x_1,\ldots,x_n \in X$, then there exists a unique generalized Euler-Lagrange type additive mapping $L : X \to Y$ such that

$$\begin{aligned}\bigl\|f(x) - L(x)\bigr\|_Y \leq \frac{C}{4-4C}\biggl\{&\varphi_{ij}\biggl(\frac{2x}{r_i},\frac{2x}{r_j}\biggr) + 2\varphi_{ij}\biggl(\frac{x}{r_i},-\frac{x}{r_j}\biggr)\\ &+ \varphi_{ij}\biggl(\frac{2x}{r_i},0\biggr) + 2\varphi_{ij}\biggl(\frac{x}{r_i},0\biggr) + \varphi_{ij}\biggl(0,\frac{2x}{r_j}\biggr) + 2\varphi_{ij}\biggl(0,-\frac{x}{r_j}\biggr)\biggr\}\end{aligned} \tag{2.15}$$

for all $x \in X$, where φ_{ij} is defined in the statement of Theorem 2.3. Moreover, $L(r_k x) = r_k L(x)$ for all $x \in X$ and $1 \leq k \leq n$.

Proof It follows from (2.10) that

$$\biggl\|f(x) - f\biggl(\frac{x}{2}\biggr)\biggr\|_Y \leq \frac{1}{2}\psi\biggl(\frac{x}{2}\biggr) \leq \frac{C}{4}\psi(x)$$

for all $x \in X$, where ψ is defined in the proof of Theorem 2.3. The rest of the proof is similar to the proof of Theorem 2.3. □

Theorem 2.6 *Let $f : X \to Y$ be a mapping with $f(0) = 0$ for which there is a function $\varphi : X^n \to [0,\infty)$ satisfying*

$$\left\|D_{u,r_1,\ldots,r_n} f(x_1,\ldots,x_n)\right\| \le \varphi(x_1,\ldots,x_n) \tag{2.16}$$

for all $x_1,\ldots,x_n \in X$ and $u \in U(A)$. If there exists $0 < C < 1$ such that

$$\varphi(x_1,\ldots,2_n) \le \frac{C}{2}\varphi(2x_1,\ldots,2x_n)$$

for all $x_1,\ldots,x_n \in X$, then there exists a unique A-linear generalized Euler-Lagrange type additive mapping $L : X \to Y$ satisfying (2.15) *for all $x \in X$. Moreover, $L(r_k x) = r_k L(x)$ for all $x \in X$ and all $1 \le k \le n$.*

Proof The proof is similar to the proof of Theorem 2.4. □

Remark 2.7 In Theorems 2.5 and 2.6, one can assume that $\sum_{k=1}^{n} r_k \ne 0$ instead of $f(0) = 0$.

3 Homomorphisms in unital C^*-algebras

In this section, we investigate C^*-algebra homomorphisms in unital C^*-algebras. We use the following lemma in the proof of the next theorem.

Lemma 3.1 [64] *Let $f : A \to B$ be an additive mapping such that $f(\mu x) = \mu f(x)$ for all $x \in A$ and $\mu \in \mathbb{S}^1_{\frac{1}{n_o}} := \{e^{i\theta}; 0 \le \theta \le 2\pi n_o\}$. Then the mapping $f : A \to B$ is $\mathbb{C}$-linear.*

Note that a $\mathbb{C}$-linear mapping $H : A \to B$ is called a *homomorphism* in C^*-algebras if H satisfies $H(xy) = H(x)H(y)$ and $H(x^*) = H(x)^*$ for all $x, y \in A$.

Theorem 3.2 *Let $f : A \to B$ be a mapping with $f(0) = 0$ for which there is a function $\varphi : A^n \to [0,\infty)$ satisfying*

$$\left\|D_{\mu,r_1,\ldots,r_n} f(x_1,\ldots,x_n)\right\|_B \le \varphi(x_1,\ldots,x_n), \tag{3.1}$$

$$\left\|f\left(2^k u^*\right) - f\left(2^k u\right)^*\right\|_B \le \varphi\big(\underbrace{2^k u,\ldots,2^k u}_{n \text{ times}}\big), \tag{3.2}$$

$$\left\|f\left(2^k ux\right) - f\left(2^k u\right)f(x)\right\|_B \le \varphi\big(\underbrace{2^k ux,\ldots,2^k ux}_{n \text{ times}}\big) \tag{3.3}$$

for all $x, x_1,\ldots,x_n \in A$, $u \in U(A)$, $k \in \mathbb{N}$ and $\mu \in \mathbb{S}^1$. If there exists $0 < C < 1$ such that

$$\varphi(2x_1,\ldots,2x_n) \le 2C\varphi(x_1,\ldots,x_n)$$

for all $x_1,\ldots,x_n \in A$, then the mapping $f : A \to B$ is a C^*-algebra homomorphism.

Proof Since $|J| \ge 3$, letting $\mu = 1$ and $x_k = 0$ for all $1 \le k \le n$ ($k \ne i, j$) in (3.1), we get

$$f\left(\frac{-r_i x_i + r_j x_j}{2}\right) + f\left(\frac{r_i x_i - r_j x_j}{2}\right) + r_i f(x_i) + r_j f(x_j) = 2f\left(\frac{r_i x_i + r_j x_j}{2}\right)$$

for all $x_i, x_j \in A$. By the same reasoning as in the proof of Lemma 2.1, the mapping f is additive and $f(r_k x) = r_k f(x)$ for all $x \in A$ and $k = i, j$. So, by letting $x_i = x$ and $x_k = 0$ for all $1 \le k \le n$, $k \neq i$, in (3.1), we get $f(\mu x) = \mu f(x)$ for all $x \in A$ and $\mu \in \mathbb{S}^1$. Therefore, by Lemma 3.1, the mapping f is $\mathbb{C}$-linear. Hence, it follows from (3.2) and (3.3) that

$$\begin{aligned}\|f(u^*) - f(u)^*\|_B &= \lim_{k\to\infty} \frac{1}{2^k} \|f(2^k u^*) - f(2^k u)^*\|_B \\ &\le \lim_{k\to\infty} \frac{1}{2^k} \varphi(\underbrace{2^k u, \dots, 2^k u}_{n \text{ times}}) \le \lim_{k\to\infty} C^k \varphi(\underbrace{u, \dots, u}_{n \text{ times}}) \\ &= 0, \\ \|f(ux) - f(u)f(x)\|_B &= \lim_{k\to\infty} \frac{1}{2^k} \|f(2^k ux) - f(2^k u)f(x)\|_B \\ &\le \lim_{k\to\infty} \frac{1}{2^k} \varphi(\underbrace{2^k ux, \dots, 2^k ux}_{n \text{ times}}) \le \lim_{k\to\infty} C^k \varphi(\underbrace{ux, \dots, ux}_{n \text{ times}}) \\ &= 0\end{aligned}$$

for all $x \in A$ and $u \in U(A)$. So, we have $f(u^*) = f(u)^*$ and $f(ux) = f(u)f(x)$ for all $x \in A$ and $u \in U(A)$. Since f is $\mathbb{C}$-linear and each $x \in A$ is a finite linear combination of unitary elements (see [65]), *i.e.*, $x = \sum_{k=1}^m \lambda_k u_k$, where $\lambda_k \in \mathbb{C}$ and $u_k \in U(A)$ for all $1 \le k \le n$, we have

$$\begin{aligned}f(x^*) &= f\left(\sum_{k=1}^m \overline{\lambda}_k u_k^*\right) = \sum_{k=1}^m \overline{\lambda}_k f(u_k^*) = \sum_{k=1}^m \overline{\lambda}_k f(u_k)^* \\ &= \left(\sum_{k=1}^m \lambda_k f(u_k)\right)^* = f\left(\sum_{k=1}^m \lambda_k u_k\right)^* = f(x)^*, \\ f(xy) &= f\left(\sum_{k=1}^m \lambda_k u_k y\right) = \sum_{k=1}^m \lambda_k f(u_k y) \\ &= \sum_{k=1}^m \lambda_k f(u_k) f(y) = f\left(\sum_{k=1}^m \lambda_k u_k\right) f(y) = f(x)f(y)\end{aligned}$$

for all $x, y \in A$. Therefore, the mapping $f : A \to B$ is a C^*-algebra homomorphism. This completes the proof. □

The following theorem is an alternative result of Theorem 3.2.

Theorem 3.3 *Let $f : A \to B$ be a mapping with $f(0) = 0$ for which there is a function $\varphi : A^n \to [0, \infty)$ satisfying*

$$\|D_{\mu, r_1, \dots, r_n} f(x_1, \dots, x_n)\|_B \le \varphi(x_1, \dots, x_n),$$

$$\left\|f\left(\frac{u^*}{2^k}\right) - f\left(\frac{u}{2^k}\right)^*\right\|_B \le \phi\bigg(\underbrace{\frac{u}{2^k}, \dots, \frac{u}{2^k}}_{n \text{ times}}\bigg), \tag{3.4}$$

$$\left\|f\left(\frac{ux}{2^k}\right) - f\left(\frac{u}{2^k}\right) f(x)\right\|_B \le \phi\bigg(\underbrace{\frac{ux}{2^k}, \dots, \frac{ux}{2^k}}_{n \text{ times}}\bigg) \tag{3.5}$$

for all $x, x_1, \dots, x_n \in A$, $u \in U(A)$, $k \in \mathbb{N}$ *and* $\mu \in \mathbb{S}^1$. *If there exists* $0 < C < 1$ *such that*

$$\varphi(x_1, \dots, 2_n) \leq \frac{C}{2} \varphi(2x_1, \dots, 2x_n)$$

for all $x_1, \dots, x_n \in A$, *then the mapping* $f : A \to B$ *is a* C^**-algebra homomorphism.*

Remark 3.4 In Theorems 3.2 and 3.3, one can assume that $\sum_{k=1}^n r_k \neq 0$ instead of $f(0) = 0$.

Theorem 3.5 *Let* $f : A \to B$ *be a mapping with* $f(0) = 0$ *for which there is a function* $\varphi : A^n \to [0, \infty)$ *satisfying* (3.2), (3.3) *and*

$$\left\| D_{\mu, r_1, \dots, r_n} f(x_1, \dots, x_n) \right\|_B \leq \varphi(x_1, \dots, x_n) \tag{3.6}$$

for all $x_1, \dots, x_n \in A$ *and* $\mu \in \mathbb{S}^1$. *Assume that* $\lim_{k\to\infty} \frac{1}{2^k} f(2^k e)$ *is invertible. If there exists* $0 < C < 1$ *such that*

$$\varphi(2x_1, \dots, 2x_n) \leq 2C\varphi(x_1, \dots, x_n)$$

for all $x_1, \dots, x_n \in A$, *then the mapping* $f : A \to B$ *is a* C^**-algebra homomorphism.*

Proof Consider the C^*-algebras A and B as left Banach modules over the unital C^*-algebra $\mathbb{C}$. By Theorem 2.4, there exists a unique $\mathbb{C}$-linear generalized Euler-Lagrange type additive mapping $H : A \to B$ defined by

$$H(x) = \lim_{k\to\infty} \frac{1}{2^k} f\left(2^k x\right)$$

for all $x \in A$. By (3.2) and (3.3), we get

$$\begin{aligned}
\left\| H\left(u^*\right) - H(u)^* \right\|_B &= \lim_{k\to\infty} \frac{1}{2^k} \left\| f\left(2^k u^*\right) - f\left(2^k u\right)^* \right\|_B \\
&\leq \lim_{k\to\infty} \frac{1}{2^k} \varphi\bigl(\underbrace{2^k u, \dots, 2^k u}_{n \text{ times}}\bigr) \\
&= 0, \\
\left\| H(ux) - H(u)f(x) \right\|_B &= \lim_{k\to\infty} \frac{1}{2^k} \left\| f\left(2^k ux\right) - f\left(2^k u\right)f(x) \right\|_B \\
&\leq \lim_{k\to\infty} \frac{1}{2^k} \varphi\bigl(\underbrace{2^k ux, \dots, 2^k ux}_{n \text{ times}}\bigr) \\
&= 0
\end{aligned}$$

for all $u \in U(A)$ and $x \in A$. So, we have $H(u^*) = H(u)^*$ and $H(ux) = H(u)f(x)$ for all $u \in U(A)$ and $x \in A$. Therefore, by the additivity of H, we have

$$H(ux) = \lim_{k\to\infty} \frac{1}{2^k} H\left(2^k ux\right) = H(u) \lim_{k\to\infty} \frac{1}{2^k} f\left(2^k x\right) = H(u)H(x) \tag{3.7}$$

for all $u \in U(A)$ and all $x \in A$. Since H is $\mathbb{C}$-linear and each $x \in A$ is a finite linear combination of unitary elements, *i.e.*, $x = \sum_{k=1}^{m} \lambda_k u_k$, where $\lambda_k \in \mathbb{C}$ and $u_k \in U(A)$ for all $1 \le k \le n$, it follows from (3.7) that

$$\begin{aligned} H(xy) &= H\left(\sum_{k=1}^{m} \lambda_k u_k y\right) = \sum_{k=1}^{m} \lambda_k H(u_k y) \\ &= \sum_{k=1}^{m} \lambda_k H(u_k) H(y) = H\left(\sum_{k=1}^{m} \lambda_k u_k\right) H(y) \\ &= H(x)H(y), \\ H(x^*) &= H\left(\sum_{k=1}^{m} \overline{\lambda}_k u_k^*\right) = \sum_{k=1}^{m} \overline{\lambda}_k H(u_k^*) = \sum_{k=1}^{m} \overline{\lambda}_k H(u_k)^* \\ &= \left(\sum_{k=1}^{m} \lambda_k H(u_k)\right)^* = H\left(\sum_{k=1}^{m} \lambda_k u_k\right)^* \\ &= H(x)^* \end{aligned}$$

for all $x, y \in A$. Since $H(e) = \lim_{k\to\infty} \frac{1}{2^k} f(2^k e)$ is invertible and

$$H(e)H(y) = H(ey) = H(e)f(y)$$

for all $y \in A$, it follows that $H(y) = f(y)$ for all $y \in A$. Therefore, the mapping $f : A \to B$ is a C^*-algebra homomorphism. This completes the proof. □

The following theorem is an alternative result of Theorem 3.5.

Theorem 3.6 *Let $f : A \to B$ be a mapping with $f(0) = 0$ for which there is a function $\varphi : A^n \to [0, \infty)$ satisfying* (3.4), (3.5) *and*

$$\left\| D_{\mu,r_1,\dots,r_n} f(x_1, \dots, x_n) \right\|_B \le \varphi(x_1, \dots, x_n)$$

for all $x_1, \dots, x_n \in A$ and $\mu \in \mathbb{S}^1$. Assume that $\lim_{k\to\infty} 2^k f(\frac{e}{2^k})$ is invertible. If there exists $0 < C < 1$ such that

$$\varphi(x_1, \dots, 2_n) \le \frac{C}{2} \varphi(2x_1, \dots, 2x_n)$$

for all $x_1, \dots, x_n \in A$, then the mapping $f : A \to B$ is a C^*-algebra homomorphism.

Remark 3.7 In Theorem 3.6, one can assume that $\sum_{k=1}^{n} r_k \ne 0$ instead of $f(0) = 0$.

Theorem 3.8 *Let $f : A \to B$ be a mapping with $f(0) = 0$ for which there is a function $\varphi : A^n \to [0, \infty)$ satisfying* (3.2), (3.3) *and*

$$\left\| D_{\mu,r_1,\dots,r_n} f(x_1, \dots, x_n) \right\|_B \le \varphi(x_1, \dots, x_n) \tag{3.8}$$

for all $x_1, \dots, x_n \in A$ *and* $\mu = i, 1$. *Assume that* $\lim_{k\to\infty} \frac{1}{2^k} f(2^k e)$ *is invertible and for each fixed* $x \in A$ *the mapping* $t \mapsto f(tx)$ *is continuous in* $t \in \mathbb{R}$. *If there exists* $0 < C < 1$ *such that*

$$\varphi(2x_1, \dots, 2x_n) \le 2C\varphi(x_1, \dots, x_n)$$

for all $x_1, \dots, x_n \in A$, *then the mapping* $f : A \to B$ *is a* C^**-algebra homomorphism.*

Proof Put $\mu = 1$ in (3.8). By the same reasoning as in the proof of Theorem 2.3, there exists a unique generalized Euler-Lagrange type additive mapping $H : A \to B$ defined by

$$H(x) = \lim_{k\to\infty} \frac{f(2^k x)}{2^k}$$

for all $x \in A$. By the same reasoning as in the proof of [58], the generalized Euler-Lagrange type additive mapping $H : A \to B$ is $\mathbb{R}$-linear. By the same method as in the proof of Theorem 2.4, we have

$$\begin{aligned} &\big\| D_{\mu,r_1,\dots,r_n} H(0, \dots, 0, \underbrace{x}_{j\text{th}}, 0, \dots, 0) \big\|_Y \\ &\quad = \lim_{k\to\infty} \frac{1}{2^k} \big\| D_{\mu,r_1,\dots,r_n} f\big(0, \dots, 0, \underbrace{2^k x}_{j\text{th}}, 0, \dots, 0\big) \big\|_Y \\ &\quad \le \lim_{k\to\infty} \frac{1}{2^k} \varphi\big(0, \dots, 0, \underbrace{2^k x}_{j\text{th}}, 0, \dots, 0\big) = 0 \end{aligned}$$

for all $x \in A$ and so

$$r_j \mu H(x) = H(r_j \mu x)$$

for all $x \in A$. Since $H(r_j x) = r_j H(x)$ for all $x \in X$ and $r_j \neq 0$,

$$H(\mu x) = \mu H(x)$$

for all $x \in A$ and $\mu = i, 1$. For each $\lambda \in \mathbb{C}$, we have $\lambda = s + it$, where $s, t \in \mathbb{R}$. Thus, it follows that

$$\begin{aligned} H(\lambda x) &= H(sx + itx) = sH(x) + tH(ix) \\ &= sH(x) + itH(x) = (s + it)H(x) \\ &= \lambda H(x) \end{aligned}$$

for all $\lambda \in \mathbb{C}$ and $x \in A$ and so

$$H(\zeta x + \eta y) = H(\zeta x) + H(\eta y) = \zeta H(x) + \eta H(y)$$

for all $\zeta, \eta \in \mathbb{C}$ and $x, y \in A$. Hence, the generalized Euler-Lagrange type additive mapping $H : A \to B$ is $\mathbb{C}$-linear.

The rest of the proof is the same as in the proof of Theorem 3.5. This completes the proof. □

The following theorem is an alternative result of Theorem 3.8.

Theorem 3.9 *Let* $f : A \to B$ *be a mapping with* $f(0) = 0$ *for which there is a function* $\varphi : A^n \to [0,\infty)$ *satisfying* (3.4), (3.5) *and*

$$\left\| D_{\mu,r_1,\ldots,r_n} f(x_1,\ldots,x_n) \right\|_B \leq \varphi(x_1,\ldots,x_n),$$

for all $x, x_1, \ldots, x_n \in A$ *and* $\mu = i, 1$. *Assume that* $\lim_{k\to\infty} 2^k f(\frac{e}{2^k})$ *is invertible and for each fixed* $x \in A$ *the mapping* $t \mapsto f(tx)$ *is continuous in* $t \in \mathbb{R}$. *If there exists* $0 < C < 1$ *such that*

$$\varphi(x_1,\ldots,2_n) \leq \frac{C}{2}\varphi(2x_1,\ldots,2x_n)$$

for all $x_1, \ldots, x_n \in A$, *then the mapping* $f : A \to B$ *is a* C^*-*algebra homomorphism*.

Proof We omit the proof because it is very similar to last theorem. □

Remark 3.10 In Theorem 3.9, one can assume that $\sum_{k=1}^n r_k \neq 0$ instead of $f(0) = 0$.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors conceived of the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

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