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# A new iterative algorithm of pseudomonotone mappings for equilibrium problems in Hilbert spaces

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**Abstract**

In this paper, we introduce a new algorithm for finding a common element of the set of fixed points of $N$ strict pseudocontractions and the set of solutions of equilibrium problems with a pseudomonotone and Lipschitz-type continuous bifunction. The scheme is motivated by the idea of extragradient methods and fixed point iteration methods. We show that the iterative sequences generated by this algorithm converge strongly to the above mentioned common element under some suitable conditions on algorithm parameters in a real Hilbert space. And also, we consider the variational inequality problems as an application.

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**Keywords:** strict pseudocontractions; pseudomonotone; Lipschitz-type continuous; equilibrium problems; fixed points

## 1 Introduction

Let $C$ be a nonempty closed convex subset of a real Hilbert space $H$ with the inner product $\langle\cdot,\cdot\rangle$ and the norm $\|\cdot\|$, and let $f$ be a bifunction from $C \times C$ into $\mathbf{R}$ such that $f(x,x) = 0$ for all $x \in C$. We consider the equilibrium problem in the sense of Blum and Oettli [1]: Find $x^* \in C$ such that

$$f(x^*, y) \geq 0 \qquad EP(f)$$

for all $y \in C$.

We denote by $\mathrm{Sol}(EP(f))$ the set of solutions of the equilibrium problem $EP(f)$.

We know that the problem $EP(f,C)$ covers many important problems in optimization and nonlinear analysis. It has also found many applications in economics, transportation and engineering (see [1, 2] and the references quoted therein). Theory and methods for solving this problem have been developed by many authors [3–7]. Alternatively, the problem of finding a common fixed point of a sequence of finite self-mappings $\{S_i\}_{i=1}^N$ $(N \geq 1)$ is described as follows: Find $x^* \in C$ such that

$$x^* \in \bigcap_{i=1}^{N} F(S_i), \qquad (FP)$$

where $F(S_i)$ is the set of fixed points of the mappings $S_i$ ($i = 1, \dots, N$) on $C$. This problem has now become a mature subject in nonlinear analysis. The theory and solution methods of this problem can be found in many research papers and monographs (see [8–10]).

We are interested in the problem of finding a common element of the set of solutions of the equilibrium problem $EP(f)$ and the set of solutions of the fixed problem ($FP$), namely: Find $x^* \in C$ such that

$$x^* \in \bigcap_{i=1}^{N} F(S_i) \cap \mathrm{Sol}\big(EP(f)\big). \tag{1.1}$$

A special case of problem (1.1) is that $f(x,y) = \langle F(x), y - x\rangle$, and this problem is reduced to finding a common element of the set of solutions of variational inequalities, *i.e.*, find $x^* \in C$ such that

$$\big\langle F\big(x^*\big), x - x^*\big\rangle \geq 0, \quad \forall x \in C, \tag{$VI(F)$}$$

and the set solutions of a fixed point problem (see [11–17]).

In this paper, we introduce a new iterative scheme for solving problem (1.1). This method can be considered to be an improvement of the viscosity approximation method in [15, 18, 19] and the iterative method in [20] via an improvement of the extragradient methods [3, 4, 21–23].

The paper is organized as follows. Section 2 recalls some concepts in equilibrium problems and fixed point problems that are used in the sequel and an iterative algorithm for solving problem (1.1). In Section 3, we prove the convergence theorems for the algorithms which are defined in Section 2 as the main results of this paper. In Section 4, we consider the variational inequality problems as an application of the main theorem.

## 2 Preliminaries

We first recall the following definitions that will be used for the main theorem.

**Definition 2.1** Let $C$ be a nonempty closed convex subset of a real Hilbert space $H$. A bifunction $f : C \times C \to \mathbf{R}$ is said to be

(a) *monotone* on $C$ if $f(x,y) + f(y,x) \leq 0$, $\forall x, y \in C$;
(b) *pseudomonotone* on $C$ if $f(x,y) \geq 0$ implies $f(y,x) \leq 0$, $\forall x, y \in C$;
(c) *Lipschitz-type continuous* on $C$ with two constants $c_1 > 0$ and $c_2 > 0$ if

$$f(x,y) + f(y,z) \geq f(x,z) - c_1\|x-y\|^2 - c_2\|y-z\|^2, \quad \forall x,y,z \in C. \tag{2.1}$$

We know that every monotone bifunction $f$ is pseudomonotone, but the converse is not true (see [24]).

**Definition 2.2** Let $C$ be a nonempty closed convex subset of a real Hilbert space $H$. A mapping $S : C \to C$ is said to be a *strict pseudocontraction* if there exists a constant $0 \leq L < 1$ such that

$$\big\|S(x) - S(y)\big\|^2 \leq \|x-y\|^2 + L\big\|(I-S)(x) - (I-S)(y)\big\|^2, \quad \forall x, y \in C,$$

where $I$ is the identity mapping on $H$. If $L = 0$, then $S$ is called *nonexpansive* on $C$.

Now, we define the projection on $C$, denoted by $\mathrm{Pr}_C(\cdot)$, *i.e.*,

$$\mathrm{Pr}_C(x) = \operatorname{argmin}\{\|y - x\| : y \in C\}, \quad \forall x \in H.$$

And we use the symbols $\rightharpoonup$ and $\to$ to denote weak convergence and strong convergence, respectively. The following proposition gives some useful properties for strict pseudocontractions.

**Proposition 2.3** [25] *Let $C$ be a nonempty closed convex subset of a real Hilbert space $H$, let $S : C \to C$ be an $L$-strict pseudocontraction, and for each $i = 1, \dots, N$, let $S_i : C \to C$ be an $L_i$-strict pseudocontraction for some $0 \le L_i < 1$. Then we have the following.*

(a) *$S$ satisfies the following Lipschitz condition*:

$$\|S(x) - S(y)\| \le \frac{1 + L}{1 - L}\|x - y\|, \quad \forall x, y \in C;$$

(b) *$(I - S)$ is demiclosed at zero. That is, if the sequence $\{x^k\}$ is in C such that $x^k \rightharpoonup \bar{x}$ and $(I - S)(x^k) \to 0$, then $(I - S)(\bar{x}) = 0$;*

(c) *The set $F(S)$ is closed and convex*;

(d) *If $\lambda_i > 0$ $(i = 1, \dots, N)$ and $\sum_{i=1}^N \lambda_i = 1$, then $\sum_{i=1}^N \lambda_i S_i$ is an $\bar{L}$-strict pseudocontraction, where $\bar{L} := \max\{L_i \mid 1 \le i \le N\}$*;

(e) *If $\lambda_i$ is the same as in* (d) *and $\{S_i \mid i = 1, \dots, N\}$ has a common fixed point, then*

$$F\left(\sum_{i=1}^N \lambda_i S_i\right) = \bigcap_{i=1}^N F(S_i).$$

Many authors studied the problem of finding a common fixed point of a finite family of mappings. For instance, Marino and Xu [26] constructed an iterative algorithm for finding a common fixed point of $N$ strict pseudocontractions $S_i$ $(i = 1, \dots, N)$. They defined the sequence $\{x^k\}$ starting from $x^0 \in H$ and taking

$$x^{k+1} = \alpha_k x^k + (1 - \alpha_k)\sum_{i=1}^N \lambda_{k,i} S_i(x^k), \tag{2.2}$$

where the control sequence of parameters $\{\lambda_k\}$ was made in order to get the guarantee for the convergence of the iterative sequence $\{x^k\}$. And they proved that the sequence $\{x^k\}$ converges weakly to the point $\bar{x} \in \bigcap_{i=1}^N F(S_i)$.

Recently, Chen *et al.* [20] introduced a new iterative scheme for finding a common element of the set of common fixed points of a sequence of strict pseudocontractions $\{\bar{S}_i\}$ and the set of solutions of the equilibrium problem $EP(f)$ in a real Hilbert space $H$. Given a starting point $x^0 \in H$, three iterative sequences $\{x^k\}$, $\{y^k\}$ and $\{z^k\}$ are generated as the following scheme:

$$\begin{cases} \text{Compute } y^k = \alpha_k x^k + (1 - \alpha_k)\bar{S}_k(x^k); \\ \text{Find } z^k \in C \text{ such that } f(z^k, y) + \frac{1}{r_k}\langle y - z^k, z^k - y^k\rangle \ge 0,\ \forall y \in C; \\ \text{Compute } x^{k+1} = \mathrm{Pr}_{C_k}(x^0),\ \text{where } C_k := \{v \in C \mid \|z^k - v\| \le \|x^k - v\|\}. \end{cases} \tag{2.3}$$

Here, two sequences $\{\alpha_k\}$ and $\{r_k\}$ are given as control parameters. The authors proved that the sequences $\{x^k\}$, $\{y^k\}$ and $\{z^k\}$ converged strongly to the same point $x^*$, under certain conditions on $\{\alpha_k\}$ and $\{r_k\}$, such that

$$x^* \in \mathrm{Pr}_{\mathrm{Sol}(EP(f))\cap F(S)}\left(x^0\right),$$

where $S$ is a nonexpansive mapping of $C$ into itself defined by

$$S(x) = \lim_{j\to\infty} \bar{S}_j(x)$$

for all $x \in C$.

The methods for finding a common element of the sets $\mathrm{Sol}(EP(f))$ and $\bigcap_{i=1}^N F(S_i)$ in a real Hilbert space have been studied in many research papers (see [7, 17, 21, 22, 27–30]).

We need the following assumptions for the main theorems.

**Assumption 2.4** The bifunction $f$ satisfies the following conditions:

(i) $f$ is pseudomonotone and weakly continuous on $C$;
(ii) $f$ is Lipschitz-type continuous on $C$;
(iii) for each $x \in C$, $f(x,\cdot)$ is convex and subdifferentiable on $C$.

**Assumption 2.5** Every $S_i$ is an $L_i$-strict pseudocontraction for some $0 \le L_i < 1$.

**Assumption 2.6** The solution set of (1.1) is nonempty, *i.e.*,

$$\bigcap_{i=1}^N F(S_i) \cap \mathrm{Sol}\big(EP(f)\big) \neq \emptyset.$$

Note that if $C \subseteq \mathrm{ri}(\mathrm{dom}(f(x,\cdot)))$, where $\mathrm{ri}(\mathrm{dom}(f(x,\cdot)))$ is the set of relative interior points of the domain of $f(x,\cdot)$, then Assumption 2.4(iii) is satisfied. Now we construct the new algorithms as follows.

**Algorithm 2.7**

Initialization: Choose positive sequences $\{\lambda_k\}$, $\{\alpha_k\}$, $\{\beta_k\}$, $\{\gamma_k\}$ and $\{\lambda_{k,i}\}$ satisfying the following conditions:

$$\begin{cases} \alpha_k + \beta_k \le 1, \quad \forall k \ge 0, \\ \liminf_{k\to\infty} \beta_k \in (0,1), \\ \liminf_{k\to\infty} \frac{\alpha_k}{\alpha_k+\beta_k} \in (\bar{L},1), \quad \text{where } \bar{L} := \max\{L_i \mid 1 \le i \le N\}, \\ \liminf_{k\to\infty}(\gamma_k + (1-\gamma_k)(\alpha_k+\beta_k)) > 0, \quad \{\gamma_k\} \subset (0,1), \\ \{\lambda_k\} \subset [a,b] \quad \text{for some } a,b \in (0,\frac{1}{L}), \text{ where } L := \max\{2c_1, 2c_2\}, \\ \sum_{i=1}^p \lambda_{k,i} = 1 \quad \text{for all } k \ge 1. \end{cases}$$

Take an initial point $x^0 \in C$ and set $k := 0$.

Iteration $k$: Carry out three steps below continuously.

- Step 1. Solve two strongly convex programs:

$$\begin{cases} y^k := \operatorname{argmin}\{\lambda_k f(x^k, y) + \frac{1}{2}\|y - x^k\|^2 \mid y \in C\}, \\ t^k := \operatorname{argmin}\{\lambda_k f(y^k, y) + \frac{1}{2}\|y - x^k\|^2 \mid y \in C\}. \end{cases}$$

- Step 2. Compute the iterations

$$\begin{cases} \bar{y}^k := (1 - \gamma_k)x^k + \gamma_k t^k, \\ z^k := (1 - \alpha_k - \beta_k)\bar{y}^k + \alpha_k t^k + \beta_k \sum_{i=1}^N \lambda_{k,i} S_i(t^k). \end{cases}$$

- Step 3. Set

$$\begin{cases} C_k := \{z \in C \mid \|z^k - z\|^2 \le \|x^k - z\|^2 - \beta_k(\frac{\alpha_k}{\alpha_k + \beta_k} - \bar{L})\|\bar{S}_k(t^k) - t^k\|^2\}, \\ \quad \text{where } \bar{S}_k := \sum_{i=1}^N \lambda_{k,i} S_i(x^k), \\ Q_k := \{z \in C \mid \langle x^k - z, x^0 - x^k\rangle \ge 0\}. \end{cases}$$

Compute $x^{k+1} := \Pr_{C_k \cap Q_k}(x^0)$.

Increase $k$ by one and go back to Step 1.

## 3 Convergence of the algorithms

In this section, we study the convergence of Algorithm 2.7. We need the following useful lemmas for the main theorems.

**Lemma 3.1** [2] *Let $C$ be a nonempty closed convex subset of a real Hilbert space $H$, and let $g : C \to \mathbf{R}$ be subdifferentiable on $C$. Then $x^*$ is a solution of the following convex problem*:

$$\min\{g(x) \mid x \in C\}$$

*if and only if*

$$0 \in \partial g(x^*) + N_C(x^*),$$

*where $\partial g(\cdot)$ denotes the subdifferential of $g$ and $N_C(x^*)$ is the* (*outward*) *normal cone of $C$ at $x^* \in C$.*

**Lemma 3.2** [8] *Let $C$ be a nonempty closed convex subset of a real Hilbert space $H$ and $x^0 \in H$. Let $\{x^k\}$ be a bounded sequence such that every weakly cluster point $\bar{x}$ of $\{x^k\}$ belongs to $C$ and*

$$\|x^k - x^0\| \le \|x^0 - \Pr_C(x^0)\|, \quad \forall k \ge 0.$$

*Then $\{x^k\}$ converges strongly to $\Pr_C(x^0)$ as $k \to \infty$.*

Now, we are in a position to prove the main theorem.

**Theorem 3.3** *Let $C$ be a nonempty closed convex subset of a real Hilbert space $H$. Suppose that Assumptions 2.4-2.6 are satisfied. Then the sequences $\{x^k\}$, $\{y^k\}$ and $\{z^k\}$ generated by Algorithm 2.7 converge strongly to the same point $x^* \in \bigcap_{i=1}^N F(S_i) \cap \mathrm{Sol}(EP(f))$, where*

$$x^* = \mathrm{Pr}_{\bigcap_{i=1}^N F(S_i)\cap \mathrm{Sol}(EP(f))}\left(x^0\right). \tag{3.1}$$

*Proof* The proof of this theorem is divided into several steps.

Step 1. Suppose that $x^* \in \bigcap_{i=1}^N F(S_i) \cap \mathrm{Sol}(EP(f))$. Then we have

$$\begin{aligned}\left\|t^k - x^*\right\|^2 \le{}& \left\|x^k - x^*\right\|^2 - (1 - 2\lambda_k c_2)\left\|t^k - y^k\right\|^2 \\ &- (1 - 2\lambda_k c_1)\left\|x^k - y^k\right\|^2, \quad \forall k \ge 0.\end{aligned} \tag{3.2}$$

Since $f(x, \cdot)$ is convex on $C$ for each $x \in C$, by Lemma 3.1, we see that

$$t^k = \mathrm{argmin}\left\{\frac{1}{2}\left\|t - x^k\right\|^2 + \lambda_k f\left(y^k, t\right) \;\middle|\; t \in C\right\}$$

if and only if

$$0 \in \partial_2\left(\lambda_k f\left(y^k, y\right) + \frac{1}{2}\left\|y - x^k\right\|^2\right)\left(t^k\right) + N_C\left(t^k\right), \tag{3.3}$$

where $N_C(x)$ is the (outward) normal cone of $C$ at $x \in C$.

Since $f(y^k, \cdot)$ is subdifferentiable on $C$, by the well-known Moreau-Rockafellar theorem (see [31]), there exists $w \in \partial_2 f(y^k, t^k)$ such that

$$f\left(y^k, t\right) - f\left(y^k, t^k\right) \ge \left\langle w, t - t^k\right\rangle, \quad \forall t \in C.$$

Substituting $t = x^*$ into this inequality, we obtain

$$f\left(y^k, x^*\right) - f\left(y^k, t^k\right) \ge \left\langle w, x^* - t^k\right\rangle. \tag{3.4}$$

And also, it follows from (3.3) that $0 = \lambda_k w + t^k - x^k + \bar{w}$, where $w \in \partial_2 f(y^k, t^k)$ and $\bar{w} \in N_C(t^k)$. By the definition of the normal cone $N_C$, we have

$$\left\langle t^k - x^k, t - t^k\right\rangle \ge \lambda_k\left\langle w, t^k - t\right\rangle, \quad \forall t \in C. \tag{3.5}$$

Substituting $t = x^* \in C$ into the last inequality, we obtain

$$\left\langle t^k - x^k, x^* - t^k\right\rangle \ge \lambda_k\left\langle w, t^k - x^*\right\rangle. \tag{3.6}$$

Combining (3.4) and (3.6), we have

$$\left\langle t^k - x^k, x^* - t^k\right\rangle \ge \lambda_k\left(f\left(y^k, t^k\right) - f\left(y^k, x^*\right)\right). \tag{3.7}$$

Since $x^* \in \mathrm{Sol}(EP(f))$, $f(x^*, y) \ge 0$ for all $y \in C$, and $f$ is pseudomonotone on $C$, we have $f(y^k, x^*) \le 0$. Hence, (3.7) implies that

$$\left\langle t^k - x^k, x^* - t^k\right\rangle \ge \lambda_k f\left(y^k, t^k\right). \tag{3.8}$$

From Lipschitz condition (2.1) for $f$ with $x = x^k$, $y = y^k$ and $z = t^k$, we have

$$f(y^k, t^k) \geq f(x^k, t^k) - f(x^k, y^k) - c_1 \|y^k - x^k\|^2 - c_2 \|t^k - y^k\|^2. \tag{3.9}$$

Combining (3.8) and (3.9), we get

$$\langle t^k - x^k, x^* - t^k \rangle \geq \lambda_k \big( f(x^k, t^k) - f(x^k, y^k) - c_1 \|y^k - x^k\|^2 - c_2 \|t^k - y^k\|^2 \big). \tag{3.10}$$

Similarly, since $y^k$ is the unique solution of the strongly convex program

$$\min \left\{ \frac{1}{2} \|y - x^k\|^2 + \lambda_k f(x^k, y) \;\Big|\; y \in C \right\},$$

we have

$$\lambda_k \big( f(x^k, y) - f(x^k, y^k) \big) \geq \langle y^k - x^k, y^k - y \rangle, \quad \forall y \in C.$$

Substituting $y = t^k \in C$ into the last inequality, we have

$$\lambda_k \big( f(x^k, t^k) - f(x^k, y^k) \big) \geq \langle y^k - x^k, y^k - t^k \rangle. \tag{3.11}$$

Since

$$2\langle t^k - x^k, x^* - t^k \rangle = \|x^k - x^*\|^2 - \|t^k - x^k\|^2 - \|t^k - x^*\|^2,$$

from (3.10), (3.11), we have

$$\begin{aligned} \|x^k - x^*\|^2 - \|t^k - x^k\|^2 - \|t^k - x^*\|^2 \geq{}& 2\langle y^k - x^k, y^k - t^k \rangle - 2\lambda_k c_1 \|x^k - y^k\|^2 \\ & - 2\lambda_k c_2 \|t^k - y^k\|^2. \end{aligned}$$

Hence, we have

$$\begin{aligned} \|t^k - x^*\|^2 \leq{}& \|x^k - x^*\|^2 - \|t^k - x^k\|^2 - 2\langle y^k - x^k, y^k - t^k \rangle \\ & + 2\lambda_k c_1 \|x^k - y^k\|^2 + 2\lambda_k c_2 \|t^k - y^k\|^2 \\ ={}& \|x^k - x^*\|^2 - \|(t^k - y^k) + (y^k - x^k)\|^2 - 2\langle y^k - x^k, y^k - t^k \rangle \\ & + 2\lambda_k c_1 \|x^k - y^k\|^2 + 2\lambda_k c_2 \|t^k - y^k\|^2 \\ \leq{}& \|x^k - x^*\|^2 - \|t^k - y^k\|^2 - \|x^k - y^k\|^2 + 2\lambda_k c_1 \|x^k - y^k\|^2 + 2\lambda_k c_2 \|t^k - y^k\|^2 \\ ={}& \|x^k - x^*\|^2 - (1 - 2\lambda_k c_1) \|x^k - y^k\|^2 - (1 - 2\lambda_k c_2) \|y^k - t^k\|^2. \end{aligned}$$

The implies that the inequality (3.2) holds.

Step 2. Next, we show that

$$\bigcap_{i=1}^{N} F(S_i) \cap \mathrm{Sol}\big(EP(f)\big) \subseteq C_k$$

for all $k \geq 0$.

Using Step 1 and $\bar{y}^k = (1-\gamma_k)x^k + \gamma_k t^k$, we have

$$\begin{aligned}\|\bar{y}^k - x^*\|^2 &= \|(1-\gamma_k)(x^k - x^*) + \gamma_k(t^k - x^*)\|^2 \\ &\le (1-\gamma_k)\|x^k - x^*\|^2 + \gamma_k\|t^k - x^*\|^2 \\ &\le (1-\gamma_k)\|x^k - x^*\|^2 + \gamma_k\{\|x^k - x^*\|^2 - (1-2\lambda_k c_1)\|x^k - y^k\|^2 \\ &\quad - (1-2\lambda_k c_2)\|y^k - t^k\|^2\} \\ &= \|x^k - x^*\|^2 - \gamma_k(1-2\lambda_k c_1)\|x^k - y^k\|^2 - \gamma_k(1-2\lambda_k c_2)\|y^k - t^k\|^2, \end{aligned} \tag{3.12}$$

where $x^* \in \mathrm{Sol}(EP(f))$.

Set

$$\bar{S}_k := \sum_{i=1}^{N} \lambda_{k,i} S_i.$$

Let $x^* \in \bigcap_{i=1}^{p} \mathrm{Fix}(S_i) \cap \mathrm{Sol}(EP(f))$, using Proposition 2.3(d), (3.12) and the relation

$$\|\lambda x + (1-\lambda)y\|^2 = \lambda\|x\|^2 + (1-\lambda)\|y\|^2 - \lambda(1-\lambda)\|x-y\|^2, \quad \forall x, y \in H, \lambda \in [0,1],$$

and $z^k = (1-\alpha_k-\beta_k)\bar{y}^k + \alpha_k t^k + \beta_k \bar{S}_k(t^k)$, we have

$$\begin{aligned}&\|z^k - x^*\|^2 \\ &\quad = \left\|(1-\alpha_k-\beta_k)(\bar{y}^k - x^*) + (\alpha_k+\beta_k)\frac{1}{\alpha_k+\beta_k}\{\alpha_k(t^k - x^*) + \beta_k(\bar{S}_k(t^k) - x^*)\}\right\|^2 \\ &\quad \le (1-\alpha_k-\beta_k)\|\bar{y}^k - x^*\|^2 + (\alpha_k+\beta_k)\left\|\frac{\alpha_k}{\alpha_k+\beta_k}(t^k - x^*) + \frac{\beta_k}{\alpha_k+\beta_k}(\bar{S}_k(t^k) - x^*)\right\|^2 \\ &\quad = (1-\alpha_k-\beta_k)\|\bar{y}^k - x^*\|^2 + \alpha_k\|t^k - x^*\|^2 + \beta_k\|\bar{S}_k(t^k) - x^*\|^2 - \frac{\alpha_k\beta_k}{\alpha_k+\beta_k}\|\bar{S}_k(t^k) - t^k\|^2 \\ &\quad \le (1-\alpha_k-\beta_k)\|\bar{y}^k - x^*\|^2 + \alpha_k\|t^k - x^*\|^2 \\ &\qquad + \beta_k(\|t^k - x^*\|^2 + \bar{L}\|(I-\bar{S}_k)(t^k) - (I-\bar{S}_k)(x^*)\|^2) - \frac{\alpha_k\beta_k}{\alpha_k+\beta_k}\|\bar{S}_k(t^k) - t^k\|^2 \\ &\quad \le (1-\alpha_k-\beta_k)\|\bar{y}^k - x^*\|^2 + (\alpha_k+\beta_k)\|t^k - x^*\|^2 + \left(\beta_k\bar{L} - \frac{\alpha_k\beta_k}{\alpha_k+\beta_k}\right)\|\bar{S}_k(t^k) - t^k\|^2 \\ &\quad \le (1-\alpha_k-\beta_k)(\|x^k - x^*\|^2 - \gamma_k(1-2\lambda_k c_1)\|x^k - y^k\|^2 - \gamma_k(1-2\lambda_k c_2)\|y^k - t^k\|^2) \\ &\qquad + (\alpha_k+\beta_k)(\|x^k - x^*\|^2 - (1-2\lambda_k c_1)\|x^k - y^k\|^2 - (1-2\lambda_k c_2)\|y^k - t^k\|^2) \\ &\qquad + \left(\beta_k\bar{L} - \frac{\alpha_k\beta_k}{\alpha_k+\beta_k}\right)\|\bar{S}_k(t^k) - t^k\|^2 \\ &\quad \le \|x^k - x^*\|^2 - m_k(1-2\lambda_k c_1)\|x^k - y^k\|^2 - m_k(1-2\lambda_k c_2)\|y^k - t^k\|^2 \\ &\qquad - \beta_k\left(\frac{\alpha_k}{\alpha_k+\beta_k} - \bar{L}\right)\|\bar{S}_k(t^k) - t^k\|^2 \\ &\quad \le \|x^k - x^*\|^2 - \beta_k\left(\frac{\alpha_k}{\alpha_k+\beta_k} - \bar{L}\right)\|\bar{S}_k(t^k) - t^k\|^2, \end{aligned} \tag{3.13}$$

where $m_k = \gamma_k + (1-\gamma_k)(\alpha_k + \beta_k)$. This means that $x^* \in C_k$. Hence

$$\bigcap_{i=1}^{N} F(S_i) \cap \operatorname{Sol}\bigl(EP(f)\bigr) \subseteq C_k, \quad \forall k \geq 0.$$

Step 3. Now, we have to prove that

$$\bigcap_{i=1}^{N} F(S_i) \cap \operatorname{Sol}\bigl(EP(f)\bigr) \subseteq C_k \cap Q_k$$

for all $k \geq 0$.

We show this assertion by mathematical induction. For $k = 0$ we have $Q_0 = C$. Hence by Step 2, we obtain

$$\bigcap_{i=1}^{N} F(S_i) \cap \operatorname{Sol}\bigl(EP(f)\bigr) \subseteq P_0 \cap Q_0.$$

Assume that for some $k \geq 0$,

$$\bigcap_{i=1}^{N} F(S_i) \cap \operatorname{Sol}\bigl(EP(f)\bigr) \subseteq C_k \cap Q_k. \tag{3.14}$$

From $x^{k+1} = \operatorname{Pr}_{C_k \cap Q_k}(x^0)$ it follows that

$$\langle x^{k+1} - x, x^0 - x^{k+1} \rangle \geq 0, \quad \forall x \in C_k \cap Q_k.$$

Using this and (3.14), we have

$$\langle x^{k+1} - x, x^0 - x^{k+1} \rangle \geq 0, \quad \forall x \in \bigcap_{i=1}^{N} F(S_i) \cap \operatorname{Sol}\bigl(EP(f)\bigr).$$

Hence we have

$$\bigcap_{i=1}^{N} F(S_i) \cap \operatorname{Sol}\bigl(EP(f)\bigr) \subseteq Q_{k+1}.$$

Then it follows from Step 2 that

$$\bigcap_{i=1}^{N} F(S_i) \cap \operatorname{Sol}\bigl(EP(f)\bigr) \subseteq C_{k+1} \cap Q_{k+1}.$$

Consequently, we have

$$\bigcap_{i=1}^{N} F(S_i) \cap \operatorname{Sol}\bigl(EP(f)\bigr) \subseteq C_k \cap Q_k, \quad \forall k \geq 0.$$

Step 4. Next, we claim that

$$\begin{aligned}\lim_{k\to\infty}\|x^{k+1}-x^k\| &= \lim_{k\to\infty}\|x^k-z^k\|\\ &= \lim_{k\to\infty}\|x^k-y^k\|\\ &= \lim_{k\to\infty}\|x^k-t^k\|\\ &= \lim_{k\to\infty}\|\bar{S}_k(t^k)-t^k\|\\ &= 0.\end{aligned}$$

It follows from Step 2 and $x^{k+1}=\mathrm{Pr}_{C_k\cap Q_k}(x^0)$ that

$$\|x^{k+1}-x^0\| \le \|\mathrm{Pr}_{\bigcap_{i=1}^N F(S_i)\cap \mathrm{Sol}(EP(f))}(x^0)-x^0\|, \quad \forall k\ge 0. \tag{3.15}$$

Hence, we get that $\{x^k\}$ is bounded. By Step 1, also the sequences $\{t^k\}$ and $\{z^k\}$ are bounded. Otherwise, we have

$$\langle x^k-x, x^0-x^k\rangle \ge 0, \quad \forall x\in Q_k,$$

and hence $x^k=\mathrm{Pr}_{Q_k}(x^0)$. Using this and $x^{k+1}\in C_k\cap Q_k\subseteq Q_k$, we have

$$\|x^k-x^0\| \le \|x^{k+1}-x^0\|, \quad \forall k\ge 0.$$

Therefore, there exists

$$A=\lim_{k\to\infty}\|x^k-x^0\|. \tag{3.16}$$

Using $x^k=\mathrm{Pr}_{Q_k}(x^0)$, $x^{k+1}\in Q_k$ and the property of projections

$$\|\mathrm{Pr}_{Q_k}(x)-x\|^2 \le \|x-y\|^2-\|\mathrm{Pr}_{Q_k}(x)-y\|^2, \quad \forall x\in H, y\in Q_k,$$

we have

$$\|x^{k+1}-x^k\|^2 \le \|x^{k+1}-x^0\|^2-\|x^k-x^0\|^2, \quad \forall k\ge 0.$$

Combining this and (3.16), we get

$$\lim_{k\to\infty}\|x^{k+1}-x^k\|=0. \tag{3.17}$$

It follows from $x^{k+1}=\mathrm{Pr}_{C_k\cap Q_k}(x^0)$ that $x^{k+1}\in C_k$, *i.e.*,

$$\|z^k-x^{k+1}\| \le \|x^k-x^{k+1}\|.$$

Hence

$$\|x^k-z^k\| \le \|x^k-x^{k+1}\|+\|x^{k+1}-z^k\| \le 2\|x^k-x^{k+1}\|, \quad \forall k\ge 0.$$

Then, by (3.17), we have

$$\lim_{k\to\infty}\|x^k - z^k\| = 0. \tag{3.18}$$

Step 2 and (3.16) imply that $\{t^k\}$ is bounded, and hence $\{\bar{S}_k(t^k) - t^k\}$ and $\{z^k\}$ are also bounded.

By (3.13), we have

$$\begin{aligned}\beta_k\left(\frac{\alpha_k}{\alpha_k+\beta_k} - \bar{L}\right)\|\bar{S}_k(t^k) - t^k\|^2 &\le \|x^k - x^*\|^2 - \|z^k - x^*\|^2 \\ &= (\|x^k - x^*\| - \|z^k - x^*\|)(\|x^k - x^*\| + \|z^k - x^*\|) \\ &\le \|x^k - z^k\|(\|x^k - x^*\| + \|z^k - x^*\|).\end{aligned}$$

From this and (3.18), we obtain

$$\lim_{k\to\infty}\|\bar{S}_k(t^k) - t^k\| = 0. \tag{3.19}$$

Using (3.13), we also have

$$m_k(1 - 2\lambda_k c_1)\|x^k - y^k\|^2 \le \|x^k - z^k\|(\|x^k - x^*\| + \|z^k - x^*\|),$$

and hence

$$\lim_{k\to\infty}\|x^k - y^k\| = 0. \tag{3.20}$$

Similarly, we have

$$\lim_{k\to\infty}\|t^k - y^k\| = 0. \tag{3.21}$$

Combining (3.20), (3.21) and $\|x^k - t^k\| \le \|x^k - y^k\| + \|y^k - t^k\|$, we have

$$\lim_{k\to\infty}\|x^k - t^k\| = 0. \tag{3.22}$$

This completes the proof of Step 4.

In Step 5 and Step 6 of this theorem, we consider weakly clusters of $\{x^k\}$. It follows from (3.15) that the sequence $\{x^k\}$ is bounded, and hence there exists a subsequence $\{x^{k_j}\}$ converging weakly to $\bar{x}$ as $j \to \infty$. By Step 4, also the sequences $\{y^{k_j}\}$, $\{t^{k_j}\}$ and $\{z^{k_j}\}$ converge weakly to $\bar{x}$.

Step 5. Claim that $\bar{x} \in \bigcap_{i=1}^N F(S_i)$.

For each $i = 1, \dots, N$, we suppose that $\{\lambda_{k_j,i}\}$ converges $\bar{\lambda}_i$ as $j \to \infty$ such that $\sum_{i=1}^p \bar{\lambda}_i = 1$. Then we have

$$S_{k_j}(x) \to S(x) := \sum_{i=1}^N \bar{\lambda}_i S_i(x) \quad (\text{as } j \to \infty), \forall x \in C.$$

Since $\sum_{i=1}^{N}\bar{\lambda}_i = 1$, from Step 4 and

$$\begin{aligned}\|t^{k_j} - S(t^{k_j})\| &\le \|t^{k_j} - \bar{S}_{k_j}(t^{k_j})\| + \|\bar{S}_{k_j}(t^{k_j}) - S(t^{k_j})\| \\ &= \|t^{k_j} - \bar{S}_{k_j}(t^{k_j})\| + \left\|\sum_{i=1}^{N}\lambda_{k_j,i}S_i(t^{k_j}) - \sum_{i=1}^{N}\bar{\lambda}_i S_i(t^{k_j})\right\| \\ &= \|t^{k_j} - \bar{S}_{k_j}(t^{k_j})\| + \left\|\sum_{i=1}^{N}(\lambda_{k_j,i} - \bar{\lambda}_i)S_i(t^{k_j})\right\| \\ &\le \|t^{k_j} - \bar{S}_{k_j}(t^{k_j})\| + \sum_{i=1}^{N}|\lambda_{k_j,i} - \bar{\lambda}_i|\,\|S_i(t^{k_j})\|,\end{aligned}$$

we obtain that $\lim_{k\to\infty}\|t^{k_j} - S(t^{k_j})\| = 0$. By Proposition 2.3(b), we have

$$\bar{x} \in F(S) = F\left(\sum_{i=1}^{N}\bar{\lambda}_i S_i\right).$$

Then, it implies that $\bar{x} \in \bigcap_{i=1}^{N} F(S_i)$ from Proposition 2.3(e).

Step 6. Now we prove that if $x^{k_j} \rightharpoonup \bar{x}$ as $j \to \infty$, then we have $\bar{x} \in \mathrm{Sol}(EP(f))$.

Since $y^k$ is the unique strongly convex problem

$$\min\left\{\frac{1}{2}\|x - x^k\|^2 + f(x^k, y) \;\middle|\; y \in C\right\},$$

from Lemma 3.1, we have

$$0 \in \partial_2\left(\lambda_k f(x^k, y) + \frac{1}{2}\|y - x^k\|^2\right)(y^k) + N_C(y^k).$$

It follows that

$$0 = \lambda_k w + y^k - x^k + \bar{w},$$

where $w \in \partial_2 f(x^k, y^k)$ and $\bar{w} \in N_C(y^k)$. The definition of the normal cone $N_C$ implies that

$$\langle y^k - x^k, y - y^k\rangle \ge \lambda_k\langle w, y^k - y\rangle, \quad \forall y \in C. \tag{3.23}$$

On the other hand, since $f(x^k, \cdot)$ is subdifferentiable on $C$, by the Moreau-Rockafellar theorem [32], there exists $w \in \partial_2 f(x^k, y^k)$ such that

$$f(x^k, y) - f(x^k, y^k) \ge \langle w, y - y^k\rangle, \quad \forall y \in C.$$

Combining this with (3.23), we have

$$\lambda_k\big(f(x^k, y) - f(x^k, y^k)\big) \ge \langle y^k - x^k, y^k - y\rangle, \quad \forall y \in C.$$

Hence

$$\lambda_{k_j}\big(f(x^{k_j}, y) - f(x^{k_j}, y^{k_j})\big) \ge \langle y^{k_j} - x^{k_j}, y^{k_j} - y\rangle, \quad \forall y \in C.$$

Then, using $\{\lambda_k\} \subset [a,b] \subset (0, \frac{1}{L})$, Step 2, $x^{k_j} \rightharpoonup \bar{x}$ as $j \to \infty$ and weak continuity of $f$, we have

$$f(\bar{x}, y) \geq 0, \quad \forall y \in C.$$

This means that $\bar{x} \in \mathrm{Sol}(EP(f))$.

Step 7. Finally, we claim that the sequences $\{x^k\}$, $\{y^k\}$, $\{z^k\}$ and $\{t^k\}$ converge strongly to the same point $x^*$, where

$$x^* = \Pr_{\bigcap_{i=1}^N F(S_i) \cap \mathrm{Sol}(EP(f))}(x^0).$$

From Step 5 and Step 6 it follows that for every weakly cluster point $\bar{x}$ of the sequence $\{x^k\}$,

$$\bar{x} \in \bigcap_{i=1}^N F(S_i) \cap \mathrm{Sol}(EP(f)).$$

On the other hand, using the definition of $Q_k$, we have

$$x^k = \Pr_{Q_k}(x^0).$$

Combining this with (3.15), we obtain

$$\|x^0 - x^k\| \leq \|x^0 - x\|$$

for all $x \in \bigcap_{i=1}^N F(S_i, C) \cap \mathrm{Sol}(EP(f))$. For $x = x^*$, we have

$$\|x^0 - x^k\| \leq \|x^0 - x^*\|.$$

By Lemma 3.2, we know that the sequence $\{x^k\}$ converges strongly to $x^*$ as $k \to \infty$, where

$$x^* = \Pr_{\bigcap_{i=1}^N F(S_i, C) \cap \mathrm{Sol}(EP(f))}(x^0).$$

We also have that $y^k, z^k, t^k \to x^*$ as $k \to \infty$ by Step 4. □

## 4 Applications

Let $C$ be a nonempty closed convex subset of a real Hilbert space $H$. Let $F$ be a function from $C$ into $H$. In this section, we consider the variational inequality problem which is presented as follows:

Find $x^* \in C$ such that

$$\langle F(x^*), x - x^* \rangle \geq 0, \quad \forall x \in C. \qquad VI(F)$$

Let $f : C \times C \to \mathbf{R}$ be defined by $f(x,y) := \langle F(x), y - x \rangle$. Then problem $EP(f)$ can be written in $VI(F)$. The set of solutions of $VI(F)$ is denoted by $\mathrm{Sol}(VI(F))$.

The function $F$ is called

- *strongly monotone* on $C$ with $\beta > 0$ if

$$\langle F(x) - F(y), x - y\rangle \geq \beta\|x - y\|^2, \quad \forall x, y \in C;$$

- *monotone* on $C$ if

$$\langle F(x) - F(y), x - y\rangle \geq 0, \quad \forall x, y \in C;$$

- *pseudomonotone* on $C$ if

$$\langle F(y), x - y\rangle \geq 0 \quad \Rightarrow \quad \langle F(x), x - y\rangle \geq 0, \quad \forall x, y \in C;$$

- *Lipschitz continuous* on $C$ with constants $L > 0$ if

$$\|F(x) - F(y)\| \leq L\|x - y\|, \quad \forall x, y \in C.$$

Since

$$\begin{aligned} y^k &= \operatorname{argmin}\left\{\lambda_k f\left(x^k, y\right) + \frac{1}{2}\left\|y - x^k\right\|^2 \,\middle|\, y \in C\right\} \\ &= \operatorname{argmin}\left\{\lambda_k\left\langle F\left(x^k\right), y - x^k\right\rangle + \frac{1}{2}\left\|y - x^k\right\|^2 \,\middle|\, y \in C\right\} \\ &= \operatorname{Pr}_C\left(x^k - \lambda_k F\left(x^k\right)\right), \end{aligned}$$

from Algorithm 2.7, we obtain the algorithm for finding a common element of the set of fixed points of $p$ strict pseudocontractions and the solution set of variational inequality problem $VI(F)$.

**Algorithm 4.1**

Initialization: Choose positive sequences $\{\lambda_k\}$, $\{\alpha_k\}$, $\{\beta_k\}$, $\{\gamma_k\}$ and $\{\lambda_{k,i}\}$ satisfying the conditions:

$$\begin{cases} \alpha_k + \beta_k \leq 1, \quad \forall k \geq 0, \\ \liminf_{k\to\infty} \beta_k \in (0, 1), \\ \liminf_{k\to\infty} \frac{\alpha_k}{\alpha_k+\beta_k} \in (\bar{L}, 1), \quad \text{where } \bar{L} := \max\{L_i \mid 1 \leq i \leq N\}, \\ \liminf_{k\to\infty}(\gamma_k + (1 - \gamma_k)(\alpha_k + \beta_k)) > 0, \quad \{\gamma_k\} \subset (0, 1), \\ \{\lambda_k\} \subset [a, b] \quad \text{for some } a, b \in (0, \frac{1}{L}), \\ \sum_{i=1}^N \lambda_{k,i} = 1 \quad \text{for all } k \geq 1. \end{cases}$$

Find an initial point $x^0 \in C$.

Iteration $k$: Perform the three steps below.

- Step 1. Solve two strongly convex programs:

$$\begin{cases} y^k := \operatorname{Pr}_C(x^k - \lambda_k F(x^k)), \\ t^k := \operatorname{Pr}_C(x^k - \lambda_k F(y^k)). \end{cases}$$

- Step 2. Compute the iterations

$$\begin{cases} \bar{y}^k := (1-\gamma_k)x^k + \gamma_k t^k, \\ z^k := (1-\alpha_k-\beta_k)\bar{y}^k + \alpha_k t^k + \beta_k \sum_{i=1}^N \lambda_{k,i} S_i(t^k). \end{cases}$$

- Step 3. Set

$$\begin{cases} C_k := \{z \in C \mid \|z^k - z\|^2 \leq \|x^k - z\|^2 - \beta_k(\frac{\alpha_k}{\alpha_k+\beta_k} - \bar{L})\|\bar{S}_k(t^k) - t^k\|^2\}, \\ Q_k := \{z \in C \mid \langle x^k - z, x^0 - x^k\rangle \geq 0\}. \end{cases}$$

Compute $x^{k+1} := \mathrm{Pr}_{C_k \cap Q_k}(x^0)$.

Increase $k$ by one and go back to Step 1.

Now, we can prove the following convergence theorem with respect to $VI(F)$ from Theorem 3.3.

**Theorem 4.2** *Let $C$ be a nonempty closed convex subset of a real Hilbert space $H$. Let $F$ be a function from $C$ into $H$ such that $F$ is pseudomonotone, weakly continuous and $L$-Lipschitz continuous on $C$. If each $i = 1, \dots, N$, $S_i : C \to C$ is $L_i$-strict pseudocontraction for some $0 \leq L_i < 1$ and*

$$\bigcap_{i=1}^N F(S_i) \cap \mathrm{Sol}\big(VI(F)\big) \neq \emptyset,$$

*then the sequences $\{x^k\}$, $\{y^k\}$ and $\{z^k\}$ generated by Algorithm 4.1 converge strongly to the same point $x^* \in \bigcap_{i=1}^p \mathrm{Fix}(S_i) \cap \mathrm{Sol}(F, C)$, where*

$$x^* = \mathrm{Pr}_{\bigcap_{i=1}^N F(S_i) \cap \mathrm{Sol}(VI(F))}\big(x^0\big).$$

**Competing interests**

The authors declare that they have no competing interests.

**Authors' contributions**

The main idea of this paper was proposed by JKK. JKK and WHL prepared the manuscript initially and performed all the steps of proof in this research. Both authors read and approved the final manuscript.

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