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A new class of general nonlinear random set-valued variational inclusion problems involving *A*-maximal *m*-relaxed *η*-accretive mappings and random fuzzy mappings in Banach spaces

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Abstract

At the present article, we consider a new class of general nonlinear random *A*-maximal *m*-relaxed *η*-accretive equations with random relaxed cocoercive mappings and random fuzzy mappings in *q*-uniformly smooth Banach spaces. By using the resolvent mapping technique for *A*-maximal *m*-relaxed *η*-accretive mappings due to Lan et al. and Chang's lemma, we construct a new iterative algorithm with mixed errors for finding the approximate solutions of this class of nonlinear random equations. We also verify that the approximate solutions obtained by the our proposed algorithm converge to the exact solution of the general nonlinear random *A*-maximal *m*-relaxed *η*-accretive equations with random relaxed cocoercive mappings and random fuzzy mappings in *q*-uniformly smooth Banach spaces.
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1 Introduction

The theory of variational inequalities was extended and generalized in many different directions because of its applications in mechanics, physics, optimization, economics and engineering sciences. For the applications, physical formulation, numerical methods and other aspects of variational inequalities (see [1-63] and the references therein). Quasi-variational inequalities are generalized forms of variational inequalities in which the constraint set depend on the solution. These were introduced and studied by Bensoussan et al. [11]. In 1991, Chang and Huang [16,17] introduced and studied some new classes of complementarity problems and variational inequalities for set-valued mappings with compact values in Hilbert spaces. An useful and important generalization of the variational inequalities is called the variational inclusions, due to Hassouni and Moudafi [34], which have wide applications in the fields of optimization and control, economics and transportation equilibrium, engineering science.

Meanwhile, it is known that accretivity of the underlying operator plays indispensable roles in the theory of variational inequality and its generalizations. In 2001, Huang and Fang [41] were the first to introduce generalized m-accretive mapping and gave the definition of the resolvent operator for generalized m-accretive mappings in Banach spaces. Subsequently, Verma [59,60] introduced and studied new notions of A-monotone and (A, η)-monotone operators and studied some properties of them in Hilbert spaces. In [52], Lan et al. first introduced the concept of (A, η)-accretive mappings, which generalizes the existing η-subdifferential operators, maximal η-monotone operators, H-monotone operators, A-monotone operators, (H, η)-monotone operators, (A, η)-monotone operators in Hilbert spaces, H-accretive mapping, generalized m-accretive mappings and (H, η)-accretive mappings in Banach spaces.

On the other hand, the fuzzy set theory which was introduced by Professor Lotfi Zadeh [62] at the university of California in 1965 has emerged as an interesting and fascinating branch of pure and applied sciences. The applications of the fuzzy set theory can be found in many branches of regional, physical, mathematical and engineering sciences (see, for example [10,32,63]). In 1989, by using the concept of fuzzy set, Chang and Zu [20] first introduced and studied a class of variational inequalities for fuzzy mappings. Since then several classes of variational inequalities with fuzzy mappings were considered by Chang and Haung [15], Ding [30], Ding and Park [31], Haung [36], Kumam and Petrot [48], Noor [55] and Park and Jeong [56,57] in Hilbert spaces. Recently, Huang and Lan [43], considered nonlinear equations with fuzzy mapping in fuzzy normed spaces and subsequently Lan and Verma [54] considered fuzzy variational inclusion problems in Banach spaces. It is worth to mention that variational inequalities with fuzzy mapping have been useful in the study of equilibrium and optimal control problem (see, for example [14]).

The random variational inequality and random quasi-variational inequality problems, random variational inclusion problems and random quasi-complementarity problems have been introduced and studied by Chang [13], Chang and Huang [18,19], Chang and Zhu [21], Cho et al. [22], Ganguly and Wadhawa [33], Huang and Cho [40], Khan et al. [47] and Lan [51], etc. Recently, Lan et al. [53] introduced and studied a class of general nonlinear random set-valued operator equations involving generalized m-accretive mappings in Banach spaces. They also established the existence theorems of the solution and convergence theorems of the generalized random iterative procedures with errors for these nonlinear random set-valued operator equations in q-uniformly smooth Banach spaces. Cho and Lan [23] considered and studied a class of generalized nonlinear random (A, η)-accretive equations with random relaxed cocoercive mappings in Banach spaces and by introducing some random iterative algorithms, they proved the convergence of iterative sequences generated by proposed algorithms. Further, by considering the concepts of random mappings and fuzzy mappings, Haung [39] was first introduced the concept of random fuzzy mapping. Subsequently, the random variational inclusion problem for random fuzzy mappings is studied by Ahmad and Bazan [4]. Very recently, Onjai-Uea and Kumam [58] introduced and studied a class of general nonlinear random (H, η)-accretive equations with random fuzzy mappings in Banach spaces and by using the resolvent mapping technique for the (H, η)-accretive mappings proved the existence and convergence theorems of the generalized random iterative algorithms for these nonlinear random equations with random fuzzy mappings in q-uniformly smooth Banach spaces.

At the present article, inspired and motivated by recent researches in this field, we shall introduce and study a new class of general nonlinear random A-maximal m-relaxed η-accretive (so called (A, η)-accretive [52]) equations with random relaxed cocoercive mappings and random fuzzy mappings in Banach spaces. By using the resolvent mapping technique for A-maximal m-relaxed η-accretive mappings due to Lan et al. [52] and Chang's lemma [12], we construct a new iterative algorithm with mixed errors for finding the approximate solutions of this class of nonlinear random equations. We also prove the existence of random solutions and the convergence of random iterative sequences generated by the our proposed algorithm in q-uniformly smooth Banach spaces. The results presented in this article improve and extend the corresponding results of [13,18,22-24,33,34,37-40,42,44,46,49,53,58] and many other recent works.

2 Preliminaries

Throughout this article, let $(\Omega, \mathcal{A}, \mu)$ be a complete σ-finite measure space and X be a separable real Banach space endowed with dual space X^*, the norm $\|.\|$ and the dual pair $\langle ., .\rangle$ between X and X^*. We denote by $\mathcal{B}(X)$, $CB(X)$ and $\hat{H}(., .)$ the class of Borel σ-fileds in X, the family of all nonempty closed bounded subsets of X and the Hausdorff metric

$$\hat{H}(A, B) = \max\left\{\sup_{x\in A}\inf_{y\in B} d(x, y), \sup_{y\in B}\inf_{x\in A} d(x, y)\right\}$$

on $CB(X)$, respectively.

The *generalized duality mapping* $j_q : X⊠X^*$ is defined by

$$J_q(x) = \{f^* \in X^* : \langle x, f^*\rangle = \|x\|^q, \|f^*\| = \|x\|^{q-1}\}, \quad \forall x \in X,$$

where $q > 1$ is a constant. In particular, J_2 is usual normalized duality mapping. It is known that, in general, $J_q(x) = \|x\|^{q-2} J_2(x)$ for all $x \neq 0$ and J_q is single-valued if X^* is strictly convex. In the sequel, we always assume that X is a real Banach space such that J_q is single-valued. If X is a Hilbert space, then J_2 becomes the identity mapping on X.

The *modulus of smoothness* of X is the function $\rho_X : [0,\infty) \to [0, \infty)$ defined by

$$\rho_X(t) = \sup\left\{\frac{1}{2}(\|x + y\| + \|x - y\|) - 1 : \|x\| \leq 1, \|y\| \leq t\right\}.$$

A Banach space X is called *uniformly smooth* if

$$\lim_{t\to 0}\frac{\rho_X(t)}{t} = 0.$$

Further, a Banach space X is called *q-uniformly smooth* if there exists a constant $c > 0$ such that

$$\rho_X(t) \leq ct^q, \quad q > 1.$$

It is well-known that Hilbert spaces, L_p(or l_p) spaces, $1 < p < \infty$, and the Sobolev spaces $W^{m, p}$, $1 < p < \infty$, are all q-uniformly smooth.

Concerned with the characteristic inequalities in q-uniformly smooth Banach spaces, Xu [61] proved the following result.

Lemma 2.1. *Let X be a real uniformly smooth Banach space. Then X is q-uniformly smooth if and only if there exists a constant $c_q > 0$ such that for all $x, y \in X$,*

$$\|x + y\|^q \leq \|x\|^q + q\langle y, J_q(x)\rangle + c_q\|y\|^q.$$

Definition 2.2. A mapping $x : \Omega \to X$ is said to be *measurable* if, for any $B \in \mathcal{B}(X)$, $\{t \in \Omega : x(t) \in B\} \in \mathcal{A}$.

Definition 2.3. A mapping $T : \Omega \times X \to X$ is called a *random* mapping if, for any $x \in X$, $T(., x)$: $\Omega \to X$ is measurable. A random mapping T is said to be *continuous* if, for any $t \in \Omega$, the mapping $T(t, .)$: $X \to X$ is continuous.

Similarly, we can define a random mapping $a : \Omega \times X \times X \to X$. We shall write $T_t(x) = T(t, x(t))$ and $a_t(x, y) = a(t, x(t), y(t))$ for all $t \in \Omega$ and $x(t), y(t) \in X$.

It is well-known that a measurable mapping is necessarily a random mapping.

Definition 2.4. A set-valued mapping $V : \Omega ⊠ X$ is said to be *measurable* if, for any $B \in \mathcal{B}(X)$, $V^{-1}(B) = \{t \in \Omega : V(t) \cap B \neq \emptyset\} \in \mathcal{A}$.

Definition 2.5. A mapping $u : \Omega \to X$ is called a *measurable selection* of a set-valued measurable mapping $V : \Omega ⊠ X$ if, u is measurable and for any $t \in \Omega$, $u(t) \in V(t)$.

Definition 2.6. A set-valued mapping $V : \Omega \times X ⊠ X$ is called a *random* set-valued mapping if, for any $x \in X$, $V(., x)$ is measurable. A random set-valued mapping $V : \Omega \times X ⊠ X$ is said to be *$\hat{H}$-continuous* if, for any $t \in \Omega$, $V(t, .)$ is continuous in the Hausdorff metric on $CB(X)$.

Definition 2.7. Let X be a q-uniformly smooth Banach space, $T, A : \Omega \times X \to X$ and $\eta : \Omega \times X \times X \to X$ be random single-valued mappings. Then

(a) T is said to be *accretive* if

$$\langle T_t(x) - T_t(y), J_q(x(t) - y(t))\rangle \geq 0, \quad \forall x(t), y(t) \in X, \quad t \in \Omega;$$

(b) T is called *strictly accretive* if T is accretive and

$$\langle T_t(x) - T_t(y), J_q(x(t) - y(t))\rangle = 0,$$

if and only if $x(t) = y(t)$ for all $t \in \Omega$;

(c) T is said to be *r-strongly accretive* if there exists a measurable function $r : \Omega \to (0, \infty)$ such that

$$\langle T_t(t) - T_t(y), J_q(x(t) - y(t))\rangle \geq r(t)\|x(t) - y(t)\|^q, \quad \forall x(t), y(t) \in X, \quad t \in \Omega;$$

(d) T is said to be *(θ, κ)-relaxed cocoercive* if there exist measurable functions $\theta, \kappa : \Omega \to (0, \infty)$ such that

$$\langle T_t(x) - T_t(y), J_q(x(t) - y(t))\rangle \geq -\theta(t)\|T_t(x) - T_t(y)\|^q + \kappa(t)\|x(t) - y(t)\|^q, \quad \forall x(t), y(t) \in X, \quad t \in \Omega;$$

(e) T is called *ϱ-Lipschitz continuous* if there exists a measurable function $\varrho : \Omega \to (0, \infty)$ such that

$$\|T_t(x) - T_t(y)\| \leq \varrho(t)\,\|x(t) - y(t)\|, \quad \forall x(t), y(t) \in X, \quad t \in \Omega;$$

(f) η is said to be *τ-Lipschitz continuous* if there exists a measurable function $\tau : \Omega \to (0, \infty)$ such that

$$\|\eta_t(x, y)\| \leq \tau(t)\,\|x(t) - y(t)\|, \quad \forall x(t), y(t) \in X, \quad t \in \Omega;$$

(g) η is said to be *μ-Lipschitz continuous in the second argument* if there exists a measurable function $\mu : \Omega \to (0, \infty)$ such that

$$\|\eta_t(x, u) - \eta_t(y, u)\| \leq \mu(t)\,\|x(t) - y(t)\|, \quad \forall x(t), y(t), u(t) \in X, \quad t \in \Omega;$$

In a similar way to part (g), we can define the Lipschitz continuity of the mapping η in the third argument.

Definition 2.8. Let X be a q-uniformly smooth Banach space, $\eta : \Omega \times X \times X \to X$ and $H, A : \Omega \times X \to X$ be three random single-valued mappings. Then set-valued mapping $M : \Omega \times X⊠X$ is said to be:

(a) *accretive* if

$$\langle u(t) - v(t), J_q(x(t) - y(t))\rangle \geq 0, \quad \forall x(t), y(t) \in X, \quad u(t) \in M_t(x), \quad v(t) \in M_t(y), \quad t \in \Omega;$$

(b) *η-accretive* if

$$\langle u(t) - v(t), J_q(\eta_t(x, y))\rangle \geq 0, \quad \forall x(t), y(t) \in X, \quad u(t) \in M_t(x), \quad v(t) \in M_t(y), \quad t \in \Omega;$$

(c) *strictly η-accretive* if M is η-accretive and the equality holds if and only if $x(t) = y(t)$, $\forall t \in \Omega$;

(d) *r-strongly η-accretive* if there exists a measurable function $r : \Omega \to (0, \infty)$ such that

$$\langle u(t) - v(t), J_q(\eta_t(x, y))\rangle \geq r(t)\|x(t) - y(t)\|^q, \quad \forall x(t), y(t) \in X, \quad u(t) \in M_t(x), \quad v(t) \in M_t(y), \quad t \in \Omega;$$

(e) *α-relaxed η-accretive* if there exists a measurable function $\alpha : \Omega \to (0, \infty)$ such that

$$\langle u(t) - v(t), J_q(\eta_t(x, y))\rangle \geq -\alpha(t)\|x(t) - y(t)\|^q, \quad \forall x(t), y(t) \in X, \quad u(t) \in M_t(x), \quad v(t) \in M_t(y), \quad t \in \Omega;$$

(f) *m-accretive* if M is accretive and $(I_t + \rho(t)M_t)(X) = X$ for all $t \in \Omega$ and for any measurable function $\rho : \Omega \to (0,\infty)$, where I denotes the identity mapping on X, $I_t(x) = x(t)$, for all $x(t) \in X$, $t \in \Omega$;

(g) *generalized m-accretive* if M is η-accretive and $(I_t + \rho(t)M_t)(X) = X$ for all $t \in \Omega$ and any measurable function $\rho : \Omega \to (0, \infty)$;

(h) *H-accretive* if M is accretive and $(H_t + \rho(t)M_t)(X) = X$ for all $t \in \Omega$ and any measurable function $\rho : \Omega \to (0, \infty)$, where $H_t(.) = H(t,.)$ for all $t \in \Omega$;

(i) *(H, η)-accretive* if M is η-accretive and $(H_t + \rho(t)M_t)(X) = X$ for all $t \in \Omega$ and any measurable function $\rho : \Omega \to (0, \infty)$;

(j) *A-maximal m-relaxed η-accretive* if M is m-relaxed η-accretive and $(A_t+\rho(t)M_t)(X) = X$ for all $t \in \Omega$ and any measurable function $\rho : \Omega \to (0, \infty)$, where $A_t(.) = A(t,.)$ for all $t \in \Omega$;

(k) *β-$\hat{H}$-Lipschitz continuous* if there exists a measurable function $\beta : \Omega \to (0, +\infty)$ such that

$$\hat{H}(M_t(x), M_t(y)) \leq \beta(t) \left\| x(t) - y(t) \right\|, \quad \forall x(t), y(t) \in X, \quad t \in \Omega.$$

Remark 2.9. (1) If $X = \mathcal{H}$ is a Hilbert space, then parts (a)-(i) of Definition 2.8 reduce to the definitions of monotone operators, η-monotone operators, strictly η-monotone operators, strongly η-monotone operators, relaxed η-monotone operators, maximal monotone operators, maximal η-monotone operators, H-monotone operators and (H, η)-monotone operators, respectively.

(2) For appropriate and suitable choices of m, A, η and X, it is easy to see that part (j) of Definition 2.8 includes a number of definitions of monotone operators and accretive mappings (see [52]).

Proposition 2.10. [52]*Let $A : \Omega \times X \to X$ be an r-strongly η-accretive mapping and $M : \Omega \times X ⊠ X$ be an A-maximal m-relaxed η-accretive mapping. Then the operator $(A_t + \rho(t)M_t)^{-1}$ is single-valued for any measurable function $\rho : \Omega \to (0, +\infty)$ and $t \in \Omega$.*

Definition 2.11. Let $A : \Omega \times X \to X$ be a strictly η-accretive mapping and $M : \Omega \times X ⊠ X$ be an A-maximal m-relaxed η-accretive mapping. Then, for any measurable function $\rho : \Omega \to (0, +\infty)$, the resolvent operator $J^{\eta_t, M_t}_{\rho(t), A_t} : X \to X$ is defined by:

$$J^{\eta_t, M_t}_{\rho(t), A_t}(u(t)) = (A_t + \rho(t)M_t)^{-1}(u(t)), \quad \forall t \in \Omega, \quad u(t) \in X.$$

Proposition 2.12. [52]*Let X be a q-uniformly smooth Banach space and $\eta : \Omega \times X \times X \to X$ be τ-Lipschitz continuous, $A : \Omega \times X \to X$ be an r-strongly η-accretive mapping and $M : \Omega \times X ⊠ X$ be an A-maximal m-relaxed η-accretive mapping. Then the resolvent operator $J^{\eta_t, M_t}_{\rho(t), A_t} : X \to X$ is $\frac{\tau^{q-1}(t)}{r(t)-\rho(t)m(t)}$-Lipschitz continuous, i.e.,*

$$\left\| J^{\eta_t, M_t}_{\rho(t), A_t}(x(t)) - J^{\eta_t, M_t}_{\rho(t), A_t}(y(t)) \right\| \leq \frac{\tau^{q-1}(t)}{r(t) - \rho(t)m(t)} \left\| x(t) - y(t) \right\|, \quad \forall x(t), y(t) \in X, \quad t \in \Omega,$$

where $\rho(t) \in \left(0, \frac{r(t)}{m(t)}\right)$ is a real-valued random variable for all $t \in \Omega$.

3 A new random variational inclusion problem and random iterative algorithm

In what follows, we denote the collection of all fuzzy sets on X by $\mathfrak{F}(X) = \{A | A : X \to [0, 1]\}$. For any set K, a mapping $\mathcal{S}$ from K into $\mathfrak{F}(X)$ is called a *fuzzy mapping*. If $\mathcal{S} : K \to \mathfrak{F}(X)$ is a fuzzy mapping, then $\mathcal{S}(x)$, for any $x \in K$, is a fuzzy set on $\mathfrak{F}(X)$ (in the sequel, we denote $\mathcal{S}(x)$ by $\mathcal{S}_x$) and $\mathcal{S}_x(y)$, for any $y \in X$, is the degree of membership of y in $\mathcal{S}_x$. For any $A \in \mathfrak{F}(X)$ and $\alpha \in [0,1]$, the set

$$(A)_\alpha = \{x \in X : A(x) \geq \alpha\}$$

is called a *α-cut* set of A.

Definition 3.1. A fuzzy mapping $\mathcal{S} : \Omega \to \mathfrak{F}(X)$ is called *measurable* if, for any $\alpha \in (0, 1]$, $(\mathcal{S}(.))_\alpha : \Omega \multimap X$ is a measurable set-valued mapping.

Definition 3.2. A fuzzy mapping $\mathcal{S} : \Omega \times X \to \mathfrak{F}(X)$ is called a *random fuzzy mapping* if, for any $x \in X$, $\mathcal{S}(., x) : \Omega \to \mathfrak{F}(X)$ is a measurable fuzzy mapping.

Now, let us introduce our main considered problem.

Suppose that $\mathcal{S}, \mathcal{T}, \mathcal{P}, \mathcal{Q}, \mathcal{G} : \Omega \times X \to \mathfrak{F}(X)$ are random fuzzy mappings, $A, p : \Omega \times X \to X$ and $\eta : \Omega \times X \times X \to X$, $N : \Omega \times X \times X \times X \to X$ are random single-valued mappings. Further, let $a, b, c, d, e : X \to [0, 1]$ be any mappings and $M : \Omega \times X \times X \multimap X$ be a random set-valued mapping such that, for each fixed $t \in \Omega$ and $z(t) \in X$, $M(t,.,z(t)) : X \multimap X$ be an A-maximal m-relaxed η-accretive mapping with $\mathrm{Im}(p) \cap \mathrm{dom}\, M(t,.,z(t)) \neq \emptyset$. Now, we consider the following problem:

For any element $h : \Omega \to X$ and any measurable function $\lambda : \Omega \to (0, +\infty)$, find measurable mappings $x, \upsilon, u, \nu, \vartheta, w : \Omega \to X$ such that for each $t \in \Omega$, $x(t) \in X$, $\mathcal{S}_{t,x(t)}(\upsilon(t)) \geq a(x(t))$, $\mathcal{T}_{t,x(t)}(u(t)) \geq b(x(t))$, $\mathcal{P}_{t,x(t)}(\nu(t)) \geq c(x(t))$, $\mathcal{Q}_{t,x(t)}(\vartheta(t)) \geq d(x(t))$, $\mathcal{G}_{t,x(t)}(w(t)) \geq e(x(t))$ and

$$h(t) \in N_t(\nu, u, \upsilon) + \lambda(t) M_t(p_t(x) - \vartheta, w), \quad \forall t \in \Omega. \tag{3.1}$$

The problem (3.1) is called *the general nonlinear random A-maximal m-relaxed η-accretive equation with random relaxed cocoercive mappings and random fuzzy mappings in Banach spaces.*

Remark 3.3. Obviously, the random fuzzy mapping includes set-valued mapping, random set-valued mapping and fuzzy mapping as the special cases. These mean that for appropriate and suitable choices of X, A, η, λ, p, M, N, $\mathcal{S}$, $\mathcal{T}$, $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{G}$ and h, one can obtain many known classes of random variational inequalities, random quasi-variational inequalities, random complementarity and random quasi-complementarity problems as special cases of the problem (3.1), (see, for example [1-3,22,23,34,37,45,49,50,53,58] and the references therein).

In the sequel, we will develop and analyze a new class of iterative methods and construct a new random iterative algorithm with mixed errors for solving the problem (3.1). For this end, we need the following lemmas.

Lemma 3.4. [12]*Let $M : \Omega \times X \to CB(X)$ be a $\hat{H}$-continuous random set-valued mapping. Then, for any measurable mapping $x : \Omega \to X$, the set-valued mapping $M(., x(.))$: $\Omega \to CB(X)$ is measurable.*

Lemma 3.5. [12]*Let $M, V : \Omega \to CB(X)$ be two measurable set-valued mappings, $\epsilon > 0$ be a constant and $x : \Omega \to X$ be a measurable selection of M. Then there exists a measurable selection $y : \Omega \to X$ of V such that, for any $t \in \Omega$,*

$$\|x(t) - y(t)\| \leq (1 + \varepsilon) \hat{H}(M(t), V(t)).$$

The following lemma offers a good approach for solving the problem (3.1).

Lemma 3.6. *The set of measurable mappings $x, \upsilon, u, \nu, \vartheta, w : \Omega \to X$ is a random solution of the problem* (3.1) *if and only if, for each $t \in \Omega$, $\upsilon(t) \in S_t(x)$, $u(t) \in T_t(x)$, $\nu(t) \in P_t(x)$, $\vartheta(t) \in Q_t(x)$, $w(t) \in G_t(x)$ and*

$$p_t(x) = \vartheta(t) + J^{\eta_t, M_t(.,w)}_{\rho(t)\lambda(t), A_t}[A_t(p_t(x) - \vartheta) - \rho(t)(N_t(\nu, u, \upsilon) - h(t))],$$

where $J^{\eta_t,M_t(.,w)}_{\rho(t)\lambda(t),A_t} = (A_t + \rho(t)\lambda(t)M_t(.,w))^{-1}$ *and* $\rho : \Omega \to (0, \infty)$ *is a measurable function.*

Proof. The fact follows directly from the definition of $J^{\eta_t,M_t(.,w)}_{\rho(t)\lambda(t),A_t}$.

In order to prove our main result, the following concepts are also needed. Let $\mathcal{S},\mathcal{T},\mathcal{P},\mathcal{Q},\mathcal{G} : \Omega \times X \to \mathfrak{F}(X)$ be five random fuzzy mappings satisfying the following condition (*): There exist five mappings $a, b, c, d, e : X \to [0,1]$ such that

$$(\mathcal{S}_{t,x(t)})_{a(x(t))} \in CB(X),\ (\mathcal{T}_{t,x(t)})_{b(x(t))} \in CB(X),\ (\mathcal{P}_{t,x(t)})_{c(x(t))} \in CB(X),$$
$$(\mathcal{Q}_{t,x(t)})_{d(x(t))} \in CB(X),\ (\mathcal{G}_{t,x(t)})_{e(x(t))} \in CB(X),\quad \forall (t,x(t)) \in \Omega \times X.$$

By using the random fuzzy mapping $\mathcal{S}$ satisfying (*) with the corresponding function $a : X \to [0,1]$, we can define a random set-valued mapping S as follows:

$$S : \Omega \times X \to CB(X),\quad (t,x(t)) \mapsto (\mathcal{S}_{t,x(t)})_{a(x(t))},\quad \forall (t,x(t)) \in \Omega \times X,$$

Where $\mathcal{S}_{t,x(t)} = \mathcal{S}(t,x(t))$. From now on, the random fuzzy mappings $\mathcal{S}$, $\mathcal{T}$, $\mathcal{P}$, $\mathcal{Q}$ and $\mathcal{G}$, are assumed to satisfying the condition (*) and we will let S, T, P, Q and G are the random set-valued mappings induced by those five random fuzzy mappings, respectively.

Now, by using Chang's lemma [12] and based on Lemma 3.6, we can construct the new following iterative algorithm for solving the problem (3.1).

Algorithm 3.7. *Let* A, p, η, M, N, $\mathcal{S}$, $\mathcal{T}$, $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{G}$, h, λ *be the same as in the problem* (3.1) *and let* S, T, P, Q, G *be* $\hat{H}$*-continuous random set-valued mappings induced by* $\mathcal{S}$, $\mathcal{T}$, $\mathcal{P}$, $\mathcal{Q}$ *and* $\mathcal{G}$, *respectively. Assume that* $\alpha : \Omega \to (0, 1]$ *is a measurable step size function. For any measurable mapping* $x_0 : \Omega \to X$, *the set-valued mappings* $S(., x_0(.))$, $T(., x_0(.))$, $P(., x_0(.))$, $Q(., x_0(.))$, $G(., x_0(.))$: $\Omega \to CB(X)$ *are measurable by Lemma* 3.4. Hence there exist measurable selections $\upsilon_0 : \Omega \to X$ of $S(., x_0(.))$, $u_0 : \Omega \to X$ of $T(., x_0(.))$, $\nu_0 : \Omega \to X$ of $P(., x_0(.))$, $\vartheta_0 : \Omega \to X$ of $Q(., x_0(.))$ and $w_0 : \Omega \to X$ of $G(., x_0(.))$ by Himmelberg [35]. For each $t \in \Omega$, set

$$x_1(t) = (1-\alpha(t))x_0(t) + \alpha(t)\{x_0(t) - p_t(x_0) + \vartheta_0(t) + J^{\eta_t,M_t(.,w_0)}_{\rho(t)\lambda(t),A_t}[A_t(p_t(x_0) - \vartheta_0) - \rho(t)(N_t(\upsilon_0,u_0,\nu_0) - h(t))]\} + \alpha(t)e_0(t) + r_0(t),$$

where $\rho(t)$ is the same as in Lemma 3.6 and e_0, $r_0 : \Omega \to X$ are measurable functions. It is easy to know that $x_1 : \Omega \to X$ is measurable. Since $\upsilon_0(t) \in S_t(x_0) \in CB(X)$, $u_0(t) \in T_t(x_0) \in CB(X)$, $\nu_0(t) \in P_t(x_0) \in CB(X)$, $\vartheta_0(t) \in Q_t(x_0) \in CB(X)$ and $w_0(t) \in G_t(x_0) \in CB(X)$, by Lemma 3.5, there exist measurable selections υ_1, u_1, ν_1, w_1, $\vartheta_1 : \Omega \to X$ of the set-valued measurable mappings $S(., x_1(.))$, $T(., x_1(.))$, $P(., x_1(.))$, $Q(., x_1(.))$ and $G(., x_1(.))$, respectively, such that, for all $t \in \Omega$,

$$\|\upsilon_0(t) - \upsilon_1(t)\| \le \left(1 + \frac{1}{1}\right)\hat{H}(S_t(x_0), S_t(x_1)),$$

$$\|u_0(t) - u_1(t)\| \le \left(1 + \frac{1}{1}\right)\hat{H}(T_t(x_0), T_t(x_1)),$$

$$\|\nu_0(t) - \nu_1(t)\| \le \left(1 + \frac{1}{1}\right)\hat{H}(P_t(x_0), P_t(x_1)),$$

$$\|\vartheta_0(t) - \vartheta_1(t)\| \le \left(1 + \frac{1}{1}\right)\hat{H}(Q_t(x_0), Q_t(x_1)),$$

$$\|w_0(t) - w_1(t)\| \le \left(1 + \frac{1}{1}\right)\hat{H}(G_t(x_0), G_t(x_1)).$$

Letting

$$x_2(t) = (1-\alpha(t))x_1(t) + \alpha(t)\{x_1(t) - p_t(x_1) + \vartheta_1(t) + J^{\eta_t, M_t(.,w_1)}_{\rho(t)\lambda(t),A_t}[A_t(p_t(x_1) - \vartheta_1) - \rho(t)(N_t(\upsilon_1, u_1, \nu_1) - h(t))]\} + \alpha(t)e_1(t) + r_1(t), \quad \forall t \in \Omega,$$

then $x_2 : \Omega \to X$ is measurable. By induction, we can define the sequences $\{x_n(t)\}$, $\{\upsilon_n(t)\}$, $\{u_n(t)\}$, $\{\nu_n(t)\}$, $\{\vartheta_n(t)\}$ and $\{w_n(t)\}$ for solving the problem (3.1) inductively satisfying

$$\begin{cases} x_{n+1}(t) = (1-\alpha(t))x_n(t) + \alpha(t)\{x_n(t) - p_t(x_n) + \vartheta_n(t) + J^{\eta_t, M_t(.,w_n)}_{\rho(t)\lambda(t),A_t}[A_t(p_t(x_n) - \vartheta_n) \\ \qquad -\rho(t)(N_t(\upsilon_n, u_n, \nu_n) - h(t))]\} + \alpha(t)e_n(t) + r_n(t), \quad \forall t \in \Omega, \\ \upsilon_n(t) \in S_t(x_n), \ \|\upsilon_n(t) - \upsilon_{n+1}(t)\| \le \left(1 + \frac{1}{1+n}\right)\hat{H}(S_t(x_n), S_t(x_{n+1})), \\ u_n(t) \in T_t(x_n), \ \|u_n(t) - u_{n+1}(t)\| \le \left(1 + \frac{1}{1+n}\right)\hat{H}(T_t(x_n), T_t(x_{n+1})), \\ \nu_n(t) \in P_t(x_n), \ \|\nu_n(t) - \nu_{n+1}(t)\| \le \left(1 + \frac{1}{1+n}\right)\hat{H}(P_t(x_n), P_t(x_{n+1})), \\ \vartheta_n(t) \in Q_t(x_n), \ \|\vartheta_n(t) - \vartheta_{n+1}(t)\| \le \left(1 + \frac{1}{1+n}\right)\hat{H}(Q_t(x_n), Q_t(x_{n+1})), \\ w_n(t) \in G_t(x_n), \ \|w_n(t) - w_{n+1}(t)\| \le \left(1 + \frac{1}{1+n}\right)\hat{H}(G_t(x_n), G_t(x_{n+1})), \end{cases} \tag{3.2}$$

where for all $n \ge 0$ and $t \in \Omega$, $e_n(t), r_n(t) \in X$ are real-valued random errors to take into account a possible inexact computation of the random resolvent operator point satisfying the following conditions:

$$\begin{cases} \lim_{n\to\infty} \|e_n(t)\| = \lim_{n\to\infty} \|r_n(t)\| = 0, \quad \forall t \in \Omega; \\ \sum_{n=0}^{\infty} \|e_n(t) - e_{n-1}(t)\| < \infty, \quad \forall t \in \Omega; \\ \sum_{n=0}^{\infty} \|r_n(t) - r_{n-1}(t)\| < \infty, \quad \forall t \in \Omega. \end{cases} \tag{3.3}$$

Remark 3.8. For a suitable and appropriate choice of the mappings $A, p, \eta, M, N, \mathcal{S}, \mathcal{T}, \mathcal{P}, \mathcal{Q}, \mathcal{G}, S, T, P, Q, G, \alpha, h, \lambda$, the sequences $\{e_n\}$, $\{r_n\}$ and the space X, Algorithm 3.7 includes many known algorithms which due to classes of variational inequalities and variational inclusions (see, for example [13,18,22-24,33,34,37-40,42,44,46,53,58]).

4 Main result

In this section, we prove the existence of solutions for the problem (3.1) and the convergence of iterative sequences generated by Algorithm 3.7 in Banach spaces.

Theorem 4.1. *Let X be a q-uniformly smooth Banach space, $A, p, \eta, M, N, \mathcal{S}, \mathcal{T}, \mathcal{P}, \mathcal{Q}, \mathcal{G}, h, \lambda$ be the same as in the problem* (3.1) and $S, T, P, Q, G : \Omega \times X \to CB(X)$ be five random set-valued mappings induced by $\mathcal{S}, \mathcal{T}, \mathcal{P}, \mathcal{Q}, \mathcal{G}$ respectively. Further, suppose that

(a) p is (γ, ϖ)-relaxed cocoercive and π-Lipschitz continuous;
(b) A is r-strongly η-accretive and σ-Lipschitz continuous;
(c) η is τ-Lipschitz continuous;
(d) S, T, P, Q and G are ξ-$\hat{H}$-Lipschitz continuous, ζ-$\hat{H}$-Lipschitz continuous, ς-$\hat{H}$-Lipschitz continuous, ϱ-$\hat{H}$-Lipschitz continuous and ι-$\hat{H}$-Lipschitz continuous, respectively;
(e) N is ϵ-Lipschitz continuous in the second argument, δ-Lipschitz continuous in the third argument and κ-Lipschitz continuous in the fourth argument;

(f) There exist the measurable functions $\mu : \Omega \to (0, +\infty)$ and $\rho : \Omega \to (0, +\infty)$ with $\rho(t) \in \left(0, \dfrac{r(t)}{\lambda(t)m(t)}\right)$, for all $t \in \Omega$, such that

$$\left\| J^{\eta_t,M_t(.,x)}_{\rho(t)\lambda(t),A_t}(z(t)) - J^{\eta_t,M_t(.,y)}_{\rho(t)\lambda(t),A_t}(z(t)) \right\| \le \mu(t)\,\|x(t) - y(t)\|, \qquad \forall t \in \Omega, x(t), y(t), z(t) \in X \tag{4.1}$$

and

$$\begin{cases} \varphi(t) = \varrho(t) + \mu(t)\iota(t) + \sqrt[q]{1 - q\varpi(t) + (q\gamma(t) + c_q)\,\pi^q(t)} < 1, \\ \sigma(t)\,(\pi(t) + \varrho(t)) + \rho(t)(\varepsilon(t)\xi(t) + \delta(t)\zeta(t) + \kappa(t)\varsigma(t)) \\ < \tau^{1-q}(t)(1 - \varphi(t))(r(t) - \rho(t)\lambda(t)m(t)), \end{cases} \tag{4.2}$$

where c_q is the same as in Lemma 2.1. Then there exists a set of measurable mappings x^*, υ^*, u^*, ν^*, ϑ^*, w^*: $\Omega \to X$ which is a random solution of the problem (3.1) and for each $t \in \Omega$, $x_n(t) \to x^*(t)$, $\upsilon_n(t) \to \upsilon^*(t)$, $u_n(t) \to u^*(t)$, $\nu_n(t) \to \nu^*(t)$, $\vartheta_n(t) \to \vartheta^*(t)$, $w_n(t) \to w^*(t)$ as $n \to \infty$, where $\{x_n(t)\}$, $\{\upsilon_n(t)\}$, $\{u_n(t)\}$, $\{\nu_n(t)\}$, $\{\vartheta_n(t)\}$ and $\{w_n(t)\}$ are the iterative sequences generated by Algorithm 3.7.

Proof. Firstly, for each $n \ge 0$, by considering (3.2) and (4.1), in view of Proposition 2.12, we see that

$$\begin{aligned}
&\|x_{n+1}(t) - x_n(t)\| \\
&\le \Big\|(1 - \alpha(t))x_n(t) + \alpha(t)\{x_n(t) - p_t(x_n) + \vartheta_n(t) + J^{\eta_t,M_t(.,w_n)}_{\rho(t)\lambda(t),A_t}[A_t(p_t(x_n) - \vartheta_n) \\
&\quad -\rho(t)(N_t(\nu_n, u_n, \upsilon_n) - h(t))]\} + \alpha(t)e_n(t) + r_n(t) - (1 - \alpha(t))x_{n-1}(t) \\
&\quad -\alpha(t)\Big\{x_{n-1}(t) - p_t(x_{n-1}) + \vartheta_{n-1}(t) + J^{\eta_t,M_t(.,w_{n-1})}_{\rho(t)\lambda(t),A_t}[A_t(p_t(x_{n-1}) - \vartheta_{n-1}) \\
&\quad -\rho(t)(N_t(\nu_{n-1}, u_{n-1}, \upsilon_{n-1}) - h(t))]\Big\} - \alpha(t)e_{n-1}(t) - r_{n-1}(t)\Big\| \\
&\le (1 - \alpha(t))\,\|x_n(t) - x_{n-1}(t)\| + \alpha(t)\,\big(\|x_n(t) - x_{n-1}(t) - (p_t(x_n) - p_t(x_{n-1}))\| \\
&\quad + \|\vartheta_n(t) - \vartheta_{n-1}(t)\| + \Big\|J^{\eta_t,M_t(.,w_n)}_{\rho(t)\lambda(t),A_t}[A_t(p_t(x_n) - \vartheta_n) - \rho(t)(N_t(\nu_n, u_n, \upsilon_n) - h(t))] \\
&\quad -J^{\eta_t,M_t(.,w_{n-1})}_{\rho(t)\lambda(t),A_t}[A_t(p_t(x_{n-1}) - \vartheta_{n-1}) - \rho(t)(N_t(\nu_{n-1}, u_{n-1}, \upsilon_{n-1}) - h(t))]\Big\|\Big) \\
&\quad +\alpha(t)\,\|e_n(t) - e_{n-1}(t)\| + \|r_n(t) - r_{n-1}(t)\| \\
&\le (1 - \alpha(t))\,\|x_n(t) - x_{n-1}(t)\| + \alpha(t)\,\big(\|x_n(t) - x_{n-1}(t) - (p_t(x_n) - p_t(x_{n-1}))\| \\
&\quad + \|\vartheta_n(t) - \vartheta_{n-1}(t)\| + \Big\|J^{\eta_t,M_t(.,w_n)}_{\rho(t)\lambda(t),A_t}[A_t(p_t(x_n) - \vartheta_n) - \rho(t)(N_t(\nu_n, u_n, \upsilon_n) - h(t))] \\
&\quad -J^{\eta_t,M_t(.,w_n)}_{\rho(t)\lambda(t),A_t}[A_t(p_t(x_{n-1}) - \vartheta_{n-1}) - \rho(t)(N_t(\nu_{n-1}, u_{n-1}, \upsilon_{n-1}) - h(t))]\Big\| \\
&\quad +J^{\eta_t,M_t(.,w_n)}_{\rho(t)\lambda(t),A_t}[A_t(p_t(x_{n-1}) - \vartheta_{n-1}) - \rho(t)(N_t(\nu_{n-1}, u_{n-1}, \upsilon_{n-1}) - h(t))] \\
&\quad -J^{\eta_t,M_t(.,w_{n-1})}_{\rho(t)\lambda(t),A_t}[A_t(p_t(x_{n-1}) - \vartheta_{n-1}) - \rho(t)(N_t(\nu_{n-1}, u_{n-1}, \upsilon_{n-1}) - h(t))]\Big\|\Big) \\
&\quad +\alpha(t)\,\|e_n(t) - e_{n-1}(t)\| + \|r_n(t) - r_{n-1}(t)\| \\
&\le (1 - \alpha(t))\,\|x_n(t) - x_{n-1}(t)\| + \alpha(t)\,\big(\|x_n(t) - x_{n-1}(t) - (p_t(x_n) - p_t(x_{n-1}))\| + \|\vartheta_n(t) - \vartheta_{n-1}(t)\| \\
&\quad +\mu(t)\,\|w_n(t) - w_{n-1}(t)\| + \frac{\tau^{q-1}(t)}{r(t) - \rho(t)\lambda(t)m(t)}\,\big(\|A_t(p_t(x_n) - \vartheta_n) - A_t(p_t(x_{n-1}) - \vartheta_{n-1})\| \\
&\quad +\rho(t)\,\|N_t(\nu_t, u_n, \upsilon_n) - N_t(\nu_{n-1}, u_{n-1}, \upsilon_{n-1})\|\big)\big) + \alpha(t)\,\|e_n(t) - e_{n-1}(t)\| + \|r_n(t) - r_{n-1}(t)\|.
\end{aligned} \tag{4.3}$$

Meanwhile, by Lemma 2.1, there exists a constant $c_q > 0$ such that

$$\begin{aligned}
&\|x_n(t) - x_{n-1}(t) - (p_t(x_n) - p_t(x_{n-1}))\|^q \\
&\le \|x_n(t) - x_{n-1}(t)\|^q - q\langle p_t(x_n) - p_t(x_{n-1}), J_q(x_n(t) - x_{n-1}(t))\rangle + c_q\|p_t(x_n) - p_t(x_{n-1})\|^q.
\end{aligned}$$

Consequently, since p is (γ, ϖ)-relaxed cocoercive and π-Lipschitz continuous, we obtain

$$\begin{aligned}
&\|x_n(t) - x_{n-1}(t) - (p_t(x_n) - p_t(x_{n-1}))\|^q \\
&\le \|x_n(t) - x_{n-1}(t)\|^q + (q\gamma(t) + c_q)\|p_t(x_n) - p_t(x_{n-1})\|^q - q\varpi(t)\|x_n(t) - x_{n-1}(t)\|^q \\
&= (1 - q\varpi(t) + (q\gamma(t) + c_q)\pi^q(t))\|x_n(t) - x_{n-1}(t)\|^q.
\end{aligned} \tag{4.4}$$

Furthermore, by ϱ-$\hat{H}$-Lipschitz continuity of Q and ι-$\hat{H}$-Lipschitz continuity of G, from (3.2) we deduce that

$$\begin{aligned} \left\|\vartheta_n(t)-\vartheta_{n-1}(t)\right\| &\le \left(1+\frac{1}{n}\right)\hat{H}(Q_t(x_n),Q_t(x_{n-1})) \\ &\le \varrho(t)\left(1+\frac{1}{n}\right)\left\|x_n(t)-x_{n-1}(t)\right\| \end{aligned} \tag{4.5}$$

and

$$\begin{aligned} \left\|w_n(t)-w_{n-1}(t)\right\| &\le \left(1+\frac{1}{n}\right)\hat{H}(G_t(x_n),G_t(x_{n-1})) \\ &\le \iota(t)\left(1+\frac{1}{n}\right)\left\|x_n(t)-x_{n-1}(t)\right\|. \end{aligned} \tag{4.6}$$

By using (4.5) together with σ-Lipschitz continuity of A, π-Lipschitz continuity of p, we obtain

$$\begin{aligned} &\left\|A_t(p_t(x_n)-\vartheta_n)-A_t(p_t(x_{n-1})-\vartheta_{n-1})\right\| \\ &\le \sigma(t)(\left\|p_t(x_n)-p_t(x_{n-1})\right\|+\left\|\vartheta_n(t)-\vartheta_{n-1}(t)\right\|) \\ &\le \sigma(t)\left(\pi(t)+\varrho(t)\left(1+\frac{1}{n}\right)\right)\left\|x_n(t)-x_{n-1}(t)\right\|. \end{aligned} \tag{4.7}$$

Moreover, since N is ϵ-Lipschitz continuous in the second argument, δ-Lipschitz continuous in the third argument, κ-Lipschitz continuous in the fourth argument and S, T, P are ξ-$\hat{H}$-Lipschitz continuous, ζ-Lipschitz continuous and ς-$\hat{H}$-Lipschitz continuous, respectively, by (3.2), we get

$$\begin{aligned} &\left\|N_t(\nu_n,u_n,v_n)-N_t(\nu_{n-1},u_{n-1},v_{n-1})\right\| \\ &\le \left\|N_t(\nu_n,u_n,v_n)-N_t(\nu_{n-1},u_n,v_n)\right\|+\left\|N_t(\nu_{n-1},u_n,v_n)-N_t(\nu_{n-1},u_{n-1},v_n)\right\| \\ &\quad+\left\|N_t(\nu_{n-1},u_{n-1},v_n)-N_t(\nu_{n-1},u_{n-1},v_{n-1})\right\| \\ &\le \varepsilon(t)\left\|\nu_n(t)-\nu_{n-1}(t)\right\|+\delta(t)\left\|u_n(t)-u_{n-1}(t)\right\|+\kappa(t)\left\|v_n(t)-v_{n-1}(t)\right\| \\ &\le \left(\varepsilon(t)\xi(t)+\delta(t)\zeta(t)+\kappa(t)\varsigma(t)\right)\left(1+\frac{1}{n}\right)\left\|x_n(t)-x_{n-1}(t)\right\|. \end{aligned} \tag{4.8}$$

Now, substitute (4.4)-(4.8) into (4.3), we get that

$$\begin{aligned} \left\|x_{n+1}(t)-x_n(t)\right\| \le\ &(1-\alpha(t)+\alpha(t)\psi(t,n))\left\|x_n(t)-x_{n-1}(t)\right\| \\ &+\alpha(t)\left\|e_n(t)-e_{n-1}(t)\right\|+\left\|r_n(t)-r_{n-1}(t)\right\|, \end{aligned} \tag{4.9}$$

where

$$\begin{aligned} \psi(t,n) &= (\varrho(t)+\mu(t)\iota(t))\left(1+\frac{1}{n}\right)+\sqrt[q]{1-q\varpi(t)+(q\gamma(t)+c_q)\pi^q(t)}+\frac{\tau^{q-1}(t)\Gamma(t,n)}{r(t)-\rho(t)\lambda(t)m(t)}, \\ \Gamma(t,n) &= \sigma(t)\left(\pi(t)+\varrho(t)\left(1+\frac{1}{n}\right)\right)+\rho(t)(\varepsilon(t)\xi(t)+\delta(t)\zeta(t)+\kappa(t)\varsigma(t))\left(1+\frac{1}{n}\right). \end{aligned}$$

Let us put

$$\theta(t,n)=1-\alpha(t)+\alpha(t)\psi(t,n),\quad \text{for each } n\ge 0,\quad t\in\Omega.$$

Then, for each $t\in\Omega$, we know that

$$\theta(t,n)\to\theta(t)=1-\alpha(t)+\alpha(t)\psi(t),\quad \text{as } n\to\infty,$$

where

$$\psi(t) = \varrho(t) + \mu(t)\iota(t) + \sqrt[q]{1 - q\varpi(t) + (q\gamma(t) + c_q)\pi^q(t)} + \frac{\tau^{q-1}(t)\Gamma(t)}{r(t) - \rho(t)\lambda(t)m(t)},$$
$$\Gamma(t) = \sigma(t)(\pi(t) + \varrho(t)) + \rho(t)(\varepsilon(t)\xi(t) + \delta(t)\zeta(t) + \kappa(t)\varsigma(t)).$$

It follows that, in view of the condition (4.2), we have $\psi(t) \in (0, 1)$ for all $t \in \Omega$. This implies $0 < \theta(t) < 1$ for all $t \in \Omega$. Hence there exist $n_0 \in \mathbb{N}$ and a measurable function $\hat{\theta} : \Omega \to (0, \infty)$ (Take $\hat{\theta}(t) = \frac{\theta(t)+1}{2} \in (\theta(t), 1)$ for each $t \in \Omega$) such that $\theta(t, n) \leq \hat{\theta}(t)$ for all $n \geq n_0$ and $t \in \Omega$. Accordingly, for all $n > n_0$, by (4.9), deduce that, for all $t \in \Omega$,

$$\begin{aligned}
&\left\|x_{n+1}(t) - x_n(t)\right\| \\
&\leq \hat{\theta}(t)\left\|x_n(t) - x_{n-1}(t)\right\| + \alpha(t)\left\|e_n(t) - e_{n-1}(t)\right\| + \left\|r_n(t) - r_{n-1}(t)\right\| \\
&\leq \hat{\theta}(t)[\hat{\theta}(t)\left\|x_{n-1}(t) - x_{n-2}(t)\right\| + \alpha(t)\left\|e_{n-1}(t) - e_{n-2}(t)\right\| + \left\|r_{n-1}(t) - r_{n-2}(t)\right\|] \\
&\quad + \alpha(t)\left\|e_n(t) - e_{n-1}(t)\right\| + \left\|r_n(t) - r_{n-1}(t)\right\| \\
&= \hat{\theta}^2(t)\left\|x_{n-1}(t) - x_{n-2}(t)\right\| + \alpha(t)[\hat{\theta}(t)\left\|e_{n-1}(t) - e_{n-2}(t)\right\| \\
&\quad + \left\|e_n(t) - e_{n-1}(t)\right\|] + \hat{\theta}(t)\left\|r_{n-1}(t) - r_{n-2}(t)\right\| + \left\|r_n(t) - r_{n-1}(t)\right\| \\
&\leq \\
&\vdots \\
&\leq \hat{\theta}^{n-n_0}(t)\left\|x_{n_0+1}(t) - x_{n_0}(t)\right\| + \sum_{i=1}^{n-n_0} \alpha(t)\hat{\theta}^{i-1}(t)\left\|e_{n-(i-1)}(t) - e_{n-i}(t)\right\| \\
&\quad + \sum_{i=1}^{n-n_0} \hat{\theta}^{i-1}(t)\left\|r_{n-(i-1)}(t) - r_{n-i}(t)\right\|.
\end{aligned} \tag{4.10}$$

By using the inequality (4.10), it follows that, for any $m \geq n > n_0$,

$$\begin{aligned}
\left\|x_m(t) - x_n(t)\right\| \leq \sum_{j=n}^{m-1}\left\|x_{j+1}(t) - x_j(t)\right\| &\leq \sum_{j=n}^{m-1}\hat{\theta}^{j-n_0}(t)\left\|x_{n_0+1}(t) - x_{n_0}(t)\right\| \\
&\quad + \sum_{j=n}^{m-1}\sum_{i=1}^{j-n_0}\alpha(t)\hat{\theta}^{i-1}(t)\left\|e_{n-(i-1)}(t) - e_{n-i}(t)\right\| \\
&\quad + \sum_{j=n}^{m-1}\sum_{i=1}^{j-n_0}\hat{\theta}^{i-1}(t)\left\|r_{n-(i-1)}(t) - r_{n-i}(t)\right\|.
\end{aligned} \tag{4.11}$$

Since $\hat{\theta}(t) < 1$ for all $t \in \Omega$, it follows from (3.3) and (4.11) that $\|x_m(t) - x_n(t)\| \to 0$ as $n \to \infty$. This means $\{x_n(t)\}$ is a Cauchy sequence in X. In view of completeness of X, there exists $x^*(t) \in X$ such that $x_n(t) \to x^*(t)$ for all $t \in \Omega$.

Consequently, by using (3.2), ξ-$\hat{H}$-Lipschitz continuity of S, ζ-$\hat{H}$-Lipschitz continuity of T, ς-$\hat{H}$-Lipschitz continuity of P, ϱ-$\hat{H}$-Lipschitz continuity of Q and ι-$\hat{H}$-Lipschitz continuity of G, we know that $\{\upsilon_n(t)\}$, $\{u_n(t)\}$, $\{\nu_n(t)\}$, $\{\vartheta_n(t)\}$ and $\{w_n(t)\}$ are also Cauchy sequences in X. Thus there are $\upsilon^*(t)$, $u^*(t)$, $\nu^*(t)$, $\vartheta^*(t)$, $w^*(t)$ in X such that, for all $t \in \Omega$, $\upsilon_n(t) \to \upsilon^*(t)$, $u_n(t) \to u^*(t)$, $\nu_n(t) \to \nu^*(t)$, $\vartheta_n(t) \to \vartheta^*(t)$ and $w_n(t) \to w^*(t)$ as $n \to \infty$. Since $\{x_n(t)\}$, $\{\upsilon_n(t)\}$, $\{u_n(t)\}$, $\{\nu_n(t)\}$, $\{\vartheta_n(t)\}$ and $\{w_n(t)\}$ are sequences of measurable mappings, we know that $x, \upsilon, u, \nu, \vartheta, w : \Omega \to X$ are also measurable. Further, for each $t \in \Omega$, we have

$$\begin{aligned} d(\nu^*(t), S_t(x^*)) &= \inf\{\|\nu^*(t) - z\| : z \in S_t(x^*)\} \\ &\leq \|\nu^*(t) - \nu_n(t)\| + d(\nu_n(t), S_t(x^*)) \\ &\leq \|\nu^*(t) - \nu_n(t)\| + \hat{H}(S_t(x_n), S_t(x^*)) \\ &\leq \|\nu^*(t) - \nu_n(t)\| + \xi(t)\|x_n(t) - x^*(t)\|. \end{aligned}$$

Notice that, the right side of the above inequality tends to zero as $n \to \infty$, this implies that $\upsilon^*(t) \in S_t(x^*)$.

Similarly, we can verify that for each $t \in \Omega$, $u^*(t) \in T_t(x^*)$, $\nu^*(t) \in P_t(x^*)$, $\vartheta^*(t) \in Q_t(x^*)$ and $w^*(t) \in G_t(x^*)$. Moreover, the condition (4.1) and $w_n(t) \to w^*(t)$, for all $t \in \Omega$, as $n \to \infty$, imply that for each $t \in \Omega$, $J^{\eta_t, M_t(.,w_n)}_{\rho(t)\lambda(t),A_t} \to J^{\eta_t, M_t(.,w^*)}_{\rho(t)\lambda(t),A_t}$ uniformly on X, as $n \to \infty$.

Now, since for each $t \in \Omega$, the mappings $J^{\eta_t, M_t(.,w_n)}_{\rho(t)\lambda(t),A_t}$ and A_t are continuous, it follows from (3.2) and (3.3) that for each $t \in \Omega$,

$$p_t(x^*) = \vartheta^*(t) + J^{\eta_t, M_t(.,w^*)}_{\rho(t)\lambda(t),A_t}[A_t(p_t(x^*) - \vartheta^*) - \rho(t)(N_t(\upsilon^*, u^*, \nu^*) - h(t))].$$

Finally, Lemma 3.6 implies that measurable mappings x^*, υ^*, u^*, ν^*, ϑ^*, w^*: $\Omega \to X$ are a random solution of the problem (3.1). This completes the proof.

Remark 4.2. If X is a 2-uniformly smooth Banach space and there exists a measurable function $\rho : \Omega \to (0, \infty)$ with $\rho(t) \in (0, \frac{r(t)}{\lambda(t)m(t)})$, for all $t \in \Omega$, such that

$$\begin{aligned} &\varphi(t) = \varrho(t) + \mu(t)\iota(t) + \sqrt{1 - 2\varpi(t) + (2\gamma(t) + c_2)\pi^2(t)} < 1, \\ &2\varpi(t) - (2\gamma(t) + c_2)\pi^2(t) < 1, \\ &\rho(t) < \frac{r(t)(1 - \varphi(t)) - r(t)\sigma(t)(\pi(t) + \varrho(t))}{\tau(t)[(\varepsilon(t)\xi(t) + \delta(t)\zeta(t) + \kappa(t)\varsigma(t)] + (1 - \varphi(t))\lambda(t)m(t)}, \end{aligned}$$

then (4.2) holds. As we know, Hilbert spaces and L_p(or l_p) spaces, $2 \leq p < \infty$, are 2-uniformly smooth.

Remark 4.3. Theorem 4.1 generalizes and improves Theorems 3.1 and 3.2 in [23], Theorems 3.1, 3.3 and 3.4 in [53] and Theorems 4.1, 4.3 and 4.4 in [58]. In brief, for an appropriate choice of the mappings $A, p, \eta, M, N, \mathcal{S}, \mathcal{T}, \mathcal{P}, \mathcal{Q}, \mathcal{G}, S, T, P, Q, G, h, \lambda$, the measurable step size function α, the sequences $\{e_n\}$, $\{r_n\}$ and the space X, Theorem 4.1 includes many known results of generalized variational inclusions as special cases (see [13,18,22-24,33,34,37-40,42,44,46,49,53,58] and the references therein).

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