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On some sufficient conditions for univalence and starlikeness

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Abstract
In this work, the conditions for univalence, starlikeness and convexity are discussed.
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1 Introduction

We shall consider the set $\mathcal{H}$ of all analytic functions in the open unit disc

$$\mathbb{D} = \{z : |z| < 1\}$$

on the complex plane $\mathbb{C}$ and

$$\mathcal{A} = \{f \in \mathcal{H} : f(z) = z + a_2z^2 + \cdots\}.$$

The class $\mathcal{S}^*_\alpha$ of starlike functions of order $\alpha < 1$ may be defined as

$$\mathcal{S}^*_\alpha = \left\{f \in \mathcal{A} : \mathfrak{Re}\frac{zf'(z)}{f(z)} > \alpha, z \in \mathbb{D}\right\}.$$

The class $\mathcal{S}^*_\alpha$ and the class $\mathcal{K}_\alpha$ of convex functions of order $\alpha < 1$

$$\begin{aligned}\mathcal{K}_\alpha &:= \left\{f \in \mathcal{A} : \mathfrak{Re}\left(1 + \frac{zf''(z)}{f'(z)}\right) > \alpha, z \in \mathbb{D}\right\}\\ &= \{f \in \mathcal{A} : zf' \in \mathcal{S}^*_\alpha\}\end{aligned}$$

were introduced by Robertson in [1]. If $\alpha \in [0;1)$, then a function in either of these sets is univalent. In particular, we denote by $\mathcal{S}^*_0 = \mathcal{S}^*$, $\mathcal{K}_0 = \mathcal{K}$ the classes of starlike and convex functions, respectively.

2 Preliminaries

Lemma 2.1 *Let $O = 0$, $P = \alpha + ia\alpha$, $Q = x + iax$ and $A \in (0, +\infty)$ be the points on the complex plane, where $0 < \alpha \leq A/2$, $\alpha < x$ and $-\infty < a < \infty$. Then we have*

$$\frac{|A - Q|}{|O - Q|} < \frac{|A - P|}{|O - P|} \leq \frac{A - \alpha}{\alpha}. \tag{2.1}$$

$$O = 0, \qquad P = \alpha + ia\alpha,$$
$$A = 2, \qquad Q = x + iax,$$
$$0 < \alpha \le A/2, \qquad \angle PAO = \psi,$$
$$\angle QOA = \varphi, \qquad \angle OAQ = \psi'.$$

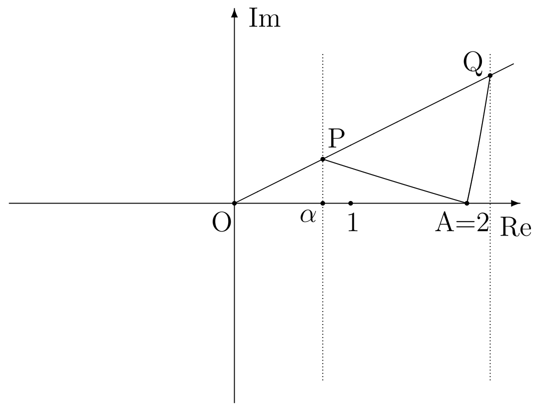

Figure 1 The lines and the points.

Proof For $a = 0$, the assertion is obvious. For $a \neq 0$, consider the triangles OAP and OAQ, see Figure 1. Both of them have the same angle φ at the point O. Let the first have the angle ψ at the point A and the second have the angle ψ' at the point A. Then the hypothesis $0 < \alpha \le A/2$ implies $\cos\varphi \le \cos\psi$. Further we have

$$\cos\varphi = \frac{\alpha}{|P|}, \qquad \cos\psi = \frac{A-\alpha}{|A-P|}.$$

This gives the second inequality of the assertion. The first one follows immediately from $\sin\psi' > \sin\psi$ and

$$\frac{|A-P|}{|P|} = \frac{\sin\varphi}{\sin\psi}, \qquad \frac{|A-Q|}{|Q|} = \frac{\sin\varphi}{\sin\psi'}. \qquad \square$$

3 Main result

Theorem 3.1 *Let* $p(z) = 1 + \sum_{n=k\ge 1}^{\infty} c_n z^n$ *be analytic in the unit disc* $\mathbb{D}$ *and* α *be a positive real number* $0 < \alpha \le 1/2$. *Then suppose that there exists a point* z_0, $|z_0| < 1$ *such that*

$$\mathfrak{Re}\{p(z)\} > \alpha \quad \textit{for } |z| < |z_0| \tag{3.1}$$

and

$$\mathfrak{Re}\{p(z_0)\} = p(z_0) = \alpha. \tag{3.2}$$

Then we have

$$\frac{z_0 p'(z_0)}{p(z_0)} \le -k(1-\alpha). \tag{3.3}$$

Proof Let us put

$$q(z) = \frac{p(z)-1}{p(z)}, \qquad q(0) = 0.$$

Then the function q is analytic in $|z| \le |z_0| < 1$ and from the hypothesis of Theorem 3.1 and Lemma 2.1, with $A = 1$, we have

$$\left|q(z)\right| < \frac{1-\alpha}{\alpha}$$

for $|z| \le |z_0|$ and

$$\left|q(z_0)\right| = \frac{1-\alpha}{\alpha}.$$

This shows that $|q(z)|$ takes its maximum at $z = z_0$ on the circle $|z| = |z_0|$. Then from Fukui-Sakaguchi [2] and Jack's [3] lemmas, there exists a real number $k \ge 1$ such that

$$\begin{aligned}\frac{z_0 q'(z_0)}{q(z_0)} &= \frac{z_0 p'(z_0)}{p(z_0)-1} - \frac{z_0 p'(z_0)}{p(z_0)} \\ &= z_0 p'(z_0)\left(\frac{1}{\alpha-1} - \frac{1}{\alpha}\right) \\ &= \frac{z_0 p'(z_0)}{\alpha(\alpha-1)} \\ &\ge k.\end{aligned}$$

This shows that $z_0 p'(z_0)$ is a negative real number and

$$\frac{z_0 p'(z_0)}{\alpha} = \frac{z_0 p'(z_0)}{p(z_0)} \le -k(1-\alpha).$$

This completes the proof of Theorem 3.1. □

Theorem 3.1 is, in a certain sense, the supplement of Nunokawa's lemma [4]. From Theorem 3.1 we have the following corollaries.

Corollary 3.2 *Let* $p(z) = 1 + \sum_{n=1}^{\infty} c_n z^n$ *be analytic in the unit disc* $\mathbb{D}$ *and* α *be a positive real number* $0 < \alpha \le 1/2$. *Suppose also that for arbitrary* r, $0 < r < 1$, p *satisfies the condition*

$$\min_{|z|\le r} \mathfrak{Re}\{p(z)\} = \min_{|z|\le r}\left|p(z)\right| \tag{3.4}$$

and

$$\mathfrak{Re}\left\{p(z) + \frac{zp'(z)}{p(z)}\right\} > 2\alpha - 1 \quad \textit{for } |z| < 1. \tag{3.5}$$

Then we have

$$\mathfrak{Re}\{p(z)\} > \alpha \quad \textit{for } |z| < 1. \tag{3.6}$$

Proof If there exists a point z_0, $|z_0| < 1$, such that

$$\mathfrak{Re}\{p(z)\} > \alpha \quad \text{for } |z| < |z_0|$$

and

$$\mathfrak{Re}\{p(z_0)\} = \alpha, \quad 0 < \alpha \le 1/2,$$

then from the hypothesis of Corollary 3.2, we have

$$\mathfrak{Re}\{p(z_0)\} = p(z_0) = \alpha.$$

Then from Theorem 3.1, we have

$$\frac{z_0 p'(z_0)}{p(z_0)} \le \alpha - 1,$$

and therefore we have

$$\mathfrak{Re}\left\{p(z_0) + \frac{z_0 p'(z_0)}{p(z_0)}\right\} \le 2\alpha - 1.$$

This contradicts the hypothesis of Corollary 3.2 and it completes the proof of Corollary 3.2. □

Corollary 3.3 *Let $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ be analytic in the unit disc $\mathbb{D}$ and α be a positive real number $0 < \alpha \le 1/2$. Suppose that for arbitrary r, $0 < r < 1$, f satisfies the condition*

$$\min_{|z| \le r} \mathfrak{Re}\left\{\frac{zf'(z)}{f(z)}\right\} = \min_{|z| \le r}\left|\frac{zf'(z)}{f(z)}\right| \tag{3.7}$$

and

$$\mathfrak{Re}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > 2\alpha - 1 \quad \text{for } |z| < 1, \tag{3.8}$$

where

$$-1 < 2\alpha - 1 \le 0.$$

Then we have

$$\mathfrak{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha \quad \text{for } |z| < 1, \tag{3.9}$$

or f is starlike of order α.

Proof Putting

$$p(z) = \frac{zf'(z)}{f(z)},$$

it follows that

$$p(z) + \frac{zp'(z)}{p(z)} = 1 + \frac{zf''(z)}{f'(z)}.$$

Then from Corollary 3.2, we have (3.9). □

Theorem 3.4 *Let $p(z) = 1 + \sum_{n=k\geq 1}^{\infty} c_n z^n$ be analytic in the unit disc $\mathbb{D}$ and α be a positive real number $1/2 < \alpha < 1$. Then suppose that there exists a point $z_0 \in \mathbb{D}$ such that*

$$\mathfrak{Re}\{p(z)\} > \alpha \quad \text{for } |z| < |z_0| \tag{3.10}$$

and

$$\mathfrak{Re}\{p(z_0)\} = p(z_0) = \alpha. \tag{3.11}$$

Then we have

$$\frac{z_0 p'(z_0)}{p(z_0)} = \mathfrak{Re}\left\{\frac{z_0 p'(z_0)}{p(z_0)}\right\} \leq -\frac{k(2-\alpha)}{2}. \tag{3.12}$$

Proof Let us put

$$q(z) = \frac{2 - p(z)}{p(z)} - 1, \qquad q(0) = 0.$$

Then from Lemma 2.1, with $A = 2$, we have that $|q(z)+1|$ takes its maximum value at $z = z_0$ on the circle $|z| = |z_0|$ or

$$\max_{|z|=|z_0|} \left|q(z) + 1\right| = \max_{|z|=|z_0|} \left|\frac{2 - p(z)}{p(z)}\right| = \frac{2-\alpha}{\alpha}.$$

Applying Jack [3], Miller-Mocanu [5] and Fukui-Sakaguchi's [2] lemmas, there exists a real number $m \geq k$ such that

$$\begin{aligned}
\frac{z_0 q'(z_0)}{q(z_0)} &= -\frac{z_0 p'(z_0)}{2 - p(z_0)} - \frac{z_0 p'(z_0)}{p(z_0)} \\
&= -z_0 p'(z_0)\left(\frac{1}{2-\alpha} + \frac{1}{\alpha}\right) \\
&= -\frac{2 z_0 p'(z_0)}{\alpha(2-\alpha)} \\
&= -\frac{z_0 p'(z_0)}{p(z_0)}\left(\frac{2}{2-\alpha}\right) \\
&\geq m \geq k.
\end{aligned}$$

This shows that

$$\frac{z_0 p'(z_0)}{p(z_0)} \leq -\frac{k(2-\alpha)}{2}.$$

This completes the proof of Theorem 3.4. □

Corollary 3.5 *Let $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ be analytic in the unit disc $\mathbb{D}$ and α be a positive real number $1/2 < \alpha < 1$. Suppose that for arbitrary r, $0 < r < 1$, f satisfies the condition*

$$\min_{|z|\leq r} \mathfrak{Re}\left\{\frac{zf'(z)}{f(z)}\right\} = \min_{|z|\leq r}\left|\frac{zf'(z)}{f(z)}\right| \tag{3.13}$$

and

$$\mathfrak{Re}\left\{1+\frac{zf''(z)}{f'(z)}\right\} > \frac{3\alpha-2}{2} \quad \textit{for } |z|<1. \tag{3.14}$$

Then we have

$$\mathfrak{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha \quad \textit{for } |z|<1. \tag{3.15}$$

Proof Applying the same method as in the proof of Corollary 3.3 and in the proof of Lemma 2.1, we can obtain Corollary 3.5. □

Corollary 3.6 *Let* $F(z) = 1/z + \sum_{n=1}^{\infty} b_n z^n$ *be analytic and not vanishing in the punctured unit disc* $0 < |z| < 1$ *and let* α *be a positive real number* $0 < \alpha < 1/2$. *Suppose also that for arbitrary* r, $0 < r < 1$, F *satisfies the following condition*:

$$\min_{|z|\le r} \mathfrak{Re}\left\{-\frac{zF'(z)}{F(z)}\right\} = \min_{|z|\le r}\left|\frac{zF'(z)}{F(z)}\right| \tag{3.16}$$

and

$$-\mathfrak{Re}\left\{1+\frac{zF''(z)}{F'(z)}\right\} < 2-\alpha \quad \textit{for } |z|<1. \tag{3.17}$$

Then we have

$$\mathfrak{Re}\left\{-\frac{zF'(z)}{F(z)}\right\} > \alpha \quad \textit{for } |z|<1. \tag{3.18}$$

Proof Putting

$$p(z) = -\frac{zF'(z)}{F(z)} = -\frac{z(-1/z^2 + \sum_{n=1}^{\infty} nb_n z^{n-1})}{1/z + \sum_{n=1}^{\infty} b_n z^n},$$

then we have

$$p(z) = 1 + \sum_{n=2}^{\infty} c_n z^n, \qquad p(0) = 1$$

and it follows that

$$p(z) - \frac{zp'(z)}{p(z)} = -\left(1+\frac{zF''(z)}{F'(z)}\right).$$

If there exists a point $z_0 \in \mathbb{D}$ such that

$$\mathfrak{Re}\{p(z)\} > \alpha \quad \text{for } |z| < |z_0|$$

and

$$\mathfrak{Re}\{p(z_0)\} = p(z_0) = \alpha,$$

then from the hypothesis, we have $p(z_0) = \alpha$. Applying Theorem 3.1, we have

$$\frac{z_0 p'(z_0)}{p(z_0)} \leq -2(1-\alpha)$$

and therefore we have

$$\begin{aligned}\mathfrak{Re}\left\{p(z_0) - \frac{z_0 p'(z_0)}{p(z_0)}\right\} &= -\mathfrak{Re}\left\{1 + \frac{z_0 F''(z_0)}{F'(z_0)}\right\} \\ &\geq \alpha + 2(1-\alpha) \\ &= 2 - \alpha.\end{aligned}$$

This contradicts the hypothesis and therefore we have

$$\mathfrak{Re}\{p(z)\} = \mathfrak{Re}\left\{-\frac{zF'(z)}{F(z)}\right\} > \alpha \quad \text{for } |z| < 1,$$

and this shows that F is meromorphic starlike of order α in the punctured unit disc $0 < |z| < 1$. □

We note the following interesting result which was published in a minor journal and so it was not well known in the public of univalent function theory but is strongly connected with the previous Corollaries 3.3, 3.5 and 3.6.

Lemma 3.7 ([6]) *Let $f(z) = z + a_2 z^2 + \cdots$ be an analytic function in $|z| < 1$ with $f(z)f'(z)/z \neq 0$ in $|z| < 1$. Then, for each α, $-1/2 < \alpha < 0$, there exists a function f which satisfies*

$$1 + \mathfrak{Re}\frac{zf''(z)}{f'(z)} > \alpha \quad \text{in } |z| < 1, \tag{3.19}$$

but f is not starlike in $|z| < 1$.

In [6] the authors pointed out that the function

$$f(z) = \frac{(1-z)^{2\alpha-1} - 1}{1 - 2\alpha} \tag{3.20}$$

satisfies the above conditions.

Lemma 3.8 *Let $p(z) = 1 + \sum_{n=m\geq 2}^{\infty} c_n z^n$ be an analytic function in $\mathbb{D}$. Suppose also that there exists a point $z_0 \in \mathbb{D}$ such that*

$$\mathfrak{Re}\{p(z)\} > 0 \quad \text{for } |z| < |z_0|$$

and

$$\mathfrak{Re}\{p(z_0)\} = 0 \quad \text{and} \quad p(z_0) \neq 0.$$

Then we have

$$\frac{z_0 p'(z_0)}{p(z_0)} = ik,$$

where k is a real number and

$$k \geq \frac{m}{2}\left(a + \frac{1}{a}\right) \geq m \geq 2 \quad \textit{when} \ \arg p(z_0) = \frac{\pi}{2}$$

and

$$k \leq -\frac{m}{2}\left(a + \frac{1}{a}\right) \leq -m \leq -2 \quad \textit{when} \ \arg p(z_0) = -\frac{\pi}{2},$$

where

$$p(z_0) = \pm ia \quad \textit{and} \quad a > 0.$$

Proof Let us put

$$\phi(z) = \frac{1 - p(z)}{1 + p(z)}, \quad |z| < 1.$$

Then we have $\phi(0) = \phi'(0) = \cdots = \phi^{(m-1)}(0) = 0$, $|\phi(z)| < 1$, for $|z| < |z_0|$ and $|\phi(z_0)| = 1$. From [2, 3] and [5], we obtain

$$\frac{z_0 \phi'(z_0)}{\phi(z_0)} = -\frac{2z_0 p'(z_0)}{1 - p^2(z_0)} = -\frac{2z_0 p'(z_0)}{1 + |p(z_0)|^2} \geq m.$$

This shows that

$$-z_0 p'(z_0) \geq \frac{m}{2}\left(1 + \left|p(z_0)\right|^2\right)$$

and $z_0 p'(z_0)$ is a negative real number. For the case $\arg p(z_0) = \pi/2$, $p(z_0) = ia$ and $0 < a$, we have

$$\mathfrak{Re}\left\{\frac{z_0 p'(z_0)}{p(z_0)}\right\} = \mathfrak{Re}\left\{\frac{z_0 p'(z_0)}{ia}\right\} = \mathfrak{Re}\left\{-\frac{iz_0 p'(z_0)}{a}\right\} = 0$$

and

$$\mathfrak{Im}\left\{\frac{z_0 p'(z_0)}{p(z_0)}\right\} = \mathfrak{Im}\left\{-\frac{iz_0 p'(z_0)}{a}\right\} \geq \frac{m}{2}\left(\frac{1 + |p(z_0)|^2}{a}\right) = \frac{m}{2}\left(a + \frac{1}{a}\right) \geq m.$$

For the case $\arg p(z_0) = -\pi/2$, $p(z_0) = -ia$ and $0 < a$, applying the same method as above, we have

$$\mathfrak{Re}\left\{\frac{z_0 p'(z_0)}{p(z_0)}\right\} = 0$$

and

$$\mathfrak{Im}\left\{\frac{z_0p'(z_0)}{p(z_0)}\right\} \leq -\frac{m}{2}\left(a+\frac{1}{a}\right) \leq -m.$$

This completes the proof. □

Theorem 3.9 *Let $f(z) = z + \sum_{n=m+1}^{\infty} a_n z^n$ be analytic in the unit disc $\mathbb{D}$. Suppose also that*

$$\mathfrak{Re}\left\{1+\frac{zf''(z)}{f'(z)}\right\} < \frac{8+m}{6} \quad \textit{for } |z|<1 \tag{3.21}$$

where $1 \leq m$. If $zf'(z)/f(z)$ is analytic in $\mathbb{D}$ and omits $4/3$, then f is starlike in the unit disc $\mathbb{D}$.

Proof Let us put

$$\frac{zf'(z)}{f(z)} = \frac{4p(z)}{1+3p(z)},$$

where $p(0) = 1$, $p'(0) = p''(0) = \cdots = p^{(m-1)}(0) = 0$. Then it follows that

$$1+\frac{zf''(z)}{f'(z)} = \frac{zf'(z)}{f(z)} + \frac{zp'(z)}{p(z)} - \frac{3zp'(z)}{1+3p(z)}.$$

If there exists a point $z_0 \in \mathbb{D}$ such that

$$\mathfrak{Re}\left\{p(z)\right\} > 0 \quad \text{for } |z| < |z_0|$$

and

$$\mathfrak{Re}\left\{p(z_0)\right\} = 0$$

and $p(z_0) \neq 0$, because $p(z_0) = 0$ contradicts the hypothesis (3.21), then from Lemma 3.8, we have

$$\frac{z_0p'(z_0)}{p(z_0)} = ik,$$

where k is a real number and $m \leq |k|$. For the case $\arg p(z_0) = \pi/2$, $p(z_0) = ia$ and $0 < a$, we have

$$\begin{aligned}
1+\frac{z_0f''(z_0)}{f'(z_0)} &= \frac{z_0f'(z_0)}{f(z_0)} + \frac{z_0p'(z_0)}{p(z_0)} - \frac{3z_0p'(z_0)}{p(z_0)}\frac{p(z_0)}{1+3p(z_0)} \\
&= \frac{4ia}{1+3ia} + ik - 3ik\frac{ia}{1+3ia} \\
&= \frac{4ia(1-3ia)}{1+9a^2} + ik + \frac{ak(1-3ia)}{1+9a^2}.
\end{aligned}$$

Therefore, we have

$$\begin{aligned}\mathfrak{Re}\left\{1+\frac{z_0f''(z_0)}{f'(z_0)}\right\} &= \frac{12a^2}{1+9a^2}+\frac{3ak}{1+9a^2}\\ &\geq \frac{12a^2}{1+9a^2}+\frac{3m(1+a^2)}{2(1+9a^2)}\\ &= \frac{24a^2+3ma^2+3m}{2(1+9a^2)}.\end{aligned}$$

Putting

$$h(x)=\frac{(8+m)x^2+m}{1+9x^2},\quad x>0,$$

we have

$$h'(x)=\frac{16(1-m)x}{(1+9x^2)^2}<0,\quad x>0.$$

This shows that

$$\mathfrak{Re}\left\{1+\frac{z_0f''(z_0)}{f'(z_0)}\right\}\geq \lim_{a\to\infty}\frac{24a^2+3ma^2+3m}{2(1+9a^2)}=\frac{24+3m}{18}. \tag{3.22}$$

This contradicts the hypothesis (3.21).

For the case $\arg p(z_0)=-\pi/2$, $p(z_0)=-ia$ and $0<a$, applying the same method as above, we also have (3.22). This is also a contradiction and therefore it completes the proof of Theorem 3.9. □

Remark 3.10 Singh and Singh obtained in [7] that if $f(z)=z+a_2z^2+\cdots$ is analytic in $\mathbb{D}$ and

$$\mathfrak{Re}\left\{1+\frac{zf''(z)}{f'(z)}\right\}<\frac{3}{2}\quad\text{for } |z|<1, \tag{3.23}$$

then f is starlike in $\mathbb{D}$. Earlier, Ozaki [8] proved the univalence of f in $\mathbb{D}$ under the same assumption (3.23).

For $m=1$, the inequality (3.21) becomes (3.23), so Theorem 3.9 is a generalization of the above result.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

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