Yaman *Journal of Inequalities and Applications* 2012, **2012**:274
http://www.journalofinequalitiesandapplications.com/content/2012/1/274

Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH Open Access

# Blow-up solution and stability to an inverse problem for a pseudo-parabolic equation

M Yaman[*]

[*]Correspondence:
myaman@sakarya.edu.tr
Department of Mathematics,
Sakarya University, Sakarya, Turkey

**Abstract**
We consider a two-fold problem for an inverse problem of pseudo-parabolic equations with a nonlinear term. Sufficient conditions for a blow-up solution are derived and a stability result is established.

**Keywords:** blow-up; inverse problem; pseudo-parabolic equation; stability

## 1 Introduction

Let us consider the following inverse problem for a pseudo-parabolic equation:

$$u_t - a\Delta u_t - \Delta u + \sum_{i=1}^{n} b_i u_{x_i} - |u|^p u = f(t)g(x), \quad x \in \Omega, t > 0, \tag{1}$$

$$u(x,t) = 0, \quad x \in \partial\Omega, t > 0, \tag{2}$$

$$u(x,0) = u_0(x), \quad x \in \Omega, \tag{3}$$

$$\int_\Omega u(x,t)g(x)\,dx = 1, \quad t > 0, \tag{4}$$

where $\Omega \subset R^n$ is a bounded domain with a sufficiently smooth boundary $\partial\Omega$, $p$ and $a$ are positive constants, $g(x)$ and $b_i(x)$ are given functions satisfying

$$\omega \in H^2(\Omega) \cap H_0^1(\Omega) \cap L^{p+2}(\Omega), \qquad \int_\Omega \omega(\omega - \Delta\omega)\,dx = 1, \tag{A1}$$

with weight function $g(x) = \omega - a\Delta\omega$, and a constant

$$B_0 = \max_{x\in\Omega} \left( \sum_{i=1}^{n} b_i^2(x) \right)^{1/2}, \quad x \in \Omega, b_i \in C(\overline{\Omega}). \tag{A2}$$

The inverse problem consists of finding a pair of functions $(u(x,t), f(t))$ satisfying (1)-(4) when

$$u_0 \in H_0^1(\Omega) \cap L^{p+2}(\Omega) \quad \text{and} \quad \int_\Omega u_0(\omega - \Delta\omega)\,dx = 1. \tag{A3}$$

Additional information about the solution to the inverse problem is given in the form of the integral overdetermination condition (4). From the physical point of view, this condition may be interpreted as measurements of the temperature $u(x,t)$ by a device averaging over the domain $\Omega$ [1].

This type of equations arises from a variety of mathematical models in engineering and physical sciences; for example, inverse scattering problems in quantum physics, an inverse problem of interest in geophysics [2].

Existence and uniqueness of solutions to an inverse problem for parabolic and pseudo-parabolic equations are studied in [3–6]. Stability of solutions is investigated by several authors [1, 7]; but less is known about blow-up solutions. Eden and Kalantarov [8] studied the same problem without a strong damping term $-\Delta u_t$. Meyvaci [9] established a blow-up result for the pseudo-parabolic equation $u_t - \Delta u_t - \Delta u + u_{x_1} u^p = |u|^{2m} u$, where $p \geq 1$ is a given integer and $m \geq 1$ is a number.

Here, we used the following notations:

$$\|u\| = \|u\|_{L_2(\Omega)}, \qquad (u, v) = \int_\Omega uv\,dx, \qquad \|u\|_p = \|u\|_{L^p(\Omega)}$$

are the arithmetic-geometric inequality and Young's inequality for $a, b > 0$ respectively;

$$ab \leq \frac{\varepsilon}{2} a^2 + \frac{1}{2\varepsilon} b^2, \qquad ab \leq \beta a^p + C(p, \beta) b^q, \tag{5}$$

with $1/p + 1/q = 1$, $C(p, \beta) = \frac{1}{q(\beta p)^{q/p}}$ and the Poincare-Friedrich inequality

$$\lambda_1 \|u\|^2 \leq \|\nabla u\|^2, \tag{6}$$

where $\lambda_1$ is the first eigenvalue of the eigenvalue problem

$$-\Delta u = \lambda u, \quad x \in \Omega; \qquad u = 0, \quad x \in \partial\Omega.$$

Multiplying both sides of (1) by $\omega$ and integrating the resulting equation over $\Omega$ lead to the following relation:

$$f(t) = -(u, \Delta\omega) + \left(\omega, \sum_{i=1}^n b_i u_{x_i}\right) - \left(\omega, |u|^p u\right), \tag{7}$$

where conditions (2), (3) and (A1) are used. Substituting (7) into (1), problem (1)-(3) yields a direct problem given by [4].

## 2 Blow-up result

Firstly, let us note the following lemma known as 'generalized concavity lemma' or 'Ladyzhenskaya-Kalantarov lemma'. It is an important tool to obtain the blow-up solutions to parabolic- and hyperbolic-type equations.

**Lemma 1** *Let $\alpha > 0$, $C_1, C_2 \geq 0$ and $C_1 + C_2 > 0$. Suppose that a positive, twice differentiable function $F(t)$ satisfies the inequality*

$$F(t)F''(t) - (1 + \alpha)\left(F'(t)\right)^2 \geq -2C_1 F(t)F'(t) - C_2 F^2(t), \quad \forall t \geq 0. \tag{8}$$

*If*

$$F(0) > 0 \quad \textit{and} \quad F'(0) + \gamma_2 \alpha^{-1} F(0) > 0, \tag{9}$$

*then $F(t)$ goes to infinity as*

$$t \to t_1 \leq t_2 = \frac{1}{2\sqrt{C_1^2 + C_2}} \ln \frac{\gamma_1 F(0) + \alpha F'(0)}{\gamma_2 F(0) + \alpha F'(0)}. \tag{10}$$

*Here,* $\gamma_1 = -C_1 + \sqrt{C_1^2 + \alpha C_2}$ *and* $\gamma_2 = -C_1 - \sqrt{C_1^2 + \alpha C_2}$.

*Proof* See [10]. □

**Theorem 1** *Assume that* (A1)-(A3) *are satisfied and suppose that the initial condition* $u_0$ *satisfies the following condition*:

$$\begin{aligned} \frac{2(2p+3)}{p+2} \|u_0\|_{p+2}^{p+2} > {} & 2\sqrt{\frac{4}{a^2} + \frac{2(p+1)^2}{ap^2}\left(2B_0^2 + K_0^2\right)\left(\|u_0\|^2 + a\|\nabla u_0\|^2\right)} \\ & - \left(1 + B_0^2\right)\|u_0\|^2 - D_1, \end{aligned} \tag{A4}$$

*where* $K_0 > 0$ *and* $D_1 = \|\Delta\omega\|^2 + B_0^2\|\omega\|^2 + \frac{2}{p+2}\|\omega\|_{p+2}^{p+2}$. *Then the solution of the problem* (1)-(4) *with the weight function* $g(x) = (\omega - a\Delta\omega)(x)$ *blows up in a finite time*.

*Proof* Multiplying (1) by $u$ and integrating over $\Omega$ give

$$\frac{1}{2}\frac{d}{dt}\left(\|u\|^2 + a\|\nabla u\|^2\right) + \|\nabla u\|^2 + \left(u, \sum_{i=1}^n b_i u_{x_i}\right) - \|u\|_{p+2}^{p+2} = f(t). \tag{11}$$

Also, multiplying (1) by $u_t$ and integrating over $\Omega$, we obtain

$$\|u_t\|^2 + a\|\nabla u_t\|^2 = -\frac{1}{2}\frac{d}{dt}\|\nabla u\|^2 - \left(u_t, \sum_{i=1}^n b_i u_{x_i}\right) + \frac{1}{p+2}\frac{d}{dt}\|u\|_{p+2}^{p+2}. \tag{12}$$

Now, let us consider the following function:

$$F(t) = \|u\|^2 + a\|\nabla u\|^2 + D_0, \tag{13}$$

where $D_0$ is a nonnegative parameter to be chosen later. It is clear that

$$F'(t) = 2(u, u_t) + 2a(\nabla u, \nabla u_t). \tag{14}$$

Using the Cauchy-Schwarz inequality, we have

$$\left(F'(t)\right)^2 \leq 4F(t)\left(\|u_t\|^2 + a\|\nabla u_t\|^2\right). \tag{15}$$

Substituting (11) into (12), we obtain

$$\begin{aligned} \|u_t\|^2 + a\|\nabla u_t\|^2 = {} & \frac{1}{2(p+2)}F''(t) - \frac{p}{2(p+2)}\frac{d}{dt}\|\nabla u\|^2 \\ & - \left(u_t, \sum_{i=1}^n b_i u_{x_i}\right) + \frac{1}{p+2}\frac{d}{dt}\left(u, \sum_{i=1}^n b_i u_{x_i}\right) - \frac{1}{p+2}\frac{d}{dt}f(t). \end{aligned} \tag{16}$$

We take the derivative of (7) with respect to $t$

$$\frac{d}{dt}f(t) = -(u_t, \Delta\omega) + \left(\omega, \sum_{i=1}^{n} b_i u_{tx_i}\right) - (p+1)\left(\omega, u^p u_t\right). \tag{17}$$

Rewrite (16) in view of (17)

$$\begin{aligned}\|u_t\|^2 + a\|\nabla u_t\|^2 = {}& \frac{1}{2(p+2)}F''(t) - \frac{p}{2(p+2)}\frac{d}{dt}\|\nabla u\|^2 - \frac{p+1}{p+2}\left(u_t, \sum_{i=1}^{n} b_i u_{x_i}\right) \\ &+ \frac{1}{p+2}\left(u, \sum_{i=1}^{n} b_i u_{tx_i}\right) + \frac{1}{p+2}(u_t, \Delta\omega) \\ &- \frac{1}{p+2}\left(\omega, \sum_{i=1}^{n} b_i u_{tx_i}\right) + \frac{p+1}{p+2}\left(\omega, |u|^p u_t\right).\end{aligned} \tag{18}$$

After applying the arithmetic-geometric inequality to estimate the terms on the right-hand side of (18), we obtain

$$\begin{aligned}&\frac{d}{dt}\|\nabla u\|^2 \le 2\int_\Omega \nabla u \cdot \nabla u_t \, dx, \\ &2\left|\int_\Omega \nabla u \cdot \nabla u_t \, dx\right| \le \frac{4}{a}\|\nabla u\|^2 + \frac{a}{4}\|\nabla u_t\|^2,\end{aligned} \tag{19}$$

$$\left(u_t, \sum_{i=1}^{n} b_i u_{x_i}\right) \le \frac{p}{8(p+1)}\|u_t\|^2 + \frac{2B_0^2(p+1)}{p}\|\nabla u\|^2, \tag{20}$$

$$\left(u, \sum_{i=1}^{n} b_i u_{tx_i}\right) \le \frac{2B_0^2}{ap}\|u\|^2 + \frac{ap}{8}\|\nabla u_t\|^2, \tag{21}$$

$$\left|(u_t, \Delta\omega)\right| \le \frac{p}{8}\|u_t\|^2 + \frac{2}{p}\|\Delta\omega\|^2, \tag{22}$$

$$\left|\left(\omega, \sum_{i=1}^{n} b_i u_{tx_i}\right)\right| \le \frac{B_0^2}{ap}\|\omega\|^2 + \frac{ap}{4}\|\nabla u_t\|^2. \tag{23}$$

For $q = \frac{2n}{n-2}$, $n \ge 3$, the following inequality is satisfied for some $K_1 > 0$:

$$\left(\int_\Omega |u|^q \, dx\right)^{1/q} \le K_1\left(\int_\Omega |\nabla u|^2 \, dx\right)^{1/2}. \tag{24}$$

We apply the Hölder inequality, with $q_1 = n$, $q_2 = 2$, $q_3 = \frac{2n}{n-2}$, to the last term in (18),

$$\left|\int_\Omega \omega |u|^p u_t \, dx\right| \le \left(\int_\Omega |\omega|^n \, dx\right)^{1/n}\left(\int_\Omega |u_t|^2 \, dx\right)^{1/2}\left(\int_\Omega |u|^{\frac{2n}{n-2}} \, dx\right)^{\frac{n-2}{2n}}. \tag{25}$$

It follows from (24) and (25) with $\|\omega\|_n \le K_2$

$$K_0\|u_t\|\|\nabla u\| \le \frac{p}{4(p+1)}\|u_t\|^2 + \frac{p+1}{p}K_0^2\|\nabla u\|^2, \tag{26}$$

where $K_0 = K_1K_2$. Substituting the estimates (19)-(23) and (26) into (18), we write

$$\begin{aligned}\frac{p+4}{2(p+2)}\left(\|u_t\|^2 + a\|\nabla u_t\|^2\right) \le{}& \frac{1}{2(p+2)}F''(t) + \frac{2B_0^2}{ap(p+2)}\|u\|^2 \\ &+ \frac{2pa^{-1} + p^{-1}(p+1)^2(2B_0^2 + K_0^2)}{a(p+2)}a\|\nabla u\|^2 \\ &+ \frac{1}{p(p+2)}\left(2\|\Delta\omega\|^2 + \frac{B_0^2}{a}\|\omega\|^2\right). \end{aligned} \tag{27}$$

Since coefficients of the term $a\|\nabla u\|^2$ are greater than those of $\|u\|^2$ on the right-hand side of (27), multiplying both sides of (27) by $2(p+2)$, we get

$$\begin{aligned}&(p+4)\left(\|u_t\|^2 + a\|\nabla u_t\|^2\right) \\ &\quad\le F''(t) + \left(\frac{4p}{a^2} + \frac{2(p+1)^2}{ap}\left(2B_0^2 + K_0^2\right)\right)\left(\|u\|^2 + a\|\nabla u\|^2\right) + D_2, \end{aligned} \tag{28}$$

where $D_2 = \frac{4}{p}\|\Delta\omega\|^2 + \frac{2B_0^2}{ap}\|\omega\|^2$. From (15) and (28), we have

$$\left(1 + \frac{p}{4}\right)F^{-1}(t)\left(F'(t)\right)^2 \le F''(t) + \beta F(t) + (D_2 - \beta D_0), \tag{29}$$

where $\beta = \frac{4p}{a^2} + \frac{2(p+1)^2}{ap}(2B_0^2 + K_0^2)$. We choose $D_0 = \beta^{-1}D_2$ in the last inequality and multiply both sides of (29) by $F(t)$, which gives

$$F(t)F''(t) - \left(1 + \frac{p}{4}\right)\left(F'(t)\right)^2 \ge -\beta\left(F(t)\right)^2. \tag{30}$$

So, inequality (8) is satisfied with $\alpha = \frac{p}{4} > 0$, $C_1 = 0$, $C_2 = \beta > 0$. Thus, the desired result is obtained by applying Lemma 1. □

## 3 Stability of problem

In this part, we consider the following inverse source problem:

$$u_t - \Delta u_t - \Delta u - \sum_{i=1}^{n} b_i u_{x_i} + |u|^p u = f(t)\omega(x), \quad x \in \Omega, t > 0, \tag{31}$$

$$u(x,t) = 0, \quad x \in \partial\Omega, t > 0, \tag{32}$$

$$u(x,0) = u_0(x), \quad x \in \Omega, \tag{33}$$

$$\int_\Omega u(x,t)\omega(x)\,dx = \varphi(t), \tag{34}$$

where $\Omega \subset R^n$ is a bounded domain with a sufficiently smooth boundary $\partial\Omega$ and $\omega$, $u_0$ and $\varphi(t)$ are given functions, $p > 0$. Assume that $\omega$ satisfies the conditions

$$\int_\Omega \omega^2\,dx = 1, \qquad \omega \in H_0^1(\Omega) \cap L^{p+2}(\Omega) \tag{A5}$$

and $u_0$ satisfies

$$u_0 \in H_0^1(\Omega) \cap L^{p+2}(\Omega) \quad \text{and} \quad \int_\Omega u_0(x)\omega(x)\,dx = \varphi(0). \tag{A6}$$

**Theorem 2** *Suppose that the conditions* (A5) *and* (A6) *are satisfied and assume that* $\varphi$ *and* $\varphi'$ *are continuous functions defined on* $[0,\infty)$ *which tend to zero as* $t \to \infty$. *Then*

$$\lim_{t\to\infty} \left(\|\nabla u\|^2 + \|u\|_{p+2}^{p+2}\right) = 0$$

*with a constant* $B_0 < \frac{2(\sqrt{1+\lambda_1}-1)}{\sqrt{\lambda_1}}$, *where* $\lambda_1$ *is constant in* (6).

*Proof* We multiply (31) by $\omega$, integrate over $\Omega$ and use (34) to obtain

$$f(t) = \varphi'(t) + (\nabla\omega, \nabla u_t) + (\nabla\omega, \nabla u) - \left(\omega, \sum_{i=1}^n b_i u_{x_i}\right) + \left(\omega, |u|^p u\right). \tag{35}$$

Inserting (35) into (31), we obtain

$$\begin{aligned} &u_t - \Delta u_t - \Delta u - \sum_{i=1}^n b_i u_{x_i} + |u|^p u \\ &\quad = \left(\varphi'(t) + (\nabla\omega, \nabla u_t) + (\nabla\omega, \nabla u) - \left(\omega, \sum_{i=1}^n b_i u_{x_i}\right) + \left(\omega, |u|^p u\right)\right)\omega(x). \end{aligned} \tag{36}$$

Now, let us multiply (36) by $u + u_t$ and integrate over $\Omega$:

$$\begin{aligned} &\frac{d}{dt}\left[\frac{1}{2}\|u\|^2 + \|\nabla u\|^2 + \frac{1}{p+2}\|u\|_{p+2}^{p+2}\right] + \|\nabla u\|^2 + \|u\|_{p+2}^{p+2} + \|u_t\|^2 + \|\nabla u_t\|^2 \\ &\quad = \left(\varphi'(t) + \varphi(t)\right)\left[\varphi'(t) + (\nabla\omega, \nabla u_t) + (\nabla\omega, \nabla u) - \left(\omega, \sum_{i=1}^n b_i u_{x_i}\right) + \left(\omega, |u|^p u\right)\right] \\ &\qquad + \left(u + u_t, \sum_{i=1}^n b_i u_{x_i}\right). \end{aligned} \tag{37}$$

Using Cauchy, Poincare and Young inequalities on the right-hand side of (37), we have

$$\left|\left(u, \sum_{i=1}^n b_i u_{x_i}\right)\right| \le \frac{B_0}{\sqrt{\lambda_1}}\|\nabla u\|^2, \tag{38}$$

$$\left|\left(u_t, \sum_{i=1}^n b_i u_{x_i}\right)\right| \le \|u_t\|^2 + \frac{B_0^2}{4}\|\nabla u\|^2, \tag{39}$$

$$\left|(\nabla\omega, \nabla u_t)\left(\varphi' + \varphi\right)\right| \le \|\nabla u_t\|^2 + \frac{1}{2}\|\nabla\omega\|^2\left(\left|\varphi'\right|^2 + |\varphi|^2\right), \tag{40}$$

$$\left|(\nabla\omega, \nabla u)\left(\varphi' + \varphi\right)\right| \le \frac{\varepsilon}{2}\|\nabla u\|^2 + \frac{1}{\varepsilon}\|\nabla\omega\|^2\left(\left|\varphi'\right|^2 + |\varphi|^2\right), \tag{41}$$

$$\left|\left(\omega, \sum_{i=1}^n b_i u_{x_i}\right)\left(\varphi' + \varphi\right)\right| \le \frac{\varepsilon}{2}\|\nabla u\|^2 + \frac{B_0^2}{\varepsilon}\|\omega\|^2\left(\left|\varphi'\right|^2 + |\varphi|^2\right), \tag{42}$$

$$\left|\left(\omega, |u|^p u\right)\left(\varphi' + \varphi\right)\right| \le \varepsilon\|u\|_{p+2}^{p+2} + C(\varepsilon, p)\|\omega\|_{p+2}^{p+2}\left(\left|\varphi'\right|^{p+2} + |\varphi|^{p+2}\right). \tag{43}$$

Rewriting (37) with estimates (38)-(43), we obtain the following inequality:

$$\begin{aligned}&\frac{d}{dt}\left[\frac{1}{2}\|u\|^2+\|\nabla u\|^2+\frac{1}{p+2}\|u\|_{p+2}^{p+2}\right]\\&\quad+\left(1-\varepsilon-\frac{B_0(4+B_0\sqrt{\lambda_1})}{4\sqrt{\lambda_1}}\right)\|\nabla u\|^2+(1-\varepsilon)\|u\|_{p+2}^{p+2}\le D(t),\end{aligned}\tag{44}$$

where

$$\begin{aligned}D(t)&=\left(\left|\varphi'\right|^2+|\varphi|^2\right)\left(\frac{1}{2}\|\nabla\omega\|^2+\frac{1}{\varepsilon}\|\nabla\omega\|^2+\frac{B_0^2}{\varepsilon}\|\omega\|^2\right)\\&\quad+\left(\varphi'(t)\right)^2+\left|\varphi'(t)\varphi(t)\right|+C(\varepsilon,p)\|\omega\|_{p+2}^{p+2}\left(\left|\varphi'\right|^{p+2}+|\varphi|^{p+2}\right).\end{aligned}$$

We choose $\varepsilon_0>0$ such that $\varepsilon_0\le\varepsilon<1-\frac{B_0(4+B_0\sqrt{\lambda_1})}{4\sqrt{\lambda_1}}$ and take

$$K_3=\min\left\{\frac{2}{3}\left(1-\varepsilon_0-\frac{B_0(4+B_0\sqrt{\lambda_1})}{4\sqrt{\lambda_1}}\right),1-\varepsilon_0\right\}.$$

So, (44) follows

$$\frac{d}{dt}\left[\frac{1}{2}\|u\|^2+\|\nabla u\|^2+\frac{1}{p+2}\|u\|_{p+2}^{p+2}\right]+K_3\left(\frac{3}{2}\|\nabla u\|^2+\|u\|_{p+2}^{p+2}\right)\le D(t).\tag{45}$$

The last term on the left-hand side of (45) can be written

$$\begin{aligned}\frac{3}{2}\|\nabla u\|^2+\|u\|_{p+2}^{p+2}&\ge\frac{\lambda_1}{2}\|u\|^2+\|\nabla u\|^2+\|u\|_{p+2}^{p+2}\\&\ge K_4\left(\frac{1}{2}\|u\|^2+\|\nabla u\|^2+\frac{1}{p+2}\|u\|_{p+2}^{p+2}\right),\end{aligned}\tag{46}$$

where $K_4=\min(\lambda_1,1)$. It follows from (45) and (46)

$$\frac{d}{dt}\eta(t)+K_5\eta(t)\le D(t).\tag{47}$$

Here, $K_5=K_3K_4$ and $\eta(t)=\frac{1}{2}\|u\|^2+\|\nabla u\|^2+\frac{1}{p+2}\|u\|_{p+2}^{p+2}$. After solving first-order differential inequality (47), it follows that

$$\|\nabla u\|^2+\|u\|_{p+2}^{p+2}\to 0\quad\text{as } t\to\infty.\qquad\square$$

**Competing interests**
The author declares that they have no competing interests.

Received: 20 February 2012 Accepted: 14 November 2012 Published: 28 November 2012

**References**

1. Vasin, V, Kamynin, L: On the asymptotic behaviour of solutions to inverse problems for parabolic equations. Sib. Math. J. **38**, 647-662 (1997)
2. Ramm, AG: Inverse Problems, Mathematical and Analytical Techniques with Applications to Engineering. Springer, New York (2005)

3. Kamynin, VL, Franchini, E: An inverse problem for a higher order parabolic equation. Math. Notes **64**(5), 590-599 (1998)
4. Prilepko, AI, Orlowskii, DG, Vasin, IA: Methods for Solving Inverse Problems in Mathematical Physics. Dekker, New York (2000)
5. Riganti, R, Savateev, E: Solution of an inverse problem for the nonlinear heat equation. Commun. Partial Differ. Equ. **19**, 1611-1628 (1994)
6. Tkachenko, DS: On an inverse problem for a parabolic equation. Math. Notes **75**(5), 729-743 (2004)
7. Guvenilir, AF, Kalantarov, VK: The asymptotic behaviour of solutions to an inverse problem for differential operator equations. Math. Comput. Model. **37**, 907-914 (2003)
8. Eden, A, Kalantarov, VK: On global behaviour of solutions to an inverse problem for nonlinear parabolic equations. J. Math. Anal. Appl. **307**, 120-133 (2005)
9. Meyvaci, M: Blow-up of solutions of pseudo-parabolic equations. J. Math. Anal. Appl. **352**, 629-633 (2009)
10. Kalantarov, VK, Ladyzhenskaya, OA: Formation of collapses in quasilinear equations of parabolic and hyperbolic types. Zap. Nauč. Semin. LOMI **69**, 77-102 (1977)

doi:10.1186/1029-242X-2012-274
**Cite this article as:** Yaman: Blow-up solution and stability to an inverse problem for a pseudo-parabolic equation. *Journal of Inequalities and Applications* 2012 **2012**:274.