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Some identities on the twisted q-Bernoulli numbers and polynomials with weight α

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Abstract

In this paper, we consider the twisted q-Bernoulli numbers and polynomials with weight α by using the bosonic q-integral on $\mathbb{Z}_p$. From the construction of the twisted q-Bernoulli numbers with weight α, we derive some identities and relations.

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1 Introduction

Let p be a fixed prime number. Throughout this paper, $\mathbb{Z}_p$, $\mathbb{Q}_p$, and $\mathbb{C}_p$ will, respectively, denote the ring of p-adic rational integers, the field of p-adic rational numbers, and the p-adic completion of the algebraic closure of $\mathbb{Q}_p$, respectively. Let ν_p be the normalized exponential valuation of $\mathbb{C}_p$ with $|p|_p = p^{\nu_p(p)} = p^{-1}$. When one talks of q-extension, q is variously considered as an indeterminate, a complex number $q \in \mathbb{C}$, or a p-adic number $q \in \mathbb{C}_p$. If $q \in \mathbb{C}$, one normally assume $|q| < 1$. If $q \in \mathbb{C}_p$, then we assume that $|1-q|_p < p^{-\frac{1}{1-p}}$, so that $q^x = \exp(x \log q)$ for $|x|_p \le 1$. The q-number is defined by $[x]_q = \frac{1-q^x}{1-q}$ (see [1–25]). Note that $\lim_{q\to 1}[x]_q = x$.

We say that f is uniformly differentiable function at a point $a \in \mathbb{Z}_p$, and denote this property by $f \in UD(\mathbb{Z}_p)$, if the difference quotient $F_f(x,y) = \frac{f(x)-f(y)}{x-y}$ has a limit $f'(a)$ as $(x,y) \to (a,a)$. For $f \in UD(\mathbb{Z}_p)$, the p-adic q-integral on $\mathbb{Z}_p$, which is called the bosonic q-integral, defined by Kim as follows:

$$I_q(f) = \int_{\mathbb{Z}_p} f(x)\, d\mu_q(x) = \lim_{N\to\infty} \frac{1}{[p^N]_q} \sum_{x=0}^{p^N-1} f(x) q^x \quad \bigl(\text{see [14]}\bigr). \tag{1}$$

From (1), we note that

$$\left| \int_{\mathbb{Z}_p} f(x)\, d\mu_q(x) \right|_p \le p \|f\|_1 \quad \bigl(\text{see [11–15]}\bigr), \tag{2}$$

where $\|f\|_1 = \sup\{|f(0)|_p, \sup_{x\neq y} |\frac{f(x)-f(y)}{x-y}|_p\}$.

In [3], Carlitz defined q-Bernoulli numbers which are called the Carlitz's q-Bernoulli numbers, by

$$\beta_{0,q} = 1 \quad \text{and} \quad q(q\beta_q + 1)^n - \beta_{n,q} = \begin{cases} 1 & \text{if } m = 1, \\ 0 & \text{if } m > 1, \end{cases} \tag{3}$$

with the usual convention about replacing $(\beta_q^{(h)})^n$ by $\beta_{n,q}^{(h)}$. In [3], Carlitz also considered the expansion of q-Bernoulli numbers as follows:

$$\beta_{0,q}^{(h)} = \frac{h}{[h]_q} \quad \text{and} \quad q^h\left(q\beta_q^{(h)} + 1\right)^n - \beta_{n,q}^{(h)} = \begin{cases} 1 & \text{if } n = 1, \\ 0 & \text{if } n > 1, \end{cases} \tag{4}$$

with the usual convention about replacing $(\beta_q^{(h)})^n$ by $\beta_{n,q}^{(h)}$. In [15], for $\alpha \in \mathbb{Q}$ and $n \in \mathbb{Z}_+$, q-Bernoulli numbers with weight α is defined by Kim as follows:

$$\beta_{0,q}^{(\alpha)} \quad \text{and} \quad q\left(q^\alpha \widetilde{\beta}_q^{(\alpha)} + 1\right)^n - \widetilde{\beta}_{n,q}^{(\alpha)} = \begin{cases} \frac{\alpha}{[\alpha]_q} & \text{if } n = 1, \\ 0 & \text{if } n > 1, \end{cases} \tag{5}$$

with the usual convention about replacing $(\beta_q^{(\alpha)})^n$ by $\beta_{n,q}^{(\alpha)}$.

Let $C_{p^n} = \{\xi \mid \xi^{p^n} = 1\}$ be the cyclic group of order p^n, and let $T_p = \lim_{n\to\infty} C_{p^n} = \bigcup_{n\geq 0} C_{p^n}$ (see [8, 13, 20–24]). Note that T_p is a locally constant space. For $\xi \in T_p$, the twisted Bernoulli numbers are defined by

$$\frac{t}{\xi e^t - 1} = e^{B_\xi t} = \sum_{n=0}^{\infty} B_{n,\xi} \frac{t^n}{n!}, \tag{6}$$

with the usual convention about replacing $(B_\xi)^n$ by $B_{n,q}$ (see [15, 19–24]).

In the view point of (6), we will try to study the twisted q-Bernoulli numbers with weight α. By using the p-adic q-integral on $\mathbb{Z}_p$, we give some identities and relations on the twisted q-Bernoulli numbers and polynomials with weight α.

2 The twisted q-Bernoulli numbers and polynomials with weight α

Let $f_n(x) = f(x + n)$. In [15, Theorem 1], Kim proved the following integral equation:

$$q^n I_q(f_n) - I_q(f) = (q-1)\sum_{l=0}^{n-1} q^l f(l) + \frac{q-1}{\log q}\sum_{l=0}^{n-1} q^l f'(l). \tag{7}$$

In particular, when $n = 1$, we have

$$qI_q(f_1) - I_q(f) = (q-1)f(0) + \frac{q-1}{\log q} f'(0). \tag{8}$$

Let $\alpha \in \mathbb{Q}$ and $n \in \mathbb{Z}_+$. The nth q-Bernoulli polynomials with weight α are defined by

$$\widetilde{\beta}_{n,\xi,q}^{\alpha}(x) = \int_{\mathbb{Z}_p} \xi^y [y + x]_{q^\alpha}^n \, d\mu_q(y). \tag{9}$$

Then, by using the bosonic p-adic q-integral on $\mathbb{Z}_p$, we evaluate the above equation (9) as follows:

$$\begin{aligned}\widetilde{\beta}^{\alpha}_{n,\xi,q}(x) &= \int_{\mathbb{Z}_p} \xi^y [y+x]^n_{q^\alpha}\, d\mu_q(y) \\ &= \frac{1}{(1-q^\alpha)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{\alpha l x} \int_{\mathbb{Z}_p} \xi^y q^{\alpha l y}\, d\mu_q(y) \\ &= \frac{1-q}{(1-q^\alpha)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l \left(\frac{\alpha l+1}{1-\xi q^{\alpha l+1}}\right) q^{\alpha l x}. \end{aligned} \tag{10}$$

In the special case $x = 0$, $\widetilde{\beta}^{\alpha}_{n,\xi,q}(0) = \widetilde{\beta}^{\alpha}_{n,\xi,q}$ are called the nth twisted q-Bernoulli numbers with weight α.

From (10), we note that

$$\begin{aligned}&\widetilde{\beta}^{\alpha}_{n,\xi,q}(x) \\ &= \frac{1-q}{(1-q^\alpha)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{\alpha l x} \left(\frac{\alpha l}{1-\xi q^{\alpha l+1}}\right) \\ &\quad + \frac{1-q}{(1-q^\alpha)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{\alpha l x} \left(\frac{1}{1-\xi q^{\alpha l+1}}\right) \\ &= \frac{n\alpha(1-q)}{(1-q^\alpha)^n} \sum_{l=1}^{n} \binom{n-1}{l-1} (-1)^l \left(\frac{1}{1-\xi q^{\alpha l+1}}\right) \\ &\quad + \sum_{m=0}^{\infty} (1-q) \frac{(1-q^{\alpha(x+m)})^n}{(1-q^\alpha)^n} \xi^m q^m \\ &= -n\alpha \frac{(1-q)}{(1-q^\alpha)^n} \sum_{m=0}^{\infty} \xi^m q^{m\alpha+m} \left(1-q^{\alpha(x+m)}\right)^{n-1} \\ &\quad + (1-q) \sum_{m=0}^{\infty} [x+m]^n_{q^\alpha} \xi^m q^m \\ &= \frac{-n\alpha}{[\alpha]_q} \sum_{m=0}^{\infty} \xi^m q^{m\alpha+m} [x+m]^{n-1}_{q^\alpha} + (1-q) \sum_{m=0}^{\infty} \xi^m q^m [x+m]^n_{q^\alpha}. \end{aligned} \tag{11}$$

From (11), when $x = 0$, we get

$$\widetilde{\beta}^{\alpha}_{n,\xi,q} = \frac{-n\alpha}{[\alpha]_q} \sum_{m=0}^{\infty} \xi^m q^{m\alpha+m} [m]^{n-1}_{q^\alpha} + (1-q) \sum_{m=0}^{\infty} \xi^m q^m [m]^n_{q^\alpha}. \tag{12}$$

Therefore, by (10) and (12), we obtain the following theorem.

Theorem 1 *Let $n \in \mathbb{N}$ and $\alpha \in \mathbb{C}$. Then we*

$$\begin{aligned}&-\frac{\widetilde{\beta}^{\alpha}_{n,\xi,q}(x)}{n} \\ &= \frac{\alpha}{[\alpha]_q} \sum_{m=0}^{\infty} \xi^m q^{m\alpha+m} [x+m]^{n-1}_{q^\alpha} + \frac{1-q}{n} \sum_{m=0}^{\infty} \xi^m q^m [x+m]^n_{q^\alpha}. \end{aligned}$$

When $x = 0$, we can obtain some identity on the twisted q-Bernoulli numbers with weight α as follows.

Corollary 2 *For $n \in \mathbb{Z}_+$ and $\alpha \in \mathbb{Q}$. Then we have*

$$-\frac{\widetilde{\beta}_{n,\xi,q}^{(\alpha)}}{n} = \frac{\alpha}{[\alpha]_q} \sum_{m=0}^{\infty} \xi^m q^{m\alpha+m} [m]_{q^\alpha}^{n-1} + \frac{1-q}{n} \sum_{m=0}^{\infty} \xi^m q^m [m]_{q^\alpha}^n.$$

For $\alpha = 1$, we note that $\widetilde{\beta}_{n,\xi,q}^{(1)}(x)$ are the twisted Carlitz's q-Bernoulli polynomials and $\widetilde{\beta}_{n,\xi,q}^{(1)}$ are the twisted Carlitz's q-Bernoulli numbers. By Corollary 2, we easily get

$$\begin{aligned}
&\int_{\mathbb{Z}_p} \xi^x e^{[x]_{q^\alpha} t} \, d\mu_q(x) \\
&= -t \frac{\alpha}{[\alpha]_q} \sum_{m=0}^{\infty} \xi^m q^{(m+1)\alpha+m} e^{[m]_{q^\alpha} t} + (1-q) \sum_{m=0}^{\infty} \xi^m q^m e^{[m]_{q^\alpha} t}.
\end{aligned} \tag{13}$$

From (13), we have

$$\sum_{n=0}^{\infty} \widetilde{\beta}_{n,\xi,q}^{(\alpha)} \frac{t^n}{n!} = -t \frac{\alpha}{[\alpha]_q} \sum_{m=0}^{\infty} \xi^m q^{(m+1)\alpha+m} e^{[m]_{q^\alpha} t} + (1-q) \sum_{m=0}^{\infty} \xi^m q^m e^{[m]_{q^\alpha} t}. \tag{14}$$

Therefore, we obtain the following corollary.

Corollary 3 *Let $\alpha \in \mathbb{Q}$ and $F_{\xi,q}^{(\alpha)}(t) = \sum_{n=0}^{\infty} \widetilde{\beta}_{n,\xi,q}^{(\alpha)} \frac{t^n}{n!}$. Then we have*

$$F_{\xi,q}^{(\alpha)}(t) = -t \frac{\alpha}{[\alpha]_q} \sum_{m=0}^{\infty} \xi^m q^{(m+1)\alpha+m} e^{[m]_{q^\alpha} t} + (1-q) \sum_{m=0}^{\infty} \xi^m q^m e^{[m]_{q^\alpha} t}.$$

Let $F_{\xi,q}^{(\alpha)}(t,x) = \sum_{n=0}^{\infty} \sum_{n=0}^{\infty} \widetilde{\beta}_{n,\xi,q}^{(\alpha)}(x) \frac{t^n}{n!}$. From Theorem 1, we have

$$\begin{aligned}
&F_{\xi,q}^{(\alpha)}(t,x) \\
&= -t \frac{\alpha}{[\alpha]_q} \sum_{m=0}^{\infty} \xi^m q^{(m+1)\alpha+m} e^{[m+x]_{q^\alpha} t} + (1-q) \sum_{m=0}^{\infty} \xi^m q^m e^{[m+x]_{q^\alpha} t} \\
&= \sum_{m=0}^{\infty} \widetilde{\beta}_{n,\xi,q}^{(\alpha)}(x) \frac{t^n}{n!}.
\end{aligned} \tag{15}$$

By simple calculation, we easily get

$$\begin{aligned}
\widetilde{\beta}_{n,\xi,q}^{(\alpha)}(x) &= \int_{\mathbb{Z}_p} \xi^y [x+y]_{q^\alpha} \, d\mu_q(y) \\
&= \sum_{l=0}^{n} \binom{n}{l} [x]_{q^\alpha}^{n-l} q^{\alpha l x} \int_{\mathbb{Z}_p} \xi^y [y]_{q^\alpha}^l \, d\mu_q(y) \\
&= \sum_{l=0}^{n} \binom{n}{l} [x]_{q^\alpha}^{n-l} q^{\alpha l x} \widetilde{\beta}_{l,\xi,q}^{(\alpha)}.
\end{aligned} \tag{16}$$

Therefore, by (12), (15), and (16), we obtain the following theorem.

Theorem 4 *For $n \in \mathbb{Z}_+$ and $\alpha \in \mathbb{Q}$, we have*

$$\begin{aligned}\widetilde{\beta}_{n,\xi,q}^{(\alpha)}(x) &= \frac{1-q}{(1-q^\alpha)^n}\sum_{l=0}^{n}\binom{n}{l}(-1)^l q^{\alpha l x}\left(\frac{\alpha l+1}{1-\xi q^{\alpha l+1}}\right)\\ &= -n\frac{\alpha}{[\alpha]_q}\sum_{m=0}^{\infty}\xi^m q^{(m+1)\alpha+m}[m+x]_{q^\alpha}^{n-1}\\ &\quad+(1-q)\sum_{m=0}^{\infty}\xi^m q^m [m+x]_{q^\alpha}^{n}. \end{aligned} \tag{17}$$

Moreover, $\widetilde{\beta}_{n,\xi,q}^{(\alpha)}(x)=\sum_{l=0}^{n}\binom{n}{l}[x]_{q^\alpha}^{n-l}q^{\alpha l x}\widetilde{\beta}_{l,\xi,q}^{(\alpha)}$.

By (7), we see that

$$\begin{aligned}&\xi^n q^n \widetilde{\beta}_{m,\xi,q}^{(\alpha)}(n)-\widetilde{\beta}_{m,\xi,q}^{(\alpha)}\\ &\quad=(q-1)\sum_{l=0}^{n-1}\xi^l q^{\alpha l}[l]_{q^\alpha}^m+m\frac{\alpha}{[\alpha]_q}\sum_{l=0}^{n-1}\xi^l q^{\alpha l+l}[l]_{q^\alpha}^{m-1}.\end{aligned} \tag{18}$$

Therefore, we obtain the following theorem.

Theorem 5 *For $n \in \mathbb{Z}_+$, $n \in \mathbb{N}$, and $\alpha \in \mathbb{Q}$, we have*

$$\begin{aligned}&\xi^n q^n \widetilde{\beta}_{m,\xi,q}^{(\alpha)}(n)-\widetilde{\beta}_{m,\xi,q}^{(\alpha)}\\ &\quad=(q-1)\sum_{l=0}^{n-1}\xi^l q^{\alpha l}[l]_{q^\alpha}^m+m\frac{\alpha}{[\alpha]_q}\sum_{l=0}^{n-1}\xi^l q^{\alpha l+l}[l]_{q^\alpha}^{m-1}.\end{aligned}$$

If we take $f(x)=\xi^x e^{[x]_{q^\alpha}t}$, by (8), then we have

$$\begin{aligned}&(q-1)+\frac{\alpha}{[\alpha]_q}t\\ &\quad=q\int_{\mathbb{Z}_p}\xi^{x+1}e^{[x+1]_{q^\alpha}t}\,d\mu_q(x)-\int_{\mathbb{Z}_p}\xi^x e^{[x]_{q^\alpha}t}\,d\mu_q(x)\\ &\quad=\xi q\int_{\mathbb{Z}_p}\xi^{x}e^{[x+1]_{q^\alpha}t}\,d\mu_q(x)-\int_{\mathbb{Z}_p}\xi^x e^{[x]_{q^\alpha}t}\,d\mu_q(x)\\ &\quad=\sum_{n=0}^{\infty}\left(\xi q\widetilde{\beta}_{n,\xi,q}^{(\alpha)}(1)-\widetilde{\beta}_{n,\xi,q}^{(\alpha)}\right)\frac{t^n}{n!}.\end{aligned} \tag{19}$$

Therefore, by Theorem 5, we obtain the following theorem.

Theorem 6 *For $\alpha \in \mathbb{Q}$ and $n \in \mathbb{Z}_+$, we have*

$$\xi q\widetilde{\beta}_{0,\xi,q}^{(\alpha)}(1)-\widetilde{\beta}_{0,\xi,q}^{(\alpha)}=\begin{cases}q-1 & \text{if } n=0,\\ \frac{\alpha}{[\alpha]_q} & \text{if } n=1,\\ 0 & \text{if } n>1.\end{cases}$$

Remark that when $n = 0$, we have $\widetilde{\beta}^{(\alpha)}_{0,\xi,q} = \frac{q-1}{\xi q-1}$. By (16) and Theorem 6, we get

$$\begin{aligned}\widetilde{\beta}^{(\alpha)}_{n,\xi,q}(x) &= \sum_{l=0}^{n}\binom{n}{l}[x]^{n-l}_{q^\alpha}q^{\alpha lx}\widetilde{\beta}^{(\alpha)}_{l,\xi,q} \\ &= \left([x]_{q^\alpha} + q^{\alpha x}\widetilde{\beta}^{(\alpha)}_{\xi,q}\right)^n, \end{aligned} \tag{20}$$

with the usual convention about replacing $(\widetilde{\beta}^{(\alpha)}_{\xi,q})^n$ by $\widetilde{\beta}^{(\alpha)}_{n,\xi,q}$. By (20) and Theorem 6, we obtain the following theorem.

Theorem 7 *For $n \in \mathbb{Z}_+$ and $\alpha \in \mathbb{Q}$, we have*

$$\xi q\left(q^\alpha\widetilde{\beta}^{(\alpha)}_{\xi,q} + 1\right)^n - \widetilde{\beta}^{(\alpha)}_{n,\xi,q} = \begin{cases} q-1 & \text{if } n = 0, \\ \frac{\alpha}{[\alpha]_q} & \text{if } n = 1, \\ 0 & \text{if } n > 1, \end{cases}$$

with the usual convention about replacing $(\widetilde{\beta}^{(\alpha)}_{\xi,q})^n$ by $\widetilde{\beta}^{(\alpha)}_{n,\xi,q}$.

From (8), we can easily derive the following equation (21). For $d \in \mathbb{N}$, we get

$$\begin{aligned}&\int_{\mathbb{Z}_p}\xi^y[x+y]^n_{q^\alpha}\,d\mu_q(y) \\ &= \frac{[d]^n_{q^\alpha}}{[d]_q}\sum_{a=0}^{d-1}q^a\int_{\mathbb{Z}_p}\xi^y\left[\frac{x+a}{d}+y\right]^n_{q^{\alpha d}} \\ &= \frac{[d]^n_{q^\alpha}}{[d]_q}\sum_{a=0}^{d-1}q^a\widetilde{\beta}^{(\alpha)}_{n,\xi,q^d}\left(\frac{x+a}{d}\right). \end{aligned} \tag{21}$$

Therefore, by (21), we obtain the following theorem.

Theorem 8 *For $n \in \mathbb{Z}_+$, $d \in \mathbb{N}$, and $\alpha \in \mathbb{Q}$, we have*

$$\widetilde{\beta}^{(\alpha)}_{n,\xi,q}(x) = \frac{[d]^n_{q^\alpha}}{[d]_q}\sum_{a=0}^{d-1}q^a\widetilde{\beta}^{(\alpha)}_{n,\xi,q^d}\left(\frac{x+a}{d}\right).$$

From (9), we note that

$$\begin{aligned}&\widetilde{\beta}^{(\alpha)}_{n,\xi^{-1},q^{-1}}(1-x) \\ &= \int_{\mathbb{Z}_p}\xi^{-y}[1-x+y]^n_{q^{-\alpha}}\,d\mu_{q^{-1}}(y) \\ &= \frac{(-1)^n q^{\alpha n}}{(1-q^\alpha)^n}\sum_{l=0}^{n}\binom{n}{l}(-1)^l q^{\alpha lx-\alpha l}\int_{\mathbb{Z}_p}\xi^{-y}q^{-\alpha ly}\,d\mu_{q^{-1}}(y) \\ &= q^{\alpha n}(-1)^n\left(\frac{1}{(1-q^\alpha)^{n-1}}\sum_{l=0}^{n-1}\binom{n}{l}(-1)^l q^{\alpha lx}\frac{\alpha l+1}{1-\xi q^{\alpha l+1}}\right) \\ &= q^{\alpha n}(-1)^n\widetilde{\beta}^{(\alpha)}_{n,\xi,q}(x). \end{aligned} \tag{22}$$

Therefore, by (22), we obtain the following theorem.

Theorem 9 *For $n \in \mathbb{Z}_+$ and $\alpha \in \mathbb{Q}$, we have*

$$\widetilde{\beta}^{(\alpha)}_{n,\xi^{-1},q^{-1}}(1-x) = (-1)^n q^{\alpha n} \widetilde{\beta}^{(\alpha)}_{n,\xi,q}(x).$$

It is easy to show that

$$\begin{aligned} &\int_{\mathbb{Z}_p} \xi^{-x}[1-x]^n_{q^{-\alpha}}\, d\mu_{q^{-1}}(x) \\ &\quad = \int_{\mathbb{Z}_p} \xi^x\big(1-[x]_{q^\alpha}\big)^n\, d\mu_q(x) \\ &\quad = (-1)^n q^{\alpha n} \int_{\mathbb{Z}_p} \xi^x [x-1]^n_{q^\alpha}\, d\mu_q(x). \end{aligned} \tag{23}$$

By (22) and (23), we obtain the following corollary.

Corollary 10 *For $n \in \mathbb{Z}_+$ and $\alpha \in \mathbb{Q}$, we have*

$$\begin{aligned} \int_{\mathbb{Z}_p} \xi^{-x}[1-x]^n_{q^{-\alpha}}\, d\mu_{q^{-1}}(x) &= \sum_{l=0}^{n} \binom{n}{l} (-1)^l \widetilde{\beta}^{(\alpha)}_{l,\xi,q} \\ &= (-1)^n q^{\alpha n} \widetilde{\beta}^{(\alpha)}_{n,\xi,q}(-1) = \widetilde{\beta}^{(\alpha)}_{n,\xi^{-1},q^{-1}}(2). \end{aligned}$$

Competing interests
The author declares that they have no competing interests.

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