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Random approximation with weak contraction random operators and a random fixed point theorem for nonexpansive random self-mappings

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Abstract

In real reflexive separable Banach space which admits a weakly sequentially continuous duality mapping, the sufficient and necessary conditions that nonexpansive random self-mapping has a random fixed point are obtained. By introducing a random iteration process with weak contraction random operator, we obtain a convergence theorem of the random iteration process to a random fixed point for nonexpansive random self-mappings.

1 Introduction

Random nonlinear analysis is an important mathematical discipline which is mainly concerned with the study of random nonlinear operators and their properties and is much needed for the study of various classes of random equations. Random techniques have been crucial in diverse areas from puremathematics to applied sciences. Of course famously random methods have revolutionised the financial markets. Random fixed point theorems for random contraction mappings on separable complete metric spaces were first proved by Spacek [1]. The survey article by Bharucha-Reid [2] in 1976 attracted the attention of several mathematician and gave wings to this theory. Itoh [3] extended Spacek's theorem to multivalued contraction mappings. Now this theory has become the full fledged research area and various ideas associated with random fixed point theory are used to obtain the solution of nonlinear random system (see [4]). Recently Beg [5,6], Beg and Shahzad [7] and many other authors have studied the fixed points of random maps. Choudhury [8], Park [9], Schu [10], and Choudhury and Ray [11] had used different iteration processes to obtain fixed points in deterministic operator theory. In this article, we study a random iteration process with weak contraction random operator and obtain the convergence theorem of the random iteration process to a random fixed point for nonexpansive random self-mapping. Our main results are the randomizations of most of the results of the recent articles by Song and Yang [12], Song and Chen [13], and Xu [14]. In particular these results extend the corresponding results of Beg and Abbas [15].

2 Preliminaries

Let X be Banach space, we denote its norm by $||\cdot||$ and its dual space by X^*. The value of $x^* \in X^*$ at $y \in X$ is denoted by $\langle y, x^*\rangle$, and the normalized duality mapping from X

into 2^{X^*} is denoted by J, that is,

$$J(x) = \{f \in X^* : \langle x, f \rangle = ||x||||f||, ||x|| = ||f||\}, \quad for \ \ all\ x \in X.$$

A Banach space X is said to be (i) *strictly convex* if $||x|| = ||y|| = 1$, $x \neq y$ implies $\left\| \frac{x+y}{2} \right\| < 1$;

(ii) *uniformly convex* if for all $\varepsilon \in [0, 2]$, there exists $\delta_\varepsilon > 0$ such that $||x|| = ||y|| = 1$ with $||x - y|| \geq \varepsilon$ implies $\left\| \frac{x+y}{2} \right\| < 1 - \delta_\varepsilon$.

It is well known that each uniformly convex Banach space X is reflexive and strictly convex. Typical examples of both uniformly convex and uniformly smooth Banach spaces are L^p spaces, where $p > 1$.

Recall that J is said to be weakly sequentially continuous, if for each $\{x_n\} \subset X$ with $x_n \rightharpoonup x$, then $J(x_n) \rightharpoonup^* J(x)$.

Recall that a Banach space X is said to satisfy *Opial's condition* if, for any $\{x_n\} \subset X$ with $x_n \rightharpoonup x$, the following inequality holds:

$$\liminf_{n\to\infty} ||x_n - x|| < \liminf_{n\to\infty} ||x_n - y||$$

for $y \in X$ with $y \neq x$. It is well known that the above inequality is equivalent to

$$\limsup_{n\to\infty} ||x_n - x|| < \limsup_{n\to\infty} ||x_n - y||$$

for $y \in X$ with $y \neq x$.

It is well known that all Hilbert spaces and l^p $(p > 1)$ spaces satisfy Opial's condition, while L^p spaces does not unless $p = 2$.

Remark 2.1. If X admits a weakly sequentially continuous duality mapping, then X satisfies Opial's condition, and X is smooth, for the details, see [16].

Let (Ω, Σ) be a measurable space (Σ-sigma algebra) and F be a nonempty subset of a Banach space X. A mapping ξ: $\Omega \to X$ is *measurable* if $\xi^{-1}(U) \in \Sigma$ for each open subset U of X. The mapping T: $\Omega \times F \to F$ is a *random map* if and only if for each fixed $x \in F$, the mapping $T(\cdot, x)$: $\Omega \to F$ is measurable, and it is *continuous* if for each $\omega \in \Omega$, the mapping $T(\omega, \cdot)$: $F \to X$ is continuous. A measurable mapping ξ: $\Omega \to X$ is *the random fixed point* of the random map T: $\Omega \times F \to X$ if and only if $T(\omega, \xi(\omega)) = \xi(\omega)$, for each $\omega \in \Omega$.

Throughout this article, we denote the set of all random fixed points of random mapping T by $RF(T)$, the set of all fixed points of $T(\omega, \cdot)$ by $F(T(\omega, \cdot))$ for each $\omega \in \Omega$, respectively.

Definition 2.1. Let F be a nonempty subset of a separable Banach space X and T: $\Omega \times F \to F$ be a random map. The map T is said to be:

(a) *weakly contractive random operator* if for arbitrary $x, y \in F$,

$$||T(\omega, x) - T(\omega, y)|| \leq ||x - y|| - \varphi(||x - y||)$$

for each $\omega \in \Omega$, where φ: $[0, \infty) \to [0, \infty)$ is a continuous and nondecreasing map such that $\varphi(0) = 0$, and $\lim_{t \to \infty} \varphi(t) = \infty$.

(b) *nonexpansive random operator* if for each $\omega \in \Omega$, such that for arbitrary $x, y \in F$ we have

$$||T(\omega, x) - T(\omega, y)|| \leq ||x - y||$$

(c) *completely continuous random operator* if the sequence $\{x_n\}$ in F converges weakly to x_0 implies that $\{T(\omega, x_n)\}$ converges strongly to $T(\omega, x_0)$ for each $\omega \in \Omega$.

(d) *demiclosed random operator* (at y) if $\{x_n\}$ and $\{y_n\}$ are two sequences such that $T(\omega, x_n) = y_n$ and $\{x_n\}$ converges weakly to x and $\{T(\omega, x_n)\}$ converges to y imply that $x \in F$ and $T(\omega, x) = y$, for each $\omega \in \Omega$.

Definition 2.2. Let T: $\Omega \times F \to F$ be a nonexpansive random operator, f: $\Omega \times F \to F$ be a weakly contractive random operator, F is a nonempty convex subset of a separable Banach space X. For each $t \in (0, 1)$, the random iteration scheme with weakly contractive random operator is defined by

$$\xi_t(\omega) = (1 - t)f(w, \xi_t(\omega)) + tT(\omega, \xi_t(\omega))$$

for each $\omega \in \Omega$. Since F is a convex set, it follows that for each $t \in (0,1)$, ξ_t is a mapping from Ω to F.

Remark 2.2. Let F be a closed and convex subset of a separable Banach space X and the random iteration scheme $\{\xi_t\}$ defined as in Definition 2.2 is pointwise convergent, that is, $\xi_t(\omega) \to q := \xi(\omega)$ for each $\omega \in \Omega$. Then closedness of F implies ξ is a mapping from Ω to F. Since F is a subset of a separable Banach space X, so, if T is a continuous random operator then by [17, Lemma 8.2.3], the map $\omega \to T(\omega, f(\omega))$ is a measurable function for any measurable function f from Ω to F. Thus $\{\xi_t\}$ is measurable. Hence ξ: $\Omega \to F$, being the limit of $\{\xi_t\}$, is also measurable.

Lemma 2.1.([15]). Let F be a closed and convex subset of a complete separable metric space X, and T: $\Omega \times F \to F$ be a weakly contractive random operator. Then T has unique random fixed point.

Lemma 2.2. Let X be a real reflexive separable Banach space which satisfies Opial's condition. Let K be a nonempty closed convex subset of X, and T: $\Omega \times K \to K$ is nonexpansive random mapping. Then, I - T is demiclosed at zero.

Proof From Zhou [18], it is easy to imply the conclusion of Lemma 2.2 is valid.

3 Main results

Theorem 3.1 Let X be a real reflexive separable Banach space which admits a weakly sequentially continuous duality mapping from X to X^*, and K be a nonempty closed convex subset of X. Suppose that T: $\Omega \times K \to K$ is nonexpansive random mapping and f: $\Omega \times K \to K$ is a weakly contractive random mapping with a function ϕ, then

(i) for each $t \in (0, 1)$, there exists a unique measurable mapping ξ_t from Ω into X such that

$$\xi_t(\omega) = tf(\omega, \xi_t(\omega)) + (1 - t)T(\omega, \xi_t(\omega))$$

for each $\omega \in \Omega$.

(ii) T has a random fixed point if and only if $\{\xi_t(\omega)\}$ is bounded as $t \to 0$ for each $\omega \in \Omega$. In this case, $\{\xi_t\}$ converges strongly to some random fixed point ξ^* of T such that ξ^* is the unique random solution in $RF(T)$ to the following variational random inequality:

$$\langle (f - I)(w, \xi^*(\omega)), j(\xi(w) - \xi^*(\omega)) \rangle \leq 0, \quad \textit{for all } \xi \in RF(T) \textit{ and each } \omega \in \Omega. \quad (3.1)$$

Proof For each $t \in (0, 1)$, we define a random mapping S_t: $\Omega \times K \to K$ by $S_t(\omega, x) = tf(\omega, x) + (1 - t)T(\omega, x)$ for each $(\omega, x) \in \Omega \times K$, then for any $x, y \in K$ and each $\omega \in \Omega$, and $\psi(\cdot) = t\phi(\cdot)$ we have

$$\begin{aligned} &||S_t(\omega, x) - S_t(\omega, y)|| \\ \leq\ & t||f(\omega, x) - f(\omega, y)|| + (1 - t)||||T(\omega, x) - T(\omega, y)|| \\ \leq\ & t||x - y|| + (1 - t)||x - y|| - t\varphi(||x - y||) \\ =\ & ||x - y|| - t\varphi(||x - y||) \\ =\ & ||x - y|| - \psi(||x - y||) \end{aligned}$$

Thus, S_t: $\Omega \times K \to K$ is a weakly contractive random mapping with a function ψ, it follows from Lemma 2.1 that there exists a unique random fixed point of S_t, say ξ_t such that for each $\omega \in \Omega$,

$$\xi_t(\omega) = tf(\omega, \xi_t(\omega)) + (1 - t)T(\omega, \xi_t(\omega)) \tag{3.2}$$

This completes the proof of (i).

Next we shall show the uniqueness of the random solution of the variational random inequality (3.1) in $RF(T)$. In fact, suppose $\xi_1(\omega), \xi_2(\omega) \in RF(T)$ satisfy (3.1), we see that

$$\left\langle (f - I)(\omega, \xi_1(\omega)), j(\xi_2(\omega) - \xi_1(\omega)) \right\rangle \leq 0 \tag{3.3}$$

$$\left\langle (f - I)(\omega, \xi_2(\omega)), j(\xi_1(\omega) - \xi_2(\omega)) \right\rangle \leq 0 \tag{3.4}$$

for each $\omega \in \Omega$. Adding (3.3) and (3.4) up, we have that

$$\begin{aligned} 0 &\geq \left\langle (I - f)(\omega, \xi_1(\omega)) - (f - I)(\omega, \xi_2(\omega)), j(\xi_1(\omega) - \xi_2(\omega)) \right\rangle \\ &= \left\langle \xi_1(\omega) - \xi_2(\omega), j(\xi_1(\omega) - \xi_2(\omega)) \right\rangle - \left\langle f(\omega, \xi_1(\omega)) - f(\omega, \xi_2(\omega)), j(\xi_1(\omega) - \xi_2(\omega)) \right\rangle \\ &\geq ||\xi_1(\omega) - \xi_2(\omega)||^2 - ||\xi_1(\omega) - \xi_2(\omega)||^2 + \varphi(||\xi_1(\omega) - \xi_2(\omega)||)||\xi_1(\omega) - \xi_2(\omega)|| \end{aligned}$$

Thus putting $\varphi(||\xi_1(\omega) - \xi_2(\omega)||) = ||\xi_1(\omega) - \xi_2(\omega)||\phi(||\xi_1(\omega) - \xi_2(\omega)||)$, we have

$$\phi(||\xi_1(\omega) - \xi_2(\omega)||) \leq 0$$

for each $\omega \in \Omega$. By the property of φ, we obtain that $\xi_1(\omega) = \xi_2(\omega)$ for each $\omega \in \Omega$ and the uniqueness is proved.

(ii) Let ξ: $\Omega \to F$ be the random fixed point of T, then we have from (3.2) that

$$\begin{aligned} &||\xi_t(\omega) - \xi(\omega)||^2 = \left\langle \xi_t(\omega) - \xi(\omega), j(\xi_t(\omega) - \xi(\omega)) \right\rangle \\ \leq\ & t\left\langle f(\omega, \xi_t(\omega)) - f(\omega, \xi(\omega)), j(\xi_t(\omega) - \xi(\omega)) \right\rangle \\ &+t\left\langle f(\omega, \xi(\omega)) - \xi(\omega), j(\xi_t(\omega) - \xi(\omega)) \right\rangle \\ &+(1 - t)\left\langle T(\omega, \xi_t(\omega)) - \xi(\omega), j(\xi_t(\omega) - \xi(\omega)) \right\rangle \\ \leq\ & ||\xi_t(\omega) - \xi(\omega)||||\xi_t(\omega) - \xi(\omega)|| + t\left\langle f(\omega, \xi(\omega)) - \xi(\omega), j(\xi_t(\omega) - \xi(\omega)) \right\rangle \\ &-t\varphi(||\xi_t(\omega) - \xi(\omega)||)||\xi_t(\omega) - \xi(\omega)|| \\ \leq\ & ||\xi_t(\omega) - \xi(\omega)||^2 + t||f(\omega, \xi(\omega)) - \xi(\omega)||||\xi_t(\omega) - \xi(\omega)|| \\ &-t\varphi(||\xi_t(\omega) - \xi(\omega)||)||\xi_t(\omega) - \xi(\omega)|| \end{aligned} \tag{3.5}$$

Therefore,

$$\varphi(||\xi_t(\omega) - \xi(\omega)||) \leq ||f(\omega, \xi(\omega)) - \xi(\omega)||$$

for each $\omega \in \Omega$, which implies that $\{\phi(||\xi_t(\omega) - \xi(\omega)||)\}$ is bounded for each $\omega \in \Omega$.It further implies $\{||\xi_t(\omega) - \xi(\omega)||\}$ is bounded by the property of ϕ for each $\omega \in \Omega$. So $\{\xi_t(\omega)\}$ is bounded for each $\omega \in \Omega$.

On the other hand, suppose that $\{\xi_t(\omega)\}$ is bounded for each $\omega \in \Omega$, and hence $\{T(\omega, \xi_t(\omega))\}$ is bounded for each $\omega \in \Omega$. By (3.2), we have that

$$\begin{aligned} ||f(\omega, \xi_t(\omega))|| &\leq \frac{1}{t}||\xi_t(\omega)|| + \frac{1-t}{t}||T(\omega, \xi_t(\omega))|| \\ &\leq \max\{\sup_{t\in(0,1)}||\xi_t(\omega)||, \sup_{t\in(0,1)}||T(\omega, \xi_t(\omega))||\} \end{aligned}$$

for each $\omega \in \Omega$. Thus $\{f(\omega, \xi_t(\omega))\}$ is bounded for each $\omega \in \Omega$. Using $t \to 0$, this implies that

$$||T(\omega, \xi_t(\omega)) - \xi_t(\omega)|| \leq \frac{t}{1-t}||\xi_t(\omega) - f(\omega, \xi_t(\omega))|| \to 0$$

for each $\omega \in \Omega$.

By reflexivity of X and boundedness of $\{\xi_t(\omega)\}$, there exists $\left\{\xi_{t_n}(\omega)\right\} \subset \{\xi_t(\omega)\}$ such that $\xi_{t_n}(\omega) \rightharpoonup \xi^*(\omega)(n \to \infty)$ for each $\omega \in \Omega$. Taken together with Banach space X with a weakly sequentially continuous duality mapping satisfying Opial's condition [[16], Theorem 1], it follows from Lemma 2.2 that $\xi^* \in RF(T)$. Therefore $RF(T) \neq \emptyset$

Furthermore, in (3.5), interchange $\xi(\omega)$ and $\xi^*(\omega)$ to obtain

$$\varphi(||\xi_{t_n}(\omega) - \xi^*(\omega)||) \leq \left\langle f(\omega, \xi^*(\omega)), j(\xi_{t_n}(\omega) - \xi^*(\omega))\right\rangle$$

Using that the duality map J is single-valued and weakly sequentially continuous from X to X^*, we get that

$$0 \leq \limsup_{n\to\infty} \varphi(||\xi_{t_n}(\omega) - \xi^*(\omega)||) \leq 0$$

and hence, $\lim_{n\to\infty}\varphi(||\xi_{t_n}(\omega) - \xi^*(\omega)||) = 0$, for each $\omega \in \Omega$, by the property of φ, $\{\xi_{t_n}\}$ strongly converges to $\xi^* \in RF(T)$.

Next we show that $\xi^*: \Omega \to F$ is a random solution in $RF(T)$ to the variational random inequality (3.1). In fact, for any $\xi: \Omega \to F \in RF(T)$, we have from (3.2),

$$\begin{aligned} &\left\langle \xi_t(\omega) - f(\omega, \xi_t(\omega)), j(\xi_t(\omega) - \xi(\omega))\right\rangle \\ &= (1-t)\left\langle T(\omega, \xi_t(\omega)) - f(\omega, \xi_t(\omega)), j(\xi_t(\omega) - \xi(\omega))\right\rangle \\ &= (1-t)\left\langle T(\omega, \xi_t(\omega)) - \xi(\omega), j(\xi_t(\omega) - \xi(\omega))\right\rangle \\ &\quad +(1-t)\left\langle \xi(\omega) - f(\omega, \xi_t(\omega)), j(\xi_t(\omega) - \xi(\omega))\right\rangle \\ &\leq (1-t)||\xi_t(\omega) - \xi(\omega)||^2 + (1-t)\left\langle \xi(\omega) - \xi_t(\omega), j(\xi_t(\omega) - \xi(\omega))\right\rangle \\ &\quad +(1-t)\left\langle \xi_t(\omega) - f(\omega, \xi_t(\omega)), j(\xi_t(\omega) - \xi(\omega))\right\rangle \\ &= (1-t)\left\langle \xi_t(\omega) - f(\omega, \xi_t(\omega)), j(\xi_t(\omega) - \xi(\omega))\right\rangle \end{aligned}$$

for each $\omega \in \Omega$. Therefore,

$$\left\langle \xi_t(\omega) - f(\omega, \xi_t(\omega)), j(\xi_t(\omega) - \xi(\omega))\right\rangle \leq 0 \tag{3.6}$$

for each $\omega \in \Omega$. Since the duality map J is single-valued and weakly sequentially continuous from X to X^*, for any $\xi \in RF(T)$, by $\xi_{t_n} \to \xi^*(t_n \to 0)$, we have from (3.6) that

$$\begin{aligned}&\left\langle f(\omega, \xi^*(\omega)) - \xi^*(\omega), j(\xi(\omega) - \xi^*(\omega))\right\rangle \\ &= \lim_{n\to\infty} \left\langle f(\omega, \xi_{t_n}(\omega)) - \xi_{t_n}(\omega), j(\xi(\omega) - \xi_{t_n}(\omega))\right\rangle \le 0\end{aligned}$$

for each $\omega \in \Omega$. That is, $\xi^* \in RF(T)$ is a random solution of the variational random inequality (3.1). To prove the entire sequence of function $\{\xi_t(\omega): 0 < t < 1\}$ strongly converges to $\xi^*(\omega)$ for each $\omega \in \Omega$. Suppose that there exists another subsequence of functions $\{\xi_{t_k}(\omega)\}$ such that $\xi_{t_k}(\omega) \rightharpoonup \eta^*(\omega)$ as $t_k \to 0$ for each $\omega \in \Omega$. Then we also have $\eta^* \in RF(T)$ and η^* is a random solution of the variational random inequality (3.1). Hence, $\eta^*(\omega) = \xi^*(\omega)$ by uniqueness for each $\omega \in \Omega$. Therefore, $\xi_t \to \xi^*$ as $t \to 0$. This complete the proof of (ii).

Remark 3.1 The above theorem is a randomization of the results by Song and Yang [12], Song and Chen [13]. In particular this result extends the corresponding result of Beg and Abbas [15].

As some Corollaries of the above theorem, we now obtain a random version of the results given in Xu [14].

Corollary 3.1 Let X, K, T be as in Theorem 3.1, f: $\Omega \times K \to K$ is a contractive random mapping, then

(i) for each $t \in (0, 1)$, there exists a unique measurable mapping ξ_t from Ω into X such that

$$\xi_t(\omega) = tf(\omega, \xi_t(\omega)) + (1 - t)T(\omega, \xi_t(\omega))$$

for each $\omega \in \Omega$.

(ii) T has a random fixed point if and only if $\{\xi_t(\omega)\}$ is bounded as $t \to 0$ for each $\omega \in \Omega$. In this case, $\{\xi_t\}$ converges strongly to some random fixed point ξ^* of T such that ξ^* is the unique random solution in $RF(T)$ to the variational random inequality (3.1).

Corollary 3.2 Let X, K, T be as in Theorem 3.1, for any given measurable mapping ξ: $\Omega \to K$, then

(i) for each $t \in (0, 1)$, there exists a unique measurable mapping ξ_t from Ω into X such that

$$\xi_t(\omega) = t\xi(\omega) + (1 - t)T(\omega, \xi_t(\omega))$$

for each $\omega \in \Omega$.

(ii) T has a random fixed point if and only if $\{\xi_t(\omega)\}$ is bounded as $t \to 0$ for each $\omega \in \Omega$. In this case, $\{\xi_t\}$ converges strongly to some random fixed point ξ^* of T such that ξ^* is the unique random solution in $RF(T)$ to the variational random inequality (3.1).

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Authors' contributions
SL and XX carried out the proof of convergence of the theorems. LL and JL carried out the check of the manuscript. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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