Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH Open Access

Some subordination and superordination results of generalized Srivastava-Attiya operator

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Abstract

In this article, we obtain some subordination and superordination-preserving results of the generalized Srivastava-Attyia operator. Sandwich-type result is also obtained.

Mathematics Subject Classification 2000: 30C45.

Keywords: analytic function, Hadamard product, differential subordination, superordination

1 Introduction

Let $H(U)$ be the class of functions analytic in $U = \{z \in \mathbb{C} : |z| < 1\}$ and $H[a, n]$ be the subclass of $H(U)$ consisting of functions of the form $f(z) = a + a_n z^n + a_{n+1} z^{n+1} + \ldots$, with $H_0 = H[0, 1]$ and $H = H[1, 1]$. Denote $A(p)$ by the class of all analytic functions of the form

$$f(z) = z^p + \sum_{n=1}^{\infty} a_{p+n} z^{p+n} (p \in \mathbb{N} = \{1, 2, 3, \ldots\}; z \in U) \tag{1.1}$$

and let $A\ (1) = A$. For $f, F \in H(U)$, the function $f(z)$ is said to be subordinate to $F(z)$, or $F(z)$ is superordinate to $f(z)$, if there exists a function $\omega(z)$ analytic in U with $\omega(0) = 0$ and $|\omega(z)| < 1 (z \in U)$, such that $f(z) = F(\omega(z))$. In such a case we write $f(z) \prec F(z)$. If F is univalent, then $f(z) \prec F(z)$ if and only if $f(0) = F(0)$ and $f(U) \subset F(U)$ (see [1,2]).

Let $\phi : \mathbb{C}^2 \times U \to \mathbb{C}$ and $h(z)$ be univalent in U. If $p(z)$ is analytic in U and satisfies the first order differential subordination:

$$\phi\left(p(z), zp'(z); z\right) \prec h(z), \tag{1.2}$$

then $p\ (z)$ is a solution of the differential subordination (1.2). The univalent function $q\ (z)$ is called a dominant of the solutions of the differential subordination (1.2) if $p(z) \prec q(z)$ for all $p(z)$ satisfying (1.2). A univalent dominant $\tilde{q}$ that satisfies $\tilde{q} \prec q$ for all dominants of (1.2) is called the best dominant. If $p(z)$ and $\varphi(p(z), zp'(z); z)$ are univalent in U and if $p(z)$ satisfies the first order differential superordination:

$$h(z) \prec \phi\left(p(z), zp'(z); z\right), \tag{1.3}$$

then $p(z)$ is a solution of the differential superordination (1.3). An analytic function $q\ (z)$ is called a subordinant of the solutions of the differential superordination (1.3) if q

$(z) \prec p(z)$ for all $p(z)$ satisfying (1.3). A univalent subordinant $\tilde{q}$ that satisfies $q \prec \tilde{q}$ for all subordinants of (1.3) is called the best subordinant (see [1,2]).

The general Hurwitz-Lerch Zeta function $\Phi(z, s, a)$ is defined by:

$$\Phi(z,\ s,\ a) = \sum_{n=0}^{\infty} \frac{z^n}{(n+a)^s}, \tag{1.4}$$

$(a \in \mathbb{C}\backslash\mathbb{Z}_0^-;\ \mathbb{Z}_0^- = \{0, -1, -2, \ldots\};\ s \in \mathbb{C}$ when $|z| < 1$; $R\{s\} > 1$ when $|z| = 1)$.

For further interesting properties and characteristics of the Hurwitz-Lerch Zeta function $\Phi(z, s, a)$ (see [3-7]).

Recently, Srivastava and Attiya [8] introduced the linear operator $L_{s,b} : A \to A$, defined in terms of the Hadamard product by

$$L_{s,b}(f)(z) = G_{s,b}(z) * f(z)(z \in U; b \in \mathbb{C}\backslash\mathbb{Z}_0^-; s \in \mathbb{C}), \tag{1.5}$$

where

$$G_{s,b} = (1+b)^s[\Phi(z,\ s,\ b) - b^{-s}](z \in U). \tag{1.6}$$

The Srivastava-Attiya operator $L_{s,b}$ contains among its special cases, the integral operators introduced and investigated by Alexander [9], Libera [10] and Jung et al. [11].

Analogous to $L_{s,b}$, Liu [12] defined the operator $J_{p,s,b} : A(p) \to A(p)$ by

$$J_{p,s,b}(f)(z) = G_{p,s,b}(z) * f(z)\ (z \in U; b \in \mathbb{C}/\mathbb{Z}_0^-; s \in \mathbb{C}; p \in \mathbb{N}), \tag{1.7}$$

where

$$G_{p,s,b} = (1+b)^s[\Phi_p(z,\ s,\ b) - b^{-s}]$$

and

$$\Phi_p(z,\ s,\ b) = \frac{1}{b^s} + \sum_{n=0}^{\infty} \frac{z^{n+p}}{(n+1+b)^s}. \tag{1.8}$$

It is easy to observe from (1.7) and (1.8) that

$$J_{p,s,b}(f)(z) = z^p + \sum_{n=1}^{\infty} \left(\frac{1+b}{n+1+b}\right)^s a_{n+p} z^{n+p}. \tag{1.9}$$

We note that

(i) $J_{p,0,b}(f)(z) = f(z)$;

(ii) $J_{1,s,b}(f)(z) = L_{s,b}f(z)\ (s \in \mathbb{C}, b \in \mathbb{C}\backslash\mathbb{Z}_0^-)$, where the operator $L_{s,b}$ was introduced by Srivastava and Attiya [8];

(iii) $J_{p,1,\nu+p-1}(f)(z) = F_{\nu,p}(f(z))\ (\nu > -p, p \in \mathbb{N})$, where the operator $F_{\nu,p}$ was introduced by Choi et al. [13];

(iv) $J_{p,\alpha,p}(f)(z) = I_p^{\alpha} f(z)\ (\alpha \geq 0,\ p \in \mathbb{N})$, where the operator I_p^{α} was introduced by Shams et al. [14];

(v) $J_{p,m,p-1}(f)(z) = J_p^m f(z)\ (m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\},\ p \in \mathbb{N})$, where the operator J_p^m was introduced by El-Ashwah and Aouf [15];

(vi) $J_{p,m,p+l-1}(f)(z) = J_p^m(l)f(z)$ $(m \in \mathbb{N}_0,\ p \in \mathbb{N},\ l \geq 0)$, where the operator $J_p^m(l)$ was introduced by El-Ashwah and Aouf [15].

It follows from (1.9) that:

$$z(J_{p,s+1,b}(f)(z))' = (b+1)J_{p,s,b}(f)(z) - (b+1-p)J_{p,s+1,b}(f)(z). \tag{1.10}$$

To prove our results, we need the following definitions and lemmas.

Definition 1 [1]*Denote by* $\mathcal{F}$ *the set of all functions* $q(z)$ *that are analytic and injective on* $\bar{U}\backslash E(q)$ *where*

$$E(q) = \left\{\zeta \in \partial U : \lim_{z \to \zeta} q(z) = \infty\right\}$$

and are such that $q'(\zeta) \neq 0$ *for* $\zeta \in \delta U \backslash E(q)$. *Further let the subclass of* $\mathcal{F}$ *for which* $q(0) = a$ *be denoted by* $\mathcal{F}(a)$, $\mathcal{F}(0) \equiv \mathcal{F}_0$ *and* $\mathcal{F}(1) \equiv \mathcal{F}_1$.

Definition 2 [2]*A function* $L(z, t)$ $(z \in U, t \geq 0)$ *is said to be a subordination chain if* $L(0, t)$ *is analytic and univalent in* U *for all* $t \geq 0$, $L(z, 0)$ *is continuously differentiable on* $[0; 1)$ *for all* $z \in U$ *and* $L(z, t_1) \prec L(z, t_2)$ *for all* $0 \leq t_1 \leq t_2$.

Lemma 1 [16]*The function* $L(z,t) : U \times [0;1) \to \mathbb{C}$ *of the form*

$$L(z,\ t) = a_1(t)z + a_2(t)z^2 + \cdots \quad (a_1(t) \neq 0; t \geq 0)$$

and $\lim_{t\to\infty} |a_1(t)| = \infty$ *is a subordination chain if and only if*

$$\mathrm{Re}\left\{\frac{z\partial L(z,t)/\partial z}{\partial L(z,t)/\partial t}\right\} > 0 \ (z \in U,\ t \geq 0).$$

Lemma 2 [17]*Suppose that the function* $\mathcal{H} : \mathbb{C}^2 \to \mathbb{C}$ *satisfies the condition*

$$\mathrm{Re}\{\mathcal{H}(is;\ t)\} \leq 0$$

for all real s *and for all* $t \leq -n(1+s^2)/2$, $n \in \mathbb{N}$. *If the function* $p(z) = 1+p_n z^n + p_{n+1}z^{n+1} + \ldots$ *is analytic in* U *and*

$$\mathrm{Re}\left\{\mathcal{H}\left(p(z); zp'(z)\right)\right\} > 0 \ (z \in U),$$

then $\mathrm{Re}\{p(z)\} > 0$ *for* $z \in U$.

Lemma 3 [18]*Let* $\kappa,\ \gamma \in \mathbb{C}$ *with* $\kappa \neq 0$ *and let* $h \in H(U)$ *with* $h(0) = c$. *If* $\mathrm{Re}\{\kappa h(z) + \gamma\} > 0$ $(z \in U)$, *then the solution of the following differential equation:*

$$q(z) + \frac{zq'(z)}{\kappa q(z) + \gamma} = h(z) \ (z \in U; q(0) = c)$$

is analytic in U *and satisfies* $\mathrm{Re}\{\kappa q(z) + \gamma\} > 0$ *for* $z \in U$.

Lemma 4 [1]*Let* $p \in \mathcal{F}(a)$ *and let* $q(z) = a + a_n z^n + a_{n+1}z^{n+1} + \ldots$ *be analytic in* U *with* $q(z) \neq a$ *and* $n \geq 1$. *If* q *is not subordinate to* p, *then there exists two points* $z_0 = r_0e^{i\theta} \in U$ *and* $\zeta_0 \in \delta U \backslash E(q)$ *such that*

$$q(U_{r_0}) \subset p(U);\ q(z_0) = p(\zeta_0) \text{ and } z_0 p'(z_0) = m\zeta_0 p'(\zeta_0) (m \geq n).$$

Lemma 5 [2]*Let* $q \in H[a; 1]$ *and* $\phi : \mathbb{C}^2 \to \mathbb{C}$. *Also set* $\phi(q(z), zq'(z)) = h(z)$. *If* $L(z, t) = \phi(q(z), tzq'(z))$ *is a subordination chain and* $q \in H[a;1] \cap \mathcal{F}(a)$, *then*

$$h(z) \prec \varphi\left(q(z),\ zq'(z)\right),$$

implies that $q(z) \prec p(z)$. *Furthermore, if* $\phi(q(z), zq'(z)) = h(z)$ *has a univalent solution* $q \in \mathcal{F}(a)$, *then q is the best subordinant.*

In the present article, we aim to prove some subordination-preserving and superordination-preserving properties associated with the integral operator $J_{p,s,b}$. Sandwich-type result involving this operator is also derived.

2 Main results

Unless otherwise mentioned, we assume throughout this section that $b \in \mathbb{C}\backslash\mathbb{Z}_0^-$, $s \in \mathbb{C}$, Re $\{b\}$, $\mu > 0$, $p \in \mathbb{N}$, $z \in \mathbb{U}$ and the powers are understood as principle values.

Theorem 1. *Let f, g* $\in A(p)$ *and*

$$\operatorname{Re}\left\{1+\frac{z\phi''(z)}{\phi'(z)}\right\} > -\delta\left(\phi(z)=\left(\frac{J_{p,s-1,b}(g)(z)}{J_{p,s,b}(g)(z)}\right)\left(\frac{J_{p,s,b}(g)(z)}{z^p}\right)^{\mu}; z\in U\right), \tag{2.1}$$

where δ *is given by*

$$\delta=\frac{1+\mu^2|b+1|^2-|1-\mu^2(b+1)^2|}{4\mu[1+\operatorname{Re}\{b\}]}(z\in U). \tag{2.2}$$

Then the subordination condition

$$\left(\frac{J_{p,s-1,b}(f)(z)}{J_{p,s,b}(f)(z)}\right)\left(\frac{J_{p,s,b}(f)(z)}{z^p}\right)^{\mu} \prec \left(\frac{J_{p,s-1,b}(g)(z)}{J_{p,s,b}(g)(z)}\right)\left(\frac{J_{p,s,b}(g)(z)}{z^p}\right)^{\mu}, \tag{2.3}$$

implies that

$$\left(\frac{J_{p,s,b}(f)(z)}{z^p}\right)^{\mu} \prec \left(\frac{J_{p,s,b}(g)(z)}{z^p}\right)^{\mu}, \tag{2.4}$$

where $\left(\frac{J_{p,s,b}(g)(z)}{z^p}\right)^{\mu}$ *is the best dominant.*

Proof. Let us define the functions $F(z)$ and $G(z)$ in U by

$$F(z)=\left(\frac{J_{p,s,b}(f)(z)}{z^p}\right)^{\mu} \text{and}\, \mathrm{G}(z)=\left(\frac{J_{p,s,b}(g)(z)}{z^p}\right)^{\mu}(z\in U) \tag{2.5}$$

and without loss of generality we assume that $G(z)$ is analytic, univalent on $\bar{U}$ and

$$G'(\zeta)\neq 0\ (|\zeta|=1).$$

If not, then we replace $F(z)$ and $G(z)$ by $F(\rho z)$ and $G(\rho z)$, respectively, with $0 < \rho < 1$. These new functions have the desired properties on $\bar{U}$, so we can use them in the proof of our result and the results would follow by letting $\rho \to 1$.

We first show that, if

$$q(z)=1+\frac{zG''(z)}{G'(z)}(z\in U), \tag{2.6}$$

then

$$\operatorname{Re}\{q(z)\} > 0\ (z\in U).$$

From (1.10) and the definition of the functions G, φ, we obtain that

$$\phi(z) = G(z) + \frac{zG'(z)}{\mu(b+1)}. \tag{2.7}$$

Differentiating both sides of (2.7) with respect to z yields

$$\phi'(z) = \left(1 + \frac{1}{\mu(b+1)}\right) G'(z) + \frac{zG''(z)}{\mu(b+1)}. \tag{2.8}$$

Combining (2.6) and (2.8), we easily get

$$1 + \frac{z\phi''(z)}{\phi'(z)} = q(z) + \frac{zq'(z)}{q(z) + \mu(b+1)} = h(z)\ (z \in U). \tag{2.9}$$

It follows from (2.1) and (2.9) that

$$\mathrm{Re}\{h(z) + \mu(b+1)\} > 0 (z \in U). \tag{2.10}$$

Moreover, by using Lemma 3, we conclude that the differential Equation (2.9) has a solution $q(z) \in H(U)$ with $h(0) = q(0) = 1$. Let

$$\mathcal{H}(u,\ v) = u + \frac{v}{u + \mu(b+1)} + \delta,$$

Where δ is given by (2.2). From (2.9) and (2.10), we obtain $\mathrm{Re}\ \{\mathcal{H}(q(z);)zq'(z))\} > 0 (z \in U)$.

To verify the condition

$$\mathrm{Re}\{\mathcal{H}(i\vartheta; t)\} \leq 0 \left(\vartheta \in \mathbb{R}; t \leq -\frac{1+\vartheta^2}{2}\right), \tag{2.11}$$

we proceed as follows:

$$\begin{aligned} \mathrm{Re}\{\mathcal{H}(i\vartheta; t)\} &= \mathrm{Re}\left\{i\vartheta + \frac{t}{\mu(b+1) + i\vartheta} + \delta\right\} = \frac{t\mu(1 + \mathrm{Re}(b))}{|\mu(b+1) + i\vartheta|^2} + \delta \\ &\leq -\frac{\Upsilon(b, \vartheta, \delta)}{2|\mu(b+1) + i\vartheta|^2}, \end{aligned}$$

where

$$\Upsilon(b,\ \vartheta,\ \delta) = [\mu(1 + \mathrm{Re}(b)) - 2\delta]\vartheta^2 - 4\delta\mu\mathrm{Im}(b)\vartheta - 2\delta|\mu(b+1)|^2 + \mu(1 + \mathrm{Re}\{b\}). \tag{2.12}$$

For δ given by (2.2), the coefficient of ϑ^2 in the quadratic expression $\Upsilon(b, \vartheta, \delta)$ given by (2.12) is positive or equal to zero. To check this, put $\mu(b + 1) = c$, so that

$$\mu(1 + \mathrm{Re}(b)) = c_1 \text{ and } \mu\mathrm{Im}(b) = c_2.$$

We thus have to verify that

$$c_1 - 2\delta \geq 0,$$

or

$$c_1 \geq 2\delta = \frac{1 + |c|^2 - |1 - c^2|}{2c_1}.$$

This inequality will hold true if

$$2c_1^2 + |1 - c^2| \geq 1 + |c|^2 = 1 + c_1^2 + c_2^2,$$

that is, if

$$|1 - c^2| \geq 1 - \mathrm{Re}(c^2),$$

which is obviously true. Moreover, the quadratic expression $\Upsilon(b, \vartheta, \delta)$by ϑ in (2.12) is a perfect square for the assumed value of δ given by (2.2). Hence we see that (2.11) holds. Thus, by Lemma 2, we conclude that

$$\mathrm{Re}\ \{q(z)\} > 0\ (z \in U),$$

that is, that G defined by (2.5) is convex (univalent) in U. Next, we prove that the subordination condition (2.3) implies that

$$F(z) \prec G(z),$$

for the functions F and G defined by (2.5). Consider the function $L(z, t)$ given by

$$L(z,\ t) = G(z) + \frac{(1+t)zG'(z)}{\mu(b+1)}(0 \leq t < \infty; z \in U). \tag{2.13}$$

We note that

$$\left.\frac{\partial L(z,t)}{\partial z}\right|_{z=0} = G^{'}(0)\left(1 + \frac{1+t}{\mu(b+1)}\right) \neq 0\,(0 \leq t < \infty;\ z \in U;\ \mathrm{Re}\ \{\mu(b+1)\} > 0).$$

This show that the function

$$L(z,\ t) = a_1(t)z + \cdots$$

satisfies the condition $a_1\ (t) \neq 0\ (0 \leq t < \infty)$. Further, we have

$$\mathrm{Re}\left\{\frac{z\partial L(z,t)/\partial z}{\partial L(z,t)/\partial t}\right\} = \mathrm{Re}\{\mu(b+1) + (1+t)q(z)\} > 0\,(0 \leq t < \infty; z \in U).$$

Since $G(z)$ is convex and Re $\{\mu(b + 1)\}$ >0. Therefore, by using Lemma 1, we deduce that $L(z, t)$ is a subordination chain. It follows from the definition of subordination chain that

$$\phi(z) = G(z) + \frac{zG'(z)}{\mu(b+1)} = L(z,\ 0)$$

and

$$L(z,\ 0) \prec L(z,\ t)\ (0 \leq t < \infty),$$

which implies that

$$L(\zeta,\ t) \notin L(U,\ 0) = \phi(U)\,(0 \leq t < \infty; \zeta \in \partial U). \tag{2.14}$$

If F is not subordinate to G, by using Lemma 4, we know that there exist two points $z_0 \in\ U$ and $\zeta_0 \in\ \partial U$ such that

$$F(z_0) = G(\zeta_0)\, and\, z_0F'(z_0) = (1+t)\zeta_0G'(\zeta_0)\,(0 \leq t < \infty). \tag{2.15}$$

Hence, by using (2.5), (2.13), (2.15) and (2.3), we have

$$\begin{aligned} L(\zeta_0,\ t) &= G(\zeta_0) + \frac{(1+t)\zeta_0 G'(\zeta_0)}{\mu(b+1)} = F(z_0) + \frac{z_0 F'(z_0)}{\mu(b+1)} \\ &= \left(\frac{J_{p,s-1,b}(f)(z)}{J_{p,s,b}(f)(z)}\right)\left(\frac{J_{p,s,b}(f)(z)}{z^p}\right)^{\mu} \in \phi(U). \end{aligned}$$

This contradicts (2.14). Thus, we deduce that $F \prec G$. Considering $F = G$, we see that the function G is the best dominant. This completes the proof of Theorem 1.

We now derive the following superordination result.

Theorem 2. *Let $f, g \in A(p)$ and*

$$\mathrm{Re}\left\{1 + \frac{z\phi''(z)}{\phi'(z)}\right\} > -\delta\left(\phi(z) = \left(\frac{J_{p,s-1,b}(g)(z)}{J_{p,s,b}(g)(z)}\right)\left(\frac{J_{p,s,b}(g)(z)}{z^p}\right)^{\mu}; z \in U\right), \quad (2.16)$$

where δ is given by (2.2) *. If the function* $\left(\frac{J_{p,s-1,b}(f)(z)}{J_{p,s,b}(f)(z)}\right)\left(\frac{J_{p,s,b}(f)(z)}{z^p}\right)^{\mu}$ is univalent in U and $\left(\frac{J_{p,s,b}(f)(z)}{z^p}\right)^{\mu} \in \mathcal{F}$ *, then the superordination condition*

$$\left(\frac{J_{p,s-1,b}(g)(z)}{J_{p,s,b}(g)(z)}\right)\left(\frac{J_{p,s,b}(g)(z)}{z^p}\right)^{\mu} \prec \left(\frac{J_{p,s-1,b}(f)(z)}{J_{p,s,b}(f)(z)}\right)\left(\frac{J_{p,s,b}(f)(z)}{z^p}\right)^{\mu}, \quad (2.17)$$

implies that

$$\left(\frac{J_{p,s,b}(g)(z)}{z^p}\right)^{\mu} \prec \left(\frac{J_{p,s,b}(f)(z)}{z^p}\right)^{\mu}, \quad (2.18)$$

where $\left(\frac{J_{p,s,b}(f)(z)}{z^p}\right)^{\mu}$ *is the best subordinant.*

Proof. Suppose that the functions F, G and q are defined by (2.5) and (2.6), respectively. By applying similar method as in the proof of Theorem 1, we get

$$\mathrm{Re}\{q(z)\} > 0 \ (z \in U).$$

Next, to arrive at our desired result, we show that $G \prec F$. For this, we suppose that the function $L(z, t)$ be defined by (2.13). Since G is convex, by applying a similar method as in Theorem 1, we deduce that $L(z, t)$ is subordination chain. Therefore, by using Lemma 5, we conclude that $G \prec F$. Moreover, since the differential equation

$$\phi(z) = G(z) + \frac{zG'(z)}{\mu(b+1)} = \varphi\left(G(z),\ zG'(z)\right)$$

has a univalent solution G, it is the best subordinant. This completes the proof of Theorem 2.

Combining the above-mentioned subordination and superordination results involving the operator $J_{p,s,b}$, the following "sandwich-type result" is derived.

Theorem 3. *Let $f, g_j \in A(p)$ $(j = 1, 2)$ and*

$$\mathrm{Re}\left\{1 + \frac{z\phi_j''(z)}{\phi_j'(z)}\right\} > -\delta\left(\phi_j(z) = \left(\frac{J_{p,s-1,b}(g_j)(z)}{J_{p,s,b}(g_j)(z)}\right)\left(\frac{J_{p,s,b}(g_j)(z)}{z^p}\right)^{\mu} (j = 1, 2); z \in U\right),$$

where δ is given by (2.2) *. If the function* $\left(\frac{J_{p,s-1,b}(f)(z)}{J_{p,s,b}(f)(z)}\right)\left(\frac{J_{p,s,b}(f)(z)}{z^p}\right)^{\mu}$ *is univalent in U and* $\left(\frac{J_{p,s,b}(f)(z)}{z^p}\right)^{\mu} \in \mathcal{F}$ *, then the condition*

$$\begin{aligned}\left(\frac{J_{p,s-1,b}(g_1)(z)}{J_{p,s,b}(g_1)(z)}\right)\left(\frac{J_{p,s,b}(g_1)(z)}{z^p}\right)^{\mu} &\prec \left(\frac{J_{p,s-1,b}(f)(z)}{J_{p,s,b}(f)(z)}\right)\left(\frac{J_{p,s,b}(f)(z)}{z^p}\right)^{\mu} \\ &\prec \left(\frac{J_{p,s-1,b}(g_2)(z)}{J_{p,s,b}(g_2)(z)}\right)\left(\frac{J_{p,s,b}(g_2)(z)}{z^p}\right)^{\mu},\end{aligned} \tag{2.19}$$

implies that

$$\left(\frac{J_{p,s,b}(g_1)(z)}{z^p}\right)^{\mu} \prec \left(\frac{J_{p,s,b}(f)(z)}{z^p}\right)^{\mu} \prec \left(\frac{J_{p,s,b}(g_2)(z)}{z^p}\right)^{\mu}, \tag{2.20}$$

where $\left(\frac{J_{p,s,b}(g_1)(z)}{z^p}\right)^{\mu}$ *and* $\left(\frac{J_{p,s,b}(g_2)(z)}{z^p}\right)^{\mu}$ *are, respectively, the best subordinant and the best dominant.*

Remark. *(i) Putting* $\mu = 1$, $b = p$ *and* $s = \alpha(\alpha = 0,\ p \in \mathbb{N})$ *in our results of this article, we obtain the results obtained by Aouf and Seoudy* [19];

(ii) Specializing the parameters s and b in our results of this article, we obtain the results for the corresponding operators $F_{\nu,p}$, I_p^{α}, J_p^m and $J_p^m(l)$ *which are defined in the introduction.*

Authors' contributions
All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

Received: 8 December 2011 Accepted: 24 May 2012 Published: 24 May 2012

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doi:10.1186/1029-242X-2012-115
Cite this article as: Aouf *et al.*: Some subordination and superordination results of generalized Srivastava-Attiya operator. *Journal of Inequalities and Applications* 2012 **2012**:115.