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# On general filtering problem of stationary processes with fixed transformation

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## Abstract

A fixed transformation are given for one-dimensional stationary processes in this paper. Based on this, we propose a general filtering problem of stationary processes with fixed transformation. Finally, on a stationary processes with no any additional conditions, we get the spectral characteristics of  $P_{H_{\eta}(t)}\xi$  in the space  $L^2(F_X(d\lambda))$ , and then we calculate the value of the best predict quantity  $Q$  of the general filtering problem.

**Keywords:** stationary processes, fixed transformation, filtering

## 1. Introduction

The Prediction theory is an important part of stationary processes, also linear filter problems is an important part of Prediction theory. The linear filtering problem of multidimensional stationary sequence and processes for a linear system are firsted studied by Rosanov in [1], and then a series of general filter problem of stationary process for a linear system are studied in [2-9]. Theoretically, this problem is a extend of the classic prediction problem. But it has high practical value, it also widely applied in communication, exploration, space technology and automatic control, etc.

## 2. Propose the problem

Let  $X(t)$ ,  $t \in \mathbf{R}$  be (simple) wide stationary process. Let

$$\begin{aligned}H_X &= \mathcal{L}\{X(t), t \in \mathbf{R}\} \\H_X(t) &= \mathcal{L}\{X(s), s \leq t, s \in \mathbf{R}\}\end{aligned}$$

Suppose the complex-value function  $b(t)$  satisfying the following conditions

1)

$$b(t) \in L^1[0, +\infty) \cap L^2[0, +\infty) \quad (2-1)$$

2)

$$b(t) = 0 \quad \text{for } t < 0 \quad (2-2)$$

Let

$$B(\lambda) = \int_0^{\infty} b(t)e^{-i\lambda t} dt \quad (2-3)$$

then  $B(\lambda)$  is boundary values analytic function  $B(z)$  in the low half plane  
 We can obtain

$$B(\lambda) \neq 0, \quad \text{a.e. Leb}$$

Let

$$E = \{\lambda : B(\lambda) = 0\}, \quad \bar{E} = (-\infty, \infty) - E \quad (2-4)$$

then

$$L(E) = 0$$

Let

$$\eta(t) = \int_0^\infty b(s) \cdot X(t-s)ds = \int_0^\infty b(s)U^{-s}X(t)ds \quad (2-5)$$

where  $U$  is the shift operator of  $X(t)$  in the space  $H_X$ .

Then, for each  $\zeta \in H_X$

1) Find out the value of  $Q$ ,

$$Q = \inf_{\tilde{\xi} \in H_X(t)} \left\| \int_0^\infty b(s)U^{-s}\tilde{\xi}ds - \tilde{\xi} \right\|^2 \quad (2-6)$$

2) Then, we will prove that

$$Q = \|P_{H_\eta(t)}\xi - \xi\|^2$$

and solve the spectral characteristics of  $P_{H_\eta(t)}$  in the space  $L^2(F_X(d\lambda))$ .

### 3. Main result

Let the random spectral measure of  $X(t)$  is  $\Phi_X(d\lambda)$ , and the spectral measure is  $F_X(d\lambda)$ , the broad spectral measure which also named the spectral measure of absolutely continuous part of  $F$  is  $f_X(\lambda)$ .

The Lebesgue decomposition of  $F_X(d\lambda)$  is

$$F_X(d\lambda) = f_X(\lambda)d\lambda + \delta(d\lambda) \quad (3-1)$$

where the Lebesgue measure of  $\delta(d\lambda)$  is singular, namely  $\delta(\lambda) = \chi_\Delta(\lambda)F_X(d\lambda)$

$$\Delta \subset (-\infty, \infty), \quad L(\Delta) = 0, \quad \bar{\Delta} = (-\infty, \infty) - \Delta$$

Let

$$A_t = \left\{ h : h = \int_0^\infty b(s)U^{-s}\gamma ds, \gamma \in H_X(t) \right\} \quad (3-2)$$

Obviously,  $A_t$  is linear set.

**Lemma 1.** Let  $X(t)$ ,  $t \in \mathbf{R}$  is stationary processes,  $F(d\lambda)$  and  $Z(d\lambda)$  are spectral measure and random spectral measure respectively.  $\forall f(\lambda), \phi(\lambda) \in L^2(F)$ , and  $|\phi(\lambda)| \leq M$ ,  $M > 0$ , where  $M$  is real number, we have

$$\left\| \int_{-\infty}^{\infty} f(\lambda)\varphi(\lambda)Z(d\lambda) \right\| \leq M \left\| \int_{-\infty}^{\infty} f(\lambda)Z(d\lambda) \right\|$$

**Proof.** According to the nature of the random integral, we have

$$\begin{aligned} & \left\| \int_{-\infty}^{\infty} f(\lambda)\varphi(\lambda)Z(d\lambda) \right\| \\ &= \left[ \int_{-\infty}^{\infty} |f(\lambda)\varphi(\lambda)|^2 F(d\lambda) \right]^{\frac{1}{2}} \\ &= \left[ \int_{-\infty}^{\infty} |f(\lambda)|^2 \cdot |\varphi(\lambda)|^2 F(d\lambda) \right]^{\frac{1}{2}} \\ &\leq M \left[ \int_{-\infty}^{\infty} |f(\lambda)|^2 \cdot F(d\lambda) \right]^{\frac{1}{2}} \\ &= M \left\| \int_{-\infty}^{\infty} f(\lambda)Z(d\lambda) \right\| \end{aligned}$$

**Lemma 2.**

$$\mathcal{L}\{A_t\} = H_{\eta}(t) \tag{3-3}$$

where  $\mathcal{L}(A_t)$  is the linear closed manifold of  $A_t$ .

**Proof.**  $\forall \eta(\tau_0), \tau_0 \leq t$ , Let  $y = X(\tau_0)$ , we have  $y \in H_X(t)$ , and

$$\eta(\tau_0) = \int_0^{\infty} b(s)U^{-s}y ds = \int_0^{\infty} b(s)U^{-s}X(\tau_0) ds$$

namely,  $\eta(\tau_0) \in A_t$ , then

$$H_{\eta}(t) \subset \mathcal{L}(A_t)$$

On the other hand,  $\forall h \in A_t$ , we have

$$h = \int_0^{\infty} b(s)U^{-s}y ds$$

where  $y \in H_X(t)$ .

Let  $z_l = \sum_{k=1}^{m_l} a_k^l X(t_k^l), t_k^l \leq t, k = 1, 2, \dots, m_l, a_k^l$ , is complex number. Let

$$\|y - z_l\| \rightarrow 0 \quad (l \rightarrow \infty)$$

while

$$\int_0^{\infty} b(s)U^{-s}z_l ds = \sum_{k=1}^{m_l} a_k^l \int_0^{\infty} b(s)U^{-s}X(t_k^l) ds \in H_{\eta}(t), \quad l = 1, 2, \dots$$

Let, the spectral characteristics of  $y$  and  $z_l$  are  $\psi_y(\lambda)$  and  $\psi_{z_l}(\lambda)$  in the space  $L^2(F_X(d\lambda))$  respectively. According to the equation (2-3)

$$|B(\lambda)| = \left| \int_0^\infty b(s)e^{-i\lambda s} ds \right| \leq \int_0^\infty |b(s)| ds \triangleq M$$

where  $M > 0$  is constant. According to the lemma 1, we get

$$\begin{aligned} & \left\| h - \int_0^\infty b(s)U^{-s}z_l ds \right\| \\ &= \left\| \int_0^\infty b(s)U^{-s}(y - z_l) ds \right\| \\ &= \left\| \int_0^\infty b(s) \int_{-\infty}^\infty e^{-is\lambda} (\psi_y(\lambda) - \psi_{z_l}(\lambda)) \Phi_X(d\lambda) ds \right\| \\ &= \left\| \int_{-\infty}^\infty (\psi_y(\lambda) - \psi_{z_l}(\lambda)) \int_0^\infty b(s)e^{-is\lambda} ds \Phi_X(d\lambda) \right\| \\ &= \left\| \int_{-\infty}^\infty (\psi_y(\lambda) - \psi_{z_l}(\lambda)) B(\lambda) \Phi_X(d\lambda) \right\| \\ &\leq M \left\| \int_{-\infty}^\infty (\psi_y(\lambda) - \psi_{z_l}(\lambda)) \Phi_X(d\lambda) \right\| \\ &= M \left[ \int_{-\infty}^\infty |\psi_y(\lambda) - \psi_{z_l}(\lambda)|^2 F(d\lambda) \right]^{\frac{1}{2}} \\ &= M \|y - z_l\| \rightarrow 0 \quad (l \rightarrow \infty) \end{aligned}$$

so,  $h \in H_\eta(t)$ , namely

$$\mathcal{L}\{A_t\} \subset H_\eta(t)$$

Then, it shows the equation (3-3) is correct.

According to the lemma 2, we have

$$\begin{aligned} Q &= \inf_{\tilde{\xi} \in H_X(t)} \left\| \int_0^\infty b(s)U^{-s}\tilde{\xi} ds - \xi \right\|^2 \\ &= \inf_{h \in A_t} \|h - \xi\|^2 \\ &= \inf_{h \in \mathcal{L}\{A_t\}} \|h - \xi\|^2 \\ &= \inf_{h \in H_\eta(t)} \|h - \xi\|^2 \\ &= \|P_{H_\eta(t)}\xi - \xi\|^2 \end{aligned} \tag{3-4}$$

According to the equation (2-5) and (2-1), we get

$$\begin{aligned} \eta(t) &= \int_0^\infty b(s)X(t-s)ds = \int_0^\infty b(s) \int_{-\infty}^\infty e^{i(t-s)\lambda} \Phi_X(d\lambda) ds \\ &= \int_{-\infty}^\infty e^{it\lambda} \left[ \int_0^\infty b(s)e^{-is\lambda} ds \right] \Phi_X(d\lambda) = \int_{-\infty}^\infty e^{it\lambda} B(\lambda) \Phi_X(d\lambda) \end{aligned}$$

on the other hand

$$\eta(t) = \int_{-\infty}^\infty e^{it\lambda} \Phi_\eta(d\lambda)$$

According to the stochastic process spectral theorem and the Relevant function spectral theorem, we have

$$\Phi_\eta(d\lambda) = B(\lambda)\Phi_X(d\lambda) \tag{3-5}$$

$$F_\eta(d\lambda) = |B(\lambda)|^2 F_X(d\lambda) \tag{3-6}$$

$$f_\eta(d\lambda) = |B(\lambda)|^2 f_X(\lambda) \tag{3-7}$$

where  $\Phi_\eta(d\lambda)$ ,  $F_\eta(d\lambda)$ ,  $f_\eta(\lambda)$  representative the random spectral measure, spectral measure and broad spectral measure of  $\eta(t)$ .

**Lemma 3.** Let  $\zeta \in H_X$ ,  $\xi = \widehat{\xi} + \widetilde{\xi}$ , where  $\widehat{\xi} \in H_\eta$ ,  $\widetilde{\xi} \perp H_\eta$ , then

$$Q = \|\widetilde{\xi}\|^2 + \|P_{H_\eta(t)}\widehat{\xi} - \widehat{\xi}\|^2 \tag{3-8}$$

If, the wold decomposition of  $\eta(t)$  is

$$\eta(t) = \eta^r(t) + \eta^s(t)$$

and

$$\xi = \xi^r + \xi^s$$

where  $\eta^r$  is regular process,  $\eta^s(t)$  is singular process, and  $\widehat{\xi}^r \perp S_\eta$ ,  $\widehat{\xi}^s \in S_\eta$ ,  $S_\eta = \bigcap_t H_\eta(t)$ , then

$$Q = \|\widetilde{\xi}\|^2 + \|P_{H_{\eta^r}(t)}\widehat{\xi}^r - \widehat{\xi}^r\|^2 \tag{3-9}$$

**Proof.** According to  $\xi = \widehat{\xi} + \widetilde{\xi}$ ,  $\widehat{\xi} \in H_\eta$ ,  $\widetilde{\xi} \perp H_\eta$ . So

$$P_{H_\eta(t)}\xi = P_{H_\eta(t)}\widehat{\xi} + P_{H_\eta(t)}\widetilde{\xi} = P_{H_\eta(t)}\widehat{\xi}$$

According to the equation (3-4),

$$\begin{aligned} Q &= \|P_{H_\eta(t)}\xi - \xi\|^2 = \|P_{H_\eta(t)}\widehat{\xi} - (\widehat{\xi} + \widetilde{\xi})\|^2 \\ &= \|(P_{H_\eta(t)}\widehat{\xi} - \widehat{\xi}) - \widetilde{\xi}\|^2 \end{aligned}$$

also, according to  $\widetilde{\xi} \perp (P_{H_\eta(t)}\widehat{\xi} - \widehat{\xi})$ , So

$$Q = \|\widetilde{\xi}\|^2 + \|P_{H_\eta(t)}\widehat{\xi} - \widehat{\xi}\|^2$$

namely, the equation (3-8) is correct.

When  $\eta(t)$  have the wold decomposition, notice

$$\widehat{\xi}^s = P_{H_\eta}\widehat{\xi}, H_\eta(t) = H_{\eta^r}(t) \oplus S_\eta$$

we get

$$P_{H_\eta(t)}\widehat{\xi}^s = P_{H_\eta(t)}(P_{S_\eta}\widehat{\xi}) = P_{S_\eta}\widehat{\xi} = \widehat{\xi}^s \tag{3-10}$$

$$P_{H_\eta(t)}\widehat{\xi}^r = P_{H_{\eta^r}(t)}\widehat{\xi}^r + P_{S_\eta}\widehat{\xi}^r = P_{H_{\eta^r}(t)}\widehat{\xi}^r \tag{3-11}$$

So

$$\begin{aligned} &\|P_{H_\eta(t)}\widehat{\xi} - \widehat{\xi}\|^2 \\ &= \|(P_{H_\eta(t)}\widehat{\xi}^r + P_{H_\eta(t)}\widehat{\xi}^s - (\widehat{\xi}^r + \widehat{\xi}^s))\|^2 \\ &= \|P_{H_{\eta^r}(t)}\widehat{\xi}^r - \widehat{\xi}^r\|^2 \end{aligned}$$

Then, according to the equation (3-8), we get that the equation (3-9) is correct.

**Lemma 4.** Let  $\zeta \in H_X$ ,  $\xi = \widehat{\xi} + \widetilde{\xi}$ , where  $\widehat{\xi} \in H_\eta, \widetilde{\xi} \perp H_\eta$ ,  $\psi(\lambda)$  is spectral characteristics of  $\zeta$  in the space of  $L^2(F_X(d\lambda))$ ,  $\widehat{\psi}(\lambda)$  is spectral characteristics of  $\widehat{\xi}$  in the space of  $L^2(F_\eta(d\lambda))$ , then

1) when  $\lambda \in \overline{E}$ ,

$$\psi(\lambda) = \widehat{\psi}(\lambda)B(\lambda), \quad a.e. \quad F_X(d\lambda) \tag{3-12}$$

2)

$$\|\widetilde{\xi}\|^2 = \int_E |\psi(\lambda)|^2 F_X(d\lambda) \tag{3-13}$$

**Proof.** 1) According to the given conditions, we have

$$\begin{aligned} \xi &= \int_{-\infty}^{\infty} \psi(\lambda) \Phi_X(d\lambda) \\ \widehat{\xi} &= \int_{-\infty}^{\infty} \widehat{\psi}(\lambda) \Phi_\eta(d\lambda) = \int_{-\infty}^{\infty} \widehat{\psi}(\lambda) B(\lambda) \Phi_X(d\lambda) \\ \eta(t) &= \int_{-\infty}^{\infty} e^{it\lambda} \Phi_\eta(d\lambda) = \int_{-\infty}^{\infty} e^{it\lambda} B(\lambda) \Phi_X(d\lambda) \end{aligned}$$

So

$$(\xi, \eta(-t)) = \int_{-\infty}^{\infty} e^{it\lambda} \psi(\lambda) \overline{B(\lambda)} F_X(d\lambda)$$

on the other hand

$$\begin{aligned} (\xi, \eta(-t)) &= (\widehat{\xi} + \widetilde{\xi}, \eta(-t)) = (\widehat{\xi}, \eta(-t)) \\ &= \int_{-\infty}^{\infty} e^{it\lambda} \widehat{\psi}(\lambda) |B(\lambda)|^2 F_X(d\lambda) \end{aligned}$$

According to the Fourier transformation, we have

$$\psi(\lambda) \overline{B(\lambda)} = \widehat{\psi}(\lambda) |B(\lambda)|^2, \quad a.e. \quad F_X(d\lambda)$$

So, when  $\lambda \in \overline{E}$ , we have

$$\psi(\lambda) = \widehat{\psi}(\lambda) B(\lambda), \quad a.e. \quad F_X(d\lambda)$$

2) According to  $\xi = \widehat{\xi} + \widetilde{\xi}$ , and  $\widehat{\xi} \perp \widetilde{\xi}$ , Thus

$$\|\xi\|^2 = \|\widehat{\xi} + \widetilde{\xi}\|^2 = \|\widehat{\xi}\|^2 + \|\widetilde{\xi}\|^2$$

$$\begin{aligned} \|\widetilde{\xi}\|^2 &= \|\xi\|^2 - \|\widehat{\xi}\|^2 \\ &= \int_{-\infty}^{\infty} |\psi(\lambda)|^2 F_X(d\lambda) - \int_{-\infty}^{\infty} |\widehat{\psi}(\lambda)|^2 F_\eta(d\lambda) \\ &= \int_{-\infty}^{\infty} |\psi(\lambda)|^2 F_X(d\lambda) - \int_{-\infty}^{\infty} |\widehat{\psi}(\lambda)| \cdot |B(\lambda)|^2 F_X(d\lambda) \\ &= \int_{-\infty}^{\infty} \chi_E(\lambda) |\psi(\lambda)|^2 F_X(d\lambda) + \int_{-\infty}^{\infty} \chi_{\overline{E}}(\lambda) |\psi(\lambda)|^2 F_X(d\lambda) - \int_{-\infty}^{\infty} \chi_{\overline{E}}(\lambda) |\widehat{\psi}(\lambda)|^2 \cdot |B(\lambda)|^2 F_X(d\lambda) \\ &= \int_{-\infty}^{\infty} \chi_E(\lambda) |\psi(\lambda)|^2 F_X(d\lambda) = \int_E |\psi(\lambda)|^2 F_X(d\lambda) \end{aligned}$$

namely, the equation (3-13) is correct.

**Theorem.** Let  $\zeta \in H_X$ ,  $\xi = \widehat{\xi} + \widetilde{\xi}$ , where  $\widehat{\xi} \in H_\eta$ ,  $\widetilde{\xi} \perp H_\eta$ ,  $\psi(\lambda)$  representative the spectral characteristics of  $\zeta$  in the space of  $L^2(F_X(d\lambda))$ ,  $\widehat{\psi}(\lambda)$  representative the spectral characteristics of  $\widehat{\xi}$  in the space of  $L^2(F_\eta(d\lambda))$ . Then

1) When  $\frac{\log f_X(\lambda)}{1+\lambda^2} \notin L^1(-\infty, \infty)$

$$Q = \int_E |\psi(\lambda)|^2 F_X(d\lambda) \tag{3-14}$$

now the spectral characteristics of  $P_{H_\eta(t)}\xi$  in the space of  $L^2(F_X(d\lambda))$  is

$$\widetilde{\psi}(\lambda) = \begin{cases} 0, & \lambda \in E, \\ \psi(\lambda), & \lambda \in \bar{E}. \end{cases} \text{ a.e. } F_X(d\lambda) \tag{3-15}$$

2) When  $\frac{\log f_X(\lambda)}{1+\lambda^2} \in L^1(-\infty, \infty)$

$$Q = \int_E |\psi(\lambda)|^2 F_X(d\lambda) + \int_t^\infty |\varphi(-s)|^2 ds \tag{3-16}$$

now the spectral characteristics of  $P_{H_\eta(t)}\xi$  in the space of  $L^2(F_X(d\lambda))$  is

$$\widetilde{\psi}(\lambda) = \begin{cases} 0, & \lambda \in E, \\ \psi(\lambda), & \lambda \in \bar{E}\Delta, \\ \frac{B(\lambda) \int_{-t}^\infty e^{-is\lambda} \varphi(s) ds}{\Gamma(\lambda)}, & \lambda \in E\Delta \end{cases} \tag{3-17}$$

where  $b(t)$ ,  $B(\lambda)$ ,  $\eta(t)$ ,  $E$  and  $\Delta$  is decided by the equation (2-1), (2-2), (2-3), (2-5), (2-4) and (3-1) respectively. Where  $\Gamma(\lambda)$  is the maximal factor boundary values of the spectral density  $f_\eta(\lambda)$ ,  $\phi(\lambda)$  is the Fourier transformation of  $\widehat{\psi}(\lambda)\Gamma(\lambda)$ , where  $\widehat{\psi}(\lambda)$  is determined by equation (3-12) and  $\widehat{\psi}(\lambda) = \psi(\lambda)/B(\lambda)$ , a.e.  $L$ .

**Proof.** 1) When  $\frac{\log f_X(\lambda)}{1+\lambda^2} \notin L^1(-\infty, \infty)$ . According to

$$f_\eta(\lambda) = |B(\lambda)|^2 f_X(\lambda) \tag{3-18}$$

$$\log |B(\lambda)| \in L^1(-\infty, \infty) \tag{3-19}$$

we know that

$$\frac{\log f_\eta(\lambda)}{1+\lambda^2} \notin L^1(-\infty, \infty)$$

Thus,  $\eta(t)$  is singular process. So

$$H_\eta = S_\eta \\ P_{H_\eta(t)}\widehat{\xi} = P_{H_\eta}\widehat{\xi} = P_{S_\eta}\widehat{\xi} = \widehat{\xi}$$

According to (3-8)

$$Q = \|\widehat{\xi}\|^2 = \int_E |\psi(\lambda)|^2 F_X(d\lambda).$$

On the other hand

$$P_{H_\eta(t)}\xi = P_{H_\eta}(\widehat{\xi} + \widetilde{\xi}) = P_{H_\eta}\widehat{\xi} = P_{S_\eta}\widehat{\xi} = \widehat{\xi}$$

The spectral characteristics of  $P_{H_\eta(t)}\xi$  in the space  $L^2(F_X(d\lambda))$  is

$$\widetilde{\psi}(\lambda) = \widehat{\psi}(\lambda)B(\lambda), \quad a.e. \quad F_X(d\lambda)$$

According to the equation (3-12), we get

$$\widetilde{\psi}(\lambda) = \begin{cases} 0, & \lambda \in E, \\ \psi(\lambda), & \lambda \in \bar{E}, \end{cases} \quad a.e. \quad F_X(d\lambda)$$

2) When  $\frac{\log f_X(\lambda)}{1+\lambda^2} \in L^1(-\infty, \infty)$ , according to (3-18) and (3-19), we get  $\frac{\log f_\eta(\lambda)}{1+\lambda^2} \in L^1(-\infty, \infty)$ .

It shows that  $\eta(t)$  is non-singular process, so  $\eta(t)$  has regular singular decomposition, and it consistent with Lebesgue-Gramer decomposition. Let the decomposition equation is

$$\eta(t) = \eta^r(t) + \eta^s(t)$$

where  $\eta^r(t)$  is regular process,  $\eta^s(t)$  is singular process

$$\begin{aligned} \eta^r(t) &= \int_{-\infty}^{\infty} e^{it\lambda} \Phi_{\eta^r}(d\lambda) = \int_{-\infty}^{\infty} e^{it\lambda} \chi_{\bar{\Delta}}(\lambda) \Phi_\eta(d\lambda) \\ \eta^s(t) &= \int_{-\infty}^{\infty} e^{it\lambda} \Phi_{\eta^s}(d\lambda) = \int_{-\infty}^{\infty} e^{it\lambda} \chi_{\Delta}(\lambda) \Phi_\eta(d\lambda) \end{aligned}$$

Let  $V(ds)$  is basic cross stochastic measure, namely

$$V(\Delta_1) = \int_{-\infty}^{\infty} \frac{e^{i\lambda t_2} - e^{i\lambda t_1}}{i\lambda} \Lambda(d\lambda)$$

where  $\Delta_1 = (t_1, t_2]$ , stochastic measure  $\Lambda(d\lambda) = \frac{1}{\Gamma(\lambda)} \Phi_{\eta^r}(d\lambda)$ , Thus

$$\eta^r(t) = \int_{-\infty}^t C(t-s)V(ds)$$

when  $C(s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{is\lambda} \Gamma(\lambda) d\lambda$ , and  $\Gamma(\lambda)$  satisfate the follow conditions

$$f(\lambda) = \frac{1}{2\pi} |\Gamma(\lambda)|^2$$

also  $\Gamma(\lambda)$  is the boundary values of the lower half plane maximum analytic functions  $\Gamma(z)$ , notice

$$\begin{aligned} \widehat{\xi}^r &= \int_{-\infty}^{\infty} \widehat{\psi}(\lambda) \chi_{\bar{\Delta}}(\lambda) \Phi_\eta(d\lambda) = \int_{-\infty}^{\infty} \widehat{\psi}(\lambda) \Phi_{\eta^r}(d\lambda) \\ &= \int_{-\infty}^{\infty} \widehat{\psi}(\lambda) \Gamma(\lambda) \Lambda(d\lambda) \\ \widehat{\xi}^s &= \int_{-\infty}^{\infty} \widehat{\psi}(\lambda) \chi_{\Delta}(\lambda) \Phi_\eta(d\lambda) = \int_{-\infty}^{\infty} \widehat{\psi}(\lambda) \Phi_{\eta^s}(d\lambda) \end{aligned}$$



Let  $\phi(s)$  is the Fourier transformation of  $\widehat{\psi}(\lambda)\Gamma(\lambda)$ , namely

$$\phi(s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{is\lambda} \widehat{\psi}(\lambda)\Gamma(\lambda)d\lambda$$

According to the Fourier transformation of random measure

$$\begin{aligned} \widehat{\xi}^r &= \int_{-\infty}^{\infty} \widehat{\psi}(\lambda)\Gamma(\lambda)\Lambda(d\lambda) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i\lambda s} \phi(s)ds\Lambda(d\lambda) \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i\lambda s} \phi(-s)ds\Lambda(d\lambda) = \int_{-\infty}^{\infty} \phi(-s)V(ds) \end{aligned}$$

Thus

$$\begin{aligned} P_{H_{\eta^r}(t)}\widehat{\xi}^r &= \int_{-\infty}^t \phi(-s)V(ds) \\ \|P_{H_{\eta^r}(t)}\widehat{\xi}^r - \widehat{\xi}^r\|^2 &= \left\| \int_t^{\infty} \phi(-s)V(ds) \right\|^2 = \int_t^{\infty} |\phi(-s)|^2 ds \end{aligned}$$

According to the equation (3-9), we get

$$Q = \int_E |\psi(\lambda)|^2 F_X(d\lambda) + \int_t^{\infty} |\phi(-s)|^2 ds$$

According to the Lemma 3

$$P_{H_{\eta}(t)}\widehat{\xi} = P_{H_{\eta}(t)}\widehat{\xi}^r + P_{H_{\eta}(t)}\widehat{\xi}^s = P_{H_{\eta^r}(t)}\widehat{\xi}^r + \widehat{\xi}^s$$

notice

$$\begin{aligned} P_{H_{\eta^r}(t)}\widehat{\xi}^r &= \int_{-\infty}^t \phi(-s)V(ds) = \int_{-\infty}^{\infty} \left[ \int_{-\infty}^t e^{i\lambda s} \phi(-s)ds \right] \Lambda(d\lambda) \\ &= \int_{-\infty}^{\infty} \left[ \int_{-t}^{\infty} e^{-i\lambda s} \phi(s)ds \right] \frac{1}{\Gamma(\lambda)} \Phi_{\eta^r}(d\lambda) \\ &= \int_{-\infty}^{\infty} \left[ \int_{-t}^{\infty} e^{-i\lambda s} \phi(s)ds \right] \frac{1}{\Gamma(\lambda)} \chi_{\overline{\Delta}}(\lambda)B(\lambda)\Phi_X(d\lambda) \\ \widehat{\xi}^s &= \int_{-\infty}^{\infty} \widehat{\psi}(\lambda)\chi_{\Delta}(\lambda)\Phi_{\eta}(d\lambda) = \int_{-\infty}^{\infty} \widehat{\psi}(\lambda)\chi_{\Delta}(\lambda)B(\lambda)\Phi_X(d\lambda) \end{aligned}$$

Thus

$$P_{H_{\eta}(t)}\widehat{\xi} = \int_{-\infty}^{\infty} \left[ \int_{-t}^{\infty} e^{-i\lambda s} \phi(s)ds \frac{B(\lambda)}{\Gamma(\lambda)} \chi_{\overline{\Delta}}(\lambda) + \widehat{\psi}(\lambda)B(\lambda)\chi_{\Delta}(\lambda) \right] \Phi_X(d\lambda)$$

So, the spectral characteristics of  $P_{H_{\eta}(t)}\widehat{\xi}$  in the space of  $L^2(F_X(d\lambda))$  is

$$\widetilde{\psi}(\lambda) = \begin{cases} 0, & \lambda \in E \\ \psi(\lambda), & \lambda \in \overline{E\Delta} \\ \frac{B(\lambda) \int_{-t}^{\infty} e^{-i\lambda s} \phi(s)ds}{\Gamma(\lambda)}, & \lambda \in \overline{E\Delta} \end{cases}$$

#### 4. Conclusions

A fixed transform is given which based on one-dimensional a stationary processes in this paper. Also, we propose a general filtering problem. Then, in the space of  $L^2(F_X(d\lambda))$ , we get the spectral characteristics of  $P_{H_{\eta}(t)}\widehat{\xi}$  with no any additional conditions. Finally, we calculate the value of the best predict quantity of the general filtering problem.

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#### Authors' contributions

The studies and manuscript of this paper was written by Longsuo Li independently.

#### Competing interests

The author declares that they have no competing interests.

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