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Schur convexity for the ratios of the Hamy and generalized Hamy symmetric functions

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Abstract

In this paper, we present the Schur convexity and monotonicity properties for the ratios of the Hamy and generalized Hamy symmetric functions and establish some analytic inequalities. The achieved results is inspired by the paper of Hara et al. [J. Inequal. Appl. **2**, 387-395, (1998)], and the methods from Guan [Math. Inequal. Appl. **9**, 797-805, (2006)]. The inequalities we obtained improve the existing corresponding results and, in some sense, are optimal.

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1 Introduction

Throughout this paper, we denote $\mathbb{R}_+^n = \{x = (x_1, x_2, \ldots, x_n) | x_i > 0, i = 1, 2, \ldots, n\}$.. For $x \in \mathbb{R}_+^n$, the Hamy symmetric function [1] is defined as

$$F_n(x, r) = F_n(x_1, x_2, \ldots, x_n; r) = \sum_{1 \le i_1 < i_2 < \cdots < i_r \le n} \left(\prod_{j=1}^{r} x_{i_j} \right)^{\frac{1}{r}}, \tag{1.1}$$

where r is an integer and $1 \le r \le n$.

The generalized Hamy symmetric function was introduced by Guan [2] as follows

$$F_n^*(x, r) = F_n^*(x_1, x_2, \ldots, x_n; r) = \sum_{i_1 + i_2 + \cdots + i_n = r} \left(x_1^{i_1} x_2^{i_2} \ldots x_n^{i_n} \right)^{\frac{1}{r}}, \tag{1.2}$$

where r is a positive integer.

In [2], Guan proved that both $F_n(x,r)$ and $F_n^*(x, r)$ are Schur concave in $\mathbb{R}_+^n$. The main of this paper is to investigate the Schur convexity for the functions $\frac{F_n(x, r)}{F_n(x, r-1)}$ and $\frac{F_n^*(x, r)}{F_n^*(x, r-1)}$ and establish some analytic inequalities by use of the theory of majorization.

For convenience of readers, we recall some definitions as follows, which can be found in many references, such as [3].

Definition 1.1. The n-tuple x is said to be majorized by the n-tuple y (in symbols $x \prec y$), if

$$\sum_{i=1}^{k} x_{[i]} \le \sum_{i=1}^{k} y_{[i]}, \quad \sum_{i=1}^{n} x_{[i]} = \sum_{i=1}^{n} y_{[i]},$$

where $1 \le k \le n - 1$, and $x_{[i]}$ denotes the ith largest component of x.

Definition 1.2. Let $E \subseteq \mathbb{R}^n$ be a set. A real-valued function $F : E \to \mathbb{R}$ is said to be Schur convex on E if $F(x) \le F(y)$ for each pair of n-tuples $x = (x_1,..., x_n)$ and $y = (y_1,..., y_n)$ in E, such that $x \prec y$. F is said to be Schur concave if $-F$ is Schur convex.

The theory of Schur convexity is one of the most important theories in the fields of inequalities. It can be used in combinatorial optimization [4], isoperimetric problems for polytopes [5], theory of statistical experiments [6], graphs and matrices [7], gamma functions [8], reliability and availability [9], optimal designs [10] and other related fields.

Our aim in what follows is to prove the following results.

Theorem 1.1. Let $x \in \mathbb{R}_+^n, 2 \le r \le n$ is an integer, then the function $\phi_r(x) = \dfrac{F_n(x, r)}{F_n(x, r-1)}$ is Schur concave in $\mathbb{R}_+^n$ and increasing with respect to x_i (i=1,2, ..., n).

Theorem 1.2. Let $x \in \mathbb{R}_+^n, 2 \le r \le n$ is an integer, then the function $\phi_r^*(x) = \dfrac{F_n^*(x, r)}{F_n^*(x, r-1)}$ is Schur concave in $\mathbb{R}_+^n$ and increasing with respect to x_i (i=1,2,... n).

Corollary 1.1. If $x_i > 0, i = 1, 2, \ldots, n,\ \sum_{i=1}^n x_i = s$ and that $c \ge s$, then

$$\frac{G_n(x)}{G_n(c-x)} = \frac{F_n(x,n)}{F_n(c-x,n)} \le \frac{F_n(x,n-1)}{F_n(c-x,n-1)} \le \cdots \le \frac{F_n(x,1)}{F_n(c-x,1)} = \frac{A_n(x)}{A_n(c-x)}$$

and

$$\frac{G_n(x)}{G_n(c+x)} = \frac{F_n(x,n)}{F_n(c+x,n)} \le \frac{F_n(x,n-1)}{F_n(c+x,n-1)} \le \cdots \le \frac{F_n(x,1)}{F_n(c+x,1)} = \frac{A_n(x)}{A_n(c+x)},$$

where $A_n(x) = \frac{1}{n}\sum_{i=1}^n x_i,\ G_n(x) = \left(\prod_{i=1}^n x_i\right)^{\frac{1}{n}}$ are the arithmetic and geo-metric means of x, respectively.

Corollary 1.2. If $x_i > 0, i = 1, 2, \ldots, n,\ \sum_{i=1}^n x_i = s$ and that $c \ge s$, then

$$\frac{F_n^*(x,r)}{F_n^*(c-x,r)} \le \frac{F_n^*(x,r-1)}{F_n^*(c-x,r-1)} \le \cdots \le \frac{F_n^*(x,2)}{F_n^*(c-x,2)} \le \frac{F_n^*(x,1)}{F_n^*(c-x,1)} = \frac{A_n(x)}{A_n(c-x)}$$

and

$$\frac{F_n^*(x,r)}{F_n^*(c+x,r)} \le \frac{F_n^*(x,r-1)}{F_n^*(c+x,r-1)} \le \cdots \le \frac{F_n^*(x,2)}{F_n^*(c+x,2)} \le \frac{F_n^*(x,1)}{F_n^*(c+x,1)} = \frac{A_n(x)}{A_n(c+x)}.$$

2 Lemmas

In order to establish our main results, we need several lemmas, which we present in this section.

Lemma 2.1 (see [3]). Let $E \subseteq \mathbb{R}^n$ be a symmetric convex set with nonempty interior int$E$ and $\phi : E \to \mathbb{R}$ be a continuous symmetric function. If ϕ is differentiable on int$E$, then ϕ is Schur convex (or Schur concave, respectively) on E if and only if

$$(x_i - x_j)\left(\frac{\partial\varphi}{\partial x_i} - \frac{\partial\varphi}{\partial x_j}\right) \geq 0 \quad (\text{or} \leq 0, \text{respectively})$$

for all $i,j = 1,2,\ldots,n$ and $x = (x_1,\ldots,x_n) \in$ int$E$.

The rth elementary symmetric function (see [11]) is defined as

$$E_n(x,r) = E_n(x_1, x_2, \ldots, x_n; r) = \sum_{1 \leq i_1 < i_2 < \cdots < i_r \leq n} \left(\prod_{j=1}^{r} x_{i_j}\right), \tag{2.1}$$

where $1 \leq r \leq n$ is a positive integer, and $E_n(x, 0) = 1$.

By (2.1) and simple computations, we have the following lemma.

Lemma 2.2. Let $x \in \mathbb{R}^n_+$, $1 \leq i \leq n$, if

$$\overline{x_i} = (x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n).$$

Then,

$$E_n(x_1, x_2, \ldots, x_n; r) = x_i E_{n-1}(\overline{x_i}, r-1) + E_{n-1}(\overline{x_i}, r). \tag{2.2}$$

Lemma 2.3 (see [11]). Let $x \in \mathbb{R}^n_+$, r is an integer and $1 \leq r \leq n - 1$.
Then,

$$(E_n(x,r))^2 > E_n(x, r-1)E_n(x, r+1). \tag{2.3}$$

Another important symmetric function is the complete symmetric function (see [3]), which is defined by

$$C_r(x) = C_r(x_1, x_2, \ldots, x_n) = \sum_{i_1 + i_2 + \cdots + i_n = r} x_1^{i_1} x_2^{i_2} \ldots x_n^{i_n},$$

where $i_1, i_2, \ldots, i_n$ are non-negative integer, $r \in \{1, 2, \ldots\}$ and $C_0(x) = 1$.

Lemma 2.4 (see [12]). Let $x_i > 0$, $i = 1, 2, \ldots, n$, and $\overline{x_i} = (x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$.
Then,

$$C_r(x) = x_i C_{r-1}(x) + C_r(\overline{x_i}).$$

Lemma 2.5 (see [13]). If $0 < r < s$, $x \in \mathbb{R}^n_+$, then

$$C_r(x)C_{s-1}(x) > C_{r-1}C_s(x).$$

Lemma 2.6 (see [14]). If $x_i > 0, i = 1, 2, \ldots, n, \sum_{i=1}^n x_i = s$ and $c \geq s$, then

(1) $$\frac{c-x}{\frac{nc}{s}-1} = \left(\frac{c-x_1}{\frac{nc}{s}-1}, \frac{c-x_2}{\frac{nc}{s}-1}, \ldots, \frac{c-x_n}{\frac{nc}{s}-1}\right) \prec (x_1, x_2, \ldots, x_n) = x,$$

(2) $$\frac{c+x}{s+nc} = \left(\frac{c+x_1}{s+nc}, \frac{c+x_2}{s+nc}, \ldots, \frac{c+x_n}{s+nc}\right) \prec \left(\frac{x_1}{s}, \frac{x_2}{s}, \ldots, \frac{x_n}{s}\right) = \frac{x}{s}.$$

3 Proof of Theorems

Proof of Theorem 1.1. It is obvious that $\varphi_r(x)$ is symmetric and has continuous partial derivatives in $\mathbb{R}^n_+$. By Lemma 2.1, we only need to prove that

$$(x_1 - x_2)\left(\frac{\partial \phi_r(x)}{\partial x_1} - \frac{\partial \phi_r(x)}{\partial x_2}\right) \le 0. \tag{3.1}$$

For any fixed $2 \le r \le n$, let $u_i = \sqrt[r]{x_i}$, $i = 1, 2, \ldots, n$ and $u = (u_1, u_2, \ldots, u_n) \in \mathbb{R}^n_+$, we have

$$\phi_r(x) = \frac{F_n(x, r)}{F_n(x, r-1)} = \frac{E_n(u, r)}{E_n(u, r-1)}.$$

Differentiating $\varphi_r(x)$ with respect to x_1 yields

$$\frac{\partial \phi_r(x)}{\partial x_1} = \frac{1}{E_n^2(u, r-1)}\left[E_n(u, r-1)\frac{\partial E_n(u, r)}{\partial u_1}\frac{\partial u_1}{\partial x_1} - E_n(u, r)\frac{\partial E_n(u, r-1)}{\partial u_1}\frac{\partial u_1}{\partial x_1}\right]. \tag{3.2}$$

Using Lemma 2.2 repeatedly, we get

$$\begin{aligned} E_n(u, r) = {} & u_1 u_2 E_{n-2}(u_3, \ldots, u_n; r-2) + (u_1 + u_2)E_{n-2}(u_3, \ldots, u_n; r-1) \\ & + E_{n-2}(u_3, \ldots, u_n; r). \end{aligned} \tag{3.3}$$

Equations (3.2) and (3.3) lead to

$$\frac{\partial \phi_r(x)}{\partial x_1} = \frac{1}{rE_n^2(u, r-1)}(u_1^{1-r}u_2 A + u_1^{1-r}B), \tag{3.4}$$

where

$$A = E_n(u, r-1)E_{n-2}(u_3, \ldots, u_n; r-2) - E_n(u, r)E_{n-2}(u_3, \ldots, u_n; r-3)$$

and

$$B = E_n(u, r-1)E_{n-2}(u_3, \ldots, u_n; r-1) - E_n(u, r)E_{n-2}(u_3, \ldots, u_n; r-2).$$

Similarly, we can deduce that

$$\frac{\partial \phi_r(x)}{\partial x_2} = \frac{1}{rE_n^2(u, r-1)}(u_1 u_2^{1-r}A + u_2^{1-r}B). \tag{3.5}$$

From (3.4) and (3.5), one has

$$\begin{aligned} & (x_1 - x_2)\left(\frac{\partial \phi_r(x)}{\partial x_1} - \frac{\partial \phi_r(x)}{\partial x_2}\right) \\ & = \frac{x_1 - x_2}{rE_n^2(u, r-1)}\left[x_1^{\frac{1}{r}}x_2^{\frac{1}{r}}(x_1^{-1} - x_2^{-1})A + (x_1^{\frac{1}{r}-1} - x_2^{\frac{1}{r}-1})B\right]. \end{aligned} \tag{3.6}$$

It follows from (3.3) and Lemma 2.3 that

$$\begin{aligned} A = {} & (u_1 + u_2)[E_{n-2}^2(u_3, \ldots, u_n; r-2) - E_{n-2}(u_3, \ldots, u_n; r-1) \\ & \times E_{n-2}(u_3, \ldots, u_n; r-3)] + E_{n-2}(u_3, \ldots, u_n; r-1)E_{n-2}(u_3, \ldots, u_n; r-2) \\ & - E_{n-2}(u_3, \ldots, u_n; r)E_{n-2}(u_3, \ldots, u_n; r-3) \\ & > 0. \end{aligned}$$

Similarly, we can get $B > 0$.

It follows from the function $x^{\frac{k-r}{r}}\ (k=0,1)$ is decreasing in (0, +∞) that

$$(x_1-x_2)\left(x_1^{\frac{k-r}{r}}-x_2^{\frac{k-r}{r}}\right)\leq 0,(k=0,1). \tag{3.7}$$

Therefore, inequality (3.1) follows from (3.6) and (3.7) together with $A > 0$ and $B > 0$.

Next, we prove that $\phi_r(x)=\frac{F_n(x,r)}{F_n(x,r-1)}$ is increasing with respect to x_i (i=1,2,...,n).

By the symmetry of $\varphi_r(x)$ with respect to x_i (i = 1, 2,..., n), we only need to prove that

$$\frac{\partial\phi_r(x)}{\partial x_1}\geq 0,$$

which can be derived directly from $A > 0$ and $B > 0$ together with Equation (3.4).

Proof of Theorem 1.2. It is obvious that $\phi_r^*(x)$ is symmetric and has continuous partial derivatives in $\mathbb{R}_+^n$. By Lemma 2.1, we only need to prove that

$$(x_1-x_2)\left(\frac{\partial\phi_r^*(x)}{\partial x_1}-\frac{\partial\phi_r^*(x)}{\partial x_2}\right)\leq 0. \tag{3.8}$$

For any fixed $2 \leq r \leq n$, let $u_i=\sqrt[r]{x_i},\ i=1,2,\ldots,n$ and $u=(u_1,u_2,\ldots,u_n)\in\mathbb{R}_+^n$. Then,

$$\phi_r^*(x)=\frac{F_n^*(x,r)}{F_n^*(x,r-1)}=\frac{C_r(u)}{C_{r-1}(u)}. \tag{3.9}$$

Differentiating $\phi_r^*(x)$ with respect to x_1, we have

$$\frac{\partial\phi_r^*(x)}{\partial x_1}=\frac{1}{C_{r-1}^2(u)}\left[C_{r-1}(u)\frac{\partial C_r(u)}{\partial u_1}\frac{\partial u_1}{\partial x_1}-C_r(u)\frac{\partial C_{r-1}(u)}{\partial u_1}\frac{\partial u_1}{\partial x_1}\right]. \tag{3.10}$$

It follows from Lemma 2.4 that

$$\begin{aligned}\frac{\partial C_r(u)}{\partial u_1}&=C_{r-1}(u)+u_1\frac{\partial C_{r-1}(u)}{\partial u_1}\\&=C_{r-1}(u)+u_1\left[C_{r-2}(u)+u_1\frac{\partial C_{r-2}(u)}{\partial u_1}\right]\\&=C_{r-1}(u)+u_1C_{r-2}(u)+u_1^2\frac{\partial C_{r-2}(u)}{\partial u_1}\\&=\cdots\cdots\\&=C_{r-1}(u)+u_1C_{r-2}(u)+u_1^2C_{r-3}(u)+\cdots+u_1^{r-2}C_1(u)+u_1^{r-1}.\end{aligned} \tag{3.11}$$

Equations (3.10) and (3.11) lead to

$$\begin{aligned}\frac{\partial\phi_r^*(x)}{\partial x_1}&=\frac{1}{C_{r-1}^2(u)}\{[C_{r-1}^2(u)-C_r(u)C_{r-2}(u)]+u_1[C_{r-1}(u)C_{r-2}(u)\\&-C_r(u)C_{r-3}(u)]+\cdots+u_1^{r-2}[C_{r-1}(u)C_1(u)-C_r(u)C_0(u)]\\&+C_{r-1}(u)u_1^{r-1}\}\frac{1}{r}u_1^{1-r}.\end{aligned} \tag{3.12}$$

Similarly, we have

$$\begin{aligned}\frac{\partial \phi_r^*(x)}{\partial x_2} = \frac{1}{C_{r-1}^2(u)}\{[C_{r-1}^2(u) - C_r(u)C_{r-2}(u)] + u_2[C_{r-1}(u)C_{r-2}(u) \\ - C_r(u)C_{r-3}(u)] + \cdots + u_2^{r-2}[C_{r-1}(u)C_1(u) - C_r(u)C_0(u)] \\ + C_{r-1}(u)u_2^{r-1}\}\frac{1}{r}u_2^{1-r}.\end{aligned} \tag{3.13}$$

From (3.12) and (3.13), one has

$$\begin{aligned}&(x_1 - x_2)\left(\frac{\partial \phi_r^*(x)}{\partial x_1} - \frac{\partial \phi_r^*(x)}{\partial x_2}\right) \\ &= \frac{x_1 - x_2}{rC_{r-1}^2(u)}\left\{[C_{r-1}^2(u) - C_r(u)C_{r-2}(u)]\left(x_1^{\frac{1-r}{r}} - x_2^{\frac{1-r}{r}}\right) + [C_{r-1}(u)C_{r-2}(u)\right. \\ &- C_r(u)C_{r-3}(u)]\left(x_1^{\frac{2-r}{r}} - x_2^{\frac{2-r}{r}}\right) + \cdots + [C_{r-1}(u)C_1(u) - C_r(u)C_0(u)] \\ &\left.\times\left(x_1^{\frac{(r-1)-r}{r}} - x_2^{\frac{(r-1)-r}{r}}\right)\right\}.\end{aligned} \tag{3.14}$$

By Lemma 2.5, we know that

$$\begin{aligned}&C_{r-1}^2(u) - C_r(u)C_{r-2}(u) > 0, \\ &C_{r-1}(u)C_{r-2}(u) - C_r(u)C_{r-3}(u) > 0, \\ &\cdots\cdots, \\ &C_{r-1}(u)C_1(u) - C_r(u)C_0(u) > 0.\end{aligned} \tag{3.15}$$

The monotonicity of the function $x^{\frac{j-r}{r}}\ (1 \le j \le r-1)$ in $(0, +\infty)$ leads to the conclusion that

$$(x_1 - x_2)(x_1^{\frac{j-r}{r}} - x_2^{\frac{j-r}{r}}) \le 0. \tag{3.16}$$

Therefore, inequality (3.8) follows from (3.14)-(3.16).

Next, we prove that $\phi_r^*(x) = \dfrac{F_n^*(x, r)}{F_n^*(x, r-1)}$ is increasing with respect to x_i (i=1,2,...,n).

From (3.12) and (3.15), we clearly see that

$$\frac{\partial \phi_r^*(x)}{\partial x_1} \ge 0. \tag{3.17}$$

Inequality (3.17) implies that $\phi_r^*(x)$ is increasing with respect to x_1, then from the symmetry of $\phi_r^*(x)$ with respect to x_i (i = 1, 2,..., n) we know that $\phi_r^*(x)$ is increasing with respect to each x_i (i = 1, 2,..., n).

Proof of Corollary 1.1. By Theorem 1.1 and Lemma 2.6, we have $\phi_r\left(\dfrac{c-x}{\frac{nc}{s}-1}\right) \ge \phi_r(x)$ and $\phi_r\left(\dfrac{c+x}{s+nc}\right) \ge \phi_r\left(\dfrac{x}{s}\right)$ which imply Corollary 1.1.

Remark 1. Let $0 < x_i \le \frac{1}{2},\ i = 1, 2, \ldots, n$, then

$$\frac{G_n(x)}{G_n(1-x)} \le \frac{A_n(x)}{A_n(1-x)}, \tag{3.18}$$

where $(1 - x) = (1 - x_1, 1 - x_2, \ldots, 1 - x_n)$, commonly referred to as Ky Fan inequality (see [15]), which has attracted the attention of a considerable number of mathematicians (see [16-20]).

Letting $\sum_{i=1}^{n} x_i \le 1$ and taking $c = 1$ in Corollary 1.1, we get

$$\frac{G_n(x)}{G_n(1-x)} = \frac{F_n(x,n)}{F_n(1-x,n)} \le \frac{F_n(x,n-1)}{F_n(1-x,n-1)} \le \cdots \le \frac{F_n(x,1)}{F_n(1-x,1)} = \frac{A_n(x)}{A_n(1-x)}. \tag{3.19}$$

It is obvious that inequality (3.19) can be called Ky Fan-type inequality.

Remark 2. Let $x_i > 0$, $i = 1, 2, \ldots, n$, the following inequalities

$$\prod_{i=1}^{n} (x_i^{-1} - 1) \ge (n-1)^n$$

and

$$\prod_{i=1}^{n} (x_i^{-1} + 1) \ge (n+1)^n$$

are the well-known Weierstrass inequalities (see [11]).

Taking $c = s = 1$ in Corollary 1.1, one has

$$\prod_{i=1}^{n} (x_i^{-1} - 1) \ge \left(\frac{F_n(1-x,n-1)}{F_n(x,n-1)}\right)^n \ge \cdots \ge \left(\frac{F_n(1-x,2)}{F_n(x,2)}\right)^n \ge (n-1)^n$$

and

$$\prod_{i=1}^{n} (x_i^{-1} + 1) \ge \left(\frac{F_n(1+x,n-1)}{F_n(x,n-1)}\right)^n \ge \cdots \ge \left(\frac{F_n(1+x,2)}{F_n(x,2)}\right)^n \ge (n+1)^n.$$

It is obvious that our inequalities can be called Weierstrass-type inequalities.

Proof of Corollary 1.2. By Theorem 1.2 and Lemma 2.6, we have $\phi_r^*\left(\frac{c-x}{\frac{nc}{s}-1}\right) \ge \phi_r^*(x)$ and $\phi_r^*\left(\frac{c+x}{s+nc}\right) \ge \phi_r^*\left(\frac{x}{s}\right)$, which imply Corollary 1.2.

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Competing interests
The author declares that he has no competing interests.

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