Rezaei and Sharifzadeh *Journal of Inequalities and Applications* 2011, **2011**:114
http://www.journalofinequalitiesandapplications.com/content/2011/1/114

Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH **Open Access**

On the super-stability of exponential Hilbert-valued functional equations

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Abstract

We generalize the well-known Baker's super-stability result for exponential mappings with values in the field of complex numbers to the case of an arbitrary Hilbert space with the Hadamard product. Then, we will prove an even more general result of this type.

2000 MSC: primary: 39B72, secondary: 46E40.

Keywords: exponential functions, stability, Hilbert-valued function

1. Introduction

The stability problem of functional equations goes back to a question of Ulam [1] concerning the stability of group homomorphisms. Hyers [2] gave a first affirmative partial answer to the question of Ulam for Banach spaces (see also [3]). Hyers's theorem was generalized by Aoki [4] for additive mappings and by Rassias [5,6] for linear mappings by considering an unbounded Cauchy difference. Baker et al. [7] have proved the super-stability of the exponential functional equation: If a function $f: \mathbb{R} \to \mathbb{R}$ is approximately exponential function, i.e., there exists a nonnegative number α such that

$$|f(x+y) - f(x)f(y)| \leq \alpha$$

for $x, y \in \mathbb{R}$, then f is either bounded or exponential. This theorem was the first result concerning the super-stability phenomenon of functional equations. Baker [8] generalized this famous result to any function $f: (G, +) \to \mathbb{C}$ where $(G, +)$ is a semigroup. The same result is also true for approximately exponential mappings with values in a normed algebra with the property that the norm is multiplicative.

Theorem 1.1. Let $(G, +)$ be a semigroup and Y be a normed algebra in which the norm is multiplicative. Then, for a function $f: G \to Y$ satisfying the inequality

$$||f(x+y) - f(x)f(y)|| \leq \alpha$$

for all $x; y \in G$ and for some $\alpha > 0$, either $||f(x)|| \leq 1/2(1+\sqrt{1+4\alpha})$ for all $x \in G$ or f is an exponential function.

In the other world every approximately exponential map $f: (G, +) \to Y$ is either bounded or exponential.

Rassias [5,6] introduced the term *mixed stability* of the function $f: E \to \mathbb{R}$, where E is a Banach space, with respect to two operations addition and multiplication among any two elements of the set $\{x, y, f(x), f(y)\}$. Especially, he raised an open problem concerning the behavior of solutions of the inequality:

$$|f(x+y) - f(x)f(y)| \leq \theta(||x||^p + ||y||^p),$$

(see also [9,10]). In connection with this open problem, Gavruta [11] gave an answer to this problem in the spirit of Rassiass approach:

Theorem 1.2 (Gavruta). Let X and Y be a real normed space and a normed algebra with multiplicative norm, respectively. If a function f: $X \to Y$ satisfies the inequality

$$||f(x+y) - f(x)f(y)|| \leq \theta(||x||^p + ||y||^p),$$

for all x; $y \in X$ and for some $p > 0$ and $\theta > 0$, then either $||f(x)|| \leq \delta||x||^p$ for all $x \in X$ with $||x|| \geq 1$ or f is an exponential function, where $\delta = 1/2(2^p + \sqrt{4p + 8\theta})$.

Baker [8] gave an example to present that the Theorem 1.1 is false if the algebra Y does not have the multiplicative norm: Given $\delta > 0$, choose an $\varepsilon > 0$ with $|\varepsilon - \varepsilon^2| = \delta$. Let $M_2(\mathbb{C})$ denote the space of 2×2 complex matrices with the usual norm and f: $\mathbb{R} \to M_2(\mathbb{C})$ is defined by $f(x) = e^x e_{11} + e^x e_{22}$ where e_{ij} is defined as the 2×2 matrix with 1 in the (i, j) entry and zeroes elsewhere. We will show that such behavior is typical for approximately exponential mappings with values in Hilbert spaces with Hadamard product which is not multiplicative.

Let H be a Hilbert space with a countable orthonormal basis $\{e_n: n \in \mathbb{N}\}$. For two vectors $x, y \in H$, we have the Hadamard product (named after French mathematician Jacques Hadamard), also known as the entrywise product on Hilbert space H as the following:

$$x * y = \sum_{n=1}^{+\infty} \langle x, e_n \rangle \langle y, e_n \rangle e_n \quad (x, y \in H).$$

The Cauchy-Schwartz inequality together with the Parseval identity insure that Hadamard multiplication is well defined. In fact,

$$||x * y|| \leq \left(\sum_{n=1}^{+\infty} |\langle x, e_n \rangle|^2 \right)^{1/2} \left(\sum_{n=1}^{+\infty} |\langle y, e_n \rangle|^2 \right)^{1/2} = ||x|| ||y||.$$

In the present paper, we state a super-stability result for the approximately exponential Hilbert-valued functional equation by Hadamard product, see Theorem 2.1 below. As a consequence, we prove if a surjective function f: $H \to H$ satisfies the inequality

$$||f(x * y) - f(x) * f(y)||_H \leq \alpha$$

for some $\alpha \geq 0$ and for all x; $y \in H$, then it must be exponential with this product, i.e.,

$$f(x * y) = f(x) * f(y).$$

Then, we will prove an even more general result of this type. We also generalized Theorem 2.1 concerning the mixed stability for Hilbert-valued functions.

2. Main results

The function $f(x) = a^x$ is said to be an exponential function, where $a > 0$ is a fixed real number. The exponent law of exponential functions is well represented by the exponential equation $f(x + y) = f(x)f(y)$. Hence, we call every solution function of the exponential equation as exponential function. A general solution of the exponential equation was introduced in [12]. In fact, a function f: $\mathbb{R} \to \mathbb{C}$ is an exponential function if and only if either $f(x) = \exp(A(x) + ia(x))$ for all $x \in \mathbb{R}$ or $f(x) = 0$ for all $x \in \mathbb{R}$; where A: $\mathbb{R} \to \mathbb{R}$ is an additive function and a: $\mathbb{R} \to \mathbb{R}$ satisfies

$$a(x+y) \equiv a(x) + a(y) \mod 2\pi \tag{1}$$

for all $x, y \in \mathbb{R}$. Indeed, a function f: $\mathbb{R} \to \mathbb{R}$ continuous at a point is an exponential function if and only if $f(x) = a^x$ for all $x \in \mathbb{R}$ or $f(x) = 0$ for all $x \in \mathbb{R}$, where $a > 0$ is a constant.

Definition 2.1. For a Hilbert space H and a semi-group $(G,.)$, a function F: $G \to H$ is said to be exponential when

$$F(x, y) = F(x) * F(y)$$

for every $x, y \in G$.

The following proposition characterizes the Hilbert-valued function satisfying the exponential equation:

Proposition 2.2. Let H be a separable complex Hilbert space and the mapping F: $\mathbb{R} \to H$ be exponential then either $F \equiv 0$ or there exist a positive integer N such that

$$F(x) = \sum_{n=1}^{N} \exp(A_n(x) + a_n(x))e_n$$

for all $x \in H$ where A_n: $\mathbb{R} \to \mathbb{R}$ is an additive function and a_n is a function satisfying (1) for $n = 1, 2, \ldots, N$.

Proof. For every integer $n \geq 1$, consider the function $e_n \otimes F$: $\mathbb{R} \to \mathbb{C}$ by

$$(e_n \otimes F)(h) = \langle F(h), e_n \rangle$$

for every $h \in H$. Since F is exponential, so is $e_n \otimes F$ for every integer $n \geq 1$. Indeed, for $n \geq 1$ and $x, y \in H$, we see that

$$\begin{aligned}
\sum_{n=1}^{+\infty} (e_n \otimes F)(x.y)e_n &= \sum_{n=1}^{+\infty} \langle F(x.y), e_n \rangle e_n \\
&= F(x.y) = F(x) * F(y) \\
&= \sum_{n=1}^{+\infty} \langle F(x), e_n \rangle \langle F(y), e_n \rangle e_n \\
&= \sum_{n=1}^{+\infty} (e_n \otimes F)(x)(e_n \otimes F)(y)e_n.
\end{aligned}$$

This yields the exponential property of $e_n \otimes F$ for every $n \geq 1$. Hence, either

$$(e_n \otimes F)(x) = \exp(A_n(x) + a_n(x)) \tag{2}$$

for all $x \in \mathbb{R}$ or $(e_n \otimes F)(x) = 0$ for all $x \in \mathbb{R}$; here A_n: $\mathbb{R} \to \mathbb{R}$ is an additive function and a_n is a function satisfying (1). The continuation of proof depend on the dimension of H. In fact, if H is infinite dimensional, since

$$(e_n \otimes F)(x) = \langle F(x), e_n \rangle \to 0$$

for every $x \in H$ as $n \to +\infty$ Equation 2 is not possible for infinitely many positive integer n and hence there exists some positive integer N such that $e_n \otimes F = 0$ for every integer $n > N$. Thus, F can be represented as

$$F(x) = \sum_{n=1}^{+\infty} \langle F(x), e_n \rangle e_n = \sum_{n=1}^{N} \langle F(x), e_n \rangle e_n = \sum_{n=1}^{N} \exp(A_n(x) + a_n(x)) e_n.$$

In the case that H is of finite dimensional type, the proof is clear.

In the following theorem, we generalize the well-known Baker's super-stability result for exponential mappings with values in the field of complex numbers to the case of an arbitrary Hilbert space with the Hadamard product.

Theorem 2.3. Let G be a semigroup and let $\alpha > 0$ be given. If a function f: $G \to H$ satisfies the inequality

$$||f(x.y) - f(x) * f(y)||_H \leq \alpha \tag{3}$$

for all x; $y \in G$, then either there exists an integer $k \geq 1$ such that

$$|\langle f(x), e_k \rangle| \leq 2^k(1 + \sqrt{1+\alpha}) \tag{4}$$

for all $x \in G$ or

$$f(x.y) = f(x) * f(y)$$

for all x; $y \in G$.

Proof. Assume that the first conclusion (i.e., (4)) is not true. Hence, for every integer $k \geq 1$, there exists a $a_k \in G$ such that

$$|\langle f(a_k), e_k \rangle| > 2^k(1 + \sqrt{1+\alpha}).$$

Let $\beta := (1 + \sqrt{1+\alpha})$, $f_k(x) = \langle f(x), e_k \rangle$, and $g_k = 2^{-k} f_k$. Then, $\beta^2 - 2\beta = \alpha$, $\beta > 2$ and $|f_k(a_k)| > 2^k\beta$ whence $|g_k(a_k)| > \beta$. By applying the Parseval identity and definition of Hadamard product with together relation (3), we find that each scalar-valued function f_k is approximately exponential, i.e.,

$$|f_k(x.y) - f_k(x)f_k(y)| < \alpha \tag{5}$$

for every integer $k \geq 1$ and x, $y \in G$. Let

$$\gamma_k = |g_k(a_k)| - \beta + 1$$

then $\gamma_k > 1$ for every integer $k \geq 1$. It follows from (5) for $x = y = a_k$ that

$$|f_k(a_k^2) - f_k(a_k)^2| \leq \alpha$$

and so

$$\begin{aligned}
|g_k(a_k^2)| &\geq |2^k g_k(a_k)^2| - |g_k(a_k^2)| - 2^k g_k(a_k)^2| \\
&= 4^k |g_k(a_k)^2| - 2^{-k}|f_k(a_k^2) - f_k(a_k)^2| \\
&\geq |g_k(a_k)^2| - |f_k(a_k^2) - f_k(a_k)^2| \\
&\geq |g_k(a_k)|^2 - \alpha \\
&= (\gamma_k + \beta - 1)^2 - \beta^2 + 2\beta \\
&= (\gamma_k - 1)^2 + 2\gamma_k\beta > 2\beta.
\end{aligned}$$

Now, make the induction hypothesis

$$|g_k(a_k^{2^n})| > (n+1)\beta. \tag{6}$$

Then, by using (5) for $x = y = a_k^{2^n}$ and (6), we observe that

$$\begin{aligned}|g_k(a_k^{2^{n+1}})| &\geq |2^k g_k(a_k^{2^n})^2| - |g_k(a_k^{2^{n+1}}) - 2^k g_k(a_k^{2^n})^2| \\ &= 4^k |g_k(a_k^{2^n})|^2 - 2^{-k} |f_k(a_k^{2^{n+1}}) - f_k(a_k^{2^n})^2| \\ &\geq |g_k(a_k^{2^n})|^2 - |f_k(a_k^{2^{n+1}}) - f_k(a_k^{2^n})^2| \\ &\geq |g_k(a_k^{2^n})|^2 - \alpha \\ &> (n+1)^2\beta^2 - \beta^2 + 2\beta > (n+2)\beta\end{aligned}$$

and (6) is established for all $n \in \mathbb{N}$. Hence, by definition of f_k and g_k, we see that

$$\left|\langle f(a_k^{2^n}), e_k\rangle\right| > 2^k(n+1)\beta. \tag{7}$$

On the other hand, for every $x, y, z \in G$, we have

$$\begin{aligned}||f(x.y) * f(z) - f(x) * f(y.z)|| &\leq ||f(x.y) * f(z) - f(x.y.z)|| \\ &\quad + ||f(x.y.z) - f(x) * f(y.z)|| \\ &\leq 2\alpha.\end{aligned}$$

Consequently, for $h(x, y) = f(x.y) - f(x) * f(y)$, one can see

$$\begin{aligned}||h(x, y) * f(z)|| &= ||f(x.y) * f(z) - f(x) * f(y) * f(z)|| \\ &\leq ||f(x.y) * f(z) - f(x) * f(y.z)|| \\ &\quad + ||f(x) * f(y.z) - f(x) * f(y) * f(z)|| \\ &\leq 2\alpha + \alpha||f(x)||.\end{aligned}$$

Now, by using Parseval identity for $h(x, y) * f(z)$ observe that

$$\begin{aligned}\left|\langle f(z), e_k\rangle\right|^2 \left|\langle h(x,y), e_k\rangle\right|^2 &\leq \sum_{k=1}^{+\infty} \left|\langle f(z), e_k\rangle\right|^2 \left|\langle h(x,y), e_k\rangle\right|^2 \\ &= ||h(x,y) * f(z)||^2 \leq 2\alpha + \alpha||f(x)||.\end{aligned}$$

Applying the last relation for $z = a_k^{2^n}$ and relation (7) to deduce that

$$\begin{aligned}4^k(n+1)^2\beta^2 \left|\langle h(x,y), e_k\rangle\right|^2 &\leq \left|\langle f(a_k^{2^n}), e_k\rangle\right|^2 \left|\langle h(x,y), e_k\rangle\right|^2 \\ &= \left|\langle f(z), e_k\rangle\right|^2 \left|\langle h(x,y), e_k\rangle\right|^2 \\ &\leq 2\alpha + \alpha||f(x)||.\end{aligned}$$

It follows that

$$||h(x,y)||^2 = \sum_{k=1}^{+\infty} \left|\langle h(x,y), e_k\rangle\right|^2 \leq \frac{2\alpha + \alpha||f(x)||}{\beta^2(n+1)^2} \sum_{k=1}^{+\infty} 4^{-k}$$

for all $x, y \in G$ and any $n \in \mathbb{N}$. Letting $n \to +\infty$, we conclude that $h(x, y) = 0$ and so $f(x.y) = f(x) * f(y)$ for all $x, y \in G$.

Notice that if f: $H \to H$ is a surjection function, then every component function $e_n \otimes f$ is unbounded. In fact, for every positive integer n, there exists some $x_n \in H$ such that $f(x_n) = ne_n$, and so $(e_n \otimes f)(x_n) = n$. This led to the following corollary:

Corollary 2.4. If a surjective function f: $H \to H$ satisfies the inequality

$$||f(x * y) - f(x) * f(y)||_H \leq \alpha$$

for some $\alpha \geq 0$ and for all x; $y \in G$, then $f(x * y) = f(x) * f(y)$ for all x; $y \in G$.

In the next theorem, we generalize the Gavruta Theorem on mixed stability for Hilbert-valued function with Hadamard product:

Theorem 2.5. Let X be a normed space and H be a separable Hilbert space. If a function f: $X \to H$ satisfies the inequality

$$||f(x + y) - f(x) * f(y)|| \leq \theta(||x||^p + ||y||^p) \tag{8}$$

for all x; $y \in X$ and for some $p > 0$ and $\theta > 0$, then either there exists an integer $k \geq 1$ such that

$$|\langle f(x), e_k\rangle| \leq 2^k \beta \tag{9}$$

for all $x \in X$ with $||x|| \geq 1$ or

$$f(x + y) = f(x) * f(y)$$

for all x; $y \in X$. where $\beta = 2^p + \sqrt{4^p + 4\theta}$.

Proof. Assume that for every integer $k \geq 1$ there exists an $x_k \in X$ with $||x_k|| \geq 1$ such that

$$|\langle f(x_k), e_k\rangle| > 2^k \beta.$$

If we set $f_k(x) := \langle f(x), e_k\rangle$ and $g_k := 2^{-k} f_k$, this is equivalent with

$$||g_k(x_k)|| > \beta ||x_k||^p.$$

It follows from Parseval identity, definition of Hadamard product and relation (8) that

$$|f_k(x + y) - f_k(x) f_k(y)| < \theta(||x||^p + ||y||^p) \tag{10}$$

for every x, $y \in X$ and $k \geq 1$. In particular, for $x = y = x_k$

$$||f(2x_k) - f(x_k)^2|| \leq 2\theta ||x_k||^p.$$

Since $\beta^2 = 2^{p+1}\beta + 2\theta$, hence

$$\begin{aligned}
|g_k(2x_k)| &\geq |2^k g_k(x_k)|^2 - |g_k(2x_k) - 2^k g_k(x_k)^2| \\
&= 4^k |g_k(x_k)|^2 - 2^{-k} |f_k(2x_k) - f_k(x_k)^2| \\
&\geq |g_k(x_k)|^2 - |f_k(2x_k) - f_k(x_k)^2| \\
&\geq \beta_2 ||x_k||^{2p} - 2\theta ||x_k||^p \\
&= (\beta^2 - 2\theta)||x_k||^p \geq 2^{p+1}\beta ||x_k||^p = 2\beta ||2x_k||^p.
\end{aligned}$$

Now, make the induction hypothesis

$$|g_k(2^n x_k)| > 2^n \beta ||2^n x_k||^p. \tag{11}$$

Then, by using (10) for $x = y = 2^n x_k$ and (11), we get

$$\begin{aligned}|g_k(2^{n+1}x_k)| &\geq |2^k g_k(2^n x_k)|^2 - |g_k(2^{n+1}x_k) - 2^k g_k(2^n x_k)^2| \\ &= 4^k |g_k(2^n x_k)|^2 - 2^{-k}|f_k(2^{n+1}x_k) - f_k(2^n x_k)^2| \\ &\geq |g_k(2^n x_k)|^2 - |f_k(2^{n+1}x_k) - f_k(2^n x_k)^2| \\ &> 2^{2n}\beta^2||2^n x_k||^{2p} - 2\theta||2^n x_k||^p \\ &\geq (2^{2n}\beta^2 - 2\theta)||2^n x_k||^p \\ &\geq 2^{2n}2^{p+1}\beta||2^n x_k||^p \geq 2^{n+1}\beta||2^{n+1}x_k||^p\end{aligned}$$

which in turn proves that the inequality (11) is true for all $n \in \mathbb{N}$. Hence, by definition of f_k and g_k, we see that

$$\left|\langle f(2^n x_k), e_k\rangle\right| > 2^{k+n}\beta||2^n x_k||^p > 2^{k+n}. \tag{12}$$

Choose x; y; $z \in X$ with $f(z) \neq 0$. It then follows from (8) that

$$\begin{aligned}||f(z) * f(x+y) - f(x) * f(y+z)|| &\leq ||f(z) * f(x+y) - f(x+y+z)|| \\ &\quad + ||f(x+y+z) - f(x) * f(y+z)|| \\ &\leq \theta(||z||^p + ||x+y||^p) + \theta(||x||^p + ||y+z||^p)\end{aligned}$$

and again by (8) we get

$$||f(x) * f(x+y) - f(x) * f(y) * f(z)|| \leq \theta||f(x)||(||x||^p + ||y||^p)$$

which together with the last relation yields

$$||f(z) * f(x+y) - f(x) * f(y) * f(z)|| \leq \theta\varphi(x, y, z), \tag{13}$$

where

$$\varphi(x, y, z) = ||x||^p + ||z||^p + ||x+y||^p + ||y+z||^p + ||f(x)||(||x||^p + ||y||^p).$$

Let $h(x, y) = f(x + y) - f(x) * f(y)$, then by (13)

$$||h(x, y) * f(z)|| \leq \theta\varphi(x, y, z)$$

and so

$$\begin{aligned}\left|\langle f(z), e_k\rangle\right|^2\left|\langle h(x, y), e_k\rangle\right|^2 &\leq \sum_{k=1}^{+\infty}\left|\langle f(z), e_k\rangle\right|^2\left|\langle h(x, y), e_k\rangle\right|^2 \\ &= ||h(x, y) * f(z)||^2 \leq 2\theta\varphi(x, y, z).\end{aligned}$$

In particular, by using the last relation for $z_k = 2^n x_k$ and by considering (12) we deduce that

$$2^{k+n}\left|\langle h(x, y), e_k\rangle\right|^2 \leq \left|\langle f(z_k), e_k\rangle\right|^2\left|\langle h(x, y), e_k\rangle\right|^2 \leq \theta\varphi(x, y, z)$$

and consequently,

$$||h(x, y)||^2 = \sum_{k=1}^{+\infty}\left|\langle h(x, y), e_k\rangle\right|^2 \leq \frac{\theta\varphi(x, y, z)}{2^n}\sum_{k=1}^{+\infty}2^{-k}$$

for all $x, y \in X$ and any $n \in \mathbb{N}$. Letting $n \to +\infty$, we conclude that $h(x, y) = 0$ and so $f(x + y) = f(x) * f(y)$ for all $x, y \in X$.

At the end of this paper, let us consider the other type multiplication in a Hilbert space. In fact, for a separable Hilbert space H and two elements $x = \sum_{n=1}^{+\infty} x_n e_n$ and $y = \sum_{n=1}^{+\infty} y_n e_n$ of H, one can define the convolution product by

$$x \bullet y = \left(\sum_{n=1}^{+\infty} \hat{x}(n) e_n\right) \bullet \left(\sum_{n=1}^{+\infty} \hat{y}(n) e_n\right) = \sum_{n=1}^{+\infty} \hat{z}(n) e_n,$$

where the numbers $\hat{z}(n)$ can be obtained by discrete convolution:

$$\hat{z}(n) = \sum_{k=1}^{n} \hat{x}(k)\hat{y}(n-k).$$

Hence, it is interesting to study and to phrase the super-stability phenomenon for functions with values in $(H, \bullet)$. For instance, it is desirable to have a sufficient condition for approximately exponential mappings with values in $(H, \bullet)$ to be exponential with the convolution product.

Authors' contributions
All authors carried out the proof. All authors conceived of the study, and participated in its design and coordination. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

Received: 24 July 2011 Accepted: 21 November 2011 Published: 21 November 2011

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doi:10.1186/1029-242X-2011-114
Cite this article as: Rezaei and Sharifzadeh: **On the super-stability of exponential Hilbert-valued functional equations.** *Journal of Inequalities and Applications* 2011 **2011**:114.