Hindawi Publishing Corporation
Journal of Inequalities and Applications
Volume 2011, Article ID 761430, 9 pages
doi:10.1155/2011/761430

Research Article

General Fritz Carlson's Type Inequality for Sugeno Integrals

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Received 18 August 2010; Revised 23 November 2010; Accepted 20 January 2011

Academic Editor: László Losonczi

Fritz Carlson's type inequality for fuzzy integrals is studied in a rather general form. The main results of this paper generalize some previous results.

1. Introduction and Preliminaries

Recently, the study of fuzzy integral inequalities has gained much attention. The most popular method is using the Sugeno integral [1]. The study of inequalities for Sugeno integral was initiated by Román-Flores et al. [2, 3] and then followed by the others [4–11].

Now, we introduce some basic notation and properties. For details, we refer the reader to [1, 12].

Suppose that Σ is a σ-algebra of subsets of X, and let $\mu : \Sigma \to [0, \infty]$ be a nonnegative, extended real-valued set function. We say that μ is a fuzzy measure if it satisfies

(1) $\mu(\emptyset) = 0$,

(2) $E, F \in \Sigma$ and $E \subset F$ imply $\mu(E) \le \mu(F)$ (monotonicity);

(3) $\{E_n\} \subset \Sigma$, $E_1 \subset E_2 \subset \cdots$ imply $\lim_{n\to\infty}\mu(E_n) = \mu(\bigcup_{n=1}^{\infty} E_n)$ (continuity from below),

(4) $\{E_n\} \subset \Sigma$, $E_1 \supset E_2 \supset \cdots$, $\mu(E_1) < \infty$, imply $\lim_{n\to\infty}\mu(E_n) = \mu(\bigcap_{n=1}^{\infty} E_n)$ (continuity from above).

If f is a nonnegative real-valued function defined on X, we will denote by $L_\alpha f = \{x \in X : f(x) \ge \alpha\} = \{f \ge \alpha\}$ the α-level of f for $\alpha > 0$, and $L_0 f = \overline{\{x \in \mathbb{B} : f(x) > 0\}} = \operatorname{supp} f$ is the support of f. Note that if $\alpha \le \beta$, then $\{f \ge \beta\} \subset \{f \ge \alpha\}$.

Let (X, Σ, μ) be a fuzzy measure space; by $\mathcal{F}_+^\mu(X)$ we denote the set of all nonnegative μ-measurable functions with respect to Σ.

Definition 1.1 (see [1]). Let (X, Σ, μ) be a fuzzy measure space, with $f \in \mathcal{F}_+^{\mu}(X)$, and $A \in \Sigma$, then the Sugeno integral (or fuzzy integral) of f on A with respect to the fuzzy measure μ is defined by

$$⨍_A f d\mu = \bigvee_{\alpha \geq 0} [\alpha \wedge \mu(A \cap \{f \geq \alpha\})], \tag{1.1}$$

where $\vee$ and $\wedge$ denote the operations sup and inf on $[0, \infty)$, respectively.

It is well known that the Sugeno integral is a type of nonlinear integral; that is, for general cases,

$$⨍(af + bg)d\mu = a ⨍ f d\mu + b ⨍ g d\mu \tag{1.2}$$

does not hold.

The following properties of the fuzzy integral are well known and can be found in [12].

Proposition 1.2. *Let (X, Σ, μ) be a fuzzy measure space, with $A, B \in \Sigma$ and $f, g \in \mathcal{F}_+^{\mu}(X)$; then*

(1) $⨍_A f d\mu \leq \mu(A)$,

(2) $⨍_A k d\mu = k \wedge \mu(A)$, *for k a nonnegative constant,*

(3) *if $f \leq g$ on A then $⨍_A f d\mu \leq ⨍_A g d\mu$,*

(4) *if $A \subset B$ then $⨍_A f d\mu \leq ⨍_A f d\mu$,*

(5) $\mu(A \cap \{f \geq \alpha\}) \geq \alpha \Rightarrow ⨍_A f d\mu \geq \alpha$,

(6) $\mu(A \cap \{f \geq \alpha\}) \leq \alpha \Rightarrow ⨍_A f\, d\mu \leq \alpha$,

(7) $⨍_A f d\mu < \alpha \Leftrightarrow$ *there exists $\gamma < \alpha$ such that $\mu(A \cap \{f \geq \gamma\}) < \alpha$,*

(8) $⨍_A f d\mu > \alpha \Leftrightarrow$ *there exists $\gamma > \alpha$ such that $\mu(A \cap \{f \geq \gamma\}) > \alpha$.*

Remark 1.3. Let F be the distribution function associated with f on A, that is, $F(\alpha) = \mu(A \cap \{f \geq \alpha\})$. By (5) and (6) of Proposition 1.2

$$F(\alpha) = \alpha \Longrightarrow ⨍_A f d\mu = \alpha. \tag{1.3}$$

Thus, from a numerical point of view, the Sugeno integral can be calculated by solving the equation $F(\alpha) = \alpha$.

Fritz Carlson's integral inequality states [13, 14] that

$$\int_0^{\infty} f(x)dx \leq \sqrt{\pi}\left(\int_0^{\infty} f^2(x)dx\right)^{1/4} \cdot \left(\int_0^{\infty} x^2 f^2(x)dx\right)^{1/4}. \tag{1.4}$$

Recently, Caballero and Sadarangani [8] have shown that in general, the Carlson's integral inequality is not valid in the fuzzy context. And they presented a fuzzy version of Fritz Carlson's integral inequality as follows.

Theorem 1.4. *Let $f : [0,1] \to [0,\infty)$ be a nondecreasing function and μ the Lebesgue measure on $\mathbb{R}$. Then,*

$$⨍_0^1 f(x)d\mu(x) \le \sqrt{2}\left(⨍_0^1 x^2 f^2(x)d\mu(x)\right)^{1/4} \cdot \left(⨍_0^1 f^2(x)d\mu(x)\right)^{1/4}. \tag{1.5}$$

In this paper, our purpose is to give a generalization of the above Fritz Carlson's inequality for fuzzy integrals. Moreover, we will give many interesting corollaries of our main results.

2. Main Results

This section provides a generalization of Fritz Carlson's type inequality for Sugeno integrals. Before stating our main results, we need the following lemmas.

Lemma 2.1 (see [11]). *Let (X, Σ, μ) be a fuzzy measure space, $f \in \mathcal{F}_+^\mu(X)$, $A \in \Sigma$, $⨍_A f d\mu \le 1$, and $s \ge 1$. Then*

$$⨍_A f^s d\mu \ge \left(⨍_A f d\mu\right)^s. \tag{2.1}$$

If the fuzzy measure μ in Lemma 2.1 is the Lebesgue measure, then $⨍_0^1 f d\mu \le 1$ is satisfied readily. Thus, by Lemma 2.1, we have the following.

Corollary 2.2 (see [8]). *Let $f : [0,1] \to [0,\infty)$ be a μ-measurable function with μ the Lebesgue measure and $s \ge 1$. Then*

$$⨍_0^1 f^s(x)d\mu(x) \ge \left(⨍_0^1 f(x)d\mu(x)\right)^s. \tag{2.2}$$

Definition 2.3. Two functions $f, g : X \to R$ are said to be comonotone if for all $(x,y) \in X^2$,

$$(f(x) - f(y))(g(x) - g(y)) \ge 0. \tag{2.3}$$

An important property of comonotone functions is that for any real numbers p, q, either $\{f \ge p\} \subset \{g \ge q\}$ or $\{g \ge q\} \subset \{f \ge p\}$.

Note that two monotone functions (in the same sense) are comonotone.

Theorem 2.4. *Let (X, Σ, μ) be a fuzzy measure space, $f, g \in \mathcal{F}_+^\mu(X)$ and f and g comonotone functions, $A \in \Sigma$ with $⨍_A f d\mu \le 1$, and $⨍_A g d\mu \le 1$. Then*

$$⨍_A f \cdot g d\mu \ge \left(⨍_A f d\mu\right) \cdot \left(⨍_A g d\mu\right). \tag{2.4}$$

Proof. If $⨍_A f d\mu = 0$ or $⨍_A g d\mu = 0$ then the inequality is obvious. Now choose α, β such that

$$1 \geq ⨍_A f d\mu > \alpha > 0, \qquad 1 \geq ⨍_A g d\mu > \beta > 0. \tag{2.5}$$

Then by (8) of Proposition 1.2, there exist $1 > \gamma_\alpha > \alpha$ and $1 > \gamma_\beta > \beta$ such that

$$\mu(A \cap \{f \geq \gamma_\alpha\}) > \alpha, \qquad \mu(A \cap \{g \geq \gamma_\beta\}) > \beta. \tag{2.6}$$

As f and g are comonotone functions, then either $\{f \geq \gamma_\alpha\} \subset \{g \geq \gamma_\beta\}$ or $\{g \geq \gamma_\beta\} \subset \{f \geq \gamma_\alpha\}$. Suppose that $\{f \geq \gamma_\alpha\} \subset \{g \geq \gamma_\beta\}$. In this case, we have the following:

$$\mu(A \cap \{fg \geq \gamma_\alpha\gamma_\beta\}) \geq \mu((A \cap \{f \geq \gamma_\alpha\}) \cap (A \cap \{g \geq \gamma_\beta\})) = \mu(A \cap \{f \geq \gamma_\alpha\}) > \alpha \geq \alpha\beta. \tag{2.7}$$

Therefore, by applying (8) of Proposition 1.2 again, we find that

$$⨍_A f \cdot g d\mu > \alpha\beta. \tag{2.8}$$

Since the values of $\alpha, \beta > 0$ are arbitrary, we obtain the desired inequality. Similarly, for the case $\{g \geq \gamma_\beta\} \subset \{f \geq \gamma_\alpha\}$ we can get the desired inequality too. □

From Theorem 2.4, we get the following.

Corollary 2.5 (see [15]). *Let μ be an arbitrary fuzzy measure on $[0, a]$ and $f, g : [0, a] \to \mathbb{R}$ be two real-valued measurable functions such that $⨍_0^a f d\mu \leq 1$ and $⨍_0^a g d\mu \leq 1$. If f and g are increasing (or decreasing) functions, then the inequality*

$$⨍_0^a f \cdot g d\mu \geq \left(⨍_0^a f d\mu\right) \cdot \left(⨍_0^a g d\mu\right) \tag{2.9}$$

holds.

If the fuzzy measure μ in Corollary 2.5 is the Lebesgue measure and $a = 1$, then $⨍_0^a f d\mu \leq 1$ and $⨍_0^a g d\mu \leq 1$ are satisfied readily. Thus, by Corollary 2.5, we obtain

Corollary 2.6 (see [2]). *Let $f, g : [0, 1] \to \mathbb{R}$ be two real-valued functions, and let μ be the Lebesgue measure on $\mathbb{R}$. If f, g are both continuous and strictly increasing (decreasing) functions, then the inequality*

$$⨍_0^1 f \cdot g d\mu \geq \left(⨍_0^1 f d\mu\right) \cdot \left(⨍_0^1 g d\mu\right) \tag{2.10}$$

holds.

The following result presents a fuzzy version of generalized Carlson's inequality.

Theorem 2.7. *Let (X, Σ, μ) be a fuzzy measure space, $f, g, h \in \mathcal{F}_+^{\mu}(X)$, f and g, and f and h are comonotone functions, respectively, $A \in \Sigma$ with $⨍_A f d\mu \leq 1$, $⨍_A g d\mu \leq 1$, $\leq ⨍_A h d\mu \leq 1$, $⨍_A fg d\mu \leq 1$, and $⨍_A fh d\mu \leq 1$. Then*

$$⨍_A f(x) d\mu(x) \leq \frac{1}{K}\left(⨍_A f^p(x) g^p(x) d\mu(x)\right)^{1/(p+q)} \cdot \left(⨍_A f^q(x) h^q(x) d\mu(x)\right)^{1/(p+q)}, \tag{2.11}$$

where $K = (⨍_A g(x) d\mu(x))^{p/(p+q)} \cdot (⨍_A h(x) d\mu(x))^{q/(p+q)}$.

Proof. By Lemma 2.1, for $p, q \geq 1$, we have the following:

$$\begin{gathered}\left(⨍_A f(x) \cdot g(x) d\mu(x)\right)^p \leq ⨍_A f^p(x) g^p(x) d\mu(x), \\ \left(⨍_A f(x) \cdot h(x) d\mu(x)\right)^q \leq ⨍_A f^q(x) h^q(x) d\mu(x).\end{gathered} \tag{2.12}$$

Multiplying these inequalities, we get that

$$\begin{aligned}&\left(⨍_A f(x) \cdot g(x) d\mu(x)\right)^p \cdot \left(⨍_A f(x) \cdot h(x) d\mu(x)\right)^q \\ &\quad\leq \left(⨍_A f^p(x) g^p(x) d\mu(x)\right) \cdot \left(⨍_A f^q(x) h^q(x) d\mu(x)\right).\end{aligned} \tag{2.13}$$

By Theorem 2.4

$$⨍_A f \cdot g d\mu \geq \left(⨍_A f d\mu\right) \cdot \left(⨍_A g d\mu\right), \qquad ⨍_A f \cdot h d\mu \geq \left(⨍_A f d\mu\right) \cdot \left(⨍_A h d\mu\right). \tag{2.14}$$

Substitutes (2.14) into (2.13), we obtain

$$\begin{aligned}&\left(⨍_A f(x) d\mu(x)\right)^{p+q} \cdot \left(⨍_A g(x) d\mu(x)\right)^p \cdot \left(⨍_A h(x) d\mu(x)\right)^q \\ &\quad\leq \left(⨍_A f^p(x) g^p(x) d\mu(x)\right) \cdot \left(⨍_A f^q(x) \cdot h^q(x) d\mu(x)\right).\end{aligned} \tag{2.15}$$

This inequality implies that (2.11) holds ☐

By Theorem 2.7, we have the following.

Corollary 2.8. *Assume that $p, q \geq 1$. Let $f, g, h : [0,1] \to [0, \infty)$ are increasing (or decreasing) functions and μ the Lebesgue measure on $\mathbb{R}$. Then be*

$$⨍_0^1 f(x) d\mu(x) \leq \frac{1}{K}\left(⨍_0^1 f^p(x) g^p(x) d\mu(x)\right)^{1/(p+q)} \cdot \left(⨍_0^1 f^q(x) h^q(x) d\mu(x)\right)^{1/(p+q)}, \tag{2.16}$$

where $K = (⨍_0^1 g(x) d\mu(x))^{p/(p+q)} \cdot (⨍_0^1 h(x) d\mu(x))^{q/(p+q)}$.

Theorem 2.9. *Let $g : [0,1] \to [0,\infty)$ be a μ-measurable function with μ the Lebesgue measure. If g^s $(s \geq 1)$ is a convex function such that, $g(0) \neq g(1)$, then*

$$\int_0^1 g(x)d\mu(x) \leq \min\left\{ \frac{\max\{g(0), g(1)\}}{(1 + |g^s(1) - g^s(0)|)^{1/s}}, 1 \right\}. \tag{2.17}$$

Proof. Firstly, we consider the case of $g^s(0) < g^s(1)$. As g^s is a convex function, we have by Theorem 1 of Caballero and Sadarangani [7] that

$$\int_0^1 g^s(x)d\mu(x) \leq \min\left\{ \frac{g^s(1)}{1 + g^s(1) - g^s(0)}, 1 \right\}. \tag{2.18}$$

By Corollary 2.2 and (2.18), we get

$$\left(\int_0^1 g(x)d\mu(x) \right)^s \leq \min\left\{ \frac{g^s(1)}{1 + g^s(1) - g^s(0)}, 1 \right\}, \tag{2.19}$$

which implies that (2.17) holds. Similarly, we can obtain (2.17) by of [7, Theorem 2] for the case of $g^s(0) > g^s(1)$. □

From Theorem 2.9 and Corollary 2.8, we have the following.

Theorem 2.10. *Assume that $p, q \geq 1$. Let $f, g, h : [0,1] \to [0,\infty)$ be increasing (or decreasing) functions and μ the Lebesgue measure on $\mathbb{R}$. If g^s ($s \geq 1$) or h^r ($r \geq 1$) is a convex function such that $g(0) \neq g(1)$ or $h(0) \neq h(1)$, then*

$$\int_0^1 f(x)d\mu(x) \leq \frac{1}{M_1^{p/p+q} K_2^{q/p+q}} \left(\int_0^1 f^p(x)g^p(x)d\mu(x) \right)^{1/(p+q)} \cdot \left(\int_0^1 f^q(x)h^q(x)d\mu(x) \right)^{1/(p+q)}, \tag{2.20}$$

where

$$M_1 = \min\left\{ \frac{\max\{g(0), g(1)\}}{(1 + |g^s(1) - g^s(0)|)^{1/s}}, 1 \right\}, \qquad K_2 = \int_0^1 h(x)d\mu(x), \tag{2.21}$$

or

$$\int_0^1 f(x)d\mu(x) \leq \frac{1}{K_1^{p/p+q} M_2^{q/p+q}} \left(\int_0^1 f^p(x)g^p(x)d\mu(x) \right)^{1/(p+q)} \cdot \left(\int_0^1 f^q(x)h^q(x)d\mu(x) \right)^{1/(p+q)}, \tag{2.22}$$

where

$$K_1 = \int_0^1 g(x)d\mu(x), \qquad M_2 = \min\left\{\frac{\max\{h(0),h(1)\}}{(1+|h^r(1)-h^r(0)|)^{1/r}},1\right\}. \tag{2.23}$$

Theorem 2.11. *Assume that $p,q \geq 1$. Let $f,g,h : [0,1] \to [0,\infty)$ be increasing (or decreasing) functions and μ the Lebesgue measure on $\mathbb{R}$. If $g^s (s \geq 1)$ and $h^r (r \geq 1)$ are two convex functions such that $g(0) \neq g(1)$ and $h(0) \neq h(1)$, then,*

$$\int_0^1 f(x)d\mu(x) \leq \frac{1}{M_1^{p/p+q} M_2^{q/p+q}} \left(\int_0^1 f^p(x)g^p(x)d\mu(x)\right)^{1/(p+q)} \cdot \left(\int_0^1 f^q(x)h^q(x)d\mu(x)\right)^{1/(p+q)}, \tag{2.24}$$

where M_1 and M_2 are as in (2.21) *and* (2.23)*, respectively.*

Straightforward calculus shows that

$$\int_0^1 x^2 d\mu(x) = \frac{3-\sqrt{5}}{2}, \qquad \int_0^1 x d\mu(x) = \frac{1}{2}, \qquad \int_0^1 1 d\mu(x) = 1. \tag{2.25}$$

If $p = q = 2$, $g(x) = x$ and $h(x) = 1$, $g(x) = x^2$ and $h(x) = x$, $g(x) = x^2$, and $h(x) = 1$, respectively, then Corollary 2.8 reduces to Theorem 1.4, and the following Corollaries 2.12 and 2.13.

Corollary 2.12. *Let $f : [0,1] \to [0,\infty)$ be a nondecreasing function and μ the Lebesgue measure on $\mathbb{R}$. Then,*

$$\int_0^1 f(x)d\mu(x) \leq \sqrt{3+\sqrt{5}} \left(\int_0^1 x^4 f^2(x)d\mu(x)\right)^{1/4} \cdot \left(\int_0^1 x^2 f^2(x)d\mu(x)\right)^{1/4}. \tag{2.26}$$

Corollary 2.13. *Let $f : [0,1] \to [0,\infty)$ be a nondecreasing function and μ the Lebesgue measure on $\mathbb{R}$. Then,*

$$\int_0^1 f(x)d\mu(x) \leq \frac{\sqrt{6+2\sqrt{5}}}{2} \left(\int_0^1 x^4 f^2(x)d\mu(x)\right)^{1/4} \cdot \left(\int_0^1 f^2(x)d\mu(x)\right)^{1/4}. \tag{2.27}$$

Remark 2.14. Corollary 2.8 is a generalization of the main result in [8, Theorem 1].

If $p = q = 1$, $g(x) = h(x) = x^2$, then Corollary 2.8 reduces to the following corollary.

Corollary 2.15. *Let $f : [0,1] \to [0,\infty)$ be a nondecreasing function and μ the Lebesgue measure on $\mathbb{R}$. Then*

$$\int_0^1 f(x)d\mu(x) \leq \frac{3+\sqrt{5}}{2} \int_0^1 x^2 f(x)d\mu(x). \tag{2.28}$$

Consider $g(x) = e^{-\sqrt{x+1}}$ on $[0,1]$. This function is nonincreasing ($g'(x) = -(1/2\sqrt{x+1})e^{-\sqrt{x+1}} < 0$), nonnegative and convex ($g''(x) = (1/4(x+1))e^{\sqrt{x+1}}(1/\sqrt{x+1}+1) \geq 0$).

Let $p = q = 1$, $g(x) = h(x) = e^{-\sqrt{x+1}}$, and $s = r = 1$. As $g(0) = 1/e > 1/e^{\sqrt{2}} = g(1)$ and $h(0) > h(1)$, we have the following

$$M_1 = M_2 = \frac{e^{\sqrt{2}-1}}{e^{\sqrt{2}} + e^{\sqrt{2}-1} - 1}. \tag{2.29}$$

Thus, by Theorem 2.11 we can get the following corollary.

Corollary 2.16. *Let $f : [0,1] \to [0,\infty)$ be a nonincreasing function and μ the Lebesgue measure on $\mathbb{R}$. Then,*

$$⨍_0^1 f(x)d\mu(x) \leq \frac{e^{\sqrt{2}} + e^{\sqrt{2}-1} - 1}{e^{\sqrt{2}-1}} ⨍_0^1 e^{-\sqrt{x+1}} f(x)d\mu(x). \tag{2.30}$$

Consider $g(x) = x - \ln(x+1)$ and $h(x) = x - \text{arc tan }\ x$ on $[0,1]$. Obviously, g and h are nonnegative, nondecreasing and convex on the interval $[0,1]$. Let $s = r = 1$, then, we have the following:

$$\begin{gathered} M_1 = \min\left\{ \frac{\max\{g(0), g(1)\}}{\left(1 + \left|g^s(1) - g^s(0)\right|\right)^{1/s}}, 1 \right\} = \frac{1 - \ln 2}{2 - \ln 2}, \\ M_2 = \min\left\{ \frac{\max\{h(0), h(1)\}}{\left(1 + |h^r(1) - h^r(0)|\right)^{1/r}}, 1 \right\} = \frac{4 - \pi}{8 - \pi}. \end{gathered} \tag{2.31}$$

Thus, by Theorem 2.11 (set $p = q = 1$) we can get the following corollary.

Corollary 2.17. *Let $f : [0,1] \to [0,\infty)$ be a nondecreasing function and μ the Lebesgue measure on $\mathbb{R}$. Then,*

$$\begin{aligned} ⨍_0^1 f(x)d\mu(x) \leq{}& \sqrt{\frac{(2 - \ln 2)(8 - \pi)}{(1 - \ln 2)(4 - \pi)}} \left(⨍_0^1 (x - \ln(x+1)) f(x)d\mu(x) \right)^{1/2} \\ & \times \left(⨍_0^1 (x - \arctan(x+1)) f(x)d\mu(x) \right)^{1/2}. \end{aligned} \tag{2.32}$$

Consider $g(x) = \sqrt{x^2 + x + 1/8}$ on $[0,1]$. Obviously, this function is nonnegative, nondecreasing ($g'(x) = ((2x+1)/2)(x^2 + x + 1/8)^{-1/2} \geq 0$), and nonconvex ($g''(x) = -(1/8)(x^2 + x + 1/8)^{-3/2} \leq 0$). But $g^2(x) = x^2 + x + 1/8$ is convex. Set $s = 2$, then we obtain

$$M_1 = \frac{\sqrt{17/8}}{\left(1 + \sqrt{17/8} - \sqrt{1/8}\right)^2} = \frac{2\sqrt{34}}{\left(\sqrt{8} + \sqrt{17} - 1\right)^2}. \tag{2.33}$$

Thus, by Theorem 2.10 (set $g = \sqrt{x^2 + x + 1/8}$, $h(x) = x$, $s = 2$, $p = 1$, $q = 2$) we can get the following corollary.

Corollary 2.18. *Let $f : [0,1] \to [0,\infty)$ be a nondecreasing function and μ the Lebesgue measure on $\mathbb{R}$. Then*

$$\begin{aligned} ⨍_0^1 f(x)d\mu(x) \le \left(\frac{\sqrt{34}(\sqrt{8}+\sqrt{17}-1)^2}{17}\right)^{1/3} \left(⨍_0^1 \sqrt{x^2+x+(1/8)}f(x)d\mu(x)\right)^{1/3} \\ \times \left(⨍_0^1 x^2 f^2(x)d\mu(x)\right)^{2/3}. \end{aligned} \tag{2.34}$$

Acknowledgments

The authors would like to thank the referees for reading this work carefully, providing valuable suggestions and comments. This work is supported by the National Natural Science Foundation of China (no. 10771212).

References

[1] M. Sugeno, *Theory of fuzzy integrals and its applications*, Ph.D. Dissertation, Tokyo Institute of Technology, 1974.
[2] A. Flores-Franulič and H. Román-Flores, "A Chebyshev type inequality for fuzzy integrals," *Applied Mathematics and Computation*, vol. 190, no. 2, pp. 1178–1184, 2007.
[3] H. Román-Flores, A. Flores-Franulič, and Y. Chalco-Cano, "A Jensen type inequality for fuzzy integrals," *Information Sciences*, vol. 177, no. 15, pp. 3192–3201, 2007.
[4] R. Mesiar and Y. Ouyang, "General Chebyshev type inequalities for Sugeno integrals," *Fuzzy Sets and Systems*, vol. 160, no. 1, pp. 58–64, 2009.
[5] H. Román-Flores, A. Flores-Franulič, and Y. Chalco-Cano, "A Hardy-type inequality for fuzzy integrals," *Applied Mathematics and Computation*, vol. 204, no. 1, pp. 178–183, 2008.
[6] H. Agahi, R. Mesiar, and Y. Ouyang, "General Minkowski type inequalities for Sugeno integrals," *Fuzzy Sets and Systems*, vol. 161, no. 5, pp. 708–715, 2010.
[7] J. Caballero and K. Sadarangani, "Hermite-Hadamard inequality for fuzzy integrals," *Applied Mathematics and Computation*, vol. 215, no. 6, pp. 2134–2138, 2009.
[8] J. Caballero and K. Sadarangani, "Fritz Carlson's inequality for fuzzy integrals," *Computers and Mathematics with Applications*, vol. 59, no. 8, pp. 2763–2767, 2010.
[9] H. Román-Flores, A. Flores-Franulič, and Y. Chalco-Cano, "The fuzzy integral for monotone functions," *Applied Mathematics and Computation*, vol. 185, no. 1, pp. 492–498, 2007.
[10] H. Román-Flores, A. Flores-Franulič, and Y. Chalco-Cano, "A convolution type inequality for fuzzy integrals," *Applied Mathematics and Computation*, vol. 195, no. 1, pp. 94–99, 2008.
[11] J. Caballero and K. Sadarangani, "A Cauchy-Schwarz type inequality for fuzzy integrals," *Nonlinear Analysis. Theory, Methods and Applications. Series A*, vol. 73, no. 10, pp. 3329–3335, 2010.
[12] Z. Wang and G. Klir, *Fuzzy Measure Theory*, Plenum Press, New York, NY, USA, 1992.
[13] F. Carlson, "Une ineqalite," *Arkiv för Matematik*, vol. 25, pp. 1–5, 1934.
[14] G. H. Hardy, "A note on two inequalities," *Journal of the London Mathematical Society*, vol. 11, pp. 167–170, 1936.
[15] Y. Ouyang, J. Fang, and L. Wang, "Fuzzy Chebyshev type inequality," *International Journal of Approximate Reasoning*, vol. 48, no. 3, pp. 829–835, 2008.