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Research Article

On Some New Impulsive Integral Inequalities

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We establish some new impulsive integral inequalities related to certain integral inequalities arising in the theory of differential equalities. The inequalities obtained here can be used as handy tools in the theory of some classes of impulsive differential and integral equations.

1. Introduction

Differential and integral inequalities play a fundamental role in global existence,uniqueness, stability, and other properties of the solutions of various nonlinear differential equations; see [1–4]. A great deal of attention has been given to differential and integral inequalities; see [1, 2, 5–8] and the references given therein. Motivated by the results in [1, 5, 7], the main purpose of this paper is to establish some new impulsive integral inequalities similar to Bihari's inequalities.

Let $0 \leq t_0 < t_1 < t_2 < \cdots$, $\lim_{k\to\infty} t_k = \infty$, $\mathbb{R}_+ = [0, +\infty)$, and $I \subset \mathbb{R}$, then we introduce the following spaces of function:

$PC(\mathbb{R}_+, I) = \{u : \mathbb{R}_+ \to I,\ u$ is continuous for $t \neq t_k$, $u(0^+),\ u(t_k^+)$, and $u(t_k^-)$ exist, and $u(t_k^-) = u(t_k),\ k = 1, 2, \ldots\}$,

$PC^1(\mathbb{R}_+, I) = \{u \in PC(\mathbb{R}_+, I) : u$ is continuously differentiable for $t \neq t_k$, $u'(0^+),\ u'(t_k^+)$, and $u'(t_k^-)$ exist, and $u'(t_k^-) = u'(t_k),\ k = 1, 2, \ldots\}$.

To prove our main results, we need the following result (see [1, Theorem 1.4.1]).

Lemma 1.1. *Assume that*

(A_0) *the sequence* $\{t_k\}$ *satisfies* $0 \leq t_0 < t_1 < t_2 < \cdots$, *with* $\lim_{k\to\infty} t_k = \infty$;

(A_1) $m \in PC^1(\mathbb{R}_+, \mathbb{R})$ *and* $m(t)$ *is left-continuous at* t_k, $k = 1, 2, \ldots$;

(A_2) *for* $k = 1, 2, \ldots,\ t \geq t_0$,

$$m'(t) \leq p(t)m(t) + q(t), \quad t \neq t_k,$$
$$m(t_k^+) \leq d_k m(t_k) + b_k, \tag{1.1}$$

where $q, p \in PC(\mathbb{R}_+, \mathbb{R})$, $d_k \geq 0$, *and* b_k *are constants.*

Then,

$$m(t) \le m(t_0) \prod_{t_0<t_k<t} d_k \exp\left(\int_{t_0}^{t} p(s)ds\right) + \int_{t_0}^{t} \prod_{s<t_k<t} d_k \exp\left(\int_{s}^{t} p(\sigma)d\sigma\right) q(s)ds$$
$$+ \sum_{t_0<t_k<t} \left(\prod_{t_k<t_j<t} d_j\right) \exp\left(\int_{t_k}^{t} p(s)ds\right) b_k, \quad t \ge t_0. \tag{1.2}$$

2. Main results

In this section, we will state and prove our results.

Theorem 2.1. *Let $u, f \in PC(\mathbb{R}_+, \mathbb{R}_+)$, $b_k \ge 1$, and $c \ge 0$ be constants. If*

$$u^2(t) \le c^2 + 2\int_0^t f(s)u(s)ds + \sum_{0<t_k<t} (b_k^2 - 1)u^2(t_k), \tag{2.1}$$

for $t \in \mathbb{R}_+$, then

$$u(t) \le c\left(\prod_{0<t_k<t} b_k\right) + \int_0^t \left(\prod_{0<t_k<t} b_k\right) f(s)ds, \tag{2.2}$$

for $t \in \mathbb{R}_+$.

Proof. Define a function $z(t)$ by

$$z(t) = (c+\varepsilon)^2 + 2\int_0^t f(s)u(s)ds + \sum_{0<t_k<t} (b_k^2 - 1)u^2(t_k), \tag{2.3}$$

where $\varepsilon > 0$ is an arbitrary small constant. For $t \ne t_k$, differentiating (2.3) and then using the fact that $u(t) \le \sqrt{z(t)}$, we have

$$z'(t) = 2f(t)u(t) \le 2f(t)\sqrt{z(t)}, \tag{2.4}$$

and so

$$\frac{d(\sqrt{z(t)})}{dt} = \frac{z'(t)}{2\sqrt{z(t)}} \le f(t). \tag{2.5}$$

For $t = t_k$, we have $z(t_k^+) - z(t_k) = (b_k^2 - 1)u^2(t_k) \le (b_k^2 - 1)z(t_k)$; thus $z(t_k^+) \le b_k^2 z(t_k)$. Let $\sqrt{z(t)} = x(t)$; it follows that

$$x'(t) \le f(t), \quad t \ne t_k,\ t \ge 0,$$
$$x(t_k^+) \le b_k x(t_k), \quad k = 1, 2 \ldots. \tag{2.6}$$

From Lemma 1.1, we obtain

$$x(t) \le x(0)\left(\prod_{0<t_k<t} b_k\right) + \int_0^t \left(\prod_{s<t_k<t} b_k\right) f(s)ds \le (c+\varepsilon)\left(\prod_{0<t_k<t} b_k\right) + \int_0^t \left(\prod_{s<t_k<t} b_k\right) f(s)ds. \tag{2.7}$$

Now by using the fact that $u(t) \le \sqrt{z(t)} = x(t)$ in (2.7) and then letting $\varepsilon \to 0$, we get the desired inequality in (2.2). This proof is complete. □

Theorem 2.2. *Let $u, f \in PC(\mathbb{R}_+, \mathbb{R}_+)$ and $b_k \geq 1$ be constants, and let c be a nonnegative constant. If*

$$u^2(t) \leq c^2 + 2\int_0^t [f(s)u^2(s) + h(s)u(s)]ds + \sum_{0<t_k<t} (b_k^2 - 1)u^2(t_k), \tag{2.8}$$

for $t \in \mathbb{R}_+$, then

$$u(t) \leq c\left(\prod_{0<t_k<t} b_k\right) \exp\left(\int_0^t f(s)ds\right) + \int_0^t \left(\prod_{s<t_k<t} b_k\right) \exp\left(\int_s^t f(\tau)d\tau\right) h(s)ds, \tag{2.9}$$

for $t \in \mathbb{R}_+$.

Proof. This proof is similar to that of Theorem 2.1; thus we omit the details here. □

Theorem 2.3. *Let $u, f, g, h \in PC(\mathbb{R}_+, \mathbb{R}_+)$, $c \geq 0$, and $b_k \geq 1$ be constants. If*

$$u^2(t) \leq c^2 + 2\int_0^t \left[f(s)u(s)\left(u(s) + \int_0^s g(\tau)u(\tau)d\tau\right) + h(s)u(s)\right]ds + \sum_{0<t_k<t} (b_k^2 - 1)u^2(t_k), \tag{2.10}$$

for $t \in \mathbb{R}_+$, then

$$u(t) \leq c\left(\prod_{0<t_k<t} b_k\right) + \int_0^t \left(\prod_{s<t_k<t} b_k\right)[f(s)a(s) + h(s)]ds, \tag{2.11}$$

for $t \in \mathbb{R}_+$, where

$$a(t) = c\left(\prod_{0<t_k<t} b_k\right) \exp\left(\int_0^t [f(\tau) + g(\tau)]d\tau\right) + \int_0^t \left(\prod_{s<t_k<t} b_k\right) \exp\left(\int_s^t [f(\tau) + g(\tau)]d\tau\right) h(s)ds. \tag{2.12}$$

Proof. Let $\varepsilon > 0$ be an arbitrary small constant, and define a function $z(t)$ by

$$z(t) = (c + \varepsilon)^2 + 2\int_0^t \left[f(s)u(s)\left(u(s) + \int_0^s g(\tau)u(\tau)d\tau\right) + h(s)u(s)\right]ds + \sum_{0<t_k<t} (b_k^2 - 1)u^2(t_k). \tag{2.13}$$

Let $\sqrt{z(t)} = x(t)$; similar to the proof of Theorem 2.1, we have

$$\begin{gathered} x'(t) \leq f(t)\left(x(t) + \int_0^t g(s)x(s)ds\right) + h(t), \quad t \neq t_k, \\ x(t_k^+) \leq b_k x(t_k), \quad k = 1, 2, \dots. \end{gathered} \tag{2.14}$$

Set $v(t) = x(t) + \int_0^t g(s)x(s)ds$; then $v(t) \geq x(t)$, and so from (2.14) we get that $x'(t) \leq f(t)v(t) + h(t)$. Thus, for $t \neq t_k$,

$$v'(t) = x'(t) + g(t)x(t) \leq f(t)v(t) + h(t) + g(t)x(t) \leq [f(t) + g(t)]v(t) + h(t), \tag{2.15}$$

and for $t = t_k$,

$$v(t_k^+) - v(t_k) = x(t_k^+) - x(t_k) \le (b_k - 1)x(t_k) \le (b_k - 1)v(t_k), \tag{2.16}$$

and so $v(t_k^+) \le b_k v(t_k)$. By Lemma 1.1, we have

$$v(t) \le (c+\varepsilon)\left(\prod_{0<t_k<t} b_k\right) \exp\left(\int_0^t [f(\tau)+g(\tau)]d\tau\right) + \int_0^t \left(\prod_{s<t_k<t} b_k\right) \exp\left(\int_s^t [f(\tau)+g(\tau)]d\tau\right) h(s)ds. \tag{2.17}$$

Let $\varepsilon \to 0$, then we obtain

$$v(t) \le a(t), \tag{2.18}$$

where $a(t)$ is defined in (2.12). Substituting (2.18) into (2.14), we have

$$\begin{aligned} x'(t) &\le f(t)a(t) + h(t), \quad t \ne t_k, \\ x(t_k^+) &\le b_k x(t_k), \quad k = 1, 2, \dots. \end{aligned} \tag{2.19}$$

Applying Lemma 1.1 again, we obtain

$$x(t) \le (c+\varepsilon)\left(\prod_{0<t_k<t} b_k\right) + \int_0^t \left(\prod_{s<t_k<t} b_k\right)[f(s)a(s) + h(s)]ds. \tag{2.20}$$

Now using $u(t) \le x(t)$ and letting $\varepsilon \to 0$, we get the desired inequality in (2.11). □

Theorem 2.4. *Let $u, f, g, h \in PC(\mathbb{R}_+, \mathbb{R}_+)$, $c \ge 0$, and $b_k \ge 1$ be constants. If*

$$u^2(t) \le c^2 + 2\int_0^t \left[f(s)u(s)\left(\int_0^s g(\tau)u(\tau)d\tau\right) + h(s)u(s)\right]ds + \sum_{0<t_k<t} (b_k^2 - 1)u^2(t_k), \tag{2.21}$$

for $t \in \mathbb{R}_+$, then

$$\begin{aligned} u(t) \le{}& c\left(\prod_{0<t_k<t} b_k\right) \exp\left(\int_0^t f(s)\left(\int_0^s g(\tau)d\tau\right)ds\right) \\ &+ \int_0^t \left(\prod_{s<t_k<t} b_k\right) \exp\left(\int_s^t f(\tau)\left(\int_0^\tau g(\omega)d\omega\right)d\tau\right) h(s)ds, \end{aligned} \tag{2.22}$$

for $t \in \mathbb{R}_+$.

Proof. Set

$$z(t) = (c+\varepsilon)^2 + 2\int_0^t \left[f(s)u(s)\left(\int_0^s g(\tau)u(\tau)d\tau\right) + h(s)u(s)\right]ds + \sum_{0<t_k<t} (b_k^2 - 1)u^2(t_k), \tag{2.23}$$

where ε is an arbitrary small constant; then $z(t)$ is nondecreasing. Let $x(t) = \sqrt{z(t)}$, then it follows for $t \ne t_k$ that

$$x'(t) \le f(t)\int_0^t g(s)x(s)ds + h(t) \le \left(f(t)\int_0^t g(s)ds\right)x(t) + h(t) \tag{2.24}$$

since $x(t)$ is nondecreasing. Also, for $t = t_k$, we have $x(t_k^+) \leq b_k x(t_k)$. Applying Lemma 1.1, we obtain

$$\begin{aligned} x(t) \leq (c+\varepsilon)c\Big(\prod_{0<t_k<t} b_k\Big)\exp\Big(\int_0^t f(s)\Big(\int_0^s g(\tau)d\tau\Big)ds\Big) \\ + \int_0^t \Big(\prod_{s<t_k<t} b_k\Big)\exp\Big(\int_s^t f(\tau)\Big(\int_0^\tau g(\omega)d\omega\Big)d\tau\Big)h(s)ds. \end{aligned} \tag{2.25}$$

Now by using the fact that $u(t) \leq x(t)$ in (2.25) and letting $\varepsilon \to 0$, we get the inequality (2.22). □

Remark 2.5. If $b_k \equiv 1$, then (2.1), (2.8), (2.10), and (2.21) have no impulses. In this case, it is clear that Theorems 2.2-2.3 improve the corresponding results of [5, Theorem 1].

Theorem 2.6. *Let $u, f \in PC(\mathbb{R}_+, \mathbb{R}_+)$, $h(t,s) \in C(\mathbb{R}_+^2, \mathbb{R}_+)$, for $0 \leq s \leq t < \infty$, $c \geq 0$, $b_k \geq 1$, and $p > 1$ be constants. Let $g \in PC(\mathbb{R}_+, \mathbb{R}_+)$ be a nondecreasing function with $g(u) > 0$, for $u > 0$, and $g(\lambda u) \geq \mu(\lambda)g(u)$, for $\lambda > 0$, $u \in \mathbb{R}$; here $\mu(\lambda) > 0$, for $\lambda > 0$. If*

$$u^p(t) \leq c + \int_0^t \Big[f(s)g(u(s)) + \int_0^s h(s,\sigma)g(u(\sigma))d\sigma\Big]ds + \sum_{0<t_k<t}(b_k - 1)u^p(t_k), \tag{2.26}$$

for $t \in \mathbb{R}_+$, then for $0 \leq t < T$,

$$u(t) \leq \Big[G^{-1}\Big(G\Big(c\prod_{0<t_k<t} b_k\Big) + \int_0^t \prod_{s<t_k<t}\frac{b_k}{\mu(b_k^{1/p})}p(s)ds\Big)\Big]^{1/p}, \tag{2.27}$$

where

$$p(t) = f(t) + \int_0^t h(t,\sigma)d\sigma, \tag{2.28}$$

$$G(r) = \int_{r_0}^r \frac{ds}{g(s^{1/p})} \quad \text{for } r \geq r_0 > 0, \tag{2.29}$$

$$T = \sup\Big\{t \geq 0 : \Big[G\Big(c\prod_{0<t_k<t} b_k\Big) + \int_0^t \prod_{s<t_k<t}\frac{b_k}{\mu(b_k^{1/p})}p(s)ds\Big] \in \operatorname{dom} G^{-1}\Big\}. \tag{2.30}$$

Proof. We first assume that $c > 0$ and define a function $z(t)$ by the right-hand side of (2.26). Then, $z(t) > 0$, $z(0) = c$, $u(t) \leq (z(t))^{1/p}$, and $z(t)$ is nondecreasing. For $t \neq t_k$,

$$\begin{aligned} z'(t) &= f(t)g(u(t)) + \int_0^t h(t,\sigma)g(u(\sigma))d\sigma \\ &\leq f(t)g\big((z(t))^{1/p}\big) + \int_0^t h(t,\sigma)g\big((z(\sigma))^{1/p}\big)d\sigma \\ &\leq g\big((z(t))^{1/p}\big)\Big[f(t) + \int_0^t h(t,\sigma)d\sigma\Big], \end{aligned} \tag{2.31}$$

and for $t = t_k$, $z(t_k^+) \le b_k z(t_k)$. As $t \in [0, t_1]$, from (2.31) we have

$$G(z(t)) - G(z(0)) = \int_{z(0)}^{z(t)} \frac{ds}{g(s^{1/p})} \le \int_0^t p(s)ds, \tag{2.32}$$

and so

$$z(t) \le G^{-1}\left(G(c) + \int_0^t p(s)ds\right). \tag{2.33}$$

Now assume that for $0 \le t \le t_n$, we have

$$z(t) \le G^{-1}\left(G\left(c \prod_{0<t_k<t} b_k\right) + \int_0^t \prod_{0<t_k<t} \frac{b_k}{\mu(b_k^{1/p})} p(s)ds\right). \tag{2.34}$$

Then, for $t \in (t_n, t_{n+1}]$, it follows from (2.32) that $G(z(t)) \le G(z(t_n^+)) + \int_{t_n}^t p(s)ds$. Using $z(t_k^+) \le b_k z(t_k)$, we arrive at

$$G(z(t)) \le G(b_n z(t_n)) + \int_{t_n}^t p(s)ds. \tag{2.35}$$

From the supposition of g, we see that

$$G(\lambda u) - G(\lambda v) = \int_0^{\lambda u} \frac{ds}{g(s^{1/p})} - \int_0^{\lambda v} \frac{ds}{g(s^{1/p})} \le \frac{\lambda}{\mu(\lambda^{1/p})}[G(u) - G(v)], \quad \text{for } u \ge v,\ \lambda > 0. \tag{2.36}$$

If $G(z(t_n)) \le G(c\prod_{k=1}^{n-1} b_k)$, then

$$G(z(t)) \le G(b_n z(t_n)) + \int_{t_n}^t p(s)ds \le G\left(c\prod_{k=1}^{n} b_k\right) + \int_0^t \prod_{s<t_k<t} \frac{b_k}{\mu(b_k^{1/p})} p(s)ds. \tag{2.37}$$

Otherwise, we have

$$G(b_n z(t_n)) - G\left(c \prod_{0<t_k<t} b_k\right) \le \frac{b_n}{\mu(b_n^{1/p})}\left[G(z(t_n)) - G\left(c\prod_{k=1}^{n-1} b_k\right)\right]. \tag{2.38}$$

This implies, by induction hypothesis, that

$$G(b_n z(t_n)) - G\left(c \prod_{0<t_k<t} b_k\right) \le \frac{b_n}{\mu(b_n^{1/p})} \int_0^{t_n} \prod_{s<t_k<t_n} \frac{b_k}{\mu(b_k^{1/p})} p(s)ds = \int_0^{t_n} \prod_{s<t_k<t} \frac{b_k}{\mu(b_k^{1/p})} p(s)ds. \tag{2.39}$$

Thus, (2.35) and (2.39) yield, for $0 < t \le t_{n+1}$,

$$G(z(t)) \le G\left(c \prod_{0<t_k<t} b_k\right) + \int_0^t \prod_{s<t_k<t} \frac{b_k}{\mu(b_k^{1/p})} p(s)ds, \tag{2.40}$$

and so

$$z(t) \le G^{-1}\left[G\left(c \prod_{0<t_k<t} b_k\right) + \int_0^t \prod_{s<t_k<t} \frac{b_k}{\mu(b_k^{1/p})} p(s)ds\right]. \tag{2.41}$$

Using (2.41) in $u(t) \le (z(t))^{1/p}$, we have the required inequality in (2.27).

If c is nonnegative, we carry out the above procedure with $c + \varepsilon$ instead of c, where $\varepsilon > 0$ is an arbitrary small constant, and by letting $\varepsilon \to 0$, we obtain (2.27). The proof is complete. □

Remark 2.7. If $\int_{r_0}^{\infty}(ds/g(s^{1/p})) = \infty$, then $G(\infty) = \infty$ and the inequality in (2.27) is true for $t \in \mathbb{R}_+$.

An interesting and useful special version of Theorem 2.6 is given in what follows.

Corollary 2.8. *Let u, f, h, c, p, and b_k be as in Theorem 2.6. If*

$$u^p(t) \le c + \int_0^t \left[f(s)u(s) + \int_0^s h(s,\sigma)u(\sigma)d\sigma \right] ds + \sum_{0<t_k<t} (b_k - 1)u^p(t_k), \tag{2.42}$$

for $t \in \mathbb{R}_+$, then

$$u(t) \le \left[\left(c \prod_{0<t_k<t} b_k \right)^{(p-1)/p} + \frac{p-1}{p} \int_0^t \prod_{s<t_k<t} b_k^{(p-1)/p} p(s)ds \right]^{p/(p-1)}, \tag{2.43}$$

for $t \in \mathbb{R}_+$, where $p(t)$ is defined by (2.28).

Proof. Let $g(u) = u$ in Theorem 2.6. Then, (2.26) reduces to (2.42) and

$$\begin{gathered} G(r) = \frac{p}{p-1}[r^{(p-1)/p} - r_0^{(p-1)/p}], \\ G^{-1}(r) = \left[\frac{p-1}{p} r + r_0^{(p-1)/p} \right]^{p/(p-1)}. \end{gathered} \tag{2.44}$$

Consequently, by Theorem 2.6, we have

$$u(t) \le \left[\left(c \prod_{0<t_k<t} b_k \right)^{(p-1)/p} + \frac{p-1}{p} \int_0^t \prod_{s<t_k<t} b_k^{(p-1)/p} p(s)ds \right]^{p/(p-1)}. \tag{2.45}$$

This proof is complete. □

3. Application

Example 3.1. Consider the integrodifferential equations

$$\begin{gathered} x'(t) - F\left(t, x(t), \int_0^t K[t,s,x(s)]ds\right) = h(t), \\ x(t_k^+) = b_k x(t_k), \quad k = 1,2,\ldots, \\ x(0) = x_0, \end{gathered} \tag{3.1}$$

where $0 = t_0 < t_1 < t_2 < \cdots$ with $\lim_{k\to\infty} t_k = \infty$; $h : \mathbb{R}_+ \to \mathbb{R}$ and $K : \mathbb{R}_+^2 \times \mathbb{R} \to \mathbb{R}$ are continuous; $F : \mathbb{R}_+ \times \mathbb{R}^2 \to \mathbb{R}$ is continuous at $t \neq t_k$; $\lim_{t\to t_k^+} F(t,\cdot,\cdot)$ and $\lim_{t\to t_k^-} F(t,\cdot,\cdot)$ exist and $\lim_{t\to t_k^-} F(t,\cdot,\cdot) = F(t,\cdot,\cdot)$; b_k are constants with $|b_k| \ge 1$ $(k = 1,2,\ldots)$. Here, we assume that the solution $x(t)$ of (3.1) exists on $\mathbb{R}_+$. Multiplying both sides of (3.1) by $x(t)$ and then integrating them from 0 to t, we obtain

$$x^2(t) = x_0^2 + 2\int_0^t \left[x(s)F\left(s, x(s), \int_0^s K[s,\tau,x(\tau)]d\tau\right) + h(s)x(s) \right] ds + \sum_{0<t_k<t} (b_k^2 - 1)x^2(t_k). \tag{3.2}$$

We assume that

$$|K(t,s,x(s))| \le f(t)g(s)|x(s)|, \qquad |F(t,x(t),v)| \le f(t)|x(t)| + |v|, \tag{3.3}$$

where $f, g \in C(\mathbb{R}_+, \mathbb{R}_+)$. From (3.2) and (3.3), we obtain

$$|x(t)|^2 \le |x_0|^2 + 2\int_0^t \left[f(s)|x(s)|\left(|x(s)| + \int_0^s g(\tau)|x(\tau)|d\tau \right) + |h(s)||x(s)| \right] ds + \sum_{0<t_k<t} \left(|b_k|^2 - 1 \right) |x(t_k)|^2. \tag{3.4}$$

Now applying Theorem 2.3, we have

$$|x(t)| \le |x_0| \left(\prod_{0<t_k<t} |b_k| \right) + \int_0^t \left(\prod_{s<t_k<t} |b_k| \right) [f(s)a(s) + h(s)] ds, \tag{3.5}$$

where

$$a(t) = |x_0| \left(\prod_{0<t_k<t} |b_k| \right) \exp\left(\int_0^t [f(\tau) + g(\tau)] d\tau \right) + \int_0^t \left(\prod_{s<t_k<t} |b_k| \right) \exp\left(\int_s^t [f(\tau) + g(\tau)] d\tau \right) h(s) ds, \tag{3.6}$$

for all $t \in \mathbb{R}_+$. The inequality (3.5) gives the bound on the solution $x(t)$ of (3.1).

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