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Research Article

Coefficient Bounds for Certain Classes of Meromorphic Functions

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Sharp bounds for $|a_1 - \mu a_0^2|$ are derived for certain classes $\Sigma^*(\phi)$ and $\Sigma_\alpha^*(\phi)$ of meromorphic functions $f(z)$ defined on the punctured open unit disk for which $-zf'(z)/f(z)$ and $(-(1-2\alpha)zf'(z)+\alpha z^2 f''(z))/((1-\alpha)f(z)-\alpha zf'(z))$ ($\alpha \in \mathbb{C} - (0,1]$; $\Re(\alpha) \geq 0$), respectively, lie in a region starlike with respect to 1 and symmetric with respect to the real axis. Also, certain applications of the main results for a class of functions defined through Ruscheweyh derivatives are obtained.

1. Introduction

Let Σ denote the class of functions of the form

$$f(z) = \frac{1}{z} + \sum_{k=0}^{\infty} a_k z^k, \tag{1.1}$$

which are *analytic* and *univalent* in the punctured open unit disk

$$\Delta^* = \{z \in \mathbb{C} : 0 < |z| < 1\} = \Delta - \{0\}, \tag{1.2}$$

where Δ is the open unit disk $\Delta = \{z \in \mathbb{C} : |z| < 1\}$.

A function $f \in \Sigma$ is said to be *meromorphic univalent starlike of order* α if

$$-\Re\frac{zf'(z)}{f(z)} > \alpha \quad (z \in \Delta;\ 0 \leq \alpha < 1), \tag{1.3}$$

and the class of all such meromorphic univalent starlike functions in Δ^* is denoted by $\Sigma^*(\alpha)$.

Recently, Uralegaddi and Desai [1] studied the class $\Sigma(\alpha,\beta)$ of functions $f \in \Sigma$ satisfying the condition

$$\left|\frac{zf'(z)/f(z)+1}{zf'(z)/f(z)+2\alpha-1}\right| \leq \beta \quad (z \in \Delta;\ 0 \leq \alpha < 1;\ 0 < \beta \leq 1). \tag{1.4}$$

Kulkarni and Joshi [2] studied the class $\Sigma(\alpha,\beta,\gamma)$ of functions $f \in \Sigma$ satisfying the condition

$$\left|\frac{zf'(z)/f(z)+1}{2\gamma\left(zf'(z)/f(z)+\alpha\right)-\left(zf'(z)/f(z)+1\right)}\right| \leq \beta \quad \left(z \in \Delta;\ 0 \leq \alpha < 1;\ 0 < \beta \leq 1;\ \frac{1}{2} < \gamma \leq 1\right). \tag{1.5}$$

Earlier, several authors [3–6] have studied similar subclasses of $\Sigma^*(\alpha)$.

Let $\mathcal{S}$ consist of functions $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$ which are analytic and univalent in Δ. Many researchers including [7–11] have obtained Fekete-Szegö inequality for analytic functions $f \in \mathcal{S}$.

In this paper, we obtain Fekete-Szegö-like inequalities for new classes of meromorphic functions, which are defined in what follows. Also, we give applications of our results to certain functions defined through Ruscheweyh derivatives.

Definition 1.1. Let $\phi(z)$ be an analytic function with positive real part on Δ with $\phi(0) = 1$, $\phi'(0) > 0$, which maps the unit disk Δ onto a region starlike with respect to 1, and is symmetric with respect to the real axis. Let $\Sigma^*(\phi)$ be the class of functions $f \in \Sigma$ for which

$$-\frac{zf'(z)}{f(z)} \prec \phi(z) \quad (z \in \Delta), \tag{1.6}$$

where $\prec$ denotes subordination between analytic functions.

The above-defined class $\Sigma^*(\phi)$ is the meromorphic analogue of the class $S^*(\phi)$, introduced and studied by Ma and Minda [8], which consists of functions $f \in \mathcal{S}$ for which $zf'(z)/f(z) \prec \phi(z),\ (z \in \Delta)$.

More generally, under the same conditions as Definition 1.1, we add a parameter.

Definition 1.2. Let $\Sigma^*_\alpha(\phi)$ be the class of functions $f \in \Sigma$ for which

$$\frac{-(1-2\alpha)zf'(z)+\alpha z^2 f''(z)}{(1-\alpha)f(z)-\alpha zf'(z)} \prec \phi(z) \quad \left(z \in \Delta;\ \alpha \in \mathbb{C}-(0,1];\ \Re(\alpha) \geq 0\right). \tag{1.7}$$

Some of the interesting subclasses of $\Sigma_{\alpha}^{*}(\phi)$ are

(1) $\Sigma_0^{*}(\phi) = \Sigma^{*}(\phi)$,

(2) $\Sigma_0^{*}((1 + (1 - 2\alpha)z)/(1 - z)) = \Sigma^{*}(\alpha)$, $(0 \leq \alpha < 1)$,

(3) $\Sigma_0^{*}((1+\beta(1-2\alpha\gamma)z)/(1+\beta(1-2\gamma)z)) = \Sigma(\alpha,\beta,\gamma)$, $(0 \leq \alpha < 1,\ 0 < \beta \leq 1,\ 1/2 \leq \gamma \leq 1)$ studied by Kulkarni and Joshi [2],

(4) $\Sigma_0^{*}((1 + Aw(z))/(1 + Bw(z))) = K_1(A,B)$, $(0 \leq B < 1;\ -B < A < B)$ studied by Karunakaran [12].

To prove our result, we need the following lemma.

Lemma 1.3 (see [13]). *If $p(z) = 1 + c_1 z + c_2 z^2 + c_3 z^3 + \cdots$ is a function with positive real part in Δ, then for any complex number μ,*

$$\left|c_2 - \mu c_1^2\right| \leq 2\max\{1, |1 - 2\mu|\}. \tag{1.8}$$

2. Coefficient bounds

By making use of Lemma 1.3, we prove the following bounds for the classes $\Sigma^{*}(\phi)$ and $\Sigma_{\alpha}^{*}(\phi)$.

Theorem 2.1. *Let $\phi(z) = 1 + B_1 z + B_2 z^2 + \cdots$. If $f(z)$ given by (1.1) belongs to $\Sigma^{*}(\phi)$, then for any complex number μ,*

$$\text{(i)}\quad \left|a_1 - \mu a_0^2\right| \leq \frac{|B_1|}{2}\max\left\{1, \left|\frac{B_2}{B_1} - (1 - 2\mu)B_1\right|\right\}, \quad B_1 \neq 0, \tag{2.1}$$

$$\text{(ii)}\quad \left|a_1 - \mu a_0^2\right| \leq 1, \quad B_1 = 0. \tag{2.2}$$

The bounds are sharp.

Proof. If $f(z) \in \Sigma^{*}(\phi)$, then there is a Schwarz function $w(z)$, analytic in Δ with $w(0) = 0$ and $|w(z)| < 1$ in Δ such that

$$-\frac{zf'(z)}{f(z)} = \phi(w(z)). \tag{2.3}$$

Define the function $p(z)$ by

$$p(z) = \frac{1 + w(z)}{1 - w(z)} = 1 + c_1 z + c_2 z^2 + \cdots. \tag{2.4}$$

Since $w(z)$ is a Schwarz function, we see that $\Re(p(z)) > 0$ and $p(0) = 1$. Therefore,

$$\begin{aligned}
\phi(w(z)) &= \phi\left(\frac{p(z)-1}{p(z)+1}\right) \\
&= \phi\left(\frac{1}{2}\left[c_1 z + \left(c_2 - \frac{c_1^2}{2}\right)z^2 + \left(c_3 + \frac{c_1^3}{4} - c_1 c_2\right)z^3 + \cdots\right]\right) \\
&= 1 + \frac{1}{2}B_1 c_1 z + \left(\frac{1}{2}B_1\left(c_2 - \frac{1}{2}c_1^2\right) + \frac{1}{4}B_2 c_1^2\right)z^2 + \cdots .
\end{aligned} \tag{2.5}$$

Now by substituting (2.5) in (2.3), we have

$$-\frac{zf'(z)}{f(z)} = 1 + \frac{1}{2}B_1 c_1 z + \left(\frac{1}{2}B_1\left(c_2 - \frac{1}{2}c_1^2\right) + \frac{1}{4}B_2 c_1^2\right)z^2 + \cdots . \tag{2.6}$$

From this equation and (1.1), we obtain

$$\begin{gathered}
a_0 + \frac{B_1 c_1}{2} = 0, \\
-a_1 = a_1 + \frac{a_0 B_1 c_1}{2} + \frac{B_1 c_2}{2} - \frac{B_1 c_1^2}{4} + \frac{B_2 c_1^2}{4}.
\end{gathered} \tag{2.7}$$

Or equivalently,

$$\begin{gathered}
a_0 = -\frac{1}{2}B_1 c_1, \\
a_1 = -\frac{1}{2}\left[\frac{1}{2}B_1 c_2 + \frac{1}{4}\left(B_2 - B_1 - B_1^2\right)c_1^2\right].
\end{gathered} \tag{2.8}$$

Therefore,

$$a_1 - \mu a_0^2 = -\frac{B_1}{4}\left\{c_2 - v c_1^2\right\}, \tag{2.9}$$

where

$$v = \frac{1}{2}\left[1 - \frac{B_2}{B_1} + (1 - 2\mu)B_1\right]. \tag{2.10}$$

Now, the result (2.1) follows by an application of Lemma 1.3. Also, if $B_1 = 0$, then $a_0 = 0$ and $a_1 = (-1/8)B_2 c_1^2$.

Since $p(z)$ has positive real part, $|c_1| \leq 2$, so that $|a_1 - \mu a_0^2| \leq |B_2|/2$. Since $\phi(z)$ also has positive real part, $|B_2| \leq 2$. Thus, $|a_1 - \mu a_0^2| \leq 1$, proving (2.2).

The bounds are sharp for the functions $F_1(z)$ and $F_2(z)$ defined by

$$-\frac{zF_1'(z)}{F_1(z)} = \phi(z^2), \quad \text{where } F_1(z) = \frac{1+z^2}{z(1-z^2)},$$
$$-\frac{zF_2'(z)}{F_2(z)} = \phi(z), \quad \text{where } F_2(z) = \frac{1+z}{z(1-z)}. \tag{2.11}$$

Clearly, the functions $F_1(z), F_2(z) \in \Sigma$.

Proceeding similarly, we now obtain the bounds for the class $\Sigma_\alpha^*(\phi)$. □

Theorem 2.2. *Let $\phi(z) = 1 + B_1 z + B_2 z^2 + \cdots$. If $f(z)$ given by* (1.1) *belongs to $\Sigma_\alpha^*(\phi)$, then for any complex number μ,*

(i) $$\left|a_1 - \mu a_0^2\right| \le \left|\frac{B_1}{2(1-2\alpha)}\right| \max\left\{1, \left|\frac{B_2}{B_1} - \left(1 - \frac{2(1-2\alpha)}{(1-\alpha)^2}\mu\right)B_1\right|\right\}, \quad B_1 \ne 0, \tag{2.12}$$

(ii) $$\left|a_1 - \mu a_0^2\right| \le \left|\frac{1}{(1-2\alpha)}\right|, \quad B_1 = 0. \tag{2.13}$$

The bounds obtained are sharp.

Proof. If $f(z) \in \Sigma_\alpha^*(\phi)$, then there is a Schwarz function $w(z)$, analytic in Δ with $w(0) = 0$ and $|w(z)| < 1$ in Δ such that

$$\frac{-(1-2\alpha)zf'(z) + \alpha z^2 f''(z)}{(1-\alpha)f(z) - \alpha z f'(z)} = \phi(w(z)), \quad (\alpha \in \mathbb{C} - (0,1], \Re(\alpha) \ge 0). \tag{2.14}$$

Now using (2.5) and (1.1) in (2.14), and comparing the coefficients, we have

$$a_0(1-\alpha) + \frac{1}{2}B_1 c_1 = 0,$$
$$-a_1(1-2\alpha) = a_1(1-2\alpha) + \frac{1}{2}a_0(1-\alpha)B_1 c_1 + \frac{1}{2}B_1 c_2 - \frac{1}{4}(B_1 - B_2)c_1^2; \tag{2.15}$$

or equivalently,

$$a_0 = -\frac{1}{2(1-\alpha)}B_1 c_1,$$
$$a_1 = -\frac{1}{2(1-2\alpha)}\left(\frac{1}{2}B_1 c_2 + \frac{1}{4}(B_2 - B_1 - B_1^2)c_1^2\right). \tag{2.16}$$

Therefore,

$$a_1 - \mu a_0^2 = -\frac{B_1}{4(1-2\alpha)}\{c_2 - v c_1^2\}, \tag{2.17}$$

where

$$v = \frac{1}{2}\left[1 - \frac{B_2}{B_1} + \left(1 - \frac{2(1-2\alpha)}{(1-\alpha)^2}\mu\right)B_1\right]. \tag{2.18}$$

Now, the result (2.12) follows by an application of Lemma 1.3. Also, if $B_1 = 0$, then $a_0 = 0$ and $a_1 = (-1/8(1-2\alpha))B_2 c_1^2$.

Since $p(z)$ has positive real part, $|c_1| \leq 2$, so that $|a_1 - \mu a_0^2| \leq |B_2|/2(1-2\alpha)$. Since $\phi(z)$ also has positive real part, $|B_2| \leq 2$. Thus, $|a_1 - \mu a_0^2| \leq |1/(1-2\alpha)|$, proving (2.13).

The bounds are sharp for the functions $F_1(z)$ and $F_2(z)$ defined by

$$\begin{aligned} \frac{-(1-2\alpha)zF_1'(z) + \alpha z^2 F_1''(z)}{(1-\alpha)F_1(z) - \alpha z F_1'(z)} &= \phi(z^2), \quad \text{where } F_1(z) = \frac{1+z^2}{z(1-z^2)}, \\ \frac{-(1-2\alpha)zF_2'(z) + \alpha z^2 F_2''(z)}{(1-\alpha)F_2(z) - \alpha z F_2'(z)} &= \phi(z), \quad \text{where } F_2(z) = \frac{1+z}{z(1-z)}. \end{aligned} \tag{2.19}$$

Clearly $F_1(z), F_2(z) \in \Sigma$. □

Remark 2.3. By putting $\alpha = 0$ in (2.12) and (2.13), we get the results (2.1) and (2.2).

3. Applications to functions defined by Ruscheweyh derivatives

In this section, we introduce two classes $\Sigma_\lambda^*(\phi)$ and $\Sigma_{\alpha,\lambda}^*(\phi)$ of meromorphic functions defined by Ruscheweyh derivatives, and obtain coefficient bounds for functions in these classes.

Let $f \in \Sigma$ be given by (2.1) and $g \in \Sigma$ be given by

$$g(z) = \frac{1}{z} + \sum_{k=0}^{\infty} b_k z^k, \tag{3.1}$$

then the Hadamard product of f and g is defined as

$$(f * g)(z) = \frac{1}{z} + \sum_{k=0}^{\infty} a_k b_k z^k = (g * f)(z). \tag{3.2}$$

In terms of the Hadamard product of two functions, the analogue of the familiar Ruscheweyh derivative [14] is defined as

$$D^\lambda f(z) := \frac{1}{z(1-z)^{\lambda+1}} * f(z) \quad (\lambda > -1;\ f \in \Sigma), \tag{3.3}$$

so that

$$D^{\lambda} f(z) = \frac{1}{z}\left(\frac{z^{\lambda+1} f(z)}{\lambda!}\right)^{(\lambda)} \quad (\lambda > -1;\ f \in \Sigma), \tag{3.4}$$

where, here and in what follows λ is an integer (> -1), that is, $\lambda \in \mathbb{N}_0 = \{0, 1, 2, \ldots\}$.

It follows from (3.3) and (3.4) that

$$D^{\lambda} f(z) = \frac{1}{z} + \sum_{k=0}^{\infty} \delta(\lambda, k) a_k z^k \quad (f \in \Sigma), \tag{3.5}$$

where $f \in \Sigma$ is given by (1.1) and

$$\delta(\lambda, k) := \binom{\lambda + k + 1}{k + 1}. \tag{3.6}$$

The above-defined operator D^{λ} for $\lambda \in \mathbb{N}_0 = \{0, 1, 2, \ldots\}$ was also studied by Cho [15] and Padmanabhan [16]. For various developments involving the operator D^{λ} for functions belonging to Σ, the reader may be referred to the recent works of Uralegaddi et al. [17–19] and others [20–22].

Using (3.5), under the same conditions as Definition 1.1, we define the classes $\Sigma_{\lambda}^{*}(\phi)$ and $\Sigma_{\alpha,\lambda}^{*}(\phi)$ as follows.

Definition 3.1. A function $f \in \Sigma$ is in the class $\Sigma_{\lambda}^{*}(\phi)$ if

$$-\frac{z\left[D^{\lambda} f(z)\right]'}{D^{\lambda} f(z)} \prec \phi(z) \quad (z \in \Delta). \tag{3.7}$$

Definition 3.2. A function $f \in \Sigma$ is in the class $\Sigma_{\alpha,\lambda}^{*}(\phi)$ if

$$\frac{-(1-2\alpha) z\left[D^{\lambda} f(z)\right]' + \alpha z^2 \left[D^{\lambda} f(z)\right]''}{(1-\alpha)\left[D^{\lambda} f(z)\right] - \alpha z\left[D^{\lambda} f(z)\right]'} \prec \phi(z), \quad \left(z \in \Delta;\ \alpha \in \mathbb{C} - (0, 1];\ \Re(\alpha) \geq 0\right). \tag{3.8}$$

For the classes $\Sigma_{\lambda}^{*}(\phi)$ and $\Sigma_{\alpha,\lambda}^{*}(\phi)$, using methods similar to those in the proof of Theorem 2.1, we obtain the following results.

Theorem 3.3. *Let $\phi(z) = 1 + B_1 z + B_2 z^2 + \cdots$. If $f(z)$ given by* (1.1) *belongs to $\Sigma_{\lambda}^{*}(\phi)$, then for any complex number μ,*

$$\text{(i)} \quad \left|a_1 - \mu a_0^2\right| \leq \left|\frac{B_1}{(\lambda+1)(\lambda+2)}\right| \max\left\{1, \left|\frac{B_2}{B_1} - \left(1 - \left(\frac{\lambda+2}{\lambda+1}\right)\mu\right) B_1\right|\right\}, \quad B_1 \neq 0, \tag{3.9}$$

$$\text{(ii)} \quad \left|a_1 - \mu a_0^2\right| \leq \left|\frac{2}{(\lambda+1)(\lambda+2)}\right|, \quad B_1 = 0. \tag{3.10}$$

The bounds are sharp.

Theorem 3.4. *Let $\phi(z) = 1 + B_1 z + B_2 z^2 + \cdots$. If $f(z)$ given by* (1.1) *belongs to $\Sigma^*_{\alpha,\lambda}(\phi)$, then for any complex number μ,*

$$\text{(i)} \quad |a_1 - \mu a_0^2| \le \left| \frac{B_1}{(1-2\alpha)(\lambda+1)(\lambda+2)} \right| \times \max\left\{1, \left| \frac{B_2}{B_1} - \left(1 - \frac{(1-2\alpha)}{(1-\alpha)^2}\left(\frac{\lambda+2}{\lambda+1}\right)\mu\right) B_1 \right| \right\}, \quad B_1 \ne 0, \tag{3.11}$$

$$\text{(ii)} \quad |a_1 - \mu a_0^2| \le \left| \frac{2}{(1-2\alpha)(\lambda+1)(\lambda+2)} \right|, \quad B_1 = 0. \tag{3.12}$$

The bounds are sharp.

Remark 3.5. For $\lambda = 0$ in (3.9), (3.11), we get the results (2.1) and (2.12), respectively. Also, for $\alpha = \lambda = 0$ in (3.11), we get the result (2.1).

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